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# Mechanism Design for Container Sharing under the Impact of Carbon Tax

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# Declaration

This statement and the accompanying publications have not previously been submitted by the candidate for a degree in this or any other university.

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# Abstract

Nowadays, appropriate management of carbon emissions is becoming one of the most urgent tasks for governments in the world. Also, the shipping industry is seriously suffering from the issue of empty container management. Hence, how to resolve the accumulation problem and improve the efficiency of empty container utilisation under the impact of government carbon tax becomes an academically interesting research question. This thesis considers three research questions:1) how a fixed carbon tax rate affects the coordination of container sharing system; 2) what is the optimal carbon tax rate that can maximise the economic benefits of container sharing system; 3) what is the optimal carbon tax rate that can maximise the social welfare for container sharing supply chains.

Three game theoretical models, with each corresponding to a sub-problem, have been developed for a container shipping system that includes the government and two liner shipping carriers. The first is a typical Newsvendor game; the second and third models extend Newsvendor games by considering more decision-making variables and different objective functions. Each of the three models have considered both centralised and decentralised decision-making mechanisms. The centralised decision-making mechanism reflects an ideal situation where two shipping carriers operate in perfect collaboration. The decentralised model considers a realistic situation, where two carriers sign a specific contract to split the container sharing costs and benefits.

This research has obtained several valuable managerial insights: (1) both Buy-back Contract and Revenue-sharing Contract can be applied to conditionally coordinate the business of empty container sharing system under the impact of government Carbon Tax impact; (2) The carbon tax can significantly affect two carriers' coordination mechanism and it should be constrained within a specific range to guarantee the coordination and (3) Two carriers' coordination can be achieved and the social welfare can be maximised if the government sets the appropriate carbon tax. These outputs provide some policy implications. For example, carriers can reach cooperation by applying Buy-back Contract and Revenue-sharing Contract to offset the negative impacts on carriers' operation when government levies carbon tax on the container with cargos. Moreover, the Revenue-sharing Contract appears to be more flexible in terms of achieving system coordination compared to the Buy-back Contract. Last but not least, government can levy a carbon tax on shipping carriers which will not interfere with the sharing of empty containers and will not significantly damage their profits. In summary, through the investigation, it is suggested that liner shipping carriers should cooperate with each other when government levies carbon tax to reduce profit loss and relieve the operation risk. Also, government

should create carbon tax scheme cautiously and appropriately. The government should not only take into consideration environmental issues, such as the cost of carbon treatment and the investment in innovative carbon recovery technologies, but also the operation of liner shipping companies, since these companies play an important role in a country's transit.

**Keywords:** Game theory; Newsvendor game; Container sharing; Carbon tax; Stackelberg Game, Revenue-sharing Contract; Buy-back Contract; Coordination

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*In loving memory of my two grandfathers*

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# List of Abbreviations

| | |
|---|---|
| $AIS$ | Automatic Identification System |
| $BBC$ | Buy-back Contract |
| $CAT$ | Cap-and-Trade |
| $CDF$ | Cumulative Density Function |
| $CH_4$ | Methane |
| $CO_2$ | Carbo Dioxide |
| $CSC$ | Cost-sharing Contract |
| $CT$ | Carbon Tax |
| $DSS$ | Decision Support System |
| $ECR$ | Empty Container Repositioning |
| $ECS$ | Empty Container Sharing |
| $EOQ$ | Economic Order Quantity |
| $ESC$ | Equal Share Contract |
| $ETS$ | Emissions Trading Systems |
| $EU$ | European Union |
| $EU-ETS$ | European Union Emission Trading System |
| $FSC$ | Fair Share Contract |
| $GIS$ | Geographic Information System |
| $GT$ | Game Theory |
| $GTAP$ | Global Trade Analysis Project |
| $GHG$ | Greenhouse Gas |
| $IDT$ | Interactive Decision Theory |
| $IMO$ | International Maritime Organization |
| $IPCC$ | Inter-governmental Panel on Climate Change |
| $LSC$ | Liner Shipping Carriers |
| $MEPC$ | Maritime Environmental Protection Committee |
| $MtCO_2e$ | Metric tons of carbon dioxide equivalent |
| $NASA$ | The National Aeronautics and Space Agency |
| $N_2O$ | Nitrous Oxides |
| $NSR$ | Northern Sea Route |
| $OM$ | Operation Management |
| $OR$ | Operation Research |
| $PhD$ | Doctor of Philosophy |
| $pdf$ | Probability Density Function |

| | |
|---|---|
| $PICTR$ | Preferred Ideal Carbon Tax Rate |
| $ppm$ | Parts per Million |
| $QCC$ | Quality-compensation Contract |
| $QFC$ | Quantity Flexibility Contract |
| $RSC$ | Revenue-sharing Contract |
| $RiSC$ | Risk-sharing Contract |
| $SPP$ | Single-period Problem |
| $TD$ | Temporal Difference |
| $TEU$ | Twenty-foot Equivalent Unit |
| $TPTV$ | Two-ports Two-voyages |
| $TTC$ | Two-part Tariffs Contract |
| $UKIERI$ | UK-India Education and Research Initiative |
| $UN$l | United Nations |
| $UNCTD$ | United Nations Conference on Trade and Development |
| $UNFCCC$ | United Nations Framework Convention on Climate Change |
| $WPC$ | Wholesale Price Contract |

# List of Notations

| | |
|---|---|
| **Random Variables** | |
| $\boldsymbol{X_i}$ | The demands of consignor for empty containers, received by $LSC\ i$ |
| $\boldsymbol{Y_i}$ | The number of empty containers generated at $LSC\ i$ |
| $\boldsymbol{\xi_i}$ | The error term with $f_i(x)$ as $pdf$; and $F_i(x)$ as $CDF$ |
| **Decision variables** | |
| $\boldsymbol{q}$ | The number of empty containers shared between the two $LSCs$ (in Chapter 4, 5 and 6) |
| $\boldsymbol{p}$ | The $CT$ rate imposed by government on each container of cargoes (where it is parameter in Chapter 4, Endogenous decision variable in Chapter 5 and Decision variable in Chapter 6, respectively) |
| **State variables** | |
| $\boldsymbol{S_i}$ | $LSC\ i$'s satisfied demands |
| $\boldsymbol{L_i}$ | $LSC\ i$'s unsatisfied demands |
| $\boldsymbol{I_i}$ | $LSC\ i$'s leftover inventory at the end of the period |
| $\boldsymbol{\emptyset_i}$ | The fraction of revenue kept by $LSC\ i$ in Revenue-sharing Contract ($RSC$) |
| $\boldsymbol{R_i}$ | $= (1-\emptyset_i)r_i$, the revenue that $LSC\ i$ transfers to the other in $RSC$ |
| $\boldsymbol{\eta_i}$ | The buy-back price in Buy-back Contract ($BBC$) paid from the supplier to the demander for every unsatisfied empty container at the end of the period |
| $\boldsymbol{w}$ | The wholesale price per container in the $RSC$ and in $BBC$ |
| $\boldsymbol{\theta}$ | The transfer payment between two $LSCs$ in decentralised model |
| $\boldsymbol{\beta_i}$ | $= n_i - \lvert q\rvert - a_i + b_i p$, the number of empty containers still held by $LSC\ i$ after meeting demands and sharing container under a $CT$ rate $p$ |
| **Parameters** | |
| $\boldsymbol{n_i}$ | Initial inventory of empty containers owned by $LSC\ i$ |
| $\boldsymbol{a_i}$ | The potential empty container demands received by $LSC\ i$ |
| $\boldsymbol{b_j}$ | The sensitivity of the consignor to the $CT$ rate on each container of cargoes |
| $\boldsymbol{r_i}$ | The amount of revenue that the $LSC\ i$ can earn per satisfied empty container |
| $\boldsymbol{h_i}$ | The holding cost per empty container at $LSC\ i$ |
| $\boldsymbol{g_i}$ | The goodwill penalty cost per unmet empty container at $LSC\ i$ |
| $\boldsymbol{c_t}$ | The cost of transporting an empty container between two $LSCs'$ terminal |
| $\boldsymbol{\alpha_i}$ | $= r_i + h_i + g_i$, the all-in-revenue for $LSC\ i$ |
| $\boldsymbol{C_g}$ | Government carbon treatment cost for each container of satisfied demands. |
| **Common solutions in Chapter 4, 5 and 6** | |
| $\boldsymbol{z_i(.)}$ | The Probability Density Function ($pdf$) of $\xi_i - Y_i$ |
| $\boldsymbol{Z_i(.)}$ | The Cumulative Density Function ($CDF$) of $\xi_i - Y_i$ |
| $\boldsymbol{\Phi_i(.)}$ | The Complementary loss function of $\xi_i - Y_i$; $d\Phi_i/\,dq = Z_i(.)$ |
| $\prod$ | The system profit in the centralised model ($\prod_{cen}$ in Chapter 6) |
| $\boldsymbol{\Delta\prod}$ | The system profit increment in the centralised model ($\Delta\prod_{cen}$ in Chapter 6) |
| $\boldsymbol{\pi_i}$ | $LSCs'$ profit in the decentralised model |
| $\boldsymbol{\Delta\pi_i}$ | $LSCs'$ profit increments in the decentralised model |
| $\boldsymbol{q_1^+}$ | The number of empty containers that $LSC$ 1 gives to $LSC$ 2 in the decentralised model |
| $\boldsymbol{q_1^-}$ | $LSC$ 1 receives the number of empty containers from $LSC$ 2 in the decentralised model |
| $\boldsymbol{q_2^+}$ | The number of empty containers that $LSC$ 2 borrows from $LSC$ 1 in the decentralised model |
| $\boldsymbol{q_2^-}$ | The number of empty containers that $LSC$ 2 intends to give $LSC$ 1 in the decentralised model |
| $\boldsymbol{\Delta S_i^e(q,p)}$ | The increment of expected satisfied demands between the scenarios with and without |

| | |
|---|---|
| | sharing $q$ empty containers under a certain government $CT$ rate $p$ |
| $\Delta \boldsymbol{S_i}(\boldsymbol{0}, \boldsymbol{p})$ | The increment of expected satisfied demands between the scenarios with and without $CT$ rate variation impact $p$ when $ECS$ is not considered |

**Other specific solutions in Chapter 4**

| | |
|---|---|
| $\boldsymbol{q}^*$ | The optimal sharing quantity of containers in the centralised model |
| $\dot{\boldsymbol{q}}$ | The optimal empty container sharing number in case 2 in the centralised model |
| $\ddot{\boldsymbol{q}}$ | The optimal empty container sharing number in case 4 in the centralised model |
| $\boldsymbol{q}^{\boldsymbol{e}}$ | The Nash equilibrium of $q$ in the decentralised model |

**Other specific solutions in Chapter 5**

| | |
|---|---|
| $\boldsymbol{q}^*$ | The optimal sharing quantity of containers in the centralised model |
| $\boldsymbol{p}^*$ | The optimal $CT$ rate in the centralised model |
| $\dot{\boldsymbol{q}}$ | The optimal empty container sharing number in case 3 in the centralised model |
| $\widehat{\boldsymbol{q}}$ | The optimal empty container sharing number in case 4 in the centralised model |
| $\check{\boldsymbol{q}}$ | The optimal empty container sharing number in case 7 in the centralised model |
| $\ddot{\boldsymbol{q}}$ | The optimal empty container sharing number in case 8 in the centralised model |
| $\overline{\boldsymbol{p}}$ | The optimal $CT$ rate in case 1 in the centralised model |
| $\dot{\boldsymbol{p}}$ | The optimal $CT$ rate in case 3 in the centralised model |
| $\boldsymbol{p}^{\boldsymbol{0}}$ | The optimal $CT$ rate in case 5 in the centralised model |
| $\ddot{\boldsymbol{p}}$ | The optimal $CT$ rate in case 8 in the centralised model |
| $\widetilde{\boldsymbol{p}}$ | The optimal $CT$ rate in case 10 in the centralised model |
| $\boldsymbol{\omega_1}$ | $= [b_1 g_1 + b_2(r_1 + g_1 - r_2 + c_t)]/[(b_1 + b_2)\alpha_1]$ |
| $\boldsymbol{\omega_1'}$ | $= [b_1 g_1 + b_2(r_1 + g_1 - r_2 - c_t)]/(b_1 + b_2)\alpha_1$ |
| $\boldsymbol{\omega_2}$ | $= [b_1(r_2 + g_2 - r_1 - c_t) + b_2 g_2]/(b_1 + b_2)\alpha_2$ |
| $\boldsymbol{\omega_2'}$ | $= [b_1(r_2 + g_2 - r_1 + c_t) + b_2 g_2]/[(b_1 + b_2)\alpha_2]$ |
| $\boldsymbol{Z_i^{-1}}(.)$ | Inverse function of $Z_i$ |
| $\boldsymbol{q}^{\boldsymbol{e}}$ | Nash equilibrium sharing number between in the decentralised model |
| $\boldsymbol{p}^{\boldsymbol{e}}$ | Nash equilibrium government $CT$ rate in the decentralised model |
| $\boldsymbol{p}^{\boldsymbol{0i}}$ | $LSCs'$ Preferred Ideal Carbon Tax Rate ($PICTRs$) in the decentralised model |

**Other specific solutions in Chapter 6**

| | |
|---|---|
| $\boldsymbol{p_s^e}$ | Stackelberg equilibrium government $CT$ rate in the centralised model |
| $\boldsymbol{q_s^e}$ | Stackelberg equilibrium sharing number in the centralised model |
| $\dot{\boldsymbol{q}}_{\boldsymbol{s}}^{\boldsymbol{e}}$ | Stackelberg equilibrium sharing number in case 2 in the centralised model |
| $\ddot{\boldsymbol{q}}_{\boldsymbol{s}}^{\boldsymbol{e}}$ | Stackelberg equilibrium sharing number in case 4 in the centralised model |
| $\boldsymbol{q_d^e}$ | Nash equilibrium of sharing number in the decentralised model |
| $\prod_{\boldsymbol{gov}}$ | The government social welfare function |
| $\boldsymbol{R_i'}$ | The lower boundary making the $LSC's$ profit increment nonnegative. |
| $\boldsymbol{R_i''}$ | The upper boundary making the $LSC's$ profit increment nonnegative. |

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# Chapter 1 Introduction

## 1.1. Global warming and climate change

With the rapid development of the economy worldwide in recent centuries, human beings increasingly are suffering the associated drawbacks of environmental damage. For instance, the global temperature is gradually increasing, and the problems of climate change are becoming serious. Global warming has become one of the most alarming environmental problems in the 21st century and for decades has led to considerable apprehension among policy makers, governments, research communities and the general public worldwide. In accordance with scientific evidence, the global temperature is rising and is having serious negative effects (Lindsey and Dahlman, 2023). The Inter-governmental Panel on Climate Change ($IPCC$) believes that the global temperature has increased by 0.8°C to 1.2°C above the pre-industrial level due to human activities as well as the emission of Greenhouse Gas ($GHG$) ($IPCC$, no date). Figure 1.1 presents evidence of the change in surface temperature shown in Geographic Information System ($GIS$) between 1880 and 2022 for each month over calendar year. Figure 1.2 further depicts the index of land-ocean temperatures (shown in Locally Weighted Scatterplot Smoothing, also called Lowess Smoothing) between 1880 and 2020. Also, in Figure 1.3, the projected increase in global temperature to 2100 is illustrated. It indicates that there is a 50% probability that global warming may exceed 2.0°C by 2100 if there is no action implemented (Climate Action Tracker, 2022).

Since the issue is becoming serious, climate change mitigation measures, such as reducing emissions of heat-trapping $GHG$ into the atmosphere, have been proposed and implemented through *The Paris Agreement, The Glasgow Climate Change Pact* and *COP27* in order to minimise the negative effects of climate change. *The Paris Agreement* aims at limiting the average increase of global temperature below 2℃, preferably to 1.5℃, compared to the pre-industrial levels (UNFCCC, 2015). The *Glasgow Climate Change Conference* in 2021 gave reassurance that the long-term worldwide purpose is to maintain the average temperature increase to below $2^oC$ above the pre-industrial level and to try to limit the temperature increase to $1.5^oC$ above the pre-industrial level (UNFCCC, 2021). It is further reaffirmed by *COP 27* in Egypt in 2022 that the world will remain within 1.5 degrees Celsius of pre-industrial temperatures (COP 27, 2022). So, most countries have pledged to make enhanced commitments to alleviate the impact of global warming by signing both The Paris Agreement and The Glasgow Climate Pact, respectively.

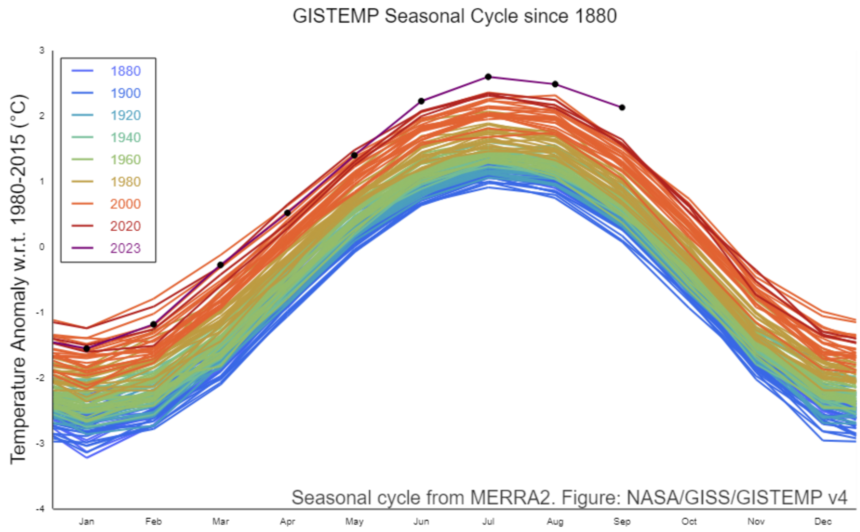


**Figure 1.1** How much warmer it was in each month between 1880 and 2022 (NASA, 2023a)

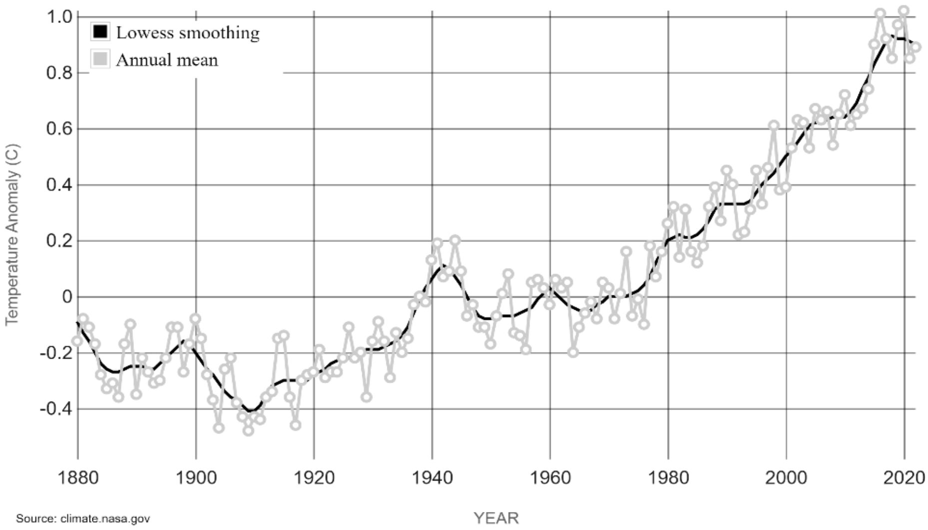


**Figure 1.2** Global land-ocean temperature index between 1880 and 2020 (NASA, 2023b)

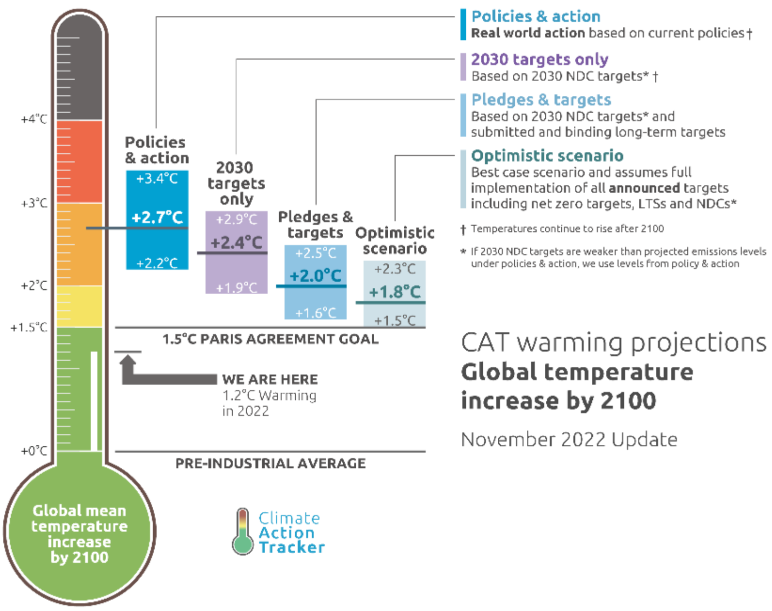


**Figure 1.3** The global temperature estimation by 2100 (Climate Action Tracker, 2022)

Reducing $GHG$ emission is one of the measures discussed in the Glasgow Climate Pact (Luo et al., 2022; Guo et al., 2021). $GHG$ is the main contributor to increasing temperatures globally and includes Carbon Dioxide ($CO_2$), Methane ($CH_4$), Nitrous Oxides ($N_2O$) and Fluorinated gases (Oertel et al., 2016). Climate Watch (2022) revealed that 48,939.71 $MtCO_2e$ $GHG$ were emitted around the world in 2018, compared to only 34,929.19 $MtCO_2e$ in 1998. Specifically, 36,441.55 $MtCO_2e$ of $CO_2$; 8,298.27 $MtCO_2e$ of $CH_4$; 3,063.75 $MtCO_2e$ of $N_2O$ were emitted in 2018 worldwide (Climate Watch, 2022). Table 1.1 illustrates the amount of emission of $CO_2$, $CH_4$, $N_2O$ and fluorinated gases in Mt$CO_2e$ globally from 2013 to 2018. Table 1.2 further illustrates the relative proportions of sources of $GHG$ emission in 2016 in Europe and different countries worldwide (Metcalf, 2021).

**Table 1.1** The amount of various $GHG$ emission globally, 2013-2018 (unit $MtCO_2e$). (Climate Watch, 2022)

| ***Source*** | ***Sum*** | ***2013*** | ***2014*** | ***2015*** | ***2016*** | ***2017*** | ***2018*** |
|---|---|---|---|---|---|---|---|
| All $\boldsymbol{GHG}$ | 283,799.02 | 46,047.13 | 46,647.29 | 46,760.47 | 47,413.95 | 47,990.47 | 48,939.71 |
| $\boldsymbol{CO_2}$ | 210,488.53 | 34,217.18 | 34,558.59 | 34,521.91 | 35,160.60 | 35,588.70 | 36,441.55 |
| $\boldsymbol{CH_4}$ | 49,102.19 | 8,001.46 | 8,161.13 | 8,240.68 | 8,172.01 | 8,228.64 | 8,298.27 |
| $\boldsymbol{N_2O}$ | 18,051.55 | 2,919.56 | 2,964.99 | 2,997.24 | 3,027.74 | 3,078.27 | 3,063.75 |
| **Fluorinated gases** | 6,156.75 | 908.93 | 962.57 | 1,000.64 | 1,053.60 | 1,094.87 | 1,136.14 |

**Table 1.2** The $GHG$ emissions' breakdown in 2016 in various countries and globally (Climate Watch, 2022)

| ***Source*** | ***World*** | ***China*** | ***EU-28*** | ***India*** | ***U.S.*** |
|---|---|---|---|---|---|
| $\boldsymbol{CO_2}$ | 74% | 83% | 80% | 70% | 83% |
| $\boldsymbol{CH_4}$ | 18% | 11% | 11% | 21% | 10% |
| $\boldsymbol{N_2O}$ | 6% | 5% | 6% | 8% | 4% |
| **Fluorinated gases** | 2% | 2% | 3% | 1% | 3% |
| **Total GHG** | 100% | 100% | 100% | 100% | 100% |

Clearly, according to Table 1.1 and Table 1.2, $CO_2$ is the main $GHG$ emission accounting for the highest proportion in every country worldwide. $CO_2$ is a gas which has a significant insulation ability and heat absorption, constantly increasing the global temperature if the concentration passes a specific threshold (Guo et al., 2021). As one of the most vital human-produced $GHG$ emissions, $CO_2$ makes a significant contribution to global warming (Salam and Noguchi, 2005). Thus, it is fair to assume that $CO_2$ is the main culprit of the greenhouse effect (Devi and Gupta, 2019). In terms of emission sources, $CO_2$ is mostly emitted in two ways: one is from natural processes and the other is due to including volcanic eruptions and respiration, and the other is due to human activity and mainly includes burning of fossil fuels and deforestation (NASA, 2022). Compared with carbon emission from natural process, in the recent two centuries, human activity, such as transportation, building and manufacturing etc. (Table 1.3), has accelerated the increase of $CO_2$ emission. Figure 1.4 shows estimated atmospheric $CO_2$ levels from 2005 to 2022 (NASA, 2022). $CO_2$ concentration peaks at 418 ppm (parts per million) in January 2022, compared to 380 ppm in 2005 (NASA, 2022). Therefore, it is necessary for governments globally and related organisation to reduce and manage $CO_2$ emission as a matter of urgency.

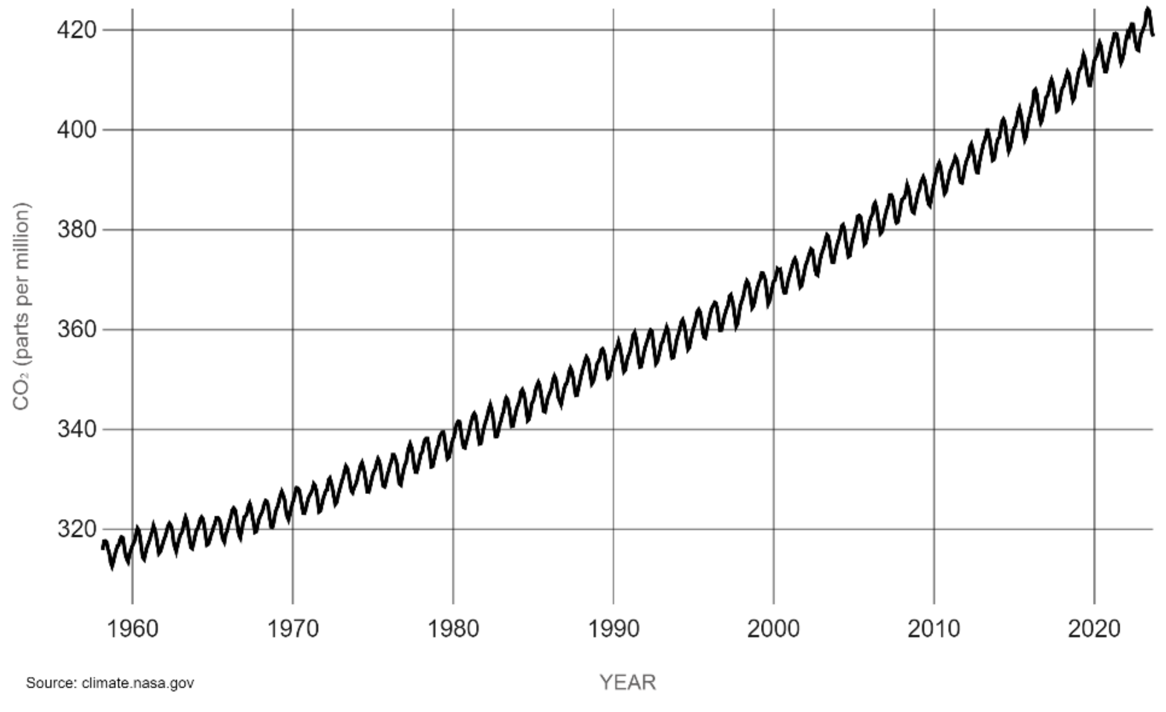


**Figure 1.4** The atmospheric $CO_2$ levels from 2005 to 2022 (NASA, 2022)

Table 1.3 shows the share of $CO_2$ emission from six different main sectors globally from 2013 to 2018. Apart from the energy and electricity/heat sector, the $CO_2$ emitted from the transportation sector and bunker fuels accounts for around 13% to 14% of total emission, a factor which should be given particular attention. Moreover, as a main component of transportation, the emitted $CO_2$ from the shipping and maritime sector cannot be ignored. The Third International Maritime Organisation ($IMO$) $GHG$ study (2014) officially pointed out that oil tankers, container ships and bulk carriers are the three main sources of the shipping industry from the perspective of $CO_2$ emission. Taking an exact number to illustrate, $IMO$ reported that there were 1,036 million tonnes of $CO_2e$[1] on average emitted from shipping activity between 2007 and 2012, which constituted 2.8% of total $CO_2e$ emission worldwide (IMO, 2014). Specifically, in 2012, approximate 938 million tonnes of $CO_2e$ was emitted from shipping activity, while 796 million tonnes of $CO_2e$ were from the international

[1] $CO_2e$: a metric measure that compares the global-warming potential of various greenhouse gases by converting other gases into carbon dioxide.

shipping activity (IMO, 2014). Some studies have focused on this topic; for example, a study at a microscopic level in Tianjin Port, which is the 9th busiest port in the world, used the Automatic Identification System ($AIS$) to collect data and it found that more than 1.97 million tons of $CO_2e$ were emitted in 2014 (Chen et al., 2016a). Rojon et al. (2021) estimated that $CO_2e$ emitted from shipping activity will increase by around $90\% - 120\%$ by 2050 compared to the level in 2008. Therefore, there is a pressing need for governments to propose effective policies to reduce the $CO_2e$ emission from international shipping activities to address their significant contribution to the issue of global warming and climate change (Luo et al., 2022).

**Table 1.3** The share of $CO_2$ emission from six different main sectors worldwide from 2013 to 2018 (unit $MtCO_2e$) (Climate Watch, 2022)

| ***Year*** | ***2018*** | ***2017*** | ***2016*** | ***2015*** | ***2014*** | ***2013*** |
|---|---|---|---|---|---|---|
| Total | 67,947.88 | 66,567.49 | 65,598.83 | 65,525.26 | 65,511.86 | 65,426.4 |
| | 100.00% | 100.00% | 100.00% | 100.00% | 100.00% | 100.00% |
| Energy | 33,746.74 | 33,070.44 | 32,620.00 | 32,608.49 | 32,627.75 | 32,601.03 |
| | 49.67% | 49.68% | 49.73% | 49.76% | 49.80% | 49.83% |
| Electricity | 15,590.95 | 15,167.36 | 14,949.51 | 15,009.83 | 15,232.52 | 15,320.55 |
| | 22.95% | 22.78% | 22.79% | 22.91% | 23.25% | 23.42% |
| Transportation | 8,257.73 | 8,078.45 | 7,878.14 | 7,717.01 | 7,498.34 | 7,373.42 |
| | 12.15% | 12.14% | 12.01% | 11.78% | 11.45% | 11.27% |
| Manufacturing | 6,158.32 | 6,174.41 | 6,188.61 | 6,315.81 | 6,360.38 | 6,324.16 |
| | 9.06% | 9.28% | 9.43% | 9.64% | 9.71% | 9.67% |
| Building | 2,882.54 | 2,796.01 | 2,737.47 | 2,689.70 | 2,660.70 | 2,712.17 |
| | 4.24% | 4.20% | 4.17% | 4.10% | 4.06% | 4.15% |
| Bunker Fuels | 1,311.60 | 1,280.82 | 1,225.10 | 1,184.42 | 1,132.17 | 1,095.07 |
| | 1.93% | 1.92% | 1.87% | 1.81% | 1.73% | 1.67% |

The initial $IMO\ GHG$ emission strategy expects to reduce $CO_2$ emissions by at least 40% by 2030 in international shipping activity (IMO, 2018). Several methods are proposed to efficiently reduce and manage the carbon emission and include Cap-and-Trade ($CAT$) system (also called "Emissions Trading Systems" - $ETS$) and Carbon Tax ($CT$) policy as the two main mechanisms to reduce carbon emission (Chen et al., 2020). The $CT$ policy is one of the essential measures to reduce carbon emission by charging tax for carbon emission activities (Calderón et al., 2016; Cao et al., 2021). Many definitions of $CT$ have been formulated. In general, $CT$ is defined as the payment given by the industries and businesses that emit the $CO_2$ to the government to protect the environment (Kagan, 2023). Metcalf (2021) defined $CT$ as a tax levied by the government on $CO_2$ pollution with the market determining the amount of pollution. Ayodele et al. (2021) further stated that $CT$ is one type of tax that the government charges for each ton of $CO_2$ emitted by the emitters (e.g., cars, ships and generator plant etc.). Parry et al. (2018) claimed that $CT$ is an extension of fuel combustion tax in the maritime sector; the government usually levies the $CT$ on the shipping fuel at the refinery gate, covering a wide range of easily identified taxpayers, such as shipping company and fuel supplier.

As an effective method to manage carbon emission in the long term, the Maritime Environmental Protection Committee ($MEPC$) of $IMO$ suggested implementing $CT$ in the maritime sector (Wang et al., 2018b). Metcalf (2021) asserted that $CT$ will tend to be implemented to reduce emissions from the shipping industry, given the current high international shipping emission production. Many countries and regions have implemented the $CT$ policy. By the end of 2021, it is reported that 27 nations and 8 sub-nations have applied or plan to conduct the $CT$ scheme to reduce carbon emission (The World Bank, 2022a), including Japan (Ashina and Nakata, 2008), British Columbia (Litman, 2009), China (Luo et al., 2022; Zhou et al., 2018), Korea (Kim et al., 2011), Russia (Orlov and Grethe, 2012) and the U.S. (Metcalf, 2019). Figure 1.5 shows the nations and sub-nations globally which have applied $CT$ policy to reduce carbon emission by the year of 2021.

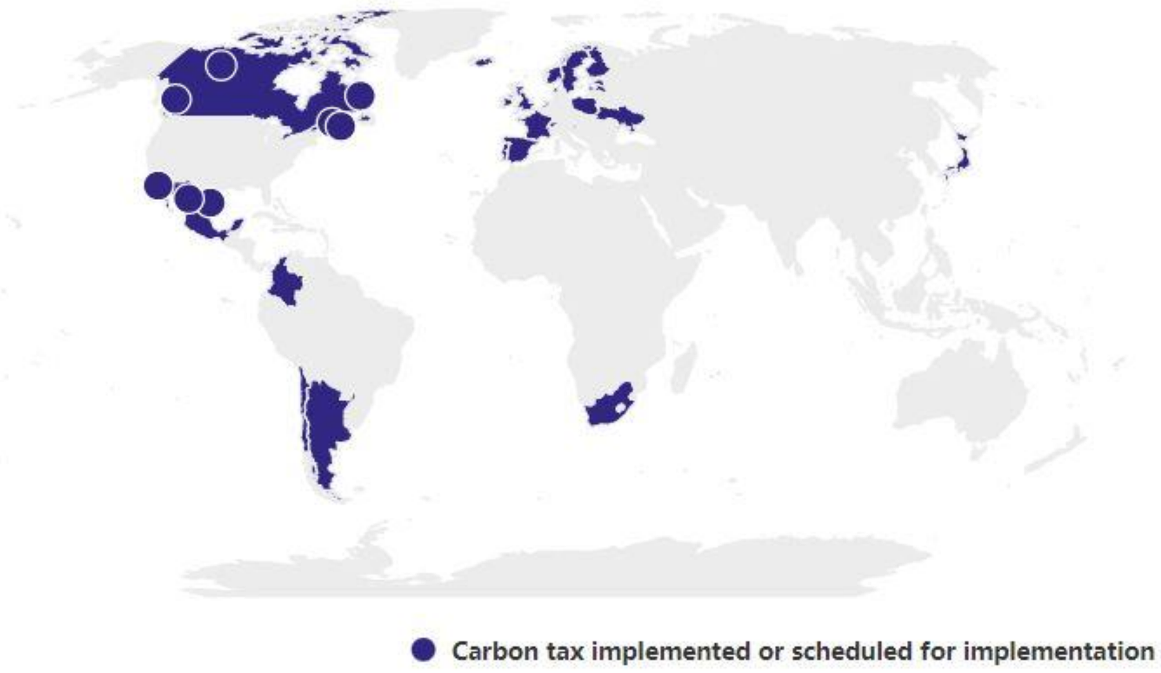


**Figure 1.5** The nations and sub-nations which implement or plan to conduct $CT$ policy by the year of 2021(The World Bank, 2022a)

The World Bank (2022a) suggested that the carbon emissions that $CT$ policy covers were in total 2.87 $GtCO_{2^e}$, occupying 5.6% of total $GHG$ worldwide by the year 2021 (The World Bank, 2022a). Table 1.4 shows various jurisdictions where the $CT$ policy has been adopted and illustrates the $CT$ rate, coverage and sector covered in different countries and states, along with the median household income in these countries. It is easy to ascertain that the $CT$ rate is relatively higher in richer countries (Farrell and Lyons, 2016).

**Table 1.4** The jurisdictions that adopt $CT$ policy (Farrell and Lyons, 2016)

| ***Country*** | ***Rate($/t$CO_2$)*** | ***Coverage*** | ***Sector covered*** | ***Median household income ($)*** |
|---|---|---|---|---|
| Japan | 2 | 70% | Transport, heat, electricity | 23,784 |
| Mexico | 1-4 | 40% | fossil fuels | 3,172 |
| Iceland | 10 | 50% | Transport, heat, electricity | 29,520 |
| UK | 15.75 | 25% | Fossil fuels for electricity | 26,623 |
| France | 20 | 35% | Transport, heat, electricity | 29,610 |
| Canada | 28 | 70% | Transport, heat, electricity | 34,422 |
| Ireland | 28 | 40% | Transport, heat | 29,214 |
| Denmark | 31 | 45% | Transport, heat, electricity | 41,705 |
| Norway | 4-69 | 50% | oil, gasoline, natural gas | 54,761 |
| Finland | 48-83 | 15% | Heat and transport | 34,683 |
| Switzerland | 68 | 30% | Heat, light and thermal | 61,036 |
| Sweden | 168 | 25% | Heat and transport | 37,846 |

For the impact of $CT$ policy on general business operation incentivises the company to find the most effective way to reduce the $CO_2$ in the company's operation by for example creating low-carbon production (Meng et al., 2018). Also, the carbon taxation can profoundly improve clean innovation investment with government subsidies (Chen et al., 2020). Quantitatively, for shipping operators, Parry et al. (2018) proposed a method to calculate the ship operators' tax liability $T_L$, which is shown in equation 1.1.

$$T_L = \tau^{CO2} \cdot F^{SHIP} \cdot \beta^{CO2} \tag{1.1}$$

Where $\tau^{CO2}$ represents the $CO_2$ emissions' tax rate; $F^{SHIP}$ is the fuel consumption of the ship and $\beta^{CO2}$ the factor of emission for the fuel being used. Moreover, by considering a variant of the $CT$ which constrains the figure of revenue raised, Parry et al. (2018) further developed the modeified operator's tax liability $MT_L$ which is shown in equation 1.2.

$$MT_L = \tau^{CO2} \cdot (F^{SHIP} \cdot \beta^{CO2} - BENCH^{SHIP}) \tag{1.2}$$

Where $BENCH^{SHIP}$ is the benchmark for the shipping operator to determine the threshold of emission assignment. In this way, the shipping operators can decide whether they should pay the tax or obtain subsidies depending on whether the emission is greater or less than the benchmark (Parry et al., 2018). Therefore, overall, not only can the carbon taxation efficiently reduce the $CO_2$ emission, it will also lead to some significant changes in ship operating costs and the shipping operators must reconsider adjusting their decision often related to the fuel consumption (Liu et al., 2021). However, as most shipping activities are usually conducted at an international level, it is difficult to set a consistent appropriate $CT$ rate globally because the taxation design involves different countries. On the other hand, the $CAT$ (or $ETS$) is another essential method to reduce the $GHG$ emission and $CO_2$ emission in many political jurisdictions. In the $CAT$ system, governments impose a limit (the "cap") on the total emission of one or more pollutants from several regulated entities over a fixed period, and then the government issues tradable allowances, each representing the right of an entity to emit a unit of pollution. The "right" is finally allocated or sold to the regulated entities. The regulated entities are legally required to have a sufficient number of allowances to cover the amount of pollution that they emit (McAllister, 2012). Simply, the $CAT$ system requires allocating carbon quotas to enterprises in the first instance then enterprises decide to sell or buy carbon quotas from others based on their actual demands (Liu et al., 2015). The $CAT$ system has developed and been accepted significantly in recent decades with 39 national and 31 sub-national jurisdictions covered by the $CAT$ system worldwide (The World Bank, 2022a).

Therefore, the $CAT$ system has been proven to be efficient in reducing emissions and has had a positive impact on companies' stable operation. However, because of the difficulty in selecting appropriate carbon quota trading price, the government's regulation efficiency could significantly fall if the carbon quota is inappropriately charged (Chen et al., 2020). Also, due to the difficulty of $CO_2$ emission tracking, the $CAT$ system cannot fully be applied in all industrial sectors such as the transportation sector (Sumner et al., 2009). In conclusion, both $CT$ policy and $CAT$ systems have their strengths and limitations. Both mechanisms are widely applied in different regions, nations and sub-nations globally. Figure 1.6 demonstrates that most developed countries and some developing countries have already applied two schemes. The application of both mechanisms in different cities, regions and countries is shown in detail in Figure 1.6 (The World Bank, 2022b).

The next section introduces the problems that the international shipping industry is encountering and include container accumulation and shortages. Also, empty container inventory management is presented and some methods to solve the current issues are proposed.

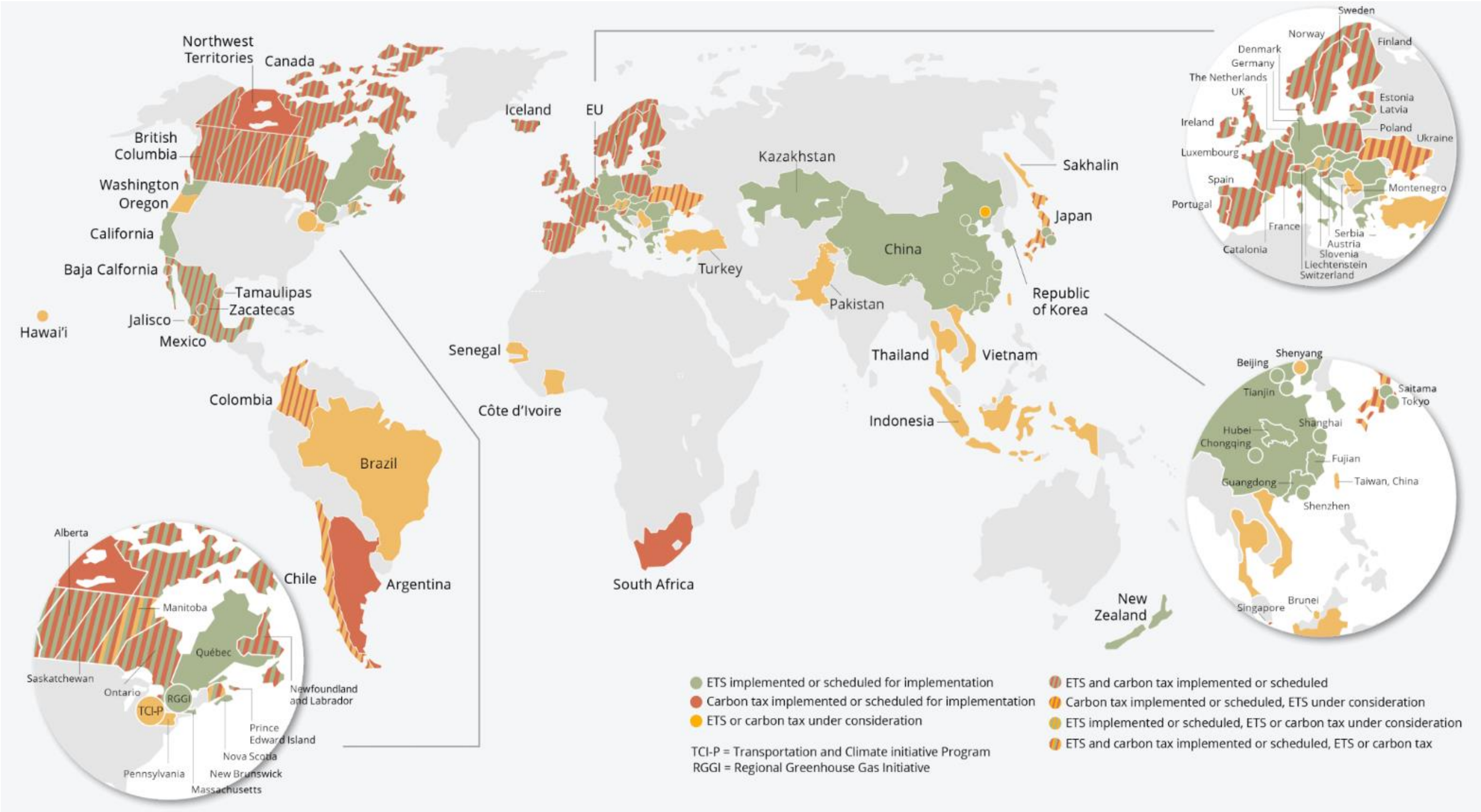


**Figure 1.6** The distribution of $CT$ and $CAT$ system application worldwide in 2021 (The World Bank, 2022b).

### 1.2. International Trade and Containerisation

The international seaborn trade has grown increasingly busy and popular in the last three decades. Figure 1.7 illustrates the seaborn trade's transport volume from 1990 to 2020 (Statista, 2022). It can be clearly seen that the seaborn trade transport volume keeps gradually rising macroscopically in the last 30 years despite there being a slight decline of the volume during the financial crisis in 2008 and the global Covid-19 pandemic since 2020. For instance, in the year of 1990, the volume was only 4.01 billion ton loaded, but it rapidly reaches a peak at 11.07 billion ton loaded in 2019. It should be noticed that, in 2020, although the trade volume worldwide contracted by 3.8% in the first half year because of the first wave of the breakout of the global Covid-19 pandemic, there was a quick recovery in the second half of the year (UNCTAD, 2021). Also, the maritime transport occupies more than 80% of the merchandise trade globally (Hoffmann et al., 2016; Wei et al., 2022; Wang et al., 2018a), mainly because the cost of maritime transport is usually less expensive compared to other transportation modes economically (Wei et al., 2022). Therefore, maritime transport is one of the most important modes of freight transportations in international commerce trade.

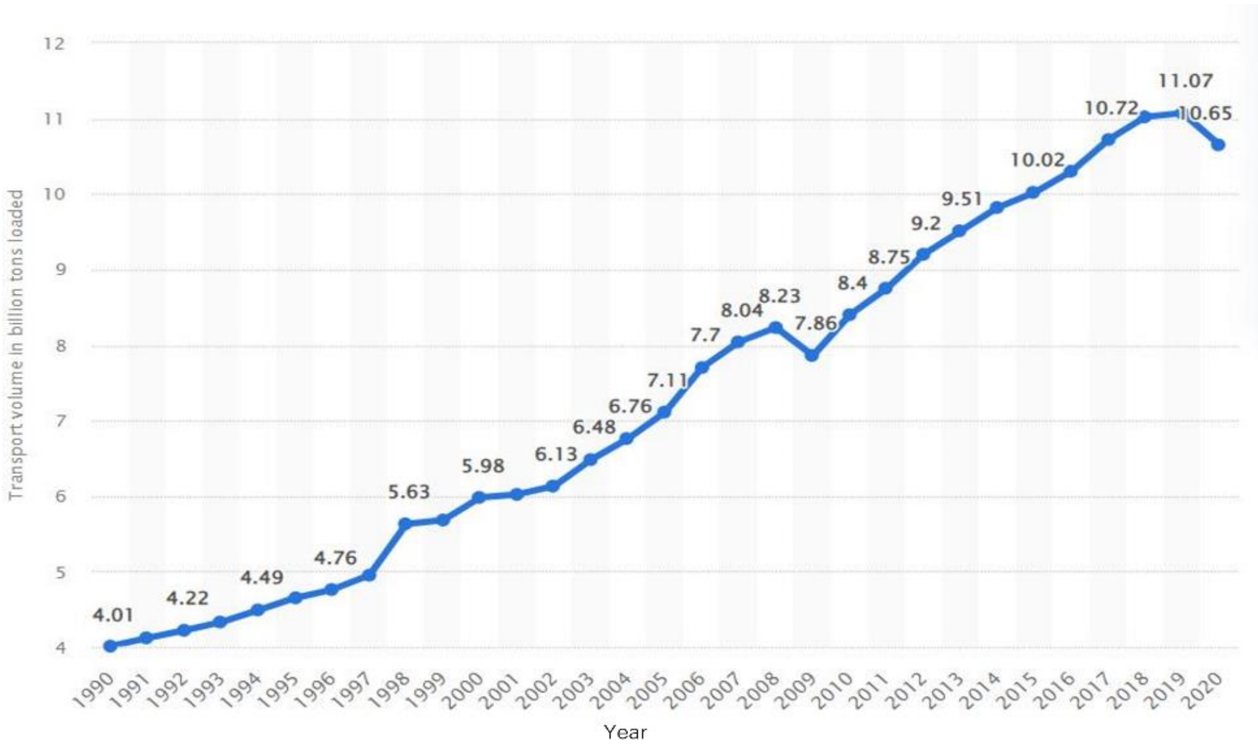


**Figure 1.7** The seaborn trade's transport volume between 1990 and 2020 (Statista, 2022).

In order to facilitate a large amount of seaborn transportation, containers are widely used in international shipping transport. A container is a large metal box for transporting freight by sea while Corten steel is the most commonly adopted material for shipping containers (Morin, 2021). The other metals used include aluminium, stainless steel, steel and fibreglass (Shen et al., 2019a). The goods and products are usually packed and loaded into containers and delivered abroad across the ocean. As soon as the container has been unloaded and collected by the consignee, it remains at the port terminal until it is leased by another consignor (Moon and Hong, 2015). Nowadays, there are several popular types of containers all of which can be applied in international ocean shipping activity. Table 1.5 shows a range of characteristics, such as width, length, height, floor area, volume and empty weight, for the four most common containers, i.e., 20 feet equivalent unit, 20 feet high cube equivalent unit and 40 feet equivalent unit.

**Table 1.5** The characteristics of the four most common containers (Shen et al., 2019a)

| *Size* | *Width (m)* | *Length (m)* | *Height (m)* | *floor area* $m^2$ | *Volume* $m^3$ | *empty weight* $kg$ |
|---|---|---|---|---|---|---|
| 20 feet equivalent unit | 2.438 | 6.069 | 2.591 | 14.860 | 33.100 | 2200.000 |
| 20 feet high cube equivalent unit | 2.438 | 6.069 | 2.900 | 14.860 | 43.090 | 2350.000 |
| 40 feet equivalent unit | 2.438 | 12.192 | 2.591 | 29.720 | 67.500 | 3800.000 |

The 20 feet equivalent container is used as the equivalent unit ($TEU$: Twenty-foot Equivalent Unit) in international shipping activity. Figure 1.8 illustrates the detailed construction of a $TEU$ container. For the other often used container types, such as a 40 feet container, can be considered as two $TEU$ (Dong and Song, 2009).

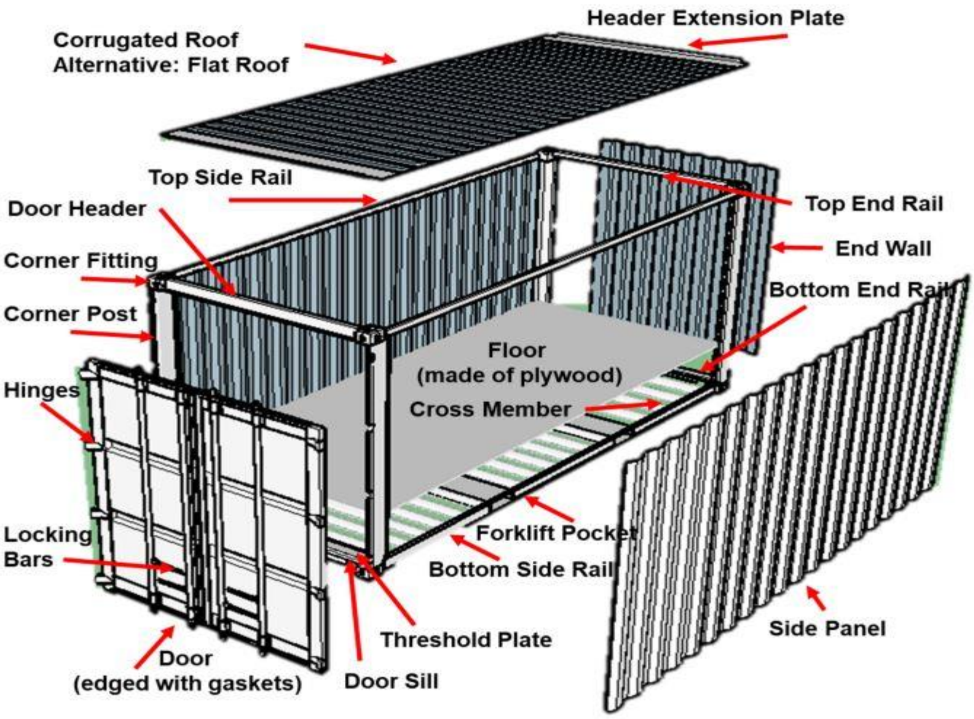


**Figure 1.8** The detailed construction of $TEU$ container (Shen et al., 2019)

Nowadays, containerisation used by most has become the standard and dedicated international transport system deployment (Legros et al., 2019). There is no doubt that the volume of global maritime trade is greatly stimulated by containerisation, which is without doubt an innovative way for international shipping activity because it ensures a shipment’s safety and efficiency (Jeong et al., 2018; Shintani et al., 2010). Specifically, the merits of containerisation use is its standardisation, ease of handling and protection against damage (Legros et al., 2019). However, although containerisation affects huge advantages for goods transport, a series of major challenges remain. For example, the movement of empty containers is one of the major operational challenges in the shipping industry because of the high cost of Empty Container Repositioning ($ECR$) and given the fact that one third of transported containers are empty globally (XChange, 2023). Therefore, in the next section, this research introduces the main reasons for empty container accumulation and why the shortage.

### 1.3. Why empty container accumulation and shortage exist?

Empty container management is one of the main issues in the shipping industry (Song et al., 2005). Fundamentally, the issue is caused by the remarkable growth of global container trade and movements in the twenty first century and the imbalance of global and regional trade (Shintani et al., 2010; Jeong et al., 2018; Olivo et al., 2005; Yu et al., 2018; Adetunji et al., 2020; Sáinz Bernat et al., 2016; Li et al., 2004; Chen et al., 2016b; Hu et al., 2020). The imbalance of international trade can cause some ports and terminals suffering container accumulation while some others have a lack of containers (Xie et al., 2017; Jeong et al., 2018). Figure1.9 illustrates the number of empty and full containers that have been transported worldwide from 1980 to 2011 (Monios and Wilmsmeier, 2015). Additionally, the chart illustrates the percentage of empty and full containers that were transported in the different years (Monios and Wilmsmeier, 2015).

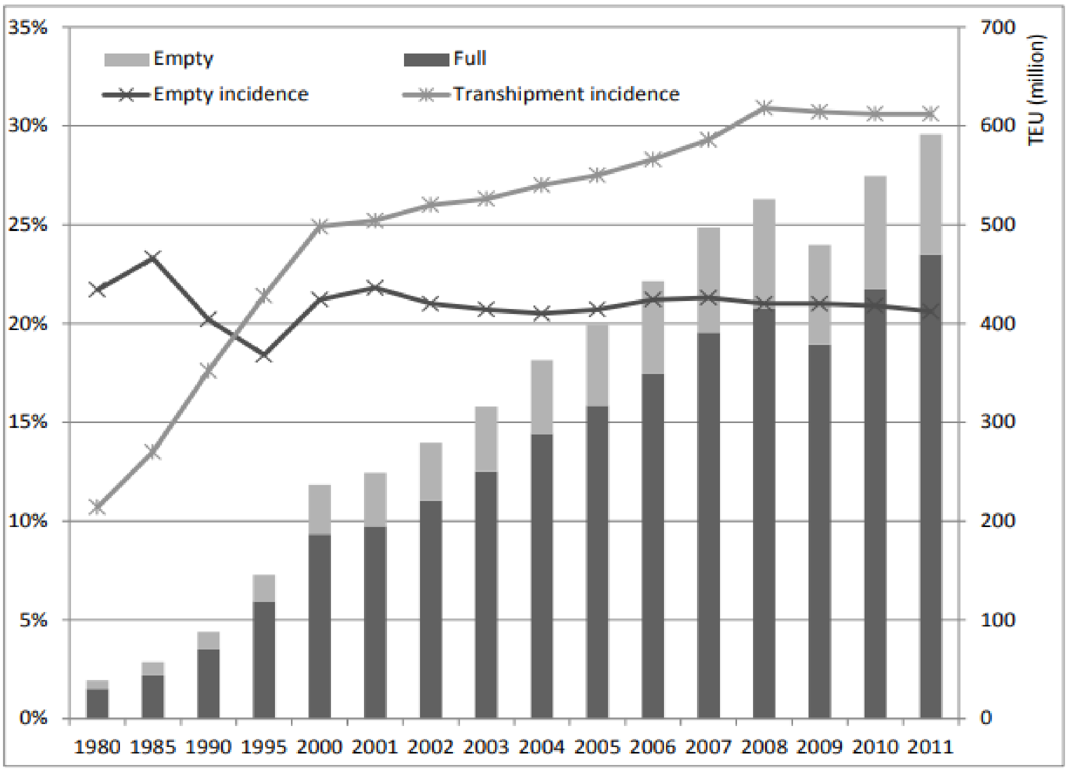


**Figure 1.9** Empty and full containers that were transported from 1980 to 2011 and their percentages.

It is evident from the chart that the movement of empty containers increased steadily from 1980 to 2011. Also, approximately 40% of inland container movements are related to empty containers (Braekers et al., 2011; Song and Dong, 2023). Consequently, the high rate of empty container movements not only increases the cost of container transportation, and reduces the efficiency of container utilisation, but also elevates the carbon emissions per ton transported. Moreover, in the last three decades, the supply and the demand of empty containers in terminal and port areas have been out of balance due to the asymmetric nature of global trade (Moon and Hong, 2015; Xie et al., 2017) and the impact of some global factors such as Covid-19 pandemic and the Ukraine war.  For example, the container imbalance between Asia and the U.S. was only 0.5 million $TEU$ in 1995; however, this number increased to the 8.2 million in 2005 and rapidly rose to 10.5 million in 2007 (Moon et al., 2010). In 2008, there were approximately 18.9 million $TEUs$ imported to the U.S. while only 8.6 million of $TEUs$ were exported to the other countries (Pérez-Rodríguez and Holguín-Veras, 2014). Thus, there must be more and more empty containers left in the U.S. On the other hand, in 2021, there was a huge number of empty containers accumulated in some U.S. ports because of massive workforce disruptions during the coronavirus pandemic restrictions in the U.S. (GORI, no date). Furthermore, the U.S. faced 40% imbalance of containers in 2021, which meant that 60 containers were accumulated and could not be exported in the U.S. ports for every 100 containers (GORI, no date) imported. In contrast, China was suffering a lack of empty containers because the empty containers could not be sent back to China from the U.S. Note that there are about 900,000 $TEUs$ on average per month transported from China to the U.S. trade route (GORI, no date). The online article ‘Container shortage: Why does the current container shortage happen?’ (Kuehne and Nagel, 2023) gave four reasons why the problem of empty container shortage continued during the Covid-19 pandemic. These were 1) decreased number of available containers, 2) congested ports, 3) a fall in the number of operational vessels and 4) goods flow changing since the pandemic, respectively.

The situation of container imbalance and empty container shortage is not confined to the ocean trading route between the U.S. and China, but a challenge for container management in other countries’ ports and terminals worldwide (Özdemir, 2018; Moon et al., 2010). Table 1.6 concludes three main containerised trade routes globally: East Asia to North America; Northern Europe and Mediterranean to East Asia; North America to Northern Europe and Mediterranean. The Asia and North America routes are globally famous for the two main export- and import-dominated districts (Jeong et al., 2018). Taking the route between Asia and North America as an example, in 2014, the containerised trade from East Asia to North America was 16.1 million $TEU$ with only 7.0 million $TEU$ for the inbound route (UNCTAD, 2021). Also, the gap between the two continually widens; for instance, in 2021, volumes were 24.1 and 7.1 million $TEU$, respectively (UNCTAD, 2021). Also, for the container trade between Northern Europe and Mediterranean and East Asia in 2021, there were 7.8 million $TEU$ delivered from Northern Europe and Mediterranean to East Asia while up to 18.5 million $TEU$ were transported in the opposite direction. All the statistics above indicate that maritime transportation is imperative because there is no alternative effective way to deliver such large volumes of freight globally and over a long-distance (Chen et al., 2016b). So, international container management should be given more attention on a global level. In summary, the huge imbalance in container trade between different regions fundamentally causes unreasonable deployment of empty containers globally (UNCTAD, 2021).

Although the international trade imbalance is the most important factor for the existing container accumulation and shortage problem, container planning issues also are a contributory factor to the whole industry (Adetunji et al., 2020). Besides, Song and Dong (2015) claimed that other factors including container size and type, uncertainty, lack of collaboration among channel members and shipping carriers' operational also diminish the efficiency of container movements and cause empty container imbalance distribution globally. Besides the additional carbon dioxide emissions and pollution, one of the negative impacts of this problem is the unprofitability and invalidity of global empty container movements. For example, not only could moving empty containers fail to generate any profits for shipping company but also it occupies space which could be used for a shipping laden container. It is easy to realise that the empty container movement does not generate any profit but only incurs extra cost (Chen et al., 2016b). Overall, appropriate empty container management and transportation route planning is necessary for Liner Shipping Carriers ($LSCs$) to decrease operational cost. Fortunately, some innovative methods have been proposed by scholars and industry to address the existing issue. They will be introduced in the next section.

**Table 1.6** Containerised trade on three East to West international trade routes (East Asia to North America; Northern Europe and Mediterranean to East Asia; North America to Northern Europe and Mediterranean) in million $TEU$ and percentage annual change, 2014–2021. (UNCTAD, 2021)

| **Year** | **Eastbound** | **Westbound** | **Total** | **Eastbound** | **Westbound** | **Total** | **Eastbound** | **Westbound** | **Total** |
|---|---|---|---|---|---|---|---|---|---|
| | East Asia-North America | North America-East Asia | | Northern Europe and Mediterranean-East Asia | East Asia-Northern Europe -Mediterranean | | North America-Northern Europe and Mediterranean | Northern Europe and Mediterranean -North America | |
| 2014 | 16.1 | 7.0 | **23.2** | 6.3 | 15.5 | **21.8** | 2.8 | 3.9 | **6.7** |
| 2015 | 17.4 | 6.9 | **24.2** | 6.4 | 15.0 | **21.3** | 2.7 | 4.1 | **6.8** |
| 2016 | 18.1 | 7.3 | **25.4** | 6.8 | 15.3 | **22.1** | 2.7 | 4.2 | **6.9** |
| 2017 | 19.3 | 7.3 | **26.6** | 7.1 | 16.4 | **23.4** | 2.9 | 4.6 | **7.5** |
| 2018 | 20.7 | 7.4 | **28.0** | 7.0 | 17.3 | **24.3** | 3.1 | 4.9 | **8.0** |
| 2019 | 19.9 | 6.8 | **26.7** | 7.2 | 17.5 | **24.8** | 2.9 | 4.9 | **7.8** |
| 2020 | 20.6 | 6.9 | **27.5** | 7.2 | 16.9 | **24.1** | 2.8 | 4.8 | **7.6** |
| 2021 | 24.1 | 7.1 | **31.2** | 7.8 | 18.5 | **26.3** | 2.8 | 5.2 | **8.0** |
| Percentage annual change | | | | | | | | | |
| 2014-2015 | 7.5 | -2.2 | **4.6** | 0.9 | -3.2 | **-2.0** | -3.1 | 5.1 | **1.7** |
| 2015-2016 | 4.3 | 6.6 | **5.0** | 6.3 | 2.4 | **3.6** | 0.2 | 3.2 | **2.0** |
| 2016-2017 | 6.6 | -0.4 | **4.6** | 4.2 | 6.8 | **6.0** | 7.3 | 8.0 | **7.7** |
| 2017-2018 | 7.1 | 1.0 | **5.4** | -0.9 | 5.7 | **3.7** | 5.3 | 7.6 | **6.7** |
| 2018-2019 | -3.6 | -7.4 | **-4.6** | 2.9 | 1.4 | **1.8** | -4.7 | -0.2 | **-1.9** |
| 2019-2020 | 3.2 | 1.6 | **2.8** | -0.1 | -3.7 | **-2.6** | -4.6 | -2.4 | **-3.2** |
| 2020-2021 | 17.1 | 2.7 | **13.5** | 8.0 | 9.5 | **9.0** | 1.4 | 9.0 | **6.2** |

## 1.4. Empty Container Management

According to the last section, it can be concluded that the pernicious accumulation or the lack of empty containers at certain terminals is mainly caused by the imbalance or difference in containerised exports and imports (Kolar et al., 2018). In addition, the container imbalance can cause some other potential problems, such as an extreme rise in empty container leasing costs and delivery issues, which also potentially could threaten liner shipping companies' prestige (Özdemir, 2018). Therefore, the current problem of container accumulation and imbalance deserves to attract attention from many academia and industry (Luo and Chang, 2019). In this section, some experiences of empty container managements will be introduced, including saving operational cost, empty container leasing, $ECR$ and Empty Container Sharing ($ECS$).

### 1.4.1 Saving operation cost

As mentioned in section 1.3, there are many factors which can cause unnecessary empty container movements and extra cost to a shipping company. So, some innovative methods have been proposed. For example, Chen et al. (2016b) claimed that filling those empty containers with wastes and scrap is to some extent a good way to recover operational cost. The waste and the scrap may include metal, plastic, paper waste and other recyclable items (Chen et al., 2016b). Such low-valued freight can be economically shipped in the container, avoiding the container being returned empty (Chen et al., 2016b). Ford (2022) pointed out that spending on shipping a container full of scrap metal from Los Angeles to China is less than the cost of transporting it to Chicago from Los Angeles. Morrison (2018) stated that U.S. exported about $5,182 million worth of waste and scrap to China in 2016, which constituted 4.5% of the total amount of exports, and this figure further rose to a value of $5,625 in 2017. The European Commission (2018) reported that there were 19 million tonnes of notified waste in 2014 transported into $EU$ countries from other countries by ship, while the figure grew to 25 million tonnes in 2018. Therefore, there is sufficient waste and scrap that can be packed into empty containers and economically delivered to other countries. Thus, shipping companies can earn some profits saving part of their costs.

### 1.4.2 Empty container leasing

In recent years, many Liner Shipping Carriers ($LSCs$) prefer to lease containers via a company as an option to improve the efficiency of empty container management (Hu et al., 2020). IICL (2022) reported that the number of 20 feet equivalent units ($TEUs$) of containers that were controlled by the container leasing companies was 6.7 million in 2001 while this number rose to 10.7 million in 2009. Furthermore, approximately 40-50% of the world container fleet were shared systematically by the container lessors from 2001 to 2007 (Dong and Song, 2012; Lloyd's List, 2022). However, the study on empty container leasing is rather limited (Dong and Song, 2012). Although some investigations have solved the leasing problems to a certain degree, most of the scholars only consider it implicitly and concentrate on $ECR$ problem (Dong and Song, 2012). In practice, the shipping companies cannot successfully find sufficient empty containers when needed because of many factors such as container distribution imbalance. These papers also claimed that shipping companies can return the containers to the lessor at any time (Dong and Song, 2012). In fact, even if a shipping company can lease sufficient empty containers, the truth is that the empty containers cannot be sent back to the lessor immediately due to several external reasons, e.g., wars and natural disaster. Therefore, many empty containers are blocked in the terminals and ports available and waiting for the next shipping activity.

Papers relating to empty container leasing include Crainic et al. (1993a); Lai et al. (1995); Cheung and Chen, (1998); Lam et al. (2007); Song (2007); Song et al. (2007) and Moon et al. (2010). Two types of cost models emerged from these papers, namely (1) one-off model and (2) time-based model. The one-off model requires a leasing company to charge a fixed one-off fee for every leased container based on the leasing-in or leasing-off activity, while in the time-based model a leasing company charges costs in proportion to the lease duration (Dong and Song, 2012). There are few investigations in the existing literature focus on empty container leasing term decisions explicitly.

Dong and Song (2012) discusses reasons why such limited research focuses on empty container leasing decisions for container shipping companies. Firstly, it is a complex issue and takes different forms, presenting challenges in their formulation in a mathematical model (Dong and Song, 2012). Secondly, if the planning horizon keeps relatively short, the long-term leased empty containers can be seen totally as shipping company's own property. For example, if a shipping company lease several empty containers for three years meanwhile it considers signing a one-year contract with the other shipping company for $ECS$, then the leased empty containers can be considered as its own property in the signed contract. Thirdly, a complicated interaction exists among the empty container leasing decision and the other decisions. Therefore, an empty container leasing decision is usually ignored and investigated implicitly Dong and Song (2012).

In conclusion, empty container leasing is an effective method to improve the efficiency of empty containers by avoiding unnecessary empty container movement and finally optimise empty container management. In spite of this, the industry still faces a number of challenges, particularly in light of the current complex external environment.

### 1.4.3 $ECR$ and $ECS$

Both $ECR$ and $ECS$ are popular in empty container management. In recent years, many international $LSCs$ have widely accepted $ECR$. However, due to external and internal factors, $ECR$ has become one of the most critical management challenges in the shipping industry (Song and Dong, 2015). Inappropriate $ECR$ strategy usually results in a huge cost to carriers. For example, the Annual Review of Maritime Transport published by United Nations Conference on Trade and Development's ($UNCTAD$) (2011) stated that the $ECR$ cost accounted for approximately \$20 billion for seaborne transportation in 2009 while it was \$10 billion for landside transportation ($UNCTAD$, 2011). Not to mention that improper $ECR$ strategy further incurs an enormous cost due to empty container stocking, handling and moving (Song and Dong, 2008).

Almost at the same time, recently, based on the concept of "sharing economy", $ECS$ has been proposed as an effective way to reduce the costs of $ECR$ increasing shippers' profits in practice (Sterzik et al., 2012). Kopfer and Sterzik (2012) first proposed the concept of container sharing to reduce the number of empty containers that have to be repositioned. $ECS$ has been proved to be an increasingly critical method to solve empty container accumulation (Xie et al., 2017). In particular, since the outbreak of the COVID-19 pandemic, this suggestion has become more important than ever, as both empty container shortages and full containers piling up have become more severe due to disruptions in container shipping systems that were not foreseen (Toygar et al., 2022).

By applying the concept of $ECS$, many $LSCs$ could work together and form shipping alliances. The empty container vessel capacities sharing is usually conducted between shipping lines in horizontal collaboration. In doing so, although different shipping carriers are in a competitive relationship, the carriers cooperate to some extent to guarantee sustainable development (Ming et al., 2014). Such a relationship is usually called "co-opetition" in supply chain management (Chen et al., 2019), which means that both competition and cooperation exist between $LSCs$ simultaneously. Furthermore, to implement $ECS$ among different $LSCs$, it is necessary to sign contracts that specify the number of containers to be shared and the split of benefits and costs resulting from the sharing of containers. The contracts available in the literature include the Wholesale Price Contract ($WPC$), Buy-back Contract ($BBC$), Revenue-sharing Contract ($RSC$), Two-part Tariffs Contract ($TTC$), Cost-sharing Contract ($CSC$), Quality-compensation Contract ($QCC$), Risk-sharing Contract ($RiSC$), Equal Share Contract ($ESC$) and Fair Share Contract ($FSC$) and Quantity Flexibility Contract ($QFC$) (Snyder and Shen, 2011; Luo and Chang, 2019). By applying such contracts above, the shipping carriers' system may have the chance to be coordinated. In supply chain management, "supply chain coordination" (or "system coordination") is usually defined so as to entice players to act to achieve a supply chain maximised by applying a certain contract (Snyder and Shen, 2011). Given the background of this research, in the maritime sector, the system coordination means that the total system's profit is maximised in an ideal centralised mode; meanwhile, the sum of individual carriers' profit in a practical decentralised mode equals the maximum system profit in the centralised mode. The centralised mode means all the $LSCs$ make perfect decisions for sharing strategy, whilst the decentralised mode is where the carrier makes the decision separately based on their own situation. The recent research by Xie et al. (2017) indicated that some contracts, e.g., $BBC$, can ensure the coordination of container sharing supply chains. However, it remains unclear whether these contracts still can achieve the coordination of the container sharing supply chains when the government introduces the $CT$ scheme.

The $CT$ policy implementation does affect the international $LSCs'$ co-opetition. The $CT$ policy can be seen as an extra expenditure of the $LSCs$ and may affect the companies' sustained competitive advantages and business profits (Kuo et al., 2016). The $CT$ policy could further affect the consignor's demand for empty containers because the tax is finally passed to the customer. According to Zhao (2011), because shipping is an energy-intensive industry, $CT$ will have a significant impact on the operations of international maritime shipping companies. Thus, $CT$ policy may encourage the customer to choose another transport mode, which negatively impacts consignors' demands and, consequently, carriers' decisions regarding container sharing. Additionally, the $CT$ policy may undermine the efficiency of the aforementioned contracts that could potentially coordinate the container supply chain system. From the perspective of $LSCs$, the $ECS$ should not only consider the $CT$ policy impact, but they must pay attention also to the interaction with the other competitors (Zhang et al., 2021). Therefore, the $CT$ rate and the market co-opetition relationship are the keys to keeping the total social welfare and business interest balanced between the policymaker (i.e., government in this thesis) and the companies (i.e., $LSCs$ in this thesis) (Chen and Hao, 2014; Chen et al., 2010) to achieve their social responsibility and profitability, respectively.

All in all, the utilisation of empty containers can be affected by factors such as government $CT$ policy and $LSCs'$ co-opetition relationship. There are some methods such as $ECR$ and $ECS$ proposed to improve empty container utilisation and transportation efficiency in current research. Therefore, both methods should be discussed and reviewed in depth to find a solution to the problem of empty container shortage and accumulation. They will be discussed further in Chapter 2.

### 1.5. Conclusions

In this chapter, the topics and the research field on which this thesis focuses were introduced. Firstly, it introduced the current issue of global warming and climate change. The environmental issues are strongly related to carbon emission and of increasing importance in recent decades. Secondly, it pointed out the problem of container accumulation and empty container management in international shipping activity which has attracted more attention during the global Covid-19 pandemic. Based on this chapter of introduction, this thesis is further developed in the following chapters.

In Chapter 2, a full literature review underpins and builds the foundations for this research and identifies the research gap. In Chapter 3, the methodology applied in this thesis will be introduced. In Chapter 4, the thesis will investigate how the government $CT$ policy influences the $ECS$ when they are bound by a $BBC$. Also, in this chapter, the government $CT$ rate is discussed as a constant parameter. Chapter 5 focuses on how the $CT$ policy affects the $LSCs$ when $CT$ is considered as a decision variable. Also, a new style contract, the $RSC$, is introduced in the model. In Chapter 6, still adopting $RSC$ as binding between two carriers, not only is government $CT$ rate assumed to be a decision variable, but also, as a player, makes $CT$ rate to $LSCs$ to maximise its utility based on its total social welfare function. Therefore, in Chapter 6, the government's social welfare function is introduced in the model, and it interacts with the $LSCs$ in a Stackelberg game. In Chapter 7, the findings are concluded, and an attempt are made to compare the results from Chapters 4, 5 and 6. Also, the limitations of this study and future research presented will be stated. In addition, a full bibliography is listed in Chapter 8 to acknowledge the contributions made by others to this research. Finally, all the mathematical calculation and proof process of Lemmas, Theorems, Corollaries, Conditions in all chapters are articulated in appendixes. Table 1.8 illustrates the topics and structures of this thesis.

In the next chapter, there will be an examination of a full literature review on the topics and the research fields that were introduced in this chapter. Also, the research gaps between this thesis and the previous literature will be highlighted.

**Table 1.8** The thesis's subjects in each chapter

| **In which chapter** | **Detail** |
|---|---|
| Chapter 2 | Mainly examining the related literature of investigation on the issue of container management, $ECR$, $ECS$ and the impact of $CT$ on supply chain management, inventory management or empty container management. Specifically, the paper of "*Empty container management and coordination in intermodal transport*" written by Xie et al. (2017) will be reviewed in detail because the models adopted in this thesis are developed from their research. |
| Chapter 3 | Presenting the methodology applied in this thesis. Firstly, a brief introduction to the concept and classification of ontology, epistemology and philosophical perspective of the research. Secondly, the research subjects of this thesis such as $OR$ and $OM$ and Game Theory (GT) will be introduced. Thirdly, the philosophical perspective adopted by this thesis will be explained. Most importantly, lastly, the research design of each research topic in Chapter 4, 5 and 6 will be presented in detail. |
| Chapter 4 | Following the research conducted by Xie et al. (2017), by introducing the factor of $CT$, this thesis will explore how government $CT$ policy as a constant factor affects $ECS$ and system coordination between two $LSCs$. The new findings will be stated and a comparison made between this research and the paper of Xie et al. (2017). |
| Chapter 5 | When $CT$ is considered as a decision endogenous variable instead of just being a constant parameter, the thesis explores how government $CT$ policy influences $ECS$ and system coordination. Also, the binding contract between the carriers will be replaced by a new type of contract compared with the research in Xie et al. (2017) to further explore the same topic. In addition, the new findings and direct comparison will be shown when two different contracts are applied in the model and considering $CT$ as a decision variable. |
| Chapter 6 | Finally, compared with the 4$^{th}$ subject, not only does the thesis assume $CT$ as an endogenous decision variable in the model, but also it will introduce government total social welfare function and establish a Stackelberg game between the $LSCs$ and the government. The thesis will explain how the government $CT$ policy affects two $LSCs$' cooperation and system coordination, given the new applied contract instead of the contract used in Xie et al. (2017). |
| Chapter 7 | The investigation of the three topics in Chapters 4, 5 and 6 respectively will be concluded in this chapter along with some important comparisons based on the new findings in each chapter. Also, some managerial insight will be demonstrated to assist the related industry's development. Finally, the potential limitations which exist in this research will be clarified and future research articulated. |

# Chapter 2 Literature Review

## 2.1. Introduction

Chapter 1 proved the background to this thesis, including climate change and empty container management. This chapter provides a full literature review to build the foundation for the research in this thesis. From section 2.2 to section 2.7, the current issue of container management and related effective solutions will be presented. Specifically, Empty Container Repositioning ($ECR$) in section 2.2 will be followed by the related research of the related research of Game Theory ($GT$), Newsvendor problem and supply chain contract design in section 2.3 and 2.4 to support the further $ECS$ research reviewing. Empty Container Sharing ($ECS$) will be fully presented in section 2.5, 2.6 and 2.7 to examine the review of how Stackelberg game, as one of the most used models, is applied in the research of container management. Next, section 2.8 presents the existing research of how and why government Carbon Tax ($CT$) policy and Cap-and-Trade ($CAT$) system are adopted to mitigate global warming and stop climate change. Most importantly, there will be a review of the research on how the application of $CT$ and $CAT$ affect supply chain, inventory and container management. Finally, a comprehensive conclusion of this chapter is made in section 2.9.

## 2.2. Empty Container Repositioning ($ECR$)

Apart from difficult long-term empty container leasing decision modelling, besides saving of cost by delivering low-valued freight is not sufficient for shipping carriers. The empty container management problem fundamentally should be solved by appropriately deploying empty containers in different ports for use as needed. Therefore, since $ECR$ was proposed, it has been fairly well investigated and attracted many scholars in recent years (Xie et al., 2017). $ECR$ is a practical method to manage empty container management (more specifically, optimising empty container storage and movement) to keep a balance between empty containers' supply and demand (Hu et al., 2020). XChange (2019) described $ECR$ as "*moving empty containers from an area with a surplus of containers to a location with a deficit*". Song and Dong (2015) pointed out that $ECR$ aims to minimise the relevant costs by efficiently and effectively repositioning empty containers while meeting customer demands at the same time. The role of $ECR$ is vital for the liner shipping industry having its potential to not only an economic impact on stakeholders in the container transport chain, but also its potential to reduce Greenhouse Gas ($GHG$) emissions and achieve sustainability because $ECR$ can decrease empty container movements and further diminish fuel consumption and reduce congestion (Song and Dong, 2015). Based on the transport modes in different research contexts, the literature relevant to the $ECR$ problem can be classified into three groups, namely 1) in seaborne shipping networks; 2) in inland or intermodal transportation networks and 3) being treated as a sub-problem or a constraint under other decision-making problems (Song and Dong, 2015).

### 2.2.1. $ECR$ problem in seaborne shipping networks

Firstly, a sole shipping line route planning or a network with some certain route structure is considered by Lai et al. (1995); Du and Hall (1997); Li et al. (2004); Song and Zhang (2010); Song (2007); Lam et al. (2007); Shi and Xu (2011); Li et al. (2007); Song and Dong (2008); Zhang et al. (2014); Feng and Chang (2008); Dong and Song, (2009); Chou et al. (2010) and Song and Dong (2011). Because of facing similar problems in terms of logistic and empty container allocation, Lai et al. (1995) applied heuristic research to investigate a policy for a shipping company in Hong Kong to identify the lowest operating cost including leasing, storage, pick-up, drop-off and other charges. They focus on the $ECR$ problem in a shipping route between the Middle East to the Far East controlled by vessel schedules and capacities. Based on inventory management theory, Du and Hall (1997) explored empty containers' decentralised stock control policies. In deriving effective operational strategies, Lam et al. (2007) proved that approximate dynamic programming approach (a powerful method for solving multistage stochastic control problems at large scales) can be successfully applied in the empty containers' relocation problem in the shipping industry. They firstly develop a dynamic stochastic model for a simple Two-ports Two-voyages ($TPTV$) system to examine the effectiveness of the approximate optimal solution obtained from temporal difference ($TD$) learning for minimising average operational cost (Lam et al., 2007). In a two-port system, Shi and Xu (2011) built a Markov Decision Process model for $ECR$ problem. They consider offline and online scenarios, while the offline scenario assumes that the demand is a random variable with known distribution and the online scenario means that the empty container demand is partly understood (Shi and Xu, 2011). They explore the optimal empty container controlling policies in both cases and they claim that the online scenario allows the possibility to conduct an online optimization using real-time information (Shi and Xu, 2011). Chou et al. (2010) conducted a similar study of the optimal $ECR$ problem management between ports, but in a two-stage model. In the first stage, they build a fuzzy backorder quantity inventory decision making model to decide the number of empty containers at a port, while in the second stage they apply an optimization mathematical programming network model to calculate the optimal allocated empty containers between ports (Chou et al., 2010). Their results show that fuzzy decision making, and optimization programming model can successfully be used to solve the $ECR$ problem (Chou et al., 2010). Song and Dong

(2008) proposed a cyclic shipping route to manage the $ECR$ problem and find the optimal solution to minimise the total cost which included demand lost-sale and inventory holding and container transportation costs as well as lifting-on and lifting-off charges in the of dynamic and stochastic situation.

More general liner shipping networks is examined by other studies (Shen and Khoong, 1995; Cheung and Chen, 1998; Cheang and Lim, 2005; Erera et al., 2009; Di Francesco et al., 2009; Moon et al., 2010; Brouer et al., 2011; Song and Dong, 2012; Epstein et al., 2012; Long et al., 2012; Di Francesco et al., 2013). Shen and Khoong (1995) proposed a Decision Support System ($DSS$) for a shipping company to manage empty containers in a large planning scale with consideration of the empty containers' multiperiod distribution (i.e., allocate the empty container to alternative location at a different stage). Cheung and Chen (1998) developed a dynamic $ECR$ problem in a two-stage stochastic network where they determine the empty container repositing and the number of leased empty containers that are required to meet customer's demand. Di Francesco et al. (2009) built a time-extended multi-scenario optimization model to solve a container repositioning problem where some parameters are uncertain and the historical data is not available to use for decision making. Furthermore, Di Francesco et al. (2013) assumed an $ECR$ problem in a shipping network under possible port disruptions. They apply a stochastic programming approach (i.e., some of the parameters of the optimisation problem are uncertain but are distributed according to known probability distributions) concerning the uncertainty of relevant influencing data to solve $ECR$ problem (Di Francesco et al., 2013). Between different ports (terminals), Moon et al. (2010) considered an $ECR$ problem where they mixed integer programming and genetic algorithms to be applied in the model with leasing and purchasing of containers between different ports (terminals) to minimise the total operational cost including handling cost, transport and holding cost, etc. Long et al. (2012) considered a two-stage stochastic programming model concerning random demand, supply and space capacity to minimize the total operational cost of the $ECR$ problem. Erera et al. (2009) explored a robust optimisation framework for dynamic $ECR$ problem by applied time-space network where the problem considers the uncertainty from future assets' forecasts of supplies and demands at different times. Although all the studies examined above are about $ECR$ problem management and solution, they still only focus on seaborne shipping networks. Next, the literature of a more complicated nature called intermodal (shipping plus rail) shipping system, rather than seaborne shipping network, is reviewed for $ECR$ problem solution.

### 2.2.2. $ECR$ problem in inland or intermodal system

The seaport terminal is always a key node of the maritime sector, and it determines the efficiency of container transportation (Yu et al., 2021). However, the empty container accumulation, the lack of handling capacity, and the low efficiency of terminal operators increasingly are becoming problems nowadays (Xie et al., 2017). Therefore, a way of building dry ports as an inland transport terminal with a high capacity for storage of empty container addressing the lack of space in the seaport is proposed. The dry port is linked with the seaport (Xie et al., 2017; Roso and Lumsden, 2009). The seaport and dry port can form an intermodal system connected with rail or road transport and the dry port also directly provides a service between the hinterland and transmarine destinations (Jaržemskis and Vasiliauskas, 2007). A wealth of research has focused on $ECR$ and management in the intermodal system (Crainic et al., 1993a; Crainic et al., 1993b; Choong et al., 2002; Caris and Janssens, 2010; Nossack and Pesch, 2013; Xie et al., 2017; Zhang et al., 2017; Kolar et al., 2018; Kuzmicz and Pesch, 2019; Zhang et al., 2020; Zhao et al., 2018; Luo and Chang, 2019; Song and Dong, 2015; Braekers et al., 2011; Erera et al., 2005; Olivo et al., 2005; Bandeira et al., 2009; Dang et al., 2013; Lee et al., 2012).

Crainic et al. (1993a) and Crainic et al.（1993b) explored the empty container allocation problem in an inland transport network which is close to a seaport. Kolar et al. (2018) summarised the literature that focuses on $ECR$ problem in the intermodal system and categorised the literature into two groups, these being intra-organisational perspective of a single company and inter-organisational company, respectively. Song and Dong (2015) also concluded some valuable investigations on $ECR$ problem in an intermodal system and they propose some effective solutions to solve the $ECR$ problem such as: intra-channel solutions, inter-channel solutions, organisational solutions and technological solutions. Song and Dong (2015) claimed that most literature focuses on the intermodal system at a regional level. Braekers et al. (2011) developed a study to improve empty container management at a regional level (among importers, exporters, inland depots and ports within a minor geographical region) and reduce empty container movements simultaneously. They also describe some management decisions (e.g., empty container allocation decision and routing planning) that should be taken at the strategic planning level, tactical planning level and operational planning level. Erera et al. (2005) concentrated on the container operators' asset management (e.g., cost and fleet sizes management) in an intermodal system and formulated the container management problem as a large-scale multi-commodity flow problem. They successfully decrease the fleet sizes and total operating costs by combining container routing optimisation and $ECR$ decisions in a single model. Olivo et al. (2005) developed a mathematical model (integer programming model) to manage empty container flow in a deterministic dynamic multimodal network where the decision of $ECR$ is made day-by-day in the network, and conduct algorithms to verify the implementation. It proved that the algorithms have a good performance in addressing $ECR$ problem. Choong et al. (2002) conducted a computational analysis of empty container management for intermodal transportation networks. Bandeira et al. (2009) addressed the problem of unbalanced export/import containers trading in a network where customers, leasing companies, harbours and warehouses exist. They manage the integrated empty and full containers' distribution by applying a heuristic method, which involves finding the best solution to a problem quickly, effectively, and efficiently (Song and Dong, 2015; Bandeira et al., 2009). Dang et al. (2013) explored the empty container positioning problem in a port district with multiple depots and they proposed three repositioning options: empty container positioning from overseas ports, empties positioning between depots and leasing empties from the container lessor, respectively. In a multi-port system with inventory-based control mechanism, Lee et al. (2012) investigated a joint problem combining container fleet sizing and $ECR$ and developed a single-level threshold policy with a repositioning rule to minimise repositioning cost. Song and Dong (2015) claimed that Lee et al. (2012) model can be appropriately seen as an intermodal or a regional network because they assume that the shipping routes are not considered clearly, and each pair of ports' travel time is less than one period length, which means that travel time is not greater than the normal travel time and it may cause empty container accumulation and port blocking. Zhao et al. (2018) conducted an $ECR$ problem between a liner and a rail firm in an intermodal system to minimise the total cost when $CO_2$ emission, stochastic demand and supply are concerned. They found that the weights of repositioning cost and $CO_2$ emission cost are the two main parts for the total cost compared with the inventory cost and leasing cost. In an intermodal system, Xie et al. (2017) explored the empty container inventory sharing problem between one liner carrier and one rail carrier. They apply a centralised and decentralised model to obtain the optimal shared empty container under system coordination. While considering customer demand switching, Luo and Chang (2019) also investigated the empty container inventory sharing game between a seaport and a dry port in an intermodal system and coordinated the system by implementing a contract, improving the empty container management performance and enhancing each participant's profit. Zhang et al. (2017) designed a mixed integer linear programming model to decide the optimal $ECR$ strategy in an intermodal system. Nossack and Pesch (2013) solved the problem of empty container accumulation by investigating a truck scheduling problem in intermodal container transportation where the containers are transported from the seaport or dry port terminal to the customers and vice versa and found that the terminal efficiency can be improved by about 4% to 30% when the total truck operating time is minimised.

In summary, the intermodal system is more complex than seaborne shipping networks (Song and Dong, 2015). Also, most related research focuses on empty container management at regional level or treat the empty container transport as a flow, as well as ignoring the individual vessels and their schedules because there is remarkable difference between inland transportation's time scale and sea transportation's time scale (Song and Dong, 2015). Next, the final type of $ECR$ problem which is usually treated as a constraint in a sub-problem instead of being the main investigation target, will be briefly introduced.

### 2.2.3. $ECR$ problem as a sub-problem or a constraint

In this subsection, the $ECR$ problem is seen as a constraint or solved as a sub-problem within the other decision-making problem (Song and Dong, 2015). This research was conducted by Jula et al. (2006); Chang et al. (2008); Imai and Rivera (2001); Zhou and Lee (2009); Shintani et al. (2007); Imai et al. (2009); Meng and Wang (2011a); Song and Dong (2013); Braekers et al. (2013); Wang (2013); Meng and Wang (2011b) and Wang and Meng (2012). Some studies concentrate on dynamic empty container reuse with regard to the $ECR$ problem. For example, the most representative is the research of Jula et al. (2006), which optimised the empty container reuse to reduce the traffic congestion in the Los Angeles and Long Beach port area with the consideration of $ECR$. In order to reduce the cost of empty container interchange, Chang et al. (2008) studied reuse between empty containers of different types. Meanwhile, some investigations, such as Meng and Wang (2011b), focus on ship fleet planning when the $ECR$ problem is also considered. They suggested a multi-period liner ship fleet planning problem for a shipping company, and they formulate the problem as a scenario-based dynamic programming model where mixed-integer linear programming are included for each single planning period (Meng and Wang, 2011b). By explicitly taking the repositioning of empty containers into account, Shintani et al. (2007) addresses the design of container liner shipping networks and shipping fleets. In addition, the $ECR$ problem also is considered as a sub-problem within other main problems such as ship fleet deployment, transport market pricing and competition and shipping service route design (Song and Dong, 2015). However, this thesis will not examine the details of these in depth because this research will discuss an alternative method, $ECS$, to address the problem of empty container management, which is quite different from $ECR$ in terms of shipping route design and cost management. This will be discussed in the section 2.5.

In summary, $ECR$ has been well studied in terms of solving empty container management. Table 2.1 summarises the details of the majority of the studies reviewed in section 2.2, including the research method, the research topic, findings, and the problem encountered. In the next two sections, there will be reviews of the research on $GT$, the Newsvendor problem and supply chain contract design prior to reviewing $ECS$ research in section 2.5, an alternative innovative method to improve the efficiency of empty container management.

**Table 2.1** The summary of research details reviewed in section 2.1 (including subsection 2.2.1, 2.2.2 and 2.2.3).

| *ECR type* | *Authors/year* | *Research method* | *Research topic or objective* | *Achievements or findings* | *Costs minimising?* |
|---|---|---|---|---|---|
| ***Seaborne shipping networks*** *(subsection 2.2.1)* | Lai et al. (1995) | Heuristic algorithm | Examining the $ECR$ problem delivered from the Middle East to the Far East's ports in accordance with the schedules and capacities of the vessels. | Achieving the lowest operating costs through the appropriate policy making for leasing, storage, pick-up, drop-off, and other services. | √, the cost of leasing, storage, pick-up, drop-off and other charges |
| | Du and Hall (1997) | Decentralised stock control | Investigating the problem of fleet sizing and empty equipment relocation. | Developing a decentralised operating policy to manage fleet sizing and $ECR$ problem | √, the cost of purchasing and equipment maintaining |
| | Lam et al. (2007) | Approximate dynamic programming | Describing how the approximate dynamic programming can be used to develop strategies for the $ECR$ problem in the sea-cargo industry. | Developing a $TPTV$ model and finding an optimal solution for the $ECR$ problem. | √, average cost minimisation |
| | Shi and Xu (2011) | Markov Decision Process | Developing a Markov Decision Process model to solve the $ECR$ problem in a two-port system. | Getting $ECR$ optimal controlling policies in offline and online case where online case indicates demand information is disclosed. | √, leasing costs, holding costs and transportation costs |
| | Chou et al. (2010) | Fuzzy decision-making model | Optimizing empty container numbers at ports to satisfy exporters' demand by repositioning empty containers. | Applying a mixed fuzzy decision-making model and optimization programming approach, the problem of $ECR$ can be solved. | ×, all the cost are the parameters and they are fixed |
| | Song and Dong (2008) | Threshold control policy | By minimising the total costs pertaining to holding costs, goodwill penalty costs, lifting-off and lifting-on charges and container transport costs, decide the optimal $ECR$ strategy under dynamic and stochastic conditions. | A three-phase threshold control policy is developed to achieve optimal $ECR$ policy and it shows that the threshold control policies show effectiveness and stability in cyclic shipping routes. | √, the costs of inventory holding, demand lost-sale, lifting-on/off, and container transport |
| | Shen and Khoong (1995) | Network optimization | Developing a plan to distribute empty containers over a multiperiodic timeframe for a shipping company. | The system can recommend cost-effective leasing-in and off-leasing decisions for containers | Not applicable |
| | Cheung and Chen (1998) | Dynamic programming | To satisfy customers' demands, repositioning empty containers as well as determine the amount of leased containers that are required over a period of time to meet their needs. | Utilising the network structure, demonstrating how a stochastic hybrid approximation procedure and a stochastic quasi-gradient method are adopted to solve the ECR problem. | ×, all the cost are deterministic parameters |
| | Di Francesco et al. (2009) | Time-extended multi-scenario optimization model | Solving the problem of container maritime repositioning in which there are several uncertain parameters and historical data are ineffective for decision-making. | The time-extended multi-scenario optimization model is effective for the $ECR$ problem under data shortage and information uncertainty. | √, cost of loading, unloading, repositioning |

| | Authors/year | Research method | Research topic or objective | Achievements or findings | Costs minimising? |
|---|---|---|---|---|---|
| | Moon et al. (2010) | Mixed integer programming; hybrid genetic algorithm | Decreasing the imbalance between container ports by proposing a plan for transporting empty containers when considering minimising cost of transportation, holding and handling. | Hybrid genetic algorithm can solve the $ECR$ problem in a larger size and it is more efficient than linear programming based genetic algorithm in the aspect of computation time. | √, transportation cost, handling cost, and holding cost |
| | Long et al. (2012) | Progressive hedging-based algorithms | Minimise $ECR$ operational costs. | In the case of $problem$, progressive hedging-based algorithms are capable of solving the large-scale shipping network effectively. | √, minimize the expected operational cost for $ECR$ |
| | Erera et al. (2009) | Integer programming | A robust optimisation framework based on time-space networks is developed for dynamic $ECR$ problems. | Performing computational experiments that demonstrate the framework's feasibility in solving the problem. | √, cost in general |
| ***Intermodal system*** *(subsection 2.2.2)* | Crainic et al. (1993a) | Dynamic deterministic formulations | Identifying the basic structure and characteristics of the container allocation problem in the intermodal system. | Developing an approach to solving problems of container allocation and distribution that considering the demand uncertainty. | √, cost of operating the land distribution and transportation system |
| | Crainic et al. (1993b) | Mixed integer program | An algorithm for balancing requirements in the container location/allocation problem is proposed using the tabu search heuristic. | Proving that tabu search is a very competitive approach for the problem of container allocation with balancing requirements. | √, the cost of operating the depots, $ECR$ between depots and customers |
| | Kolar et al. (2018) | - | Providing a review of literature dealing with $ECR$ problem, initiated by a qualitative data analysis according to the semi-structured interviews with ocean carriers' representatives. | | Not applicable |
| | Song and Dong (2015) | - | Concluding the reasons for the $ECR$ problem and literature of network-based modelling of the $ECR$; Considering the logistics channel scope when addressing the problem. | | √, minimise the cost of laden/empty container, and goodwill penalty. |
| | Braekers et al. (2011) | - | Discussing the $ECR$ problem in three different decision planning levels, which are strategic, tactical and operational. | | Not applicable |
| | Erera et al. (2005) | Multi-commodity network flow model | A time-discretized network is used to develop the container management problem as a flow problem of large-scale multi-commodity. | The cost of total operations and fleet size can be decreased by integrating container routing and repositioning into a single model. | √, minimise the cost of laden/empty container, and depot cost. |
| | Olivo et al. (2005) | Integer programming | Developing an approach to managing empty containers based on mathematical programming. | The implementation of hourly time-steps in a dynamic network and algorithms present a good efficiency for the $ECR$ problem. | √, cost in general |
| | Choong et al. (2002) | Integer programming | Minimising the total cost of $ECR$, while meeting the requirements for the movement of loaded containers. | Empty container management is analysed in intermodal networks based on planning horizon length. Cheaper transport modes are encouraged by extending the planning range | √, empty container transportation cost and shortage cost |

| | Authors/year | Research method | Research topic or objective | Achievements or findings | Costs minimising? |
|---|---|---|---|---|---|
| | Bandeira et al. (2009) | Heuristics network model | Presenting a decision support system ($DSS$) that integrates empty and full container transshipment operations. | It is proved that the DSS system is flexible, and it can configure several parameters in empty and full containers distribution. | √, the costs of moving and handling of full/empty containers |
| | Dang et al. (2013) | Genetic algorithm | Identifying the optimal empty containers' location within a port area with multiple depots, subject to minimising inventory holding cost, overseas and inland positioning cost, and container leasing cost. | A simulation-based genetic algorithm is formulated to solve empty container replenishment strategy and it is proved that the proposed algorithm can reduce the total cost by between 28% and 46%. | √, inventory holding, overseas positioning, inland positioning and leasing costs. |
| | Lee et al. (2012) | Non-linear programming; | Optimizing fleet size and threshold policy parameters to minimise total cost per period. | Optimising computational efficiency using infinitesimal perturbation analysis. | √, minimising the repositioning cost |
| | Zhao et al. (2018) | Nonlinear integer programming | A sea-rail intermodal transport system with stochastic demand and supply is considered in order to investigate the $ECR$ problem, subject to minimising the expected value of weighted cost. | Emission costs rise due to stochastic demand and supply. Leasing and inventory costs rise when parameters are uncertain. Emission-related costs greatly influence total costs. | √, $CO_2$ emission-related cost, inventory cost and leasing cost |
| | Zhang et al. (2017) | Mixed integer linear programming | By taking into consideration both standard and foldable containers in the Belt and Road Initiative intermodal transportation network, an $ECR$ model is developed. | Exploring Artificial Bee Colony algorithms to find the solution for large-scale $ECR$ problems and demonstrating the proposed algorithms' efficacy through numerical experiments. | Not applicable |
| | Nossack and Pesch (2013) | Full-truck load pickup and delivery Problem | Maximizing the efficiency of all trucks in use by minimizing their total operating time in the $ECR$ problem. | A two-stage heuristic is better in terms of computational efficiency when solving the problem. | Not applicable |
| ***ECR* problem as a sub-problem or a constraint** *(subsection 2.2.3)* | Jula et al. (2006) | Dynamic optimisation | Aiming to reduce congestion by optimizing the reuse of empty containers in Los Angeles and Long Beach ports. | Reusing empty containers in the area can result in significant reductions in costs and congestion, simulation suggests. | √, the cost of dynamic empty container movements. |
| | Meng and Wang (2011b) | Dynamic/integer linear programming model | The problem of planning a liner containership fleet for a liner shipping company over multiple periods is presented in a more realistic manner. | It finds that although it is cheaper than buying ships short-term, chartering ships might not be the best policy for long-term planning. | √, cost of voyage, daily lay-up, chartering in a particular ship |
| | Chang et al. (2008) | Heuristics network model | Multi-commodity substitution problem | According to computational tests, heuristic methods are relatively fast at yielding a sub-optimal integer solution that is of high quality. | √,empty container interchange. |

## 2.3. Game theory ($GT$)

In the last section, this thesis has fully reviewed the previous research related to the $ECR$ problem. Next, in prior to reviewing $ECS$ problem, it is necessary to introduce and review of $GT$, the Newsvendor problem and supply contract design.

$GT$ is a major topic in the field of economics, management, applied mathematics, biology, psychology and computer science (Shoham and Leyton-Brown, 2009). The history of using the idea from $GT$ can be found as early as the 17th and 18th century. It is said that the $GT$ was proposed to solve gambling issues of the indolent French nobility (Kelly, 2011). However, acknowledging the contributions of mathematicians, such as Zermelo (1871–1953), Borel (1871–1956) and Neumann (1903–1957), the development of $GT$ was intensified in 1920s (Osborne, 2000; Kelly, 2011). For example, given the condition that both players in a competitive game fully know its counterpart's preference and information, Zermelo (no data) proves that there exists an optimal strategy for both players in the game (Kelly, 2011). In the 1950s, the related $GT$ model and knowledge was initially applied in economics, government management and political science (Osborne, 2000). Psychologists also started to explore human behaviour in experimental gaming by using $GT$ and biologists applied it to evolutionary biology (Osborne, 2000). Finally, after the 1970s, the game theoretic methods became widely used and popular in microeconomic theory and other social science fields (Osborne, 2000).

Many scholars have given a definition of $GT$. Kelly (2011) defined $GT$ as *how to make independent and interdependent decision among different game players*. Ungureanu (2018a) and Tadelis (2013) were of the same opinion. They treat $GT$ as a mathematical tool to conduct the decision-making process in a situation of conflict and cooperation among rational and intelligent players. Ungureanu (2018a) also gave the definition of "rational" and "intelligent" in the context of $GT$. The "rational" means the players in the game always pursue their maximum pay-off while "intelligent" stands for the player gaining with sufficient comprehensive knowledge to make the decision individually (Ungureanu, 2018a). Similarly, Osborne (2000) stated that $GT$ aims to help people understand the circumstances where decision-makers interact. Shoham and Leyton-Brown (2009) said that $GT$ mathematically investigates the interactions among independent and self-interested subjects. All the definitions are fairly similar and the common words of these definitions are "interaction", "mathematics" and "decision-making". Thus, scholars such as Aumann (1989) claimed that "Interactive Decision Theory ($IDT$)" should be an appropriate name to replace the name of "game theory".

Also, Gilles (2010) believed that $GT$ is an *implement to solve and analyse social interactive problems and make the optimal decision*. Thus, it is neither a single theory nor an unified knowledge system; instead, it is a collection of different subfields and approaches to solve various social interactive decision situations (Gilles, 2010). Gilles (2010) concluded that there were three common features of $GT$ namely:

- Multiple decision makers (at least 2) are involved in the social interactive decision situations;
- Principally, each decision maker has the full right to control their own decision in these situations; and
- The decision made by one of the decision makers could be made collaboratively with others.

It should be noted that the third feature is vital. It shows that the decision that a decision maker makes not only depends on this decision maker's strategy, it also depends on the other decision maker's strategy which this decision maker cannot control (Vella, 2021). Moreover, it means that each decision maker should make the best decision for themselves and simultaneously taking into account the other decision makers' decision since the others' strategy more or less could affect the decision maker's decision (Vella, 2021).

In $GT$, people usually abstract the complex problems in the real world to simple and understandable models, which are called "strategic game". In each strategic game, decision-makers are referred to as "players" and each player possesses several options from which to choose. The choices made by the players are referred to as the "action" and each player has its own preference action at different stages of the game. Then, a player's preference action can be identified by building a pay-off function (Osborne, 2000). Also, generally speaking, the time is not usually considered in $GT$, which means that all players in the game choose the action at once and simultaneously. However, there are some specific game models that consider the time and sequence of action that players make, such as the Stackelberg game, which will be introduced later in section 2.6.

In summary, $GT$ is a fairly complicated yet popular topic in many research fields. There are many classic strategic games designed by many scholars to better understand the idea of $GT$ such as "the Prisoner's Dilemma", "Bach or Stravinsky?", "Matching Pennies" and "the Stag Hunt". For sake of convenience, "the Prisoner's Dilemma" (van Dijk, 2015) and "Matching Pennies" (Brock, 2020) as two examples are presented below to demonstrate how $GT$ is applied for players' decision making.

### 2.3.1. The Prisoner's dilemma

The most popular and typical strategic game in $GT$ is the "Prisoner's dilemma". This assumes that there are two prisoners in jail and each prisoner has two strategies:

*__Strategy 1__-Expose the other prisoner's crime and __Strategy 2__-Keep silence.*

Table 2.2 demonstrates an example of the game. If one of the prisoners confesses the other prisoner's crime, then the prisoner could only be detained for 2 years, but the other prisoner receives 6 years (note that the prisoners are separated, and one cannot know the other's strategy). If both prisoners expose the crime to each other, both get 4 years detention. If both prisoners keep their silence, then both can get 1 year detention. Each player has to choose a strategy in the game and the strategies that both players choose respectively to form an "outcome" (Snyder and Shen, 2011). A "payoff" exists for every player in each outcome (Snyder and Shen, 2011).

**Table 2.2** The payoff for each outcome in the assumed game

| **Player No.** | **Player B** | | |
|---|---|---|---|
| **Player A** | | strategy 1 | strategy 2 |
| | strategy 1 | $(-4,-4)$ | $(-2,-6)$ |
| | strategy 2 | $(-6,-2)$ | $(-1,-1)$ |

Let us help the two prisoners to analyse the optimal outcome. If both prisoners choose strategy 1 (confess), then the outcome reaches the Nash equilibrium (-4, -4, shown in Table 2.2) as no one wants to get "profit" diminished in the game and both sides would wish to selfishly maximise their own "profit". In $GT$, it is usually referred to this outcome as Nash equilibrium, which means no player can vary his or her strategy to enhance his or her payoff unilaterally (Snyder and Shen, 2011). However, clearly, it is easy to see that this Nash equilibrium is not the optimal outcome in this game because both prisoners can be better off if they choose the other strategy (i.e., keep silence; -1,-1, shown in Table 2.2) simultaneously. Also, on the other hand, if the game is played to the infinite number of repetitions, the Nash equilibrium will move to the Pareto optimal, and the two players tend to choose to cooperate. This is because each player could have the opportunity to punish the other player for not cooperating in the previous round and the cheating motivation could be overcome by the threat of punishment (i.e., -6,-2 or -2, -6, shown in Table 2.2). Therefore, the cooperation starts to emerge as a balanced status and eventually there is no player who can get a better payoff without hurting the other (Snyder and Shen, 2011). In summary, the Pareto optimal strategy is better than the Nash equilibrium strategy in Prisoner's dilemma because it provides a mutually optimal solution.

### 2.3.2. Matching pennies

In the game of "Prisoner's dilemma", it is easy to see that both players could choose to cooperate with each other or act selfishly. In that game, two players could simultaneously receive an optimal pay-off as a whole when they choose to work together instead of pleading guilty unilaterally. However, in the game of "Matching pennies", two players must fully compete with each other (Kelly, 2011).

Assuming that there are some coins on a table, some of them are valued £2 and the others are valued £1. Two players (he and she) need to each pick one coin. The rule is demonstrated as follows:

- If two players choose the same value coin (e.g., £1), then all the coins are given to him;

- If the value of the coins that two players choose is different (e.g., he chooses £1and she chooses £2), then all the coins are given to her.

Naturally, the player only selfishly cares if he (or she) can own all the coins, so both of them want to maximise their pay-off. Therefore, in this game, two players have a completely opposite preference action and their actions are totally conflicting. For example, he always wants to exactly follow her action, meanwhile she always tends to act oppositely with him so that she can gain a maximum pay-off. Therefore, two players are in a completely competitive game.

Kelly (2011) stated that the matching game problem can be easily applied in some models in the real world such as a new production's appearance selection between an established company and a new-born company in a fixed sized market. Normally, the established company prefers to choose an appearance which is different from the appearance that the new-born company selects so that the customers could easily recognise its products and subsequently makes a purchase (Kelly, 2011). However, the new-born company would like to choose the appearance which is similar to the established firms' choice as it could promote sales (Kelly, 2011).

All in all, nowadays, $GT$ is increasingly popular in many fields, including the field of supply chain management. $GT$ usually involves multiple players, who compete with each other (Snyder and Shen, 2011). The Newsvendor model (which will be introduced in detail in section 2.4) is a traditional game played between the newsvendor and supplier. Both newsvendor and supplier should make a decision based on their own interest. On the one hand, the newsvendor tends to pay less wholesale cost to the supplier, while the supplier is likely to charge a large wholesale cost (Snyder and Shen, 2011). If the newsvendor and the supplier act selfishly, then one of the players may receive profits unilaterally because they compete mutually. However, similar to the Pareto optimal outcome in the Prisoner's dilemma, considering the profits of both players as a whole, maximising the whole supply chain's total profit seems a "fairer" solution. In supply chain management, it is usually called "*supply chain coordination*". It means that each player tends to accept a mechanism which is binding them, and it can maximise the total supply chain profit (Snyder and Shen, 2011). In supply chain coordination, all players make a decision according to the mechanism and the mechanism can entice all players to make a decision based on the maximum supply chain profit. In industry, this mechanism usually is delivered and achieved through a certain form of contract agreed and signed by all players. The details of supply chain coordination will be reviewed in next section.

Although both games shown in subsection 2.3.1 and 2.3.2 and other games mentioned previously are important, they remain belonging to the thought experiment and are quite simple and basic. In the next section, I will introduce the very classic and practical model in $GT$, called the Newsvendor problem and the application of contract in supply chain coordination also will be presented.

### 2.4. Newsvendor problem and supply chain contract design

In $GT$, the Newsvendor model has been widely investigated and has a long history in inventory management (Silver et al., 2006; Qin et al., 2011; Chang et al., 2021). The Newsvendor model also is developed from Single-period Problem ($SPP$) or the "newsboy problem" (Gaspars-Wieloch, 2016). It was originally found by economist Edgeworth in 1888 (Chang et al., 2021; Petruzzi and Dada, 1999). However, similar to other models built in the field of Operation Research ($OR$) and Operation Management ($OM$), the Newsvendor model was developed quickly and became popular in management science applications by academicians in the early 1950s (Petruzzi and Dada, 1999). In the simplest version, there are two players in the model; one is the product retailer and the other is the product supplier. A simple example to illustrate the general Newsvendor model assumes a perishable product (e.g., fresh meat, milk or fish) is ordered by a retailer and the retailer further sells it to the customer. As the retailer's goal is to pursue the maximum profit or minimise the cost, one of the most important problems that the retailer needs to be concerned with is the quantity that should be ordered from the supplier because the customer demand is uncertain. If the retailer's order is greater than the real demand, then there would be some leftover products which can lead to extra holding cost at the end of period. If the order amount is not sufficient, then the demand cannot be fully satisfied, and the goodwill penalty (or alternatively "lost sales") could be generated.

In brief, a classic Newsvendor model states that a retailer orders the product quantity to sell them under the situation of random demand and tries to maximise its expected profit or minimise its expected operation cost in a certain single period with appropriate inventory management (Gaspars-Wieloch, 2016). Also, as in the example provided above, the retailer usually does not know the exact demand distribution in the real world, so the stochastic demand of a certain product is usually the common assumption in the Newsvendor model (Mahdavi and Olsen, 2017). In addition, in the Newsvendor model, it is usual to denote the unit of inventory holding cost as an *overage cost* if some products are unsold, and the unit of cost as *underage cost* if some demands are not met (Mahdavi and Olsen, 2017). The *overage cost* is a tangible cost while the *underage cost* is less tangible because the former describes the loss of leftover products and the latter stands for the loss due to lost sales (Mahdavi and Olsen, 2017).

Furthermore, the Newsvendor model can be specifically divided into two categories, these being *price-taking* Newsvendor model and *price-setting* Newsvendor model (Arikan, 2018). For *price-taking* Newsvendor model, the retailer is usually treated as a small player in a perfectly competitive market and he cannot decide the product selling price while the price is decided centrally by the supplier (Arikan, 2018). Therefore, the selling price is only an exogenous variable for the newsvendor in *price-taking* Newsvendor model, and he needs to determine the order quantity based on the selling price and purchasing cost per unit simultaneously (Arikan, 2018). Meanwhile, for *price-setting* Newsvendor, the retailer has the power to decide the product price and the price can affect the customer's demand, so the product price is a decision variable in this case and it should be determined by the retailer with order quantity simultaneously (Arikan, 2018). Whitin (1955) was the first scholar who investigated the order quantity determination in a Newsvendor model, where the product demand is assumed to be dependent on product price. He found the optimal order quantity, whilst giving a closed-form expression for optimal selling price when a uniform demand distribution is considered to be related to a price dependent mean (Arikan, 2018). Cachon (2003) further developed the Newsvendor model where he assumed that the retailer positively disburses to expand customer demand.

In summary, the Newsvendor model in different cases has been explored in-depth and has a long history. As the $ECS$ game in this thesis is developed from Newsvendor model, the literature related to the $ECS$ game will be given in the section 2.5.

In supply chain management, the contract always is designed and applied in the Newsvendor model. The common purpose of different contract applications is to make both supplier and retailer in the Newsvendor model decide the optimal order quantity so that the supply chain profit can be maximised. When this situation occurs, the supply chain can be seen as being "coordinated" and the profit can be distributed between supplier and retailer (Becker-Peth, 2019). Cachon (2003) claimed that it is necessary to explore three questions in supply chain coordination when a certain contract is designed.

- First, how the supply chain can be coordinated by which contract?

Cachon (2003) claimed that a coordination is a state that optimal actions exist in the supply chain, otherwise, the actions are suboptimal, and the total supply chain profit cannot be maximised. In this section's context, the coordination means the retailer's order quantity in Newsvendor model.

- Second, which contract has full flexibility to let the profit to be shared arbitrarily between both sides through appropriately adjusting contract parameters (Cachon, 2003)?

Although coordination can guarantee that a retailer can make the optimal action, the share of profit increment for the retailer can still be adjusted by appropriately making a contract because a good contract could flexibly distribute the profit between retailer and supplier under coordination.

- The final question is which contract can be accepted more easily?

Such contracts with the function of coordination and flexibility usually cost a great deal, so the retailer usually tends to choose a simple contract with high efficiency even if the supply chain performance is not optimised (Cachon, 2003). Note that the definition of "efficiency" in this context means "the ratio of supply chain profit with the certain contract to the supply chain's optimal profit" (Cachon, 2003).

In the Newsvendor game, the process of contract design and application are generally described as follows:

**First** - the supplier proposes a contract to the retailer (it does not matter whether the supplier or retailer proposes the contract first, it is simply for expositional convenience (Cachon, 2003);

**Second** - the retailer considers choosing to accept or reject the contract. If the contract is accepted by the retailer, then the retailer orders a certain amount from the supplier and the supplier produces the desired amount of the product and transports it to the retailer;

**Finally** - the increment profit is made and distributed to the supplier and the retailer according to the signed contract (Cachon, 2003). In contrast, if the retailer rejects the contract made by the supplier, then both sides gain profit individually (Cachon, 2003).

In conclusion, the Newsvendor model has been widely investigated in $GT$ and in supply chain management. The $ECS$ model that is developed in this thesis has evolved from the Newsvendor model. So, in section 2.5, the application of different contracts on $ECS$ will be reviewed in detail. Also, another classic game in $GT$ called Stackelberg game will be introduced and reviewed in section 2.6.

## 2.5. Empty container sharing ($ECS$)

The method of $ECR$ to ameliorate empty container movements and improve empty container utilisation was introduced. Within previous sections, however, sometimes the application of the $ECR$ strategy is not sufficient to optimise empty container management, as was the case during the Covid-19 pandemic. Therefore, to perfectly solve the problem of empty container imbalance and shortage, and improve terminal operation efficiency, the different terminals and ports should cooperate to enhance the system's profit (Yu et al., 2021). Also, nowadays, more and more shipping companies tend to exchange or share empty containers to promote empty container usage and further save on operational costs and environmental impact when they suffer an empty container imbalance. Sterzik et al. (2012) proposed that shipping carriers share containers in order to reduce the operational costs of those companies (Sterzik et al., 2012). However, little research has focused on exploring the $ECS$ strategy and most studies still emphasise the $ECR$ problem. Tang et al. (2021) conducted a study on decreasing the operational costs and promoting a container sharing strategy by optimising empty container transfer and rent when container sharing is applied. Xue et al. (2015) considered the $ECS$ problem among a range of customers to enhance the usage, efficiency and decrease the operational costs.

Also, many studies also focus on empty container truck sharing in terminals and ports. Sterzik et al. (2012) discussed the benefits that the truck companies take from $ECS$ of empty or laden containers within seaport hinterland transportation and designed two scenarios. The first scenario assumes that the truck company can only use their own containers, while the second scenario assumes the empty containers can be inter-changed among various truck companies (Sterzik et al., 2012). Other research that concentrates on this topic includes Islam and Olsen, (2014); Islam et al. (2019); Islam et al. (2013); Islam (2017a); Islam (2017b).

Nevertheless, the methodology used in the $ECR$ problem can be applied in the $ECS$ game, specifically when there is only one stakeholder in the shipping network to make the decision. Under this circumstance, the $ECR$ problem and the $ECS$ game sometimes share the same idea based on empty container movements. However, on the other hand, if there are multiple stakeholders owning empty containers and operating container movements, then stakeholders could work together in part and promote empty container utilisation, which is a different situation to the shipping network with single stakeholder. So, the literature related to the two types of $ECR$ game will be reviewed in subsection of 2.5.1 and 2.5.2 as follows.

### 2.5.1. $ECS$ problem involving single stakeholder

In the specific domain of $ECS$, the early optimisation models mainly focus on a single stakeholder's decision. These models assume that ocean carriers or container lessors fully cooperate in an integrated way and effectively operate as a single organisation under a centralised planning system (Xie et al., 2017; Luo and Chang, 2019). Therefore, empty containers are not shared between stakeholders in this mode (Song and Carter, 2009). The common research methods applied in the single stakeholder $ECS$ problem are mixed integer linear, dynamic and stochastic programming as well as heuristic methods, all of which could help shipping company alliances achieve an optimal or suboptimal result in the centralised model (Xie et al., 2017). This has been extensively explored previously (Song and Zhang, 2010, Meng and Wang, 2011a; Song and Dong, 2012; Shintani et al., 2007; Ng et al., 2012; Zheng et al., 2015; Song, 2007; Long, Lee and Chew, 2012; Zhang et al., 2014; Lam et al., 2007; Li et al., 2004; Li et al., 2007; Cheung and Chen, 1998; Shen and Khoong, 1995; Young Yun et al., 2011). Table 2.3 summarises the methodology that has been applied in these studies.

Specifically, Shen and Khoong (1995) proposed $DSS$ (mentioned in section 2.2) to solve a problem for the distribution of empty containers in different stages in different ports by a shipping company. Zhang et al. (2014) investigated the problem of $ECS$ with stochastic demand by developing a Heuristic method (called polynomial-time algorithm, an algorithm to determine the threshold of imported) to minimise the total operating cost for multiple ports over multiple periods. By formulating the problem as a stochastic dynamic program, Ng et al. (2012) solved the optimal control problem of transferring empty containers between two depots to minimise the total cost comprising inventory holding costs, empty container transfer costs and demand backlog costs in a multiperiod planning horizon. Using the dynamic programming, Song (2007) proposed the optimal stationary policy of $ECR$ for a periodic-review shuttle service system. Young Yun et al. (2011) proposed an efficient inventory policy $(s, S)$ control for the movement of empty containers from surplus areas to deficit areas easily, which alleviate the empty container accumulation in the surplus area. Meng and Wang (2011a) proposed a design problem on a liner shipping service network combining hub-and-spoke, multi-port-calling operations and $ECR$ (mentioned in section 2.2). They use a mixed-integer linear programming model for the problem in their paper (Meng and Wang, 2011a). Song and Dong (2012) considered multiple deployed vessels on multiple service routes and multiple regular voyages to minimise the total operational costs and investigated the problem of $ECR$ and joint cargo routing by using Integer programming in a centralised model. Shintani et al. (2007) developed an algorithm-based heuristic to design a container liner shipping service network for $ECR$, where the deploying of ships and containers are considered in the problem simultaneously (mentioned in section 2.2). Zheng et al. (2015) explored the empty container allocation problem among various liner carriers in a centralised optimisation solution from a horizontal perspective (i.e., among different ports). Long et al. (2012) explored a two-stage stochastic programming model to minimise the total operational cost of the $ECR$ problem with the random variable of demand, supply, along with ship weight and space capacity (mentioned in section 2.2). Lam et al. (2007) explored effective operational strategies to manage empty container relocation in the containerized sea-cargo industry by using a dynamic stochastic model in a simple $TPTV$ system (mentioned in section 2.2). The studies mentioned above focus on a single stakeholder perspective. Combined with the research review in section 2.2, evidence that the single stakeholder $ECS$ problem can be equivalent to the $ECR$ problem. However, in real-world situations, multiple stakeholders are involved in the process of container management, a factor that will be given adequate attention in this thesis.

**Table 2.3** The summary of methodology used in $ECS$ (or $ECR$) problem with single stakeholder's decision.

| *Author* | *Research topic or objective* | *Methodology* | *Application/ case study?* | *Findings* |
|---|---|---|---|---|
| *Zhang et al. (2014)* | Cost minimising in container holding, stockouts, imports, and exports | Heuristic method | Assumed the random variable of exported and imported containers follow normal distribution and discrete uniform distribution. Both show good performance. | The approximate $ECS$ algorithm performs effectively |
| *Ng et al. (2012)* | Investigating the movement of empty containers between two ports/depots with backlogging. | Stochastic dynamic program | A numerical case is examined to demonstrate the efficiency of the approach on $ECS$ between two ports. The incurred cost is $59.87, which accounts for 0.93% of all cost | Established a useful of the optimal $ECS$ policy to share empty container in a shipping network |
| *Li et al. (2007)* | Investigating the empty container allocation problem in mult-port case | Heuristic algorithm | The algorithm is estimated using two examples: inland and global line. There is a mutual import and export agreement on contianer transport between inland ports. Three ports are in the global line but their status on contianer transport are unequal. | With the heuristic algorithm, the costs of inland lines are close to the lower bound but performance is not satisfied in global lines among ports |
| *Song (2007)* | Finding the optimal stationary policy of $ECS$ by minimizing container leasing cost, inventory cost and reposition cost. | Dynamic program; Markov decision process approach | Presenting the optimal control parameters and optimal costs when empty repositioning and container leasing costs change through a numerical example. | It is possible to obtain a stationary distribution of the states of the system ($ECS$) by using the special structure of the optimal policy |
| *Song and Zhang (2010)* | Optimal repositioning policies for empty containers are determined | Dynamic program | An assumed numerical example is given to verify how to obtain inventory-based control policies and how to adjust the empty container inventory level when the system is changed. | The simple inventory-based control policies are optimal potentially, based on the results of numerical case. As the parameters of the system change, the safety inventory level changed |
| *Cheung and Chen, 1998* | Calculating the number of leased containers needed to meet customer demand while considering the dynamic allocation of empty containers | Different algorithms (SHAPE and DETM) based on stochastic dynamic programming | The dynamic container allocation problem is evaluated using a theoretical numerical example comparing a two-stage stochastic model with a two-stage deterministic model. Additionally, the proposed algorithms were compared in terms of their efficiency. | The two algorithms both perform well in the empty container allocation. However, the soultions are very dependent on and senstive to the length of planning horization. |
| *Meng and Wang, 2011a* | An analysis is conducted of a hub-and-spoke and multiple port-calling operations combined with $ECR$ in a liner shipping service network. | Mixed-integer linear programming | The proposed mixed-integer linear programming model is evaluated for computational efficiency and practical implications by generating 24 instances based on the Asia–Europe–Oceania shipping service network. All data are derived from an international liner shipping company | CPLEX (an optimisation software) has successfully solved real-case problems based on Asian-European-Ocean shipping operations. Demonstrating cost savings over hub-and-spoke or multi-port-calling networks, or networks without $ECR$. |

| *Author* | *Research topic or objective* | *Methodology* | *Application/ case study?* | *Findings* |
|---|---|---|---|---|
| *Song and Dong, 2012* | In the context of an operational shipping network with a multitude of service routes, a multitude of deployed vessels and of regular voyages, solving the problem of joint cargo routing and $ECR$. Container lifting costs at ports and demurrage costs at transshipment ports is minimised in the planning | 1. Integer programming based on shortest paths in two stages.<br>2. Integer programming based on heuristic rules in two stages | A simple hypothetical shipping network is used in the first case (A small-scale network); and a more complex realistic shipping network is used in the second (A large-scale network). | There is a substantial difference between the performance of two solution methods and the practical policy, as determined by the results. It is preferred to use shortest-path based methods for relatively small-scale problems since they yield better results than heuristic-rule based methods. |
| *Shintani et al., 2007* | By explicitly considering ECR problem in the design of container shipping service networks. | Genetic algorithm-based heuristic | Considering several impact factors (e.g., ship size, calling frequency and storage cost at each port) on the problem formulation of a shipping route with a focus on container transportation in Southeast Asia. | By taking into consideration the $ECR$ problem when formulating the problem, the algorithm becomes stronger, and the solution becomes more insightful. |
| *Long et al., 2012* | Stochastic programming is used to formulate a two-stage model, which includes ship weight capacity, supply, random demand, and ship space capacity. $ECR's$ operational costs are minimised by applying this model. | Stochastic program; Sample Average Approximation; Incorporating progressive hedging into heuristic algorithms | To solve the Sample Average Approximation problem for $ECR$, two heuristic algorithms are developed based on the progressive hedging strategy. Numerical cases are conducted to examine the performance of algorithms and the method of Sample Average Approximation | Scenario-based methods perform well in numerical simulations of $ECR$ problems with uncertainties. |
| *Lam et al., 2007* | A dynamic programming approach is applied to a containerized sea cargo industry study to demonstrate the success of generating effective operational strategies for the relocation of empty containers. | Approximate dynamic programming; CVI algorithm | By using a configuration of two-ports and two-voyages system, 16 imbalanced demand scenarios are generated. Based on the CVI algorithm developed by Bertsekas (1998), the exact dynamic programming methodology for obtaining the optimal average cost solution is used. | The CVI algorithm based on Dynamic programming approach have a good performance on $ECR$ problem and can minimise the total cost. |

### 2.5.2. $ECS$ problem involving multiple stakeholders

Meanwhile, in recent years, in empty container management, applying $ECS$ between different stakeholders is the trend to reduce empty container movements and improve empty container utilisation. Thus, researchers and practitioners recently have begun to pay attention to the idea of sharing empty containers between multiple stakeholders. A study by Song and Carter (2009) indicated that internet-based support systems increasingly are becoming more prevalent because they offer a platform to facilitate container sharing among shippers, forwarders and shipping companies. In their study, they examined the $ECS$ between carriers and multiple inland transportation companies in three major routes, including Trans-Pacific, Trans-Atlantic, Europe–Asia. $ECS$ is more practical in empty container management because it allows the $LSC$ to decide the $ECS$ strategy independently, but it requires stakeholders to sign certain contracts to determine the allocation of benefits and costs. Therefore, the different stakeholders can usually form a game in container sharing activity.

Several contracts are usually applied in the $ECS$ game, some of them being very famous such as Wholesale Price Contract ($WPC$), Buy-back Contract ($BBC$), Revenue-sharing Contract ($RSC$) and Quantity Flexibility Contract ($QFC$). Devlin et al. (2017) defined that a supplier in the $WPC$ charges the price to the retailer per each unit and there are no other obligations for the suppler to fulfil after the sale. However, this means that the retailer must bear all the risk of demand uncertainty, so the retailer usually tends to order a lower quantity of the product than the optimal quantity that should be ordered (Cachon, 2003). Based on $WPC$, $BBC$ is proposed and asks the supplier to share part of the risks generated from the customer demand uncertainty. The supplier usually pays the retailer for each unsold inventory or subsidises the retailer's loss at the end-of-season (Devlin et al., 2017; Pasternack, 1985 and Tsay, 2001). On the other hand, an $RSC$ does not allow the retailer to own the whole revenue; instead, the retailer has to share part of the revenue with the supplier, which may cause behavioural effects, e.g., the impact of order quantity etc. (Becker-Peth and Thonemann, 2016). In the case of the $QFC$, this regulates that the retailer is capable of reserving a certain capacity to satisfy future demand and there is no need for the retailer to pay any fee; in this case, the manufacturer builds the capacity based on the retailer's reservation and its own market expectation (Li et al., 2021). Finally, retailers can order at the wholesale price level when the actual demand is known; however, the retailer must pay the penalty if its order is beyond that of its reservation number; in contrast, if the manufacturer is not able to satisfy the retailer's order, the retailer has the right to ask for payment from the manufacturer for that part within the unsatisfied range.

The contracts mentioned above have recently been adopted in the $ECS$ problem by some scholars. Tan et al. (2018) investigated the excess $RSC$ mechanisms for container sharing in a shipping alliance involving an ocean carrier and an inland shipping company with the presence of vertical-horizontal[2] competition. They explored the three strategies: vertical separation, vertical–horizontal competition and alliance, and further compared the outcome (Tan et al., 2018). They claimed that if the negotiation cost of the alliance is excluded, the vertical-horizontal competition strategy clearly outperforms vertical separation (Tan et al., 2018). Xie et al. (2017) designed a $BBC$ to coordinate a container sharing system for a rail company based at a dry port and a liner company based at a seaport. They analysed the Nash equilibrium of the container inventory sharing game and identified the equilibrium quantity for container sharing under the $BBC$ (Xie et al., 2017). They concluded that the container sharing system under a decentralised decision-making mechanism can be coordinated if contract parameters are appropriately set (Xie et al., 2017). As the idea and applied method in the paper of Xie et al. (2017) is the basis of this thesis, it will be fully reviewed in the next subsection (2.5.3). The reason why their paper is the basis of this thesis is that their paper perfectly demonstrates how two shipping carriers achieve cooperation under a certain contract and achieve system coordination. Luo and Chang (2019) investigated the effect of repositioning price on the optimal inventory level of empty containers at a seaport and a dry port with containers shared between the two sites under a $RSC$. They concluded that the dry port does not need to store excessive empty containers to address the issue of inventory shortages and that both companies can achieve a win-win situation through coordination. Huang and Zhao (2019) built a shared container transportation mode system to demonstrate the restrictions and positive impact of the implementation in a sharing economy in the maritime sector. They design a framework for $ECS$ including a new business process, a new waiting mode and a software platform system design. Table 2.4 summarises the details of the research on $ECS$ problem involving multiple stakeholders such as research method, contract applied and research objects.

When multiple stakeholders are involved in the $ECS$ game, a decentralised decision-making mode is formed. In this mechanism, each shipping carrier has their own $ECS$ strategy. However, the various strategies can reach the Nash equilibrium eventually by elimination after several negotiations within the framework of a particular contract (Xie et al., 2017). In doing so, the $ECS$ in the decentralised decision-making mode is actually an empty container inventory sharing game (Luo and Chang, 2019). For example, in an intermodal system, Luo and Chang (2019) explored an empty container inventory sharing game in a decentralised model while considering customer demand switching and coordinated the system by applying a $RSC$. They found that

[2] vertical-horizontal competition: vertical competition exists in the container shipping chain among shipping company, freight trucking firm and terminal operator etc., while horizontal competition means the competition among different shipping companies.

the $RSC$ can help liner firms and rail companies achieve a win-win situation. As introduced previously, in supply chain management, “coordination” is a situation where the quantity of retailer’s order equals the quantity that the supplier tends to offer and also equal to the quantity that can simultaneously maximise the total supply chain profit in supply chain management (Snyder and Shen, 2011). In order to improve terminal operation efficiency and minimise the opportunity cost and holding cost of the empty container, Yu et al. (2018) explored the decision of the container free detention time decided by the ocean carrier and the time decision to transport the empty container back to the ocean terminal from the hinterland by developing a two-stage game model. The result was to derive the optimal empty container delivery plan. However, they found that the system is not always coordinated in the decentralised model.

All in all, most studies were found to only focus on inventory sharing in a general supply chain management (Rudi et al., 2001; Lee and Whang, 2002; Cachon, 2003; Giannoccaro and Pontrandolfo, 2004; Wong et al., 2009; Zhao and Atkins, 2009; Olsson, 2010; Zou et al., 2010; Lee et al., 2011; Shao et al., 2011; Lee et al., 2013a; Oliveira et al., 2013; Zhao et al., 2013; Govindan and Popiuc, 2014; Zhao et al., 2014; Kim et al., 2015; Wang et al., 2015; Zhao et al., 2015; Hu et al., 2016; Chen et al., 2017; Hu and Feng, 2017; Nouri et al., 2018; Heydari et al., 2020; Liu et al., 2020; Sošić, 2006; Zhao et al., 2006; Avinadav, 2020; Vafa Arani et al., 2016) instead of investigating the empty container inventory sharing game in practice. This is because most of the related research only focuses on a centralised decision-making system, which ideally assumes that all the container operators belong to the same firm, instead of a decentralised decision-making system where the operators run the firm independently but act upon, and follow the contract that they sign (Xie et al., 2017; Luo and Chang, 2019). Therefore, upon investigating the $ECS$ problem further by involving multiple stakeholders and applying various contracts is valuable. Next, the paper published by Xie et al. (2017) will be reviewed in more detail in the next subsection because it forms the foundation to this thesis, which also will present how this work further develops their research in a different dimension.

**Table 2.4** The summary of the research on the $ECS$ problem involving multiple stakeholders.

| ***Author and Time*** | ***Research objectives*** | ***Research method*** | ***Contract applied*** | ***Coordination?*** | ***Main findings*** |
|---|---|---|---|---|---|
| *Tan et al. (2018)* | Studying the cooperation and competition strategies between a liner shipping company and an inland carrier in a vertical-horizontal shipping chain | Bi-objective programming | $RSC$ | × | If the negotiation cost of the alliance is excluded, the vertical-horizontal competition strategy clearly outperforms vertical separation |
| *Xie et al. (2017)* | Investigating the $ECS$ game in an intermodal system and achieving the system coordination by adopting $BBC$ | Inventory management; game theory | $BBC$ | √ | They achieved the optimal $ECS$ policy in the centralised model; they obtain the equilibrium quantity of $ECS$ in the decentralised model; they coordinate the system |
| *Luo and Chang (2019)* | A study of the repositioning of empty containers in the intermodal transport system when the customer demand switches. By applying contract coordination theory, the profit of each participant will be improved as a result. | game theory | $RSC$ | √ | Through coordination, both companies can achieve a win-win situation by storing fewer empty containers to address inventory shortages. |
| *Huang and Zhao (2019)* | Examining the benefits and limitations of sharing economy in maritime transportation through an analysis framework for a shared container transportation mode | Sharing economy concept | Not applicable | × | In the Yangtze River, a shared container transportation idea is proposed; To obtain the container sharing idea logically feasible in a business process based on mobile internet technology, a new waiting mode is proposed, as well as a new business process and software platform design. |

### 2.5.3. Reviewing "Empty container management and coordination in intermodal transport"

Xie et al. published the paper in 2017 and pointed out that the problem of empty container accumulation in some transport terminals was caused by global trade asymmetry. They proposed an $ECS$ coordination problem in an intermodal system. They analyse the $ECS$ problem between two shipping companies in centralised and decentralised model, respectively. They also successfully coordinate the system. Therefore, their investigation is a perfect basis and guide for this research. In the system, one seaport and one dry port are included with one liner firm operating at the seaport and one rail firm at the dry port. Both firms own empty containers, and the empty containers are distributed both in the seaport and the dry port. The liner firm and rail firm make the internal and external decisions. Also, the internal makes decision to reposition their own empty container in various terminals, while the external makes the decision for sharing empty containers between the two firms. If one of the firms is still lacking in empty containers when they finish the internal decision, the extra empty containers could be borrowed from the other firm and delivered through a railway connecting the seaport and the dry port. The action of "borrow" can be based on a certain agreement. Figure 2.1 conceptually describes the model showing how empty containers are shared in both an export and import direction between two firms.

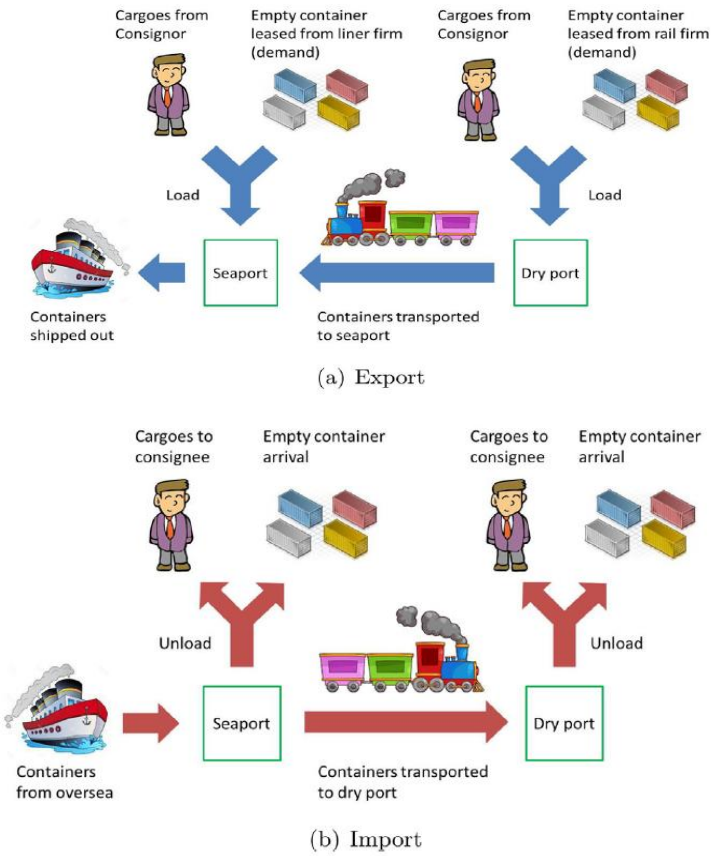


**Figure 2.1** Schematic diagram for the model of (Xie et al., 2017)

Mainly, two problems are solved in their paper. Firstly, they tried to prove that there exists a Nash equilibrium between liner firm and rail company in terms of deciding the number of empty containers for sharing. Secondly, they adopted a biliteral $BBC$ to coordinate the $ECS$ system. Also, under this coordination, they allocated the system profit for the two firms according to the $BBC$ contract under system coordination.

Specifically, they firstly built an ideal model in which two firms completely work together in order to obtain the maximum system's profit increment during the sharing activity. In this model, both firms fully and selflessly share empty containers to maximise the total system's profit instead of only caring for their own profit earned in the sharing activity. Thus, this model is considered to be a central planner between the two firms who owns the full information regards $ECS$ and always makes a perfect decision. This model is called a "centralised model", which only exists theoretically because neither the liner nor rail firm can act selflessly in a business operation. However, although a theoretical model, it remains valuable in the context of this thesis because when two firms act as one company it offers a maximum profit increment value, which can be used as a total value of profit allocation between the two firms under system coordination. Secondly, they built another model which is opposite to the centralised model, called the decentralised model. In this model, instead of assuming perfect collaboration between two firms in $ECS$ activity, a bilateral $BBC$ is applied to regulate that two firms can borrow empty containers from each other, but the empty container owner should pay a buy-back price to the leasee for each leftover empty container (only applied to the shared empty container) as compensation for the holding cost. By adopting the $BBC$, Xie et al. (2017) built a more practical model to conduct $ECS$ activity between two firms and compared the result with the centralised model.

In the decentralised model, Xie et al. (2017) further proved that there exists a unique Nash equilibrium in the $ECS$ game, as long as no firm adopts a weakly dominated strategy[3], and the equilibrium is called the Pareto optimal. This means that no firm can improve its profit without detriment to the profit of another firm. In addition, Xie et al. (2017) successfully coordinated the system when parameters (i.e., the wholesale price and the buyback price) in the $BBC$ are appropriately selected to make two shipping carriers' profit nonnegative. Under system coordination, the sum of two firms' profit increment earned between the scenario with and without $ECS$ activity in the decentralised model equals the maximum system profit increment that is calculated in the centralised model. When the contract parameters such as the $BBC$ are adjusted in the feasible range, two firms in the decentralised model can be allocated a non-negative profit increment from the system profit increment in the centralised model. Thus, under this circumstance, the utility of $ECS$ activity between two firms in the decentralised model is equivalent to that in the centralised model and the system can be seen as

[3] Weakly dominated strategy: the potential optimal empty container sharing strategy for one shipping company but it is not the optimal strategy for the system.

coordinated. Finally, Xie et al. (2017) also provided the range of two firms' profit allocation (including the upper and lower margins) under system coordination, and both profit allocations are nonnegative. In addition, a numerical case was conducted by them to examine the process and the rationality of the results. In particular, showing the process of how to coordinate the system, provided a biliteral $BBC$ between the two firms.

This subsection specifically reviews the paper published by Xie et al. (2017) and provides the basic idea and method adopted in this thesis. However, it does not consider the other factors such as the impact of $CT$ when the method tries to coordinate the system. Instead, whilst focusing on the system coordination, this thesis includes the impact of government $CT$ policy on shipping carriers' operation and on $ECS$ activity. Moreover, apart from using the $BBC$ applied by Xie et al. (2017), $RSC$ will be introduced also in this thesis to examine whether $RSC$ could perform the system coordination. The next section will review how the Stackelberg game, another the game-theoretical models adopted in this thesis, is applied in container management.

### 2.6. Stackelberg game

Stackelberg game, which in full is called "Generalised Stackelberg games" (Ungureanu, 2018b), is one of the most important models in $GT$. It was first proposed and investigated in depth by Stackelberg (1934). Stackelberg game is a sequential game which asks the players to act in a sequential process to make a decision (Ungureanu, 2018b). It describes a scenario in which a hierarchy of actions are made between two players (Yang et al., 2013), but it usually includes multiple players. However, unlike other games, the Stackelberg game relatively is special as players have a different status, and they choose their strategy asynchronously. Including two players in a Stackelberg game as an example, one player is assigned leader and the other one is assigned as follower, and they compete with each other for finite market resources. Under this circumstance, the leader is usually referred to as the market leader and has the power to make the decision (to act) first, then the follower decides its optimal strategy, which can maximise its utility. When the leader understands the follower's strategy, then the leader further strategically adjusts his/her strategy again to maximise its utility. Finally, the leader's optimal strategy and the follower's optimal response strategy both reach the Stackelberg equilibrium of the game (Yang et al., 2013). In short, Stackelberg game includes three basic elements, which are 1) the leader; 2) the followers and 3) game rules that all players should follow (Yang et al., 2022). Also, it is a dynamic game and the players in the game reach the equilibrium successively instead of simultaneously (Yang et al., 2022).

Stackelberg game is compared always with the Cournot game. Both Stackelberg game and Cournot game are applied in the duopoly market, where the duopoly market was first investigated by Antoine Augustin

Cournot in 1838 (Cournot, 1897), and it usually contains two companies dominating the market, such as Pepsi and Coca Cola or Boeing and Airbus (Ibrahim, 2019). Cournot game refers to a duopoly model which involves two oligopoly companies in a single homogeneous market. In the Cournot game, the companies have already formed an assumption about the other's output prior to deciding its own production quantity to maximise its utility (Ibrahim, 2019). Also, in contrast to the Stackelberg game, both companies make decisions at the same time in the Cournot game instead of sequentially. Based on these assumptions, Cournot game explores how one of the company's production strategy interacts with the other's strategy when both companies are in a competing and uncoordinated relationship. It was proved that both strategies of two companies in the Cournot game can reach the Nash equilibrium (Ibrahim, 2019). However, unlike Nash equilibrium in the Cournot game, Stackelberg equilibrium only refers to the best (or optimal) response strategy to the leader. This is easy to understand because the followers act based on the leader's strategy and their strategy correspond to the leader's best response strategy. Therefore, some constraints and conditions should be further made if the Stackelberg equilibrium is maintained. For example, the leader should recognise that the followers always observe and understand its action. In addition, the follower should always implement its Stackelberg action, and the leader also fully understands follower's implementation so that the leader can give the optimal response strategy according to the follower's Stackelberg action. In doing so, the $SE$ can be kept.

The Stackelberg game has been used in many practical fields such as energy sharing management of a microgrid. For instance, Erol and Filik (2022) claimed that the Stackelberg game has been applied in smart microgrids' energy sharing management to build a system of energy trading between the service supplier and users with the structure of leader–follower pattern. Some other papers also have focused on the similar topic such as Yu and Hong (2015); Meng and Zeng (2013); Chen et al. (2012); Maharjan et al. (2013); Matamala and Feijoo (2021). However, these are not discussed in this thesis for the sake of convenience because the characteristics of microgrid are quite different from the features of this research in maritime sector. In the next section, there is an examination of the literature on how Stackelberg game is applied in container management.

### 2.7. The application of Stackelberg game in container management

Although Stackelberg game is a vital and effective method to investigate sequential decision-making problems among different stakeholders in economic and politics, unfortunately, it is still not widely applied in container management in the maritime industry. Nevertheless, there is some limited but valuable research on this topic. For example, to promote the empty container leasing between port authority and terminal operator, Zhou and Kim (2019) developed a method of designing an optimal concession contract, using different $RSC$ schemes with quantity discounts, between the two shipping terminal operators and a port authority. Their model involves defining a Stackelberg two-stage game, where the port authority decides the parameters of the $RSC$ scheme so as to maximise the total revenue in the first stage, whilst two terminal operators compete for the maximum profit by determining the terminal handling charge in the second stage (Zhou and Kim, 2019). Based on the research conducted by Wang et al. (2014), three game-theoretical models were proposed to evaluate the shipping competition between two shipping carriers in a new emerging market for liner container shipping, one of which was the Stackelberg game. Wang et al. (2018) developed a generalised Nash equilibrium model to examine the feasibility of using Arctic routes as a "relief valve" for intercontinental container transportation. The Stackelberg form was developed where the $LSCs$ represent the leaders and the customers represent the followers (Wang et al., 2018). Thus, it is not difficult to find that the Stackelberg game is only adopted and explored in the different $LSCs'$ operation and competition generally and not in international shipping sector. The study conducted by Zhou and Kim (2019) involved the terminal operators and port authority but did not consider the other factors such as the impact of $CT$ levied by government in the game. Therefore, in this thesis, the Stackelberg game model is applied to investigate how government $CT$ policy affects $ECS$ between $LSCs$.

The next section reviews the literature on how $CT$ systems affect supply chain, inventory and container management in detail. Moreover, as another vital system applied to decrease carbon emission, $CAT$ system, will be reviewed and its impact on supply chain, inventory and container management will be discussed also.

## 2.8. Carbon emission and its impact on supply chain, inventory, or container management

Some taxation systems were designed to reduce international carbon emission, whereas two systems, $CAT$ and $CT$ are the two proven effective systems (ITF, 2022). This section reviews how two systems work in reducing carbon emission and how the application of them affects supply chain, inventory, and container management.

### 2.8.1. Cap-and-Trade system ($CAT$)

In order to mitigate the adverse effects of global warming on ecosystems and human development, $CAT$ systems were devised as one of the most vital mechanisms to reduce carbon emissions. The European Union ($EU$) launched the first international $CAT$ programme in the world to reduce carbon emissions in 2005 (Kenton, 2020). Subsequently, in 2009, across the 27 $EU$-member countries, the European Union Emission Trading System ($EU-ETS$) carried out a mandatory $CAT$ system (Hua et al., 2011). The $CAT$ system is widely adopted by many governments and organisations including the United Nations ($UN$) and $EU$ (Hua et al., 2011).

The $CAT$ system requires that governments allocate a cap or limit on carbon emissions to firms (Dong et al., 2014). The word "cap" is important in this context because it is a limit set by the government on how much emissions a particular industry is allowed to emit (Kenton, 2020). If more than the prescribed capacity is produced, a firm must purchase the right to emit excess carbon; otherwise, it may sell its excess carbon credits (Dong et al., 2014; Hua et al., 2011). In other words, carbon $CAT$ allocates a certain amount of carbon emissions to firms, and then selling or buying those permits on the carbon trading markets if they want more permits or have surplus permits (Chai et al., 2018). Therefore, it is easy for the $CAT$ system to be referred to as a market system. In other words, emissions are assigned an exchange value (Kenton, 2020).

As introduced in section 1.1, a $CAT$ system is in place in 38 national jurisdictions worldwide (The World Bank, 2022a). Hua et al. (2011) claimed that carbon trading systems are now available on more than 20 different platforms throughout the world. European Climate Exchange, Chicago Climate Exchange, and Australia Climate Exchange have become the most famous carbon trading markets in Europe, the United States, and Australia, respectively (Zhang and Xu, 2013). Nevertheless, domestic carbon emission markets are still being developed in Australia, Canada, Japan, and the United States (Hua et al., 2011). Over the past twenty years, $CAT$ systems have proven to be effective mechanisms for reducing carbon emissions footprints. Cushing et al. (2018) claimed that in order to reduce $GHG$ emissions from large stationary sources, $CAT$ systems are the most common regulatory mechanisms. Also, the $CAT$ mechanism is applied not only to reducing carbon emission footprints, but also to controlling the emission of other pollutants, including industrial waste and wastewater (Zhang and Xu, 2013). Raymond (2019) believed that for pricing carbon, $CAT$ regulations continue to be the most common mechanism.

Many studies have investigated the impact of $CAT$ system implementation on supply chain management and inventory management. For the impact on supply chain management, Chaabane et al. (2012) presented a mixed-integer linear programming framework for designing sustainable supply chains and suggested that a meaningful environmental strategy cannot be achieved without strengthening and harmonizing existing laws as well as the $CAT$ system at a global level (Chaabane et al., 2012). Jaber et al. (2013) presented a two-level (vendor–buyer) supply chain model as well as a mechanism for coordinating activities while incorporating climate change mitigation measures. They tried to consider different emission trading schemes, and demonstrated how to combine them Jaber et al. (2013). As a result, the model could jointly minimise both inventory-related and $GHG$ emissions costs in supply chains when penalties are applied for exceeding emission limits (Jaber et al., 2013). Xu et al. (2017) examined how a manufacturer and a retailer under a $CAT$ system determine their production and emission abatement strategies. Moreover, they explored the supply chain coordination under a $CAT$ system when wholesale price and cost sharing contracts are considered (Xu et al., 2017). Considering $CAT$ regulations, Xu et al. (2018) examined how low-carbon preferences and channel substitution drive decision-making and coordination in the dual channel supply chain and designed a better $RSC$. To facilitate effective coordination between manufacturers and retailers (Xu et al., 2018). According to the results, the government should implement $CAT$ legislation so that carbon emissions can be reduced efficiently, and that environmental and economic development can be coordinated (Xu et al., 2018).

Also, for the impact on inventory management, under the carbon emission trading mechanism, Hua et al. (2011) examined how firms manage carbon footprints when managing inventory. They did research including determining the optimal quantity of orders, as well as analysing and numerically examining the effects of carbon trade, carbon price and cap on the order quantity decisions, the carbon emissions, and the total costs of the project (Hua et al., 2011). An investigation was presented in Song and Leng (2012) in which a classical single-period newsvendor problem was investigated under a $CAT$ scheme. Their study concluded that emissions capacity should be set so that the company's marginal profit is no more than the cost of purchasing carbon credits under a $CAT$ policy (Song and Leng, 2012). To analyse the optimal dynamic production strategy over a finite time period, a continuous optimal control pattern was applied by Ma et al. (2014) to consider inventory-dependent demand and the carbon emission sources associated with both production and inventory management. Using the classical Economic Order Quantity ($EOQ$) model, Shu et al. (2017) incorporated carbon emissions from production activities and product transportation, as well as developing a carbon-friendly inventory cost model. A study conducted by García-Alvarado et al. (2017) examined the impact of environmental legislation on inventory control policies resulting in two significant contributions. Firstly, they compared conventional and green inventory policies in terms of their cost and environmental performance; and second, they provided managerial insight into green inventory policies from an organisational perspective.

Overall, the $CAT$ system, widely applied and accepted by governments and organisations worldwide to alleviate and control carbon emissions, is a well-developed and effective system, which influences the supply chain and inventory management. Next, compared to the $CAT$ system, another important mechanism for managing carbon emission, $CT$ system, and its impact on supply chain, inventory, and container management.

### 2.8.2. Carbon tax ($CT$)

Based on The World Bank (2014), $CT$ determines the rate of taxation on $GHG$ emissions or the carbon content of fossil fuels to place a price on carbon. Fang et al. (2013) claimed that taking steps to reduce $CO_2$ and energy intensity can be achieved effectively through the imposition of a $CT$ at the appropriate time. The $CT$ policy or a similar carbon pricing scheme has been implemented in around 40 countries and in more than 20 cities, states and provinces (Farrell, 2017). For example, the introduction of $CO_2$ taxes has been undertaken by Finland since 1990, in Sweden since 1991, in Denmark since 1991, Norway since 1992 and Ireland since 2010 (Hammar and Sjöström, 2011). Nowadays, the $CT$ system, among the variables affecting carbon emissions, has become one of the most effective economic measures for reducing carbon emissions (Fang et al., 2013).

Although $CT$ has been widely applied in various industry sectors by many governments worldwide, the vast majority of the existing studies only focus on the impact of $CT$ on general supply chains and inventory games rather than the container sharing supply chains. Zhao et al. (2020) investigated the optimal ordering decision for a risk-neutral supplier and a risk-averse retailer under a call option contract subject to carbon taxation. They proved that the call option contract can be financially positive for both supplier and retailer, promote the whole supply chain system's performance and reduce unnecessary carbon emission. Nagurney et al. (2006) designed a computational framework to determine the optimal $CT$ rate for electric power supply chains that involves power generation, distribution and consumption. Their framework provides the optimal $CT$ for each power generator while ensuring their assigned emission limit is not exceeded. Halat et al. (2021) developed four structures (decentralised, vertical downward, upward vertical, and horizontal cooperation) in inventory games of multi-echelon supply chains to investigate the influence of $CT$ on costs, emissions, and cooperation savings. They found that appropriate $CT$ policy not only decreases the carbon emissions, but also reduces coalition costs and emissions savings (Halat et al., 2021). Hasan et al. (2021) optimised the technology investment and the inventory level under a $CT$ policy, strict carbon limit regulations and $CAT$ system, respectively. They found that the $CT$ scheme significantly affects the total system profit, and falling emissions level results in more profits, however the carbon cap and limit do not influence the total profit dramatically. Shen et al. (2019b) investigated the inventory problem subject to a $CT$ scheme in supply chains. Their main target was deciding the optimal policies in terms of production, delivery, ordering, and investment for the buyer and vendor so that the joint total profit per unit time can be maximised under the $CT$ policy. Benjaafar et al. (2013) developed a relatively simple model to demonstrate how carbon emission care can be considered in operational decision-making such as production, procurement and inventory management. In addition, they also offered some vital insights into the influence of operational decisions on carbon emission. Hammami et al. (2015) explored a production-inventory model which considers carbon emission, and they investigated the influence of $CT$ policy on inventory decision-making. Yu et al. (2020) optimised the inventory management problems, including perishable products under the $CT$ scheme and $CAT$ policy. They supposed that perishable products' market demand is deterministic and simultaneously linked to selling price and stock level, as well as perishable products' ordering and storing can create carbon emissions. Lin and Sarker (2017) established a inventory model, which considers $CT$ policy and incomplete quality items, in which the buyer exerts power over its suppliers. It is a new inventory model because they also examined the impact of different $CT$ systems on the model's performance. Chen et al. (2013) developed an $EOQ$ model to obtain an optimal inventory strategy given the impact of $CT$ policy and $CAT$ policy, and they further compared the impact of two policies on the inventory strategy. The first inventory policy is paying the tax resulting from the order quantity that minimizes the operating cost and then continue doing business as usual while the second strategy is that the company can reduce their operational and emission costs by adjusting their order quantity. Similarly, Qin et al. (2015) also published inventory policies under $CT$ policy and carbon $CAT$ policy, considering the carbon emissions of delivering, storage and purchasing. They concluded that when the retailer is capable of determining credit policies, carbon regulations have a better performance on carbon emission reduction than with exogenous credit terms. Hua et al. (2011) determine the optimal order quantity and conduct an analytical and numerical analysis to determine the impact of carbon price, cap on order decisions and carbon emissions. They found that the carbon emission can be reduced but the total cost is increased. Choi (2013) studied the optimal supplier selection based on the $BBC$ for the fashion apparel supply chain, considering charging $CT$. Chen and Hao (2014) proposed the best pricing strategy and production policy to achieve sustainable development for two competing firms considering an emissions tax policy. They claimed that to reach a desired carbon emissions reduction rate, the imposed $CT$ on a high-efficiency company should be more than the levied $CT$ on a low-efficiency company (Chen and Hao, 2014). Fahimnia et al. (2015) explored a tactical supply chain planning model with a combination of economic and carbon emission objectives and adopted a modified Cross-Entropy solution method for solving nonlinear supply chain planning model. Yu and Han (2017) investigated how the $CT$ affects the carbon emission of a supply chain involving a manufacturer and a retailer and applied the modified wholesale price and the modified cost-sharing contract to successfully coordinate the supply chain (Yu and Han, 2017). By considering the emission reduction penalty mechanism and taking into account consumers' low-carbon preferences, Wang et al. (2019) explored the emission reduction level, based on consumers' low-carbon preferences, for a retailer and a manufacturer in the supply chain.

The aforementioned studies have not considered carbon emission in the maritime sector and container shipping operations, in spite of the fact that the $MEPC$ of $IMO$ has proposed to levy $CT$ on port operators and shipping companies in the long term (Wang et al., 2018b). Some studies have focused on how $CT$ affects the maritime sector and shipping operators (Rojon et al., 2021; Liu et al., 2021; Lee et al., 2013b; Tiwari et al., 2021; Ding et al., 2020). Rojon et al. (2021) investigated the impact of carbon taxation on transport costs in the maritime sector, especially in developing countries and found that the $CT$ policy can significantly influence shipping costs claiming that freight costs would increase by between 0.4% and 16% (Rojon et al., 2021). By considering the $CT$ charging system and the liner companies' alliances formation, Liu et al. (2021) studied the joint optimisation problem of the shipping network-transaction mechanism. They concluded that the $CT$ can directly affect liner companies' operation strategy and that the liner companies' total cost is related to the $CT$ rate (Liu et al., 2021). They further suggested that the liner firms should increase ship numbers and reduce the sailing speed to counter the operation cost brought by the $CT$ (Liu et al., 2021). Lee et al. (2013b) claimed that the $CT$ could affect the competitiveness of shipping lines, and they adopt an energy–environmental version of $GTAP$ (Global Trade Analysis Project) to analyse the impact of the $CT$ on freight cost. Tiwari et al. (2021) minimised the carbon emission and total transport cost at the same time, given the $CT$ policy. Interestingly, they concluded that the shipment containerisation strategy behaves better under the $CT$ regulation, and both the total transport cost and the carbon emissions decrease (Tiwari et al., 2021). Ding et al. (2020) proposed two kinds of $CT$ schemes (fixed and progressive) to examine the economic viability of the Northern Sea Route ($NSR$) for containerships. They found that the viability depends on the specific $CT$ scheme and fuel choice (Ding et al., 2020). Cui and Notteboom (2017) investigated the $CT$ effect on port privatisation using the Cournot and Bertrand competition. Gao et al. (2022) developed a model for container shipping network design to maximise a liner company's revenue by considering the government $CT$ policy.

For the definition of $CT$ in the maritime sector, Tiwari et al. (2021) claimed that the $CT$ is a kind of fossil combustion tax levied by the government involving motor gasoline, diesel, jet fuel, etc., to alleviate environmental pollution. So, the $CT$ is usually regarded as an extension of the fuel combustion tax levied by the government (Parry et al., 2018). They also suggested that the shipping companies' tax liability is dependent on a ship's fuel consumption, $CO_2$ emission and the emission factor for the fuel being consumed. Since the $CT$ is usually passed on to the charterers and consumers (Adamopoulos, 2020), the government's $CT$ rate does affect the customers' demands for empty containers. $MEPC$ suggested that an initial $CT$ rate of 10 dollars per tonne should be applied, and then the rate should be lifted to a higher level between 50 dollars and 75 dollars per tonne (Poter, 2019). However, it is a challenging task for governments to appropriately set the $CT$ rate, as they need to maintain the stability of the ocean freight market and also employment in the maritime sector.

In summary, the $CT$ system and the $CAT$ system are both widely adopted by many countries and states around the world. Both systems have proved to be effective in alleviating carbon emissions. Table 2.5 concludes the details of the key references that are mentioned in this section. It is necessary to mention that $CT$ is adopted as the system implemented by the government to investigate how $CT$ affects the $ECS$ game between two $LSCs$ in this thesis. In the next section, the research gaps between this and previous research will be clarified. Furthermore, based on the research, the research objectives also will be determined.

**Table 2.5** The summary of the research on carbon emission policy and their impact on supply chain, inventory, or container management

| *Authors and year* | *Research objective and topic* | *CT?* | *Applied in Maritime?* | *Main findings* |
|---|---|---|---|---|
| *Nagurney et al. (2006)* | An analysis of the electric power supply chain network in the context of optimal carbon taxes is presented using a modelling and computational framework. | √ | × | It is evident from the results of numerical example that the carbon taxation scheme achieve their desired goal, i.e., they do not exceed the imposed bounds on carbon emissions in the electric power supply chain network. |
| *Hua et al. (2011)* | Using $CAT$ system as an example, it examines how companies manage their carbon footprints in their inventory management. | × $CAT$ | × | The optimal order quantity is calculated and the effects of carbon trading, carbon price, and carbon cap are analysed analytically to determine the carbon emissions, overall cost and order decisions. |
| *Benjaafar et al. (2013)* | Demonstrates how the management of production, procurement and inventory can be incorporated with carbon emission concerns using popular and relatively simple models. | √ | × | A significant reduction in emissions could often be achieved without a significant increase in costs with operational adjustments. |
| *Chen et al. (2013)* | By modifying order quantities, offers a condition in which emissions is reduced using the $EOQ$ model. Discusses the factors that impact the magnitude of the difference between emission decrease and cost increases based on conditions under which emissions decline relative to cost increases. | × | × | Results show that the cost function is quite flat about the optimal in both facility location and newsvendor models, allowing significant emissions decrease without cost increases under certain conditions. |
| *Choi (2013)* | The problem of choosing suppliers in the fashion apparel supply chain under $CT$ is studied. | √ | × | Multi-stage stochastic dynamic programming is applied to choose the best supplier through a two-phase optimal supplier selection scheme. Various carbon taxation schemes is investigated along with their impacts. |
| *Fang et al. (2013)* | A novel four-dimensional system of emission-reduction and energy-saving with $CT$ constraints is investigated to determine the effects of $CT$ on economic growth and energy intensity. | √ | × | The power of the four-dimensional system could be better controlled as the tax levy point for carbon tax grows larger. Carbon emissions could be controlled more easily if the carbon tax is implemented at a more appropriate time, if the tax's growth rate is higher, and if policies and laws are better adapted to the situation. |
| *Lee et al. (2013b)* | A maritime $CT$ policy is analysed using an energy–environmental version of the Global Trade Analysis Project, highlighting the significant role containerisable commodities play in international trade to see what quantitative impact it has on the global economy. | √ | √ | Unless the marine $CT$ is high, the introduction of a maritime $CT$ policy on international container shipping is unlikely to have a remarkable economic influence. China would lose around 0.02% of its GDP when global maritime $CT$ reach \$90/t$CO_2$. |
| *Chen and Hao (2014)* | It explores how carbon taxation policy influences a company's sustainable pricing and production policies. In this paper, a comparison is made between two competing companies producing a similar end product with different operational efficiencies. | √ | × | When two competing firms are charged the same $CT$, both will set a higher retail price than if they were not charged $CT$, and the high-efficiency firm shows a smaller amount of profits decrease and carbon emissions reduction than the low-efficiency firm. |

| *Authors and year* | *Research objective and topic* | *CT?* | *Applied in Maritime?* | *Main findings* |
|---|---|---|---|---|
| *Fahimnia et al. (2015)* | A $CT$ policy scheme is applied to integrate economic and carbon emission objectives for tactical supply-chain planning. The nonlinear supply-chain model is solved using a modified Cross-Entropy method. | √ | × | Using the numerical results, organisations and policy makers can gain valuable insights regarding (1) the financial impacts and emissions decrease of $CT$ at tactical planning level, (2) how to make investment decisions using cost/emission trade-off analysis, and (3) how to price carbon for maximum environmental returns. |
| *Hammami et al. (2015)* | A deterministic optimisation model incorporating carbon emissions is developed based on a multi-echelon and production-inventory model with lead time constraints. | √ | × | The longer the customer queue, the more emissions will be generated even if carbon taxes are in place. The number of emissions may be reduced, however, if orders increase in frequency. |
| *Qin et al. (2015)* | Using a $CAT$ policy and a $CT$ regulation model, examines inventory policies and sustainable trade credit that consider consumer environmental sensitivity. | √ and $CAT$ | × | The credit period is adversely affected by $CAT$ and $CT$. Those retailers who follow carbon $CAT$ regulation have a higher level of motivation to follow regulations than those who follow $CT$ regulation. |
| *Cui and Notteboom (2017)* | This paper explores the implications of a government $CT$ on vessels and port operations for pollution control in ports by using game theory. | √ | √ | When ports cooperate, environmental protection must be improved more and more strongly than when they compete with each other withoutco-operation, and the optimal $CT$ rate should always be less than the marginal emission effect. |
| *Lin and Sarker (2017)* | An inventory model with carbon taxes and imperfect quality items is developed in this paper. | √ | × | As shown by the numerical examples, (1) the quantity discounts and $CT$ system affect shipments; (2) the operating costs increase if $CT$ are imposed; (3) unlike a flat $CT$ system, a progressive $CT$ system provides flexibility when it comes to shipping sizes and numbers; |
| *Farrell (2017)* | A case study of Irish carbon tax-related inequality has been investigated here by decomposing it by socioeconomic factors. | √ | × | Using the concentration index methodology, carbon tax incidence inequality is quantified across income spectrums. This inequality of incidence is quantified through a subsequent multivariate decomposition. |
| *Yu and Han (2017)* | In a two-echelon supply chain with a manufacturer and a retailer, the study examines the impact of carbon taxes on carbon emissions and retail prices. In order to promote the efficiency of the supply chain, two types of contracts are used: modified wholesale prices and modified cost-sharing contracts. | √ | × | The optimal level of emission reduction and the optimal retail price both increase with the rise of $CT$, and then remain unchanged. Wholesale contracts and cost-sharing contracts are not beneficial to the manufacturer once the supply chain has been coordinated. A two-part tariff contract and a fixed fee can be paid by the retailer to ensure a win-win solution for the supply chain. It integrates optimal decisions from the wholesale contract and the cost-sharing contract to guarantee a win-win solution for all parties. |

| *Authors and year* | *Research objective and topic* | *CT?* | *Applied in Maritime?* | *Main findings* |
|---|---|---|---|---|
| *Wang et al. (2018b)* | The purpose is to investigate integrated berth and crane allocation problems in ports with regard to different $CT$ policies: a unitary $CT$ policy and a piecewise $CT$ policy. | √ | √ | The carbon emission taxation policy is found to have substantially reduced carbon emissions and has a significant positive effect on berth plans. |
| *Shen et al. (2019b)* | Based on a collaborative preservation technology investment model and a carbon tax policy, examining a production inventory model for deteriorating goods. | √ | × | For the buyer and vendor, tries to determine the most efficient production, delivery, ordering, and investment policies in the face of carbon tax policies. |
| *Wang et al. (2019)* | Consumers' low-carbon preferences, stochastic market demand, and $CT$ policy is considered in order to determine the level of supply chain emission reduction. In a centralised supply chain, the revenue model for retailers and manufacturers is derived by introducing the emission reduction penalty mechanism and using reverse derivation, when the supply chain decreases emissions or is not affected by stochastic market demand. | √ | × | Supply chain emission reduction levels are determined by stochastic market demands, consumer preferences for low-carbon products and carbon tax policies. Retailers and manufacturers can calculate revenue models for decentralised and centralised supply chains using the emission decrease penalty mechanism and reverse derivation when emissions are reduced, or stochastic market demand is not affecting the whole supply chain. |
| *Ding et al. (2020)* | Two proposed $CT$ schemes, a fixed $CT$ and a progressive $CT$, are evaluated in comparison with the Northern Sea Route against the Suez Canal Route. | √ | √ | According to the results, Northern Sea Route is more economically viable regardless of fuel type or whether $CT$ are levied on either route. |
| *Yu et al. (2020)* | Under $CT$ policy, considers the problem of optimising inventory of perishable products. Modelling properties are displayed numerically, sensitivity analyses of parameters also are implemented, and the impact of parameters on retailer inventory policies is explained. | √ | × | Under two carbon policies, $CT$ policy and $CAT$ system, two inventory models are established. As a result of these models, the optimal selling price, and preservation technology investment was determined. The first model assists retailers operating under the $CT$ policy in maximising their profits. Under a $CT$ policy, the second model supports retailers solve the inventory problem, which shows good performance in practice. |
| *Zhao et al. (2020)* | With option contracts, examines how to determine the optimal operational decisions for both suppliers and retailers in an iron and steel supply chain under the $CT$ regulation. | √ | × | A call option contract benefits both retailers and suppliers, improves supply chain performance, and reduces invalid carbon emissions. A retailer also can mitigate the effects of carbon emissions tax and risk aversion through the introduction of call option contracts. |
| *Halat et al. (2021)* | A multi-echelon supply chain inventory game is considered as a model for $CT$ policy. Investigates the influence of the carbon taxation scheme on costs, emissions, and savings by each cooperation scheme, finding the solutions of inventory and comparing strategies. | √ | × | Supply chains can reduce both costs and carbon emissions through cooperation, regardless of inventory parameters or carbon emission parameters. Also, $CT$ could reduce coalition costs and lower carbon emissions. |

| *Authors and year* | *Research objective and topic* | *CT?* | *Applied in Maritime?* | *Main findings* |
|---|---|---|---|---|
| *Hasan et al. (2021)* | In the context of a $CT$ policy and $CAT$ system, and strict carbon limits, optimised the inventory and technology investment. Several examples are presented, followed by sensitivity analysis of the inventory level for different scenarios of $CT$ and $CAT$. | √ and $CAT$ | × | Whilst green technology promotes a rise in demand, green technology investments are proportional to profits and carbon emission reductions. Also revealed that the carbon price played a significant role in the total profit for the $CT$ system policy. |
| *Liu et al. (2021)* | Examines the joint design problem between a liner shipping network and a transaction mechanism. Liner alliance cooperation strategies are discussed in relation to the $CT$ levy on liner companies. An optimisation model is developed to solve the above problem. The alliance liner shipping network design scheme as well as the alliance transaction mechanism are optimised to maximise the total profit of the alliance. | √ | √ | Using actual China import and export transport data, numerical experiments are conducted. Proves that using the proposed model and algorithm, the joint design problem of liner alliances could be effectively solved and decision support for the alliance's operation is provided. |
| *Rojon et al. (2021)* | Identifying the determinants of maritime transport costs and evaluating their relevance to trade and economic growth. Examining the economic effects and transmission channels of a $CT$ on maritime transport. | √ | √ | $CT$ policy has a limited influence on maritime transportation costs for the country in general. However, Small Island States and Least Developed Countries tend to be negatively affected by $CT$ policy in terms of maritime transport costs |
| *Tiwari et al. (2021)* | Demonstrates the effectiveness of various carbon footprint schemes on both cost and emissions by incorporating environmental factors into the Freight Consolidation and Containerisation Problem model. | √ | √ | Found that compared with business as usual, shipment containerisation under $CT$ regulation achieves a better result in terms of total transport costs and carbon emissions. |
| *Gao et al. (2022)* | Examines the problem of designing an ocean container shipping network. The $CT$ and the cargo owners' choice inertia factors are considered simultaneously because the governments have proposed carbon-neutral initiatives. | √ | √ | Profitability of a liner company is directly related to its shipping network design scheme. As an example, shipping carriers should adjust their shipping network according to the off-peak and peak seasons for shipping activity between China and Europe. Additionally, liner companies should be able to design capacity, fleet size, and speed schemes based on the characteristics of different time periods. |

## 2.9. Research gaps and objectives

In this Chapter, I have examined the literature of empty container management problem and government $CT$ scheme implementation. Thus, this thesis will be developed regard to these two main research fields. The research gaps addressed in this thesis developed and evolved from the paper of Xie et al. (2017). Although they produced exceptional work in $ECS$, some drawbacks remains and the gaps should be filled. For example, firstly, they only focus on applying $BBC$ as the binding agreement between two firms to investigate the Nash equilibrium and system coordination and other contracts such as $WPC$ and $RSC$ are not included in their paper. Not even to mention the comparison in terms of system coordination achievement when different contracts are applied. Secondly, Xie et al. (2017) focused on the $ECS$ in an intermodal system. Normally, the dry port is located on hinterland which is often far away from the seaport, and it is usually connected by a railway, so the high transportation costs generated between the dry port and seaport should not be ignored. However, the transportation cost could be massively reduced if the empty container is well managed and shared among different $LSCs$ in different terminals in one port. Similarly, there could be several terminals in one seaport and the distance between the terminals is relatively short so the transportation cost can be rationally ignored among different terminals compared with the transportation cost generated between dry port and seaport. Therefore, in this thesis, the focus will be on the $ECS$ problem between two $LSCs$ in two terminals in one seaport.

On the other hand, with the increasing importance of controlling carbon emission in maritime sector, governments have introduced the $CT$ to international shipping activity. However, all current studies concentrate on how $CT$ affects liner shipping network design, operation strategy adjustment or transportation cost (Liu et al., 2021; Rojon et al., 2021 and Gao et al., 2022). These studies mainly examined the impact of $CT$ levy macroscopically on liner shipping operation rather than investigating how government controls carbon emission and how $LSCs$ based at the terminal respond to government $CT$ policy and further maximise their business profit. In particular, no studies focus on the problem of $ECS$ and system coordination among $LSCs$ when $CT$ is levied by government for ocean shipping activity.

Finally, in the previous research, when $CT$ is introduced, the influence of fluctuating government $CT$ policy usually is excluded from the empty container management or liner shipping network design. In other words, the $CT$ is included often as a constant parameter instead of a decision-variable. However, a government has its own concern to achieve maximum social welfare, which means that it should set $CT$ rate appropriately according to the maximum social welfare principle. Moreover, the $CT$ could be adjusted by government all the time because of many factors such as economy, carbon emission targets changing or during a financial crisis.

Thus, as the $CT$ policy maker, the government should be included in the model as a participant when $ECS$ among different $LSCs$ is explored. Unfortunately, no previous research considers this collaboration. In conclusion, to the best of our knowledge, the research gaps are:

- No studies have been carried out research to investigate the impact of $CT$ rate on the coordination of container sharing systems among $LSCs$ bound by a $RSC$ and $BBC$.
- For the studies relating to general supply chain coordination with the presence of $CT$, the $CT$ rate is generally considered as a fixed constant parameter instead of a variable.
- The previous literature does not consider the government as a player and involves the government $CT$ rate as a decision variable in the container sharing game. In other words, the previous research only focuses on the impact of static $CT$ rate on container sharing problem instead of investigating the influence of a dynamic $CT$ rate scheme on the same topic. The system coordination needs to be explored when the government's maximum total social welfare and the $LSCs'$ maximum business profit are considered simultaneously.

In summary, this thesis will endeavour to address gaps by considering the carbon emission tax rate as a decision variable when the government is considered as a player in the container sharing game. Also, how container sharing coordination may be affected by respectively applying the $RSC$ and $BBC$ will be investigated. To completely achieve the final target, this thesis will be conducted in three steps, which will be demonstrated in three sequential Chapters (4, 5 and 6). For clarity, the main ideas, features and settings of the three chapters and the comparison to the important research of Xie et al. (2017) are summarised in Table 2.6.

**Table 2.6** The main ideas, features and settings in three chapters.

| | ***Contract applied*** | ***CT setting*** | ***Model applied*** | ***Gov included?*** |
|---|---|---|---|---|
| Xie et al., (2017) | $BBC$ | Not included | Container-sharing | No |
| Chapter 3 | $BBC$ | Constant parameter | Container-sharing | No |
| Chapter 4 | $RSC$ | Exogenous variable | Container-sharing | No |
| Chapter 5 | $RSC$ | Decision variable | Stackelberg game | Yes |

## 2.10. Conclusion

In summary, in this chapter, I summarised the current knowledge and identified the research gap which led to my research. I also clarified the impact of $CT$ rate on the empty container sharing among $LSCs$ bound by a $RSC$ and $BBC$. Moreover, I discussed the how could carbon tax rate can be treated as a variable rather than a fixed constant parameter.

Specifically, firstly, the literature related to the $ECR$ was fully reviewed and found that the $ECR$ has been investigated thoroughly in international shipping industry. However, it still cannot perfectly solve the empty container imbalance and shortage problem. Secondly, this chapter reviewed the $GT$, Newsvendor problem and supply chain contract design, which are the foundation for reviewing the previous research of $ECS$. Thirdly, whilst $ECS$ as a valuable method to alleviate the container accumulation has been studied, previous research did not explore sufficiently for example ECS between LCSs and failed to consider carbon emission reduction. Nevertheless, $ECR$ has been studied in some previous research and proven to be effective in empty container management being adopted by many $LSCs$. Moreover, based on the concept of "sharing economy", $ECS$ is proposed to help in solving the empty container accumulation problem and save $LSCs'$ operational cost. Fourthly, an important game model, Stackelberg game, was introduced for exploring how government $CT$ policy may affect $LSCs'$ $ECS$ activity in a Stackelberg game, which have not been examined previously. Moreover, a review of how the $CT$ and $CAT$ system has been applied to decrease carbon emissions was carried out and found that many scholars concentrated on the topic of how $CT$ system affects the management of the general supply chain and inventory. However, one factor that has not been examined in the current literature is whether $CT$ policy will affect $LSCs'$ $ECS$ practices. Finally, I addressed the research gap between this thesis and the other research particularly identified the gaps with the research of Xie et al. (2017).

In the next chapter, the methodology adopted in this thesis, including the thesis's philosophical standpoint, research subjects and study design, will be presented.

# Chapter 3 Methodology

## 3.1. Introduction

The previous chapter reviewed the literature of empty container management and the impact of carbon tax ($CT$) on container and inventory management. In this chapter, the methodology will be described. Firstly, in section 3.2, a brief review of the concept of ontology, epistemology and philosophical perspective of the research will be presented. Secondly, in section 3.3, the research subjects going from a macro to a micro perspective of this thesis including Operation Research ($OR$), Operation Management ($OM$) and Game theory ($GT$) will be introduced. Thirdly, in section 3.4, the philosophical standpoint of this research, including the ontology, epistemology and philosophical perspective that adopted in this research will be demonstrated. Most importantly, in section 3.5, the research design for Chapter 4, 5 and 6 and based on three steps described at the end of section 2.9 will be presented. The research design includes model setting, contract applied, notation illustration, centralised decision-making and decentralised decision-making mechanism design. The decision, random and state variable which will be used in this thesis are detailed. The research design will be conceptualised in figures and finally, there will be a brief summary of this chapter in section 3.6.

In the next section, the research method in the field of philosophy will be reviewed in brief so that I can determine the related ontology, epistemology and philosophical perspective of this thesis at a later point.

## 3.2. Research method review

Choosing an appropriate philosophical perspective is an important component of a PhD dissertation as a philosophical perspective (or philosophical stance) refers to a methodologically relevant and pragmatically justified way of seeing (Boucher, 2014). More precisely, a philosophical perspective is the system of beliefs that guides our actions through a generalised view of the world (Spirkin, 1983 and Guba, 1990).

The two main concepts of philosophical perspective are ontology and epistemology. In brief, the ontology refers to what actually exists in the world that humans can know and understand (i.e., the study of being) (Moon and Blackman, 2014). The term ontology was first used about 2000 years ago. Objects, their properties, and how they are similar or different from one another were all topics that ancient Greek philosophers studied in order to understand the origin and nature of the universe (Moon and Blackman, 2014 and Spirkin, 1983). The second concept is epistemology. This differs from ontology in that essentially, epistemology is the study of how people come to know what they know and what they are capable of knowing (i.e., the study of knowledge) (Moon and Blackman, 2014). Several scholars have argued that ontology and epistemology are inseparable and intimate: talking of meaning is talking of meaningful reality (Crotty, 1998). In the subsection 3.2.1 and 3.2.2, I will briefly introduce the details of ontology and epistemology so that I can determine the research method of this thesis in section 3.4.

### 3.2.1. Ontology

There are different kinds of ontological positions existing in philosophy (Johnson and Gray, 2010) such as realism, internal realism, relativism and nominalism. As far as realism is concerned, the belief is that the only “truth” exists objectively in the world and its existence is independent of human’s experience (Moses and Knutsen, 2019). The truths and the facts in the world can be experienced, observed, understood and described (Moses and Knutsen, 2019). Realism believes that the world can be studied, it is therefore always adopted by scientists. This is because a falsification principle and a correspondence theory of truth can be used to empirically test observations, or experimental statements based on these regularities (Moses and Knutsen, 2019). In addition, realism holds that the scientific research should focus on the general nomothetic rather than on the idiographic circumstances (Moses and Knutsen, 2019). In summary, realist ontology believes there is only a single truth, and that facts exist and can be revealed through experiments.

Another important ontological position is relativism. Those who subscribe to relativism are of the opinion that reality is constructed in the mind of the individual, meaning that there does not exist a single true reality; rather, reality is relative to each individual who experiences it at a particular time and place (Moon and Blackman, 2014). In other words, in the view of relativists, reality only exists within the mind, with each individual creating their own version of it (Moon and Blackman, 2014). Therefore, relativist ontology believes that there are many truths for a subject in this world, and facts are also dependent on the viewpoint of the observer.

However, from a broad ontological standpoint, individuals differ in their confidence that they are capable of defining reality (Moon and Blackman, 2014). For example, internal realism considers that in spite of the fact that the world exists, it almost is impossible to examine it directly. It holds that truth exists, but it remains elusive, and facts are concrete, but they cannot always be uncovered. Furthermore, some people believe in nominalism, which suggests that reality is entirely the product of human beings, and that there is no objective "truth" to be found. For the sake of clarity, I create Table 3.1 to illustrate the features of the various ontological positions.

**Table 3.1** The features of various ontology. (Loux, no date; Moon and Blackman, 2014)

| *Ontology* | *Claims* | *Believed Truth* | *Believed Facts* |
|---|---|---|---|
| **Realism** | As a result of examining and observing the world, science is able to determine the nature of reality. | Only a single truth exists | Experiments enable the discovery of facts |
| **Internal realism** | There is no doubt that the world exists, but the ability to observe it directly is almost impossible. | Truth exists; however, it is obscure | Although facts are concrete, they cannot always be revealed |
| **Relativism** | The laws of science are primarily created by people in order to accommodate their views of reality | Many truths exist | Observer's view determines what is true and what is false |
| **Nominalism** | There is no such thing as an external "truth", because reality is entirely created by humans | No truth exists in the world | Humans create all facts. |

In the next section, various epistemologies will be introduced.

### 3.2.2. Epistemology

In epistemology, all aspects of the validity, scope, and methods of acquiring knowledge are considered. For instance, what constitutes a knowledge claim, and how it is created or acquired, as well as how it is applied (Moon and Blackman, 2014). Crotty (1998) divided the epistemology into three categories, which are objectivism, constructionism and subjectivism.

The objectivist view is that meaning, and by extension, meaningful reality, exists independently of any conscious action (Crotty, 1998). Objectivism believes the view of “what it means to know” and they also hold that objectification of understandings and values enables us to discover the objective truth about the group of people we are studying, if we approach it in the correct manner (Crotty, 1998). Also, as far as the objectivist is concerned, the researcher can only achieve objective, scientific knowledge by acting as a detached observer in the research situation (Dreyer, 1998). However, another epistemological position, constructionism, does not agree with this opinion. It asserts that there is no objective "truth" in the real world that we are capable of discovering (Crotty, 1998; Moon and Blackman, 2014). Truth is more likely to emerge from our interaction with reality, or meaning, rather than from a preconceived notion (Crotty, 1998; Moon and Blackman, 2014). Constructionist always holds that the knowledge can be constructed, and the world can be built and given meaning with the human mind. For instance, Schwandt (2003) believed that, as opposed to being passive, knowledge involves the active imprinting of sense data on the mind; the mind usually forms abstractions or concepts based on this data. From the perspective of the individual, constructionism claims that the way an individual interacts with and understands his or her world is dependent on the cultural, historical, and social context in which he or she lives, and so meaning emerges from human interactions (Moon and Blackman, 2014). The final epistemological position is subjectivism. In subjectivism, meaning does not emerge through the interaction between the subject and the object, but rather is imposed by the subject upon the object (Crotty, 1998), which in this case means that objects do not contribute to the generation of meaning, and reality is pluralistic (Moon and Blackman, 2014). There have previously been many debates between objectivism and subjectivism (Burrell and Morgan, 1979). By comparison, subjectivists normally reject the notion that organisations, behaviours, and causal relations exist independently from human knowledge. Objectivists accept the existence of organisational entities, behaviours, and causal relationships independent of human knowledge (Powell, 2001). Although this is debated, in Berkeley's subjective immaterialism of the 18th century, subjectivism finds its philosophical roots in platonic idealism (Powell, 2001). Table 3.2 concludes the three epistemological positions.

**Table 3.2** Three epistemological position claims (Moon and Blackman, 2014)

| ***Epistemology*** | ***Claims*** |
|---|---|
| Objectivism | Objects possess meaning in the sense that an objective reality exists in them independent from their subjects. |
| Constructionism | Subject and object interact to create meaning: subject constructs reality for object |
| Subjectivism | The subject imposes meaning on the object while meaning is inherent in the subject |

In the next section, based on the introduction of classification of ontology and epistemology, I introduce different philosophical perspectives in philosophy.

### 3.2.3. Philosophical perspective

Philosophical perspective, which also is called paradigms (Guba and Lincoln 1994) and worldviews (Creswell 2009), can be described as “what is the relationship of thinking to being” (Moon and Blackman, 2014). Also, based on Guba (1990), philosophical perspective refers to the basic set of beliefs that guide human behaviour. Evely et al. (2008) claimed that philosophies are systems of values adhered to by individuals. They can be decided by holding different position of epistemology and ontology. Philosophical perspective is the fundamental for scientific research, including natural science and social science. There are some critical philosophical perspectives such as positivism and interpretivism. I will demonstrate introduction for each of them.

It is the belief of positivism that there is an objective reality that is independent of human behaviour, thus objective reality is not a product of the human mind (Evely et al., 2008). During the late eighteenth and nineteenth centuries, advocates of a science of society accepted and embraced the concept of positivism in science (Keat, 1979). Generally, natural scientists adhere to positivism when they conduct natural scientific experiments. For social science research, the positivist viewpoint also holds that social research should be scientific in the sense that it adopts the methods and approaches of natural science and frames explanations in terms of laws which may enable us to anticipate the course of events (Dyson and Brown, 2006). Dyson and Brown (2006) also claimed that the second requirement of positivism is that knowledge should be observable, thus eliminating theoretical concepts from the category of knowledge. Moreover, the positivist view of science has further been applied to a substantial amount of statistical work in the social sciences. Therefore, it is necessary to take into consideration the more general implications of debates regarding positivism and social science when evaluating the potential uses of statistics (Keat., 1979).

Contrary to the positivist position, interpretivism seeks to understand and explain human and social reality in a more creative and open-minded manner (Crotty, 1998). Interpretivism holds that in order to understand this world of meaning, it is necessary to interpret it (Schwandt, 2003). Fixed facts and detached entities are usually rejected by the paradigm (Irshaidat, 2022). Also, human behaviour is constructed as purposeful by interpretivism (Schwandt, 2003). Irshaidat (2022) thought that the researcher could not be detached from the subject matter in order to fully comprehend a certain phenomenon, and a researcher's involvement in the research process was closely linked to its detailed elements. In contrast to positivism, interpretivists believe that the subject matter of the social sciences differs fundamentally from that of the natural sciences who adopt a "different logic of research procedure" taking into account human distinctiveness rather than that of nature (Bryman, 2008). Furthermore, an important difference between positivism and interpretivism is that interpretivist research results are based on scientists' interactions with participants, and all interpretations are contextually dependent on the history and culture that influence how people interpret their own world (Moon and Blackman, 2014). In other words, interpretivism, in contrast to positivism, seeks to understand reality based on experiential learning, therefore, building a connection between the researcher and the subject of study (Irshaidat, 2022). Thus, in conclusion, interpretivism offers a means of establishing profound understanding of meaning by concentrating on minor details without presupposing or making scientific claims (Irshaidat, 2022).

There are other philosophical perspectives in philosophy which can guide the research of natural scientist and social scientists, such as post-positivism, critical theory, post-structuralism, post modernism and pragmatism. For the sake of simplicity and clarity, these have been summarised in Table 3.3.

**Table 3.3** The details of various philosophical perspectives (Moon and Blackman, 2014)

| ***philosophical perspectives*** | ***Details*** |
|---|---|
| Post-positivism | As all methods are imperfect, multiple techniques are required to identify a valid belief or concept. |
| positivism | According to positivism, reality exists independently of human behavior, therefore it cannot be created by humans |
| interpretivism | A more creative and open-minded approach to understanding and explaining human and social reality is embodied in interpretivism. According to interpretivism, understanding this world of meaning requires interpretation. |
| Critical theory | By challenging, revealing conflict and oppression and making changes to make a difference. |
| Post-structuralism | The world is divided and given meaning by different languages and discourses. |
| Post-modernism | Different methods of determining truth are equally distrusted; it may not be possible to define reality definitively. |
| pragmatism | The research problem should be understood using all necessary approaches. |

In the next section, the research subjects of this thesis will be clarified.

### 3.3. Research subjects

This section outlines the research subjects of this thesis from a macro to a micro perspective. Firstly, taking a macro perspective, this thesis focuses on the subject of a management problem. On the one hand, this thesis investigates the empty container management problem, optimise the $LSCs'$ profit, and reduce the operational cost. On the other hand, this thesis also helps governments to devise a $CT$ policy in the maritime sector. For public policy makers in all governments around the world, how to manage carbon emission has become a challenging issue, particularly in recent years. The topic of this thesis falls into the management field due to the fact that management is the process of establishing a strategy or policy for an organisation or administrative institution, through the application of available resources, in order to accomplish the organisation's objectives. Also, a management system is, based on Soni (2020) cited by Van Fleet and Peterson, a set of activities designed to maximise the efficient use of resources for the achievement of a specific goal or objective. The features and description mentioned above indicate that the scope of this thesis matches the field of management. Therefore, management is the macroscope subject of this thesis.

Furthermore, from a more specific perspective, this research explores the problem of $OR$ and $OM$. Generally speaking, $OR$ is a method of analysing problems and making decisions in order to improve organisational performance (Lewis, 2019). Using mathematical analysis, $OR$ solves problems in defined steps by breaking them down into basic components (Lewis, 2019). Meanwhile, $OM$ is responsible for organizing business processes within an organisation so that they are as efficient as possible; and to maximise the profit of an organisation, efficient conversion of materials and labour is required (Hayes, 2023). Normally, in business, a corporate operations manager's objective is to maximise net operating profits by balancing costs and revenues (Hayes, 2023), which could be efficiently achieved under $OM$.

In this thesis, mathematical analysis and simulation will be used to explore how, given the impact of government $CT$ policy, $LSCs$ share empty containers in a port area and thereby improve their performance. I also model the problem mathematically, and then conduct numerical cases. I will maximise the total system profit, and offer a contract to the carriers which they will have no reason to reject. In addition, I aim to help government to achieve maximum social welfare. Although government is not an entity which pursues the business profit, it has a responsibility to optimise different social resources and maximise social welfare. In the context of this thesis, firstly, to mitigate the negative impact of climate change and global warming, I consider that government should regulate and make $CT$ policy to control carbon emissions. Secondly, I believe that the government should be responsible for maintaining the stability of maritime sector operations since financial stability for $LSCs$ and steady international shipping activities are critical for the country's logistic, supply chain management, and even macroeconomic growth. For instance, Cebr (2017) estimated that the maritime sector generated revenue of £91.9 billion in 2015, employed 957,300 people, and paid £21.0 billion in employee compensation. The maritime sector contributes billions of pounds to the UK Exchequer, and international exports goods and services contribute significantly to UK trade (Cebr, 2017). Thus, I adopt the method of $OM$ to help the government maximise total social welfare. In conclusion, it is reasonable to model the issues in this thesis as $OM$ and $OR$ problems so that I can find a scientific and satisfactory solution.

Finally, this thesis also concentrates on $GT$ investigation. As this research have stated in section 1.3, the goal of $GT$ is to determine how teams of players make independent and dependent decisions (Kelly, 2011). In this thesis, from the perspective of $LSCs$, I help them make decisions independently on the number of empty containers that are shared. I design different contracts (e.g., $BBC$ and $RSC$) to enhance the profits and performance of two carriers in empty container management, where neither carrier has reason to reject the contract because their profit will be increased. Most importantly, under the mechanism, the whole system profit is maximised (i.e., system coordinated). Under this circumstance, the shared empty containers will reach the equilibrium level. It should be mentioned that I will eventually apply the Stackelberg game to the model between the government and $LSCs$, where the government is the leader and $LSCs$ are the followers in the game, and they make their own decision in sequence.

In conclusion, this thesis focuses on the problem of $OR$, $OM$ and $GT$. In the next section, I will introduce the research approach and method adopted by this thesis.

### 3.4. The philosophical standpoint of this thesis

Macroscopically speaking, this thesis investigates the management problems in social science. I adopt positivism as the philosophical perspective because the author thinks that the subjects on which this thesis concentrates are independent of human behaviour and the human mind. Two reasons are given to support this position. First, the author believes that social science research should be scientific. For example, the $LSCs'$ $ECS$ strategy and the government's $CT$ policy should be made scientifically and cautiously. On the one hand, both $ECS$ strategy and $CT$ policy making are affected by external subjective factors such as the carriers' level of empty container inventory and the sensitivity of the government to carbon emissions in the maritime sector. On the other hand, the $LSCs'$ $ECS$ strategy and the government's $CT$ policy making are affected also by internal interaction between $LSCs$ and government. For instance, the government's $CT$ policy fluctuation could influence the $LSCs'$ $ECS$ strategy and vice versa. However, neither the $LSCs'$ $ECS$ strategy nor the government's $CT$ policy depends on the subjective decision of a human (e.g., manager and government sector). The second reason for choosing positivism in this thesis is the methods and results accumulated in the previous literature. Based on the review in Chapter 2, some scholars have scientifically and objectively investigated and found that such $ESC$ mechanisms in general supply chains enhance the business profits and performance. In international shipping activity, it has been proved that $ECS$ can effectively reduce unnecessary empty container movements and empty container accumulation in port fields and inland depots. Moreover, in recent decades, with $CT$ policy increasingly applied by many countries globally, the impact of $CT$ levying on enterprises' business has been explored significantly. All the research has indicated that the government $CT$ policy does affect the $ECS$ mechanism, $LSCs'$ profits and performance. Therefore, all the knowledge can be studied scientifically in this thesis, which also satisfies the idea of positivism.

Additionally, the author believes that the research results presented in this thesis exist objectively and cannot be influenced by the author's or others' observations. The existing results and conclusion in this thesis are decided only by the external factors (e.g., $CT$ level) and internal decision making (e.g., $ECS$ strategy). Thus, this indicates that the idea of realism is applied in this thesis. Also, for epistemology, objectivism is used in this thesis because the author holds that the reality of the research results is independent of any conscious action. Furthermore, in this research situation, the author is able to act as a detached observer to investigate the problems. Figure 3.1 give a diagram which illustrates the philosophical perspective of this thesis.

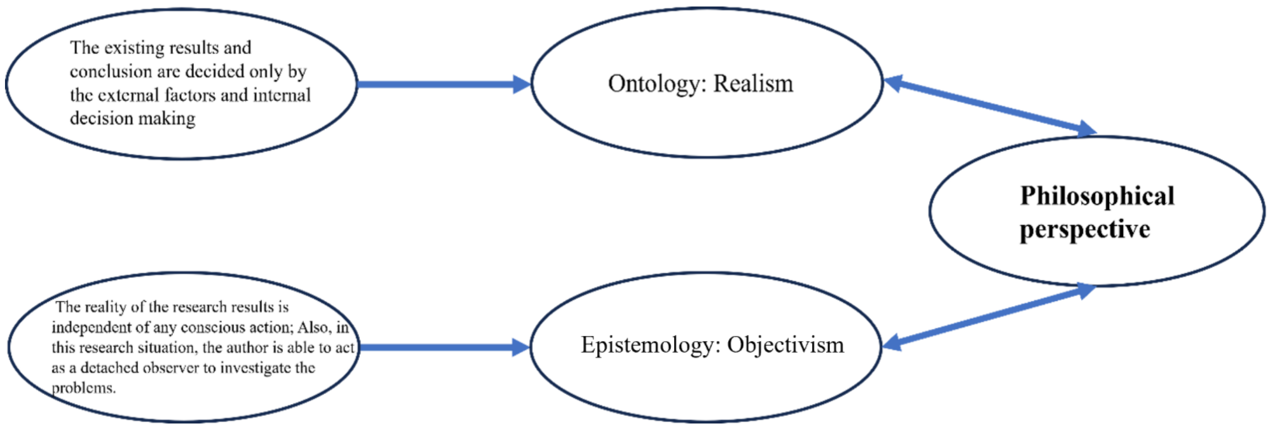


**Figure 3.1** The philosophical perspective of this thesis.

Thus, given the philosophical perspective guidance above, without loss of generality, the author firstly abstracts the complicated problem in the real world into a relatively simple and understandable problem. Then, the author models the problem mathematically and conducts the analysis on how the government $CT$ policy affects the $ECS$ strategy among $LSCs$. Lastly, for the sake of rigor, the author applies the numerical example analysis to examine the rationality and scientificity of the conclusions.

In the next section, the three steps (see section 2.9) of the research design of this thesis is presented and, subsequently detailed in the next three chapters.

### 3.5. Research Design

Generally speaking, all three chapters consider a stylised container sharing supply chain where two $LSCs$ share empty containers under a government carbon taxation scheme. The two $LSCs$ operate in a port area (denoted as $LSC$ 1 and 2), and each is based at a shipping terminal (terminal 1 and 2). Containers are provided to customers by the $LSCs$ for packing their cargoes and $LSCs$ are responsible for delivering and receiving empty and ladened containers to and from overseas. It should be noted that this cargo transport can only be completed if a sufficient number of empty containers is available. If, however, one of the $LSCs$ does not have sufficient empty containers, the other $LSC$ may be able to help by supplying empty containers. Thus, a co-opetition relationship exists between the two $LSCs$. If an empty container supplier has met all of its own demand, it may consider renting out surplus empty containers to meet empty container demand. By doing so, both $LSCs$ reduce their risk of a shortage of empty containers, which enables them to increase their profits with consequential benefit to the environment.

For the sake of convenience and consistency, the $TEU$ container is assumed and applied in this thesis as the stylised container. The ladened ship from overseas can randomly choose $LSC$ 1 or $LSC$ 2 to unload the container, regardless of the ownership of the container. For example, assuming that a ship with full laden containers arrives at terminal 1 (or terminal 2), if the containers belong to $LSC$ 2 (or $LSC$ 1), then the empty containers will be firstly transported back to $LSC$ 2's terminal (or $LSC$ 1's terminal) to satisfy $LSC$ 2's (or $LSC$ 1's) demand. For the customers, those who wish to ship their cargo overseas may choose the $LSC$ according to their preferences. However, they should transport the cargo to the $LSC$ storage yard first. Once they pay the shipping fee, the cargo will be put into the empty container, and then delivered to the terminal by the $LSC$. Finally, the container is loaded and can be transported overseas. It should be noted that the empty container leasing fee has been included in the shipping fee for convenience. Also, in this thesis, it is assumed that the $LSCs$ own the empty container, and they do not need to lease an empty container from container leasing companies.

$CT$ rates are determined by the government for $LSCs'$ transportation activities. As stated in Chapter 2, the $CT$ in the maritime sector is usually seen as an extension of a fuel combustion pollution tax (Parry et al., 2018). Thus, this thesis assumes that the government charges the tax based on the $LSCs'$ transport fuel combustion. Then, the $LSCs$ equally divide the tax between each ladened container and the charge is passed on to the customers. So, the government levies the $CT$ to the $LSCs$, and the $LSCs$ pass the tax to the customer. In other words, it is the consignor who actually pays the $CT$ to the government rather than $LSCs$. Under this circumstance, the government $CT$ policy does not directly affect the profits of the two $LSCs$. Instead, it has impact on the consignors' transport requirements and, subsequently, on the $LSCs'$ requirements for empty containers. Then, the $CT$ policy further influences the $LSCs'$ profit. Therefore, carbon emission taxes affect the $LSCs'$ decisions on $ECR$ activity, because the tax can influence the consignors' transport demands. Figure 3.2 shows the $ECS$ process when $CT$ is levied on $LSCs$.

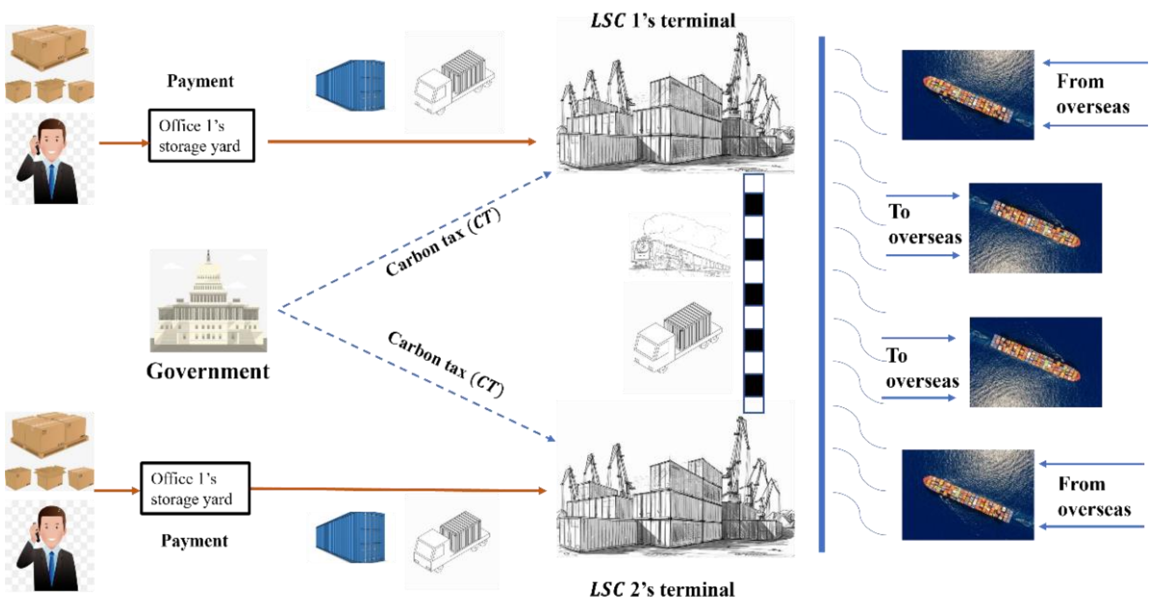


**Figure 3.2** $ECS$ process when $CT$ is levied on $LSCs$

Additionally, the government pays attention, not only to the reduction of carbon emissions, but also to the stabilisation of the shipping market and to improving employment prospects, as the maritime sector plays a significant role in international transportation and the global economy. Bear this mind because this idea will form the basis of the research in Chapter 6.

In this thesis, $p$ is used to denote the $CT$ rate imposed by the government on each cargo container. Following an earlier study (Zhang et al., 2010), the following relationship between the government $CT$ rate $p$ and consignors' demand $X_i$ for empty containers to transport cargo is proposed:

$$X_i = a_i - b_j p + \xi_i \quad (i = 1, 2)$$

Where $a_i > 0$ is the intercept of $X_i$, which represents the potential empty container demands received by $LSC\ i$; $b_j > 0$ is the slope of $X_i$ on $p$, which measures the sensitivity of the consignor to the government's levied $CT$ rate $p$ on every container of cargoes. $\xi_i$ is the error term for $LSC\ i$. I further denote $\xi_i$'s Probability Density Function ($pdf$) and Cumulative Density Function ($CDF$) as $f_i$ and $F_i$, respectively. The equation above indicates that the two $LSCs's$ empty container demands decrease with the levied $CT$ rate.

Other notations including random variable, decision variables, state variables, parameters and solutions are concluded in Table 3.4. $q$ denotes the share of empty container between two $LSCs$. $LSC\ i$'s initial inventory of empty containers is denoted by $n_i$. It should be noted that $q \in [-n_2, n_1]$. Meanwhile, $Y_i$ indicates that $LSC\ i$ receives the number of containers from the other ports. $Y_i$ includes directly imported empty containers and imported ladened containers that become empty after unloading. Furthermore, I assume a transport link, e.g., road or rail, connects the two $LSCs'$ container terminals, enabling empty containers to be shared between two $LSCs$. If $LSC$ 1's on-hand inventory of empty containers $n_1$ plus the number of imported empty containers $Y_1$ cannot satisfy its demands, it can request empty containers $q$ from $LSC$ 2, and vice versa. In this research, I define that $q$ is positive ($q^+$) if the empty containers are transferred from $LSCs$ 1 to 2; otherwise, $q$ is negative ($q^-$).

I let $r_i$ denote the amount of revenue that the $LSC$ can earn per satisfied empty container. I also let $h_i$ denote $LSC\ i$'s empty container holding cost per container, denote $g_i$ as the goodwill penalty cost when $LSC\ i$ fails to meet one container, and $c_t$ is the transportation cost for sharing an empty container between the two $LSCs$ through the road or rail link. Additionally, for keeping consistency, it is assumed that the empty container supplier pays the transportation cost to meet the demand for empty container. Finally, it should be mentioned that this thesis does not consider the internal road transport cost between the $LSC's$ storage yard and its terminal because the cost is much less than the alternative cost including shipping cost, holding cost and goodwill penalty cost.

**Table 3.4** The summary of notations ($i = 1, 2$) confirmed

| | |
|---|---|
| **RANDOM VARIABLE** | |
| $\boldsymbol{X_i}$ | The demands of consignor for empty containers, received by $LSC\ i$ |
| $\boldsymbol{\xi_i}$ | The error term with $f_i(x)$ as $pdf$; and $F_i(x)$ as $CDF$ |
| $\boldsymbol{Y_i}$ | The number of empty containers generated at $LSC\ i$ |
| **DECISION VARIABLE** | |
| $\boldsymbol{p}$ | The $CT$ rate imposed by government on each container of cargoes |
| $\boldsymbol{q}$ | The number of empty containers shared between the $LSCs$ (in Chapter 4, 5 and 6) |
| **STATE VARIABLES** | |
| $\boldsymbol{S_i}$ | $LSC\ i$'s satisfied demands |
| $\boldsymbol{L_i}$ | $LSC\ i$'s unsatisfied demands |
| $\boldsymbol{I_i}$ | $LSC\ i$'s leftover inventory at the end of the period |
| $\emptyset_i$ | The fraction of revenue kept by $LSC\ i$ in $RSC$ |
| $\boldsymbol{R_i}$ | $= (1 - \emptyset_i)r_i$, the revenue that $LSC\ i$ transfers to the other in $RSC$ |
| $\boldsymbol{\eta_i}$ | The buyback price in $BBC$ paid from the supplier for every unsatisfied empty container |
| $\boldsymbol{w}$ | The wholesale price per container in the $RSC$ and in $BBC$ |
| $\boldsymbol{\theta}$ | The transfer payment between two $LSCs$ in decentralised model |
| $\boldsymbol{\beta_i}$ | $= n_i - \lvert q \rvert - a_i + b_i p$; intermediate static variables |
| **PARAMETERS** | |
| $\boldsymbol{n_i}$ | Initial inventory of empty containers owned by $LSC\ i$ |
| $\boldsymbol{a_i}$ | The potential empty container demands received by $LSC\ i$ |
| $\boldsymbol{b_j}$ | The sensitivity of the consignor to the $CT$ rate on each container of cargoes |
| $\boldsymbol{r_i}$ | The amount of revenue that the $LSC\ i$ can earn per satisfied empty container |
| $\boldsymbol{h_i}$ | The holding cost per empty container at $LSC\ i$ |
| $\boldsymbol{g_i}$ | The goodwill penalty cost per unmet empty container at $LSC\ i$ |
| $\boldsymbol{c_t}$ | The cost of transporting an empty container between two $LSCs'$ terminal |
| $\boldsymbol{\alpha_i}$ | $= r_i + h_i + g_i$, the all-in-revenue for $LSC\ i$ |
| $\boldsymbol{C_g}$ | Government carbon treatment cost for each container of satisfied demands. |
| **COMMON SOLUTIONS IN CHAPTER 4, 5 AND 6** | |
| $\boldsymbol{z_i}(.)$ | The Probability Density Function ($pdf$) of $\xi_i - Y_i$ |
| $\boldsymbol{Z_i}(.)$ | The Cumulative Density Function ($CDF$) of $\xi_i - Y_i$ |
| $\boldsymbol{\Phi_i}(.)$ | The Complementary loss function of $\xi_i - Y_i$; $d\Phi_i / dq = Z_i(.)$ |
| $\prod$ | The system profit in the centralised model ($\prod_{cen}$ in Chapter 6) |
| $\boldsymbol{\Delta}\prod$ | The system profit increment in the centralised model ($\Delta\prod_{cen}$ in Chapter 6) |
| $\boldsymbol{\pi_i}$ | $LSCs'$ profit in the decentralised model |
| $\boldsymbol{\Delta\pi_i}$ | $LSCs'$ profit increments in the decentralised model |
| $\boldsymbol{q_1^+}$ | The number of empty containers that $LSC\ 1$ gives to $LSC\ 2$ in the decentralised model |
| $\boldsymbol{q_1^-}$ | $LSC\ 1$ receives the number of empty containers from $LSC\ 2$ in the decentralised model |
| $\boldsymbol{q_2^+}$ | The number of empty containers that $LSC\ 2$ borrows from $LSC\ 1$ in the decentralised model |
| $\boldsymbol{q_2^-}$ | The number of empty containers that $LSC\ 2$ intends to give $LSC\ 1$ in the decentralised model |
| $\boldsymbol{\Delta S_i^e(q,p)}$ | The increment of expected satisfied demands between the scenarios with and without sharing $q$ empty containers under a certain government $CT$ rate $p$ |
| $\boldsymbol{\Delta S_i(0,p)}$ | The increment of expected satisfied demands between the scenarios with and without $CT$ rate variation impact $p$ when $ECS$ is not considered |

**Table 3.4** The summary of notations ($i = 1, 2$) confirmed (continuing from the above table)

| | |
|---|---|
| **OTHER SPECIFIC SOLUTIONS IN CHAPTER 4** | |
| $\boldsymbol{q^*}$ | The optimal sharing quantity of containers in the centralised model |
| $\boldsymbol{\dot{q}}$ | The optimal empty container sharing number in case 2 in the centralised model |
| $\boldsymbol{\ddot{q}}$ | The optimal empty container sharing number in case 4 in the centralised model |
| $\boldsymbol{q^e}$ | The Nash equilibrium of $q$ in the decentralised model |
| **OTHER SPECIFIC SOLUTIONS IN CHAPTER 5** | |
| $\boldsymbol{q^*}$ | The optimal sharing quantity of containers in the centralised model |
| $\boldsymbol{p^*}$ | The optimal $CT$ rate in the centralised model |
| $\boldsymbol{\dot{q}}$ | The optimal empty container sharing number in case 3 in the centralised model |
| $\boldsymbol{\hat{q}}$ | The optimal empty container sharing number in case 4 in the centralised model |
| $\boldsymbol{\breve{q}}$ | The optimal empty container sharing number in case 7 in the centralised model |
| $\boldsymbol{\ddot{q}}$ | The optimal empty container sharing number in case 8 in the centralised model |
| $\boldsymbol{\bar{p}}$ | The optimal $CT$ rate in case 1 in the centralised model |
| $\boldsymbol{\dot{p}}$ | The optimal $CT$ rate in case 3 in the centralised model |
| $\boldsymbol{p^0}$ | The optimal $CT$ rate in case 5 in the centralised model |
| $\boldsymbol{\ddot{p}}$ | The optimal $CT$ rate in case 8 in the centralised model |
| $\boldsymbol{\tilde{p}}$ | The optimal $CT$ rate in case 10 in the centralised model |
| $\boldsymbol{\omega_1}$ | $= [b_1 g_1 + b_2(r_1 + g_1 - r_2 + c_t)]/[(b_1 + b_2)\alpha_1]$; intermediate variable |
| $\boldsymbol{\omega_1'}$ | $= [b_1 g_1 + b_2(r_1 + g_1 - r_2 - c_t)]/(b_1 + b_2)\alpha_1$; intermediate variable |
| $\boldsymbol{\omega_2}$ | $= [b_1(r_2 + g_2 - r_1 - c_t) + b_2 g_2]/(b_1 + b_2)\alpha_2$; intermediate variable |
| $\boldsymbol{\omega_2'}$ | $= [b_1(r_2 + g_2 - r_1 + c_t) + b_2 g_2]/[(b_1 + b_2)\alpha_2]$ ; intermediate variable |
| $\boldsymbol{Z_i^{-1}}(.)$ | Inverse function of $Z_i$ |
| $\boldsymbol{q^e}$ | Nash equilibrium sharing number between in the decentralised model |
| $\boldsymbol{p^e}$ | Nash equilibrium government $CT$ rate in the decentralised model |
| $\boldsymbol{p^{0i}}$ | $LSCs$' Preferred Ideal Carbon Tax Rate ($PICTRs$) in the decentralised model |
| **OTHER SPECIFIC SOLUTIONS IN CHAPTER 6** | |
| $\boldsymbol{p_s^e}$ | Stackelberg equilibrium government $CT$ rate in the centralised model |
| $\boldsymbol{q_s^e}$ | Stackelberg equilibrium sharing number in the centralised model |
| $\boldsymbol{\dot{q}_s^e}$ | Stackelberg equilibrium sharing number in case 2 in the centralised model |
| $\boldsymbol{\ddot{q}_s^e}$ | Stackelberg equilibrium sharing number in case 4 in the centralised model |
| $\boldsymbol{q_d^e}$ | Nash equilibrium of sharing number in the decentralised model |
| $\boldsymbol{\prod_{gov}}$ | The government social welfare function |
| $\boldsymbol{R_i'}$ | The lower boundary making the $LSC's$ profit increment nonnegative. |
| $\boldsymbol{R_i''}$ | The upper boundary making the $LSC's$ profit increment nonnegative. |

The proposed methodology is conducted in three steps to explore how the government $CT$ scheme affects the $ECS$ and the system coordination when the $RSC$ and the $BBC$ are applied. The three steps are specifically investigated respectively in Chapter 4, 5 and 6. Firstly, in Chapter 4, taking the government $CT$ rate as a parameter, applying the $BBC$, and exploring how the government $CT$ rate affects two $LSCs$' the $ECS$ strategy. In Chapter 4, the decision variable is the number of empty containers shared between two $LSCs$. Secondly, in Chapter 5, the government $CT$ rate is introduced as an endogenous variable. How the $RSC$ should be made, and how the appropriate $CT$ rate is set to achieve the system coordination is explored. In Chapter 5, the number of shared empty containers between two $LSCs$ is still the decision variable. Note that the government $CT$ is the endogenous variable and the drawback is that the government's social welfare function is not considered. Therefore, in Chapter 6, still applying the $RSC$, the government social welfare function is proposed, and forms a Stackelberg game with two $LSCs$. In Chapter 6, the decision variables are the number of shared empty containers and the government $CT$ rate. I explore how to coordinate the system and maximise social welfare.

### 3.5.1. Research design in Chapter 4

In chapter 4, I assume that the government imposes a constant $CT$ on two $LSCs$ for their shipping activity. In the $ECS$ activity, two $LSCs$ sign a $BBC$ to determine the number of shared empty containers. Two modes are built sequentially, one is the centralised model, which exists in theory, the other is the decentralised model, which is much more practical. Under the centralised model, two $LSCs$ cooperate completely and perfectly. In this model, it is assumed that the $ECS$ strategy that maximises the total profit of the system can be determined by a virtual central planner. Thus, the centralised model makes the perfect and ideal collaboration between two $LSCs$. By solving the centralised model, given the constant $CT$ impact, the optimal number of shared empty containers and the system's maximum profit is obtained. In practice, however, there is no central planner who has access to complete information to help decide the optimal sharing number. Instead, the two $LSCs$ usually make decisions independently in a decentralised mode. They need to sign a contract, to decide the number of empty containers to be shared and how the costs and benefits will be spilt. Therefore, in this chapter, given the constant $CT$ impact, I assume that two $LSCs$ sign a $BBC$ to decide the equilibrium number of shared empty containers and the related benefit. The $BBC$ stipulates that the empty container supplier charges the wholesale price for every empty container the demander borrows; however, the supplier should pay a buy-back price for every empty container the demander does not lease out at the end of the period (Snyder and Shen, 2011).

The maximum system profit obtained in the centralised model offers the upper boundary of the sum of two $LSCs$ profit in the decentralised model under the system coordination. The system is coordinated when the optimal number of shared empty containers in the centralised model equals the equilibrium number of that in the decentralised model, and two $LSCs'$ profit increment is non-negative. In this context, system coordination means that the two $LSCs$ in the decentralised model act in such a way that the purpose of making the sum of two carriers' profits equals the maximum profit obtained in the centralised model. Therefore, Chapter 4 investigates:

- How the government $CT$ affects two $LSCs'$ $ECS$ strategy in the decentralised model when $BBC$ is applied.
- Given the $CT$ impact, how $ECS$ affects two $LSCs'$ profit when $BBC$ is applied in the decentralised model.
- The condition for the system coordination when the constant $CT$ rate is imposed on $LSCs$. Also, it determines the profit allocation of two $LSCs$ under the system coordination.

Figure 3.3 shows conceptualise the $ECS$ activity when $BBC$ is applied and the impact of $CT$ to be presented in Chapter 4.

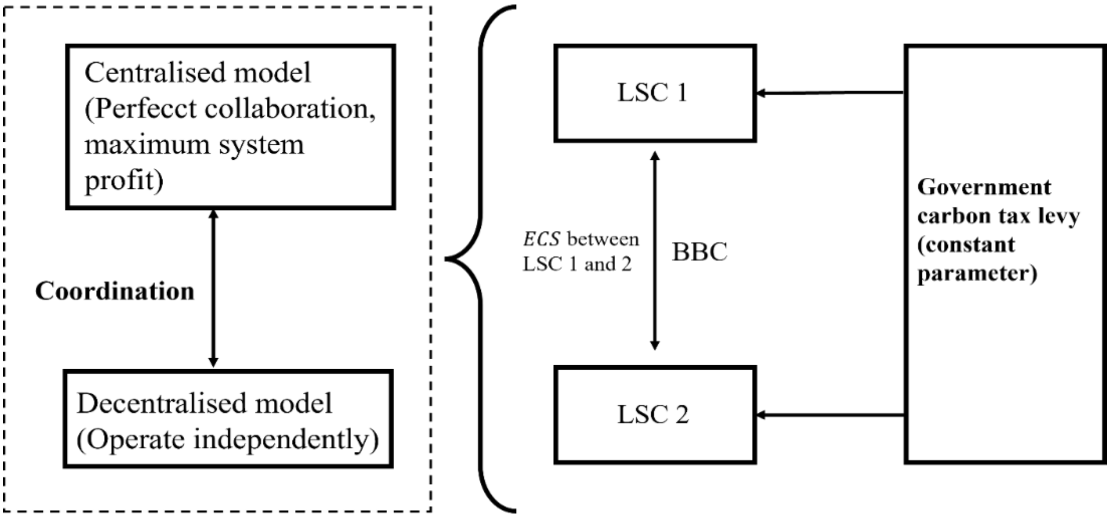


**Figure 3.3** Conceptualise diagram of the $ECS$ activity when $BBC$ is applied and the impact of constant $CT$.

### 3.5.2. Research design in Chapter 5

As with Chapter 4, in chapter 5, two decision-making modes: centralised model and decentralised model, are considered. However, in chapter 5, the government $CT$ is considered as an endogenous variable. In other words, in Chapter 5, the government $CT$ rate is seen as the factor that can affect two $LSCs'$ $ECS$ strategy and the system coordination. Also, a new contract, $RSC$, is introduced between two $LSCs$ to replace the $BBC$. The centralised model is developed in the first instance. By solving the centralised model, the maximum system profits measured by the sum of the two carriers' profits minus the transport cost for the shared empty containers between the carriers is obtained. Then, another model is proposed for the decentralised model. The maximum system profits obtained in the centralised model can be used as a baseline to analyse the performance of the $RSC$ and the impact of the government $CT$ rate variation in the decentralised model. Similarly, in Chapter 5, the system is deemed coordinated as long as two $LSCs$ ensure 1) that the sum of two $LSCs$' profits in the decentralised model equals the maximum profits obtained in the centralised model; 2) that each $LSC$ can be allocated a non-negative profit in the decentralised model.

In summary, Chapter 5 tries to:

- Investigate how $ECS$ strategy between two $LSCs$ in a port hinterland is affected by a fluctuated $CT$ rate under the $RSC$, and simultaneously explore the conditions of container sharing system coordination.
- Examine how much profits each $LSC$ can obtain in a coordinated container sharing system when two $LSCs$ adopt the $RSC$ to share empty containers and the imposed government $CT$ is fluctuated.

Figure 3.4 shows the conceptualise $ECS$ activity process and the impact of $CT$ on the sharing to be presented in Chapter 5.

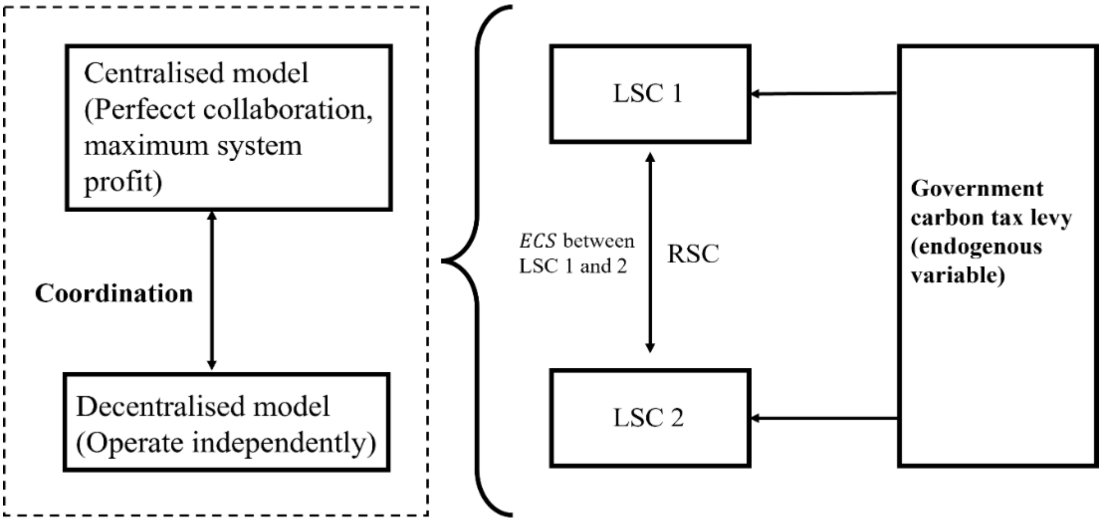


**Figure 3.4** Conceptualise diagram of the $ECS$ activity when $RSC$ is applied and the impact of $CT$ as an endogenous variable.

### 3.5.3. Research design in Chapter 6

In Chapter 6, I propose a Stackelberg game model to explore how $ECS$ may be affected by the $CT$ imposed by governments. I define the Stackelberg game as the game that is played between two $LSCs$ and government where government makes decision on $CT$ rate firstly then two carriers determine the $ECS$ strategy subsequently. The $RSC$ is adopted in this chapter to regulate two $LSCs$' sharing strategy, but the government social welfare function is considered also. Therefore, in this chapter, the model involves three players: the government and two $LSCs$. As the leader in the game, the government is required to set up a $CT$ rate that maximises social welfare; as the followers, $LSCs$ seek to design a mechanism for sharing empty containers to facilitate container supply chain coordination under the government's $CT$. They make decisions sequentially in the Stackelberg game. The sequential decision-making process in the game is illustrated in Table 3.5.

**Table 3.5** The sequential decision-making process in the Stackelberg game

| ***Step*** | ***Details*** |
|---|---|
| 1 | The government proposes a $CT$ rate $p$ per container transportation. |
| 2 | To react $p$, the ideal virtual planner decides the optimal $ECS$ strategy $q^*$ to maximise the system profit in the centralised model. |
| 3 | The government adjusts $p$ if social welfare can be further improved. Otherwise, the government keeps $p$ and the game reaches the Stackelberg equilibrium. The equilibrium rate is denoted as $p^e$. |
| 4 | To react $p^e$, the virtual planner decides the Stackelberg equilibrium $q^e$ to maximise system profit. |

Specifically, in this chapter, I assume that each $LSC$ owns a terminal with a storage yard. The terminal is the place where they can transport or receive the empty or laden container to or from overseas. As with the previous assumption, in chapter 6 two $LSCs$ as the commercial subjects, are in a co-opetition relationship, meaning that they want to act to maximise individual profit selfishly, but they still tend to cooperatively share the empty containers to save loss if there exists an imbalance between supply and demand of empty containers. For example, if one of the $LSC$ lacks empty containers, it could conditionally borrow some empty containers from the other. By doing so, both $LSCs$ can retrieve the loss to some extent and maximise the individual profit by sharing empty containers. On the other hand, the government is the regulator deciding the $CT$ rate on the $LSCs'$ international shipping activity. Unlike in Chapter 4 and 5, in Chapter 6, I further consider that, not only should the government take responsibility to reduce carbon emissions, but it should also consider the total social welfare including the $LSCs'$ stable operation because an inappropriate carbon taxation scheme could create huge costs for $LSCs$. There is no doubt that the $LSCs$ play a critical role in the international transport industry and in the global economy. Therefore, I presume that the government imposes the $CT$ on $LSC$ based on a concern for total optimisation of social welfare. It is easy to understand that the $LSCs'$ $ECS$ decision depends on the $CT$ rate because it can fluctuate the consignor's demand for empty containers.

In summary, Chapter 6 explores the impact of the $CT$ on the $ECS$ strategy made by two $LSCs$, and the system coordination when the government social welfare model is maximised. Therefore, in Chapter 6, I investigate:

- The impact of $CT$ on the $ECS$ strategy when the government's social welfare is maximised.
- The impact of $CT$ on the system coordination when the government adopts the imposition of the optimal $CT$ based on the maximising of social welfare.
- How two $LSCs$ adjust contract parameters (i.e., the wholesale price and the revenue sharing price) to keep the system coordinated when the government imposes $CT$ in a Stackelberg game.

To explore the three objectives, three steps will be designed in Chapter 6. Firstly, the system profit functions of two $LSCs$, as a whole, in the centralised decision-making model, and the government's total social welfare function are built, respectively. In the centralised decision-making model, two $LSCs$ are assumed to cooperate perfectly, and that there is a virtual planner who has the complete information to decide the optimal $ECS$ strategy to maximise the total system profit. The government is the dominator deciding the $CT$ rate per empty container in international transport activity. Thus, the virtual planner should decide the optimal $ECS$ strategy according to the government $CT$ rate. Then, according to the virtual planner's optimal $ECS$ strategy, the government should determine the $CT$ rate based on the goal that maximise the total social welfare. After a few rounds playing, as a result, the $LSCs'$ $ECS$ strategy and the government $CT$ rate both reach the Stackelberg equilibrium eventually. Secondly, for three specific reasons which will be demonstrated in subsection 6.3, the government imposes the equilibrium $CT$ rate obtained in the Stackelberg game on the $LSCs$. By applying the $RSC$, two $LSCs$ cooperate in a decentralised decision-making mechanism. Their profit functions in the decentralised decision-making model are developed. In a decentralised decision-making mechanism, the two $LSCs$ sharing strategies eventually reach the Nash equilibrium. Finally, given the Stackelberg equilibrium $CT$ impact, I try to coordinate the system by appropriately selecting the parameters of $RSC$ (i.e., the wholesale price and the revenue sharing price). In this Chapter, the “system coordination” means that the Nash equilibrium $ECS$ strategy in the decentralised model is equivalent to the Stackelberg equilibrium $ECS$ strategy obtained in the centralised model. Figure 3.5 and Figure 3.6 show the schematic diagram of the centralised and decentralised decision-making model respectively, in Chapter 6.

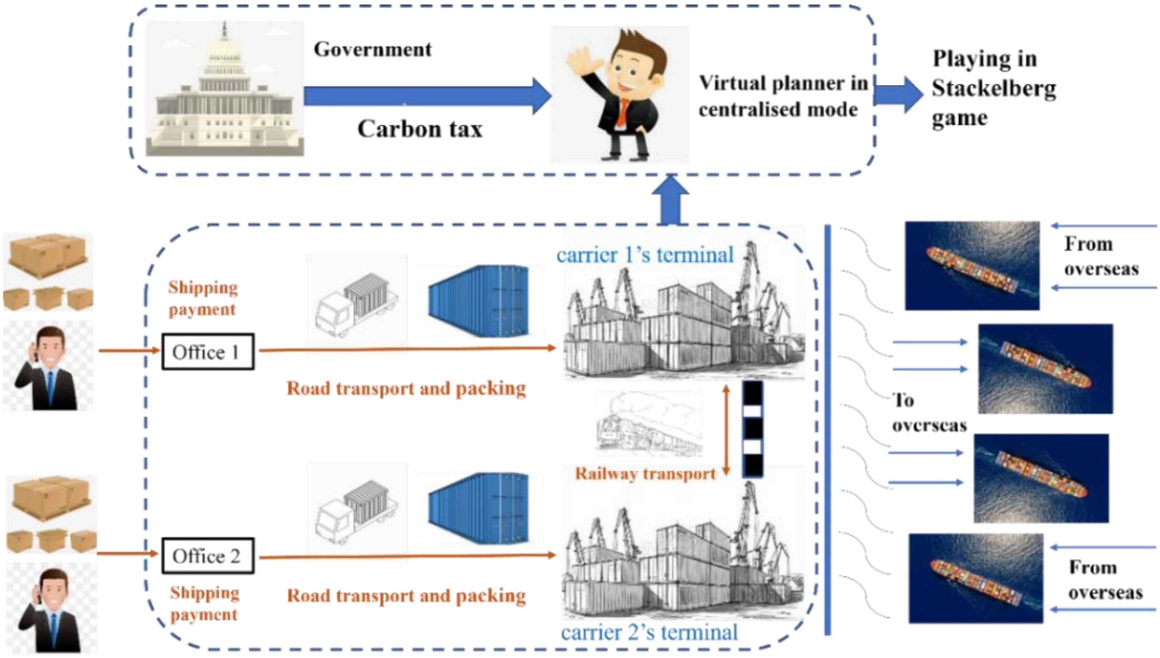


**Figure 3.5** The centralised decision-making model in Chapter 6

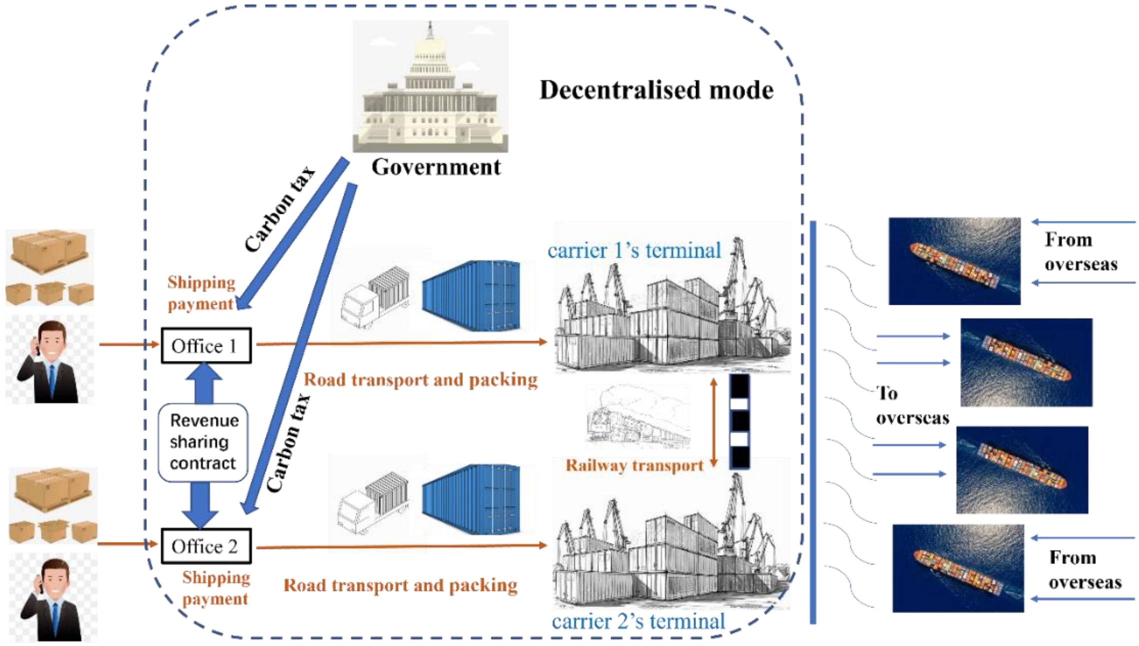


**Figure 3.6** The decentralised decision-making model in Chapter 6

Section 3.6 generally concludes the key points in the chapter of methodology, which includes the research subjects and philosophical standpoint of this thesis as well as the research design for the next three chapters.

### 3.6. Conclusion

In this chapter, I introduced the methodology of this thesis. In particular, I firstly presented the research subjects, which involve the fields of $OR$, $OM$ and $GT$. Then, I introduced the philosophical standpoint of this thesis, which includes the ontology, epistemology and philosophical perspective. These are realism, objectivism and positivism, respectively. Lastly, I showed the research design for the next three chapters (4, 5 and 6) according to the three steps by which I proposed to investigate how the $CT$ system affects $ECS$ between two $LSCs$. Specifically, I comprehensively clarified the research method and purposes for the next three chapters, including the model setting, contracts applied, $CT$ imposing mechanisms, centralised decision-making, and decentralised decision-making mechanism design. Also, some figures are given to illustrate the research process. Meanwhile, all the notations used in this thesis are listed in this chapter. Next, in Chapter 4, I will initially explore how a constant $CT$ rate affects two $LSCs'$ $ECS$ when a $BBC$ is applied.

# Chapter 4 How does Carbon Tax Affect Empty Container Management and Coordination under a Buy-back Contract?

## 4.1. Introduction

This chapter mainly investigates how a constant government Carbon Tax ($CT$) affects two Liner Shipping Carriers' ($LSCs'$) Empty Container Sharing ($ECS$) strategy and how to coordinate the system when a Buy-back Contract ($BBC$) is adopted to bind two $LSCs$. First, an ideal centralised decision-making model, where two $LSCs$ fully cooperate to maximise the whole system profit, is built, taking $CT$ as a constant parameter imposed on the two $LSCs$. As a result, the optimal $ECS$ strategy is obtained. Second, still considering the $CT$ influence, a more realistic decentralised decision-making model is built to decide the equilibrium number of sharing empty containers between two $LSCs$ by introducing a $BBC$. Finally, given the $CT$ impact, consideration is given to how to choose the appropriate contract parameters to achieve the system coordination. Under the system coordination, the two $LSCs$ can be better off in the sense that each carrier can obtain a greater profit than non-coordinated cases and the system profit can be maximised. Also, I will further analyse the impact of $ECR$ activity and the $CT$ on the profits of the two $LSCs$.

The structure of this chapter is as follows. In subsection 4.2, the centralised and decentralised decision-making models will be established. In subsection 4.3, the design of the mechanism for the system coordination will be conducted while simultaneously analysing how a constant $CT$ rate would affect the system coordination. Finally, in subsection 4.4, a numerical case will be conducted to examine the results.

## 4.2. The model

In this section, given the constant government $CT$ impact, a stylised Empty Container Shipping ($ECS$) system is built where two $LSCs$ in the system share empty containers. Firstly, the optimal $ECS$ strategy is explored under the impact of the imposition of the government $CT$ on the centralised decision-making model. In this model, a virtual central planner exists who has all information needed to make a perfect $ECS$ strategy and achieve the maximum system profit. Secondly, still taking into considerion the constant $CT$ impact, the decentralised decision-making model is obtained where the two $LSCs$ make decisions independently but are bound to a pre-determined $BBC$. By exploring a decentralised decision-making model, the equilibrium $ECS$ quantity can be obtained, leading to the sum of the two $LSCs'$ profits to be the same as the maximum system profit in the centralised decision-making model as long as the $BBC$ is appropriately made. Thirdly, the system is coordinated and obtains the profit increment for the two $LSCs$ in the decentralised decision-making model under the system coordination. Figure 3.1 shows the details of model.

### 4.2.1. The centralised decision-making model

As I stated above, the centralised decision-making model assumes that a central planner owns the complete information about the optimal $ECS$ strategy to maximise the system profit for all parties. Ideally, the two $LSCs$ work together perfectly regardless of their individual interest. Although it is an idealised model, it does provide an upper limit on the total system profit. The sum of the two $LSCs'$ profit would not exceed this maximum profit in any other circumstances. To build the centralised decision-making model, the terms of the two $LSCs'$ satisfied demand of empty container $S_i(q,p)$, the leftover inventory of empty container $I_i(q,p)$ and the unsatisfied demand of empty container $L_i(q,p)$, which are the components of the centralised decision-making model, should be defined:

$$S_1(q,p) = \boldsymbol{E}min[X_1, n_1 - q + Y_1] = \boldsymbol{E}min[a_1 - b_1 p + \xi_1, n_1 - q + Y_1] \quad 4.1$$

$$I_1(q,p) = (n_1 - q + Y_1 - X_1)^+ = (n_1 - q + Y_1 - a_1 + b_1 p - \xi_1)^+ \quad 4.2$$

$$L_1(q,p) = (X_1 - n_1 + q - Y_1)^+ = (a_1 - b_1 p + \xi_1 - n_1 + q - Y_1)^+ \quad 4.3$$

$$S_2(q,p) = \boldsymbol{E}min[X_2, n_2 + q + Y_2] = \boldsymbol{E}min[a_2 - b_2 p + \xi_2, n_2 + q + Y_2] \quad 4.4$$

$$I_2(q,p) = (n_2 + q + Y_2 - X_2)^+ = (n_2 + q + Y_2 - a_2 + b_2 p - \xi_2)^+ \quad 4.5$$

$$L_2(q,p) = (X_2 - n_2 - q - Y_2)^+ = (a_2 - b_2 p + \xi_2 - n_2 - q - Y_2)^+ \quad 4.6$$

Considering the transport cost $T$ per shared empty container between the two $LSCs'$ terminal, the centralised decision-making model is formulated as:

$$\begin{aligned}\prod(q,p) &= r_1 S_1 + h_1 \boldsymbol{E}I_1 + g_1 \boldsymbol{E}L_1 + r_2 S_2 + h_2 \boldsymbol{E}I_2 + g_2 \boldsymbol{E}L_2 - c_t|q| \\ &= \alpha_1[\beta_1 - \Phi_1(\beta_1)] + \alpha_2[\beta_2 - \Phi_2(\beta_2)] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) \\ &\quad - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p) - c_t|q|\end{aligned} \quad 4.7$$

The transformation details of equation 4.7 are shown in Appendix A, where $\alpha_i$ stands for the all-in-revenue that $LSC\ i$ could obtain for every satisfied empty container ($\alpha_i = r_i + h_i + g_i$). I also define $\beta_i = n_i - |q| - a_i + b_i p$. $\Phi_i(.)$ denoted as the complementary loss function and the $pdf$ and $CDF$ of the random variable $\xi_i - Y_i$ are denoted as $z_i$ and $Z_i$, respectively. (where $\frac{d\Phi_i}{dq} = Z_i(.); i = 1, 2$). Next, give $p$, ***Lemma 4.1*** is given to demonstrate $\prod(q,p)$ is strictly concave in $q$. (Appendix B)

***Lemma 4.1*** *Given p, $\prod(q,p)$ is strictly concave in $q$.*

Given $p$, ***Lemma 4.1*** illustrates that $\prod(q,p)$ can always be maximised when the $ECS$ strategy is optimised. It indicates that the two $LSCs$ could find an optimal $ECS$ strategy in the centralised decision-making model. However, ***Lemma 4.1*** is incomplete because it does not consider the constraint of $q$. Therefore, due to $q \in [-n_2, n_1]$, **Theorem 4.1** explores the comprehensive solution of the optimal sharing number $q^*$ between the two $LSCs$ in the centralised decision-making model. (Appendix C).

***Theorem 4.1*** *Given* $q \in [-n_2, n_1]$ *and* $p$*,* $q^*$ *in different cases is shown in Figure 4.1 and Table 4.1*

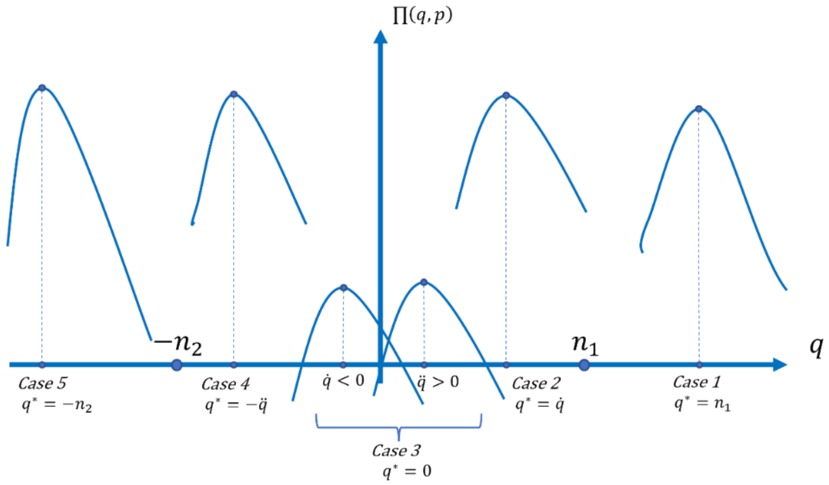


**Figure 4.1** The optimal $ECS$ strategy in the centralised decision-making model in different cases

**Table 4.1** The optimal $ECS$ strategy in the centralised decision-making model in different case

| ***Case*** | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** |
|---|---|---|---|---|---|
| $\boldsymbol{q^*}$ | $n_1$ | $\dot{q}$ | 0 | $\ddot{q}$ | $-n_2$ |

**Theorem 4.1** further provides the full solution of $q^*$ when the constraint of $q$ is considered. It indicates that the virtual planner could get a maximum profit in the centralised decision-making model given any government $CT$ rate. For example, in case 1 (or case 5), the optimal number of empty containers that could be shared from $LSC$ 1 to $LSC$ 2 (or $LSC$ 2 to $LSC$ 1) is $n_1$ (or $-n_2$). This is because the optimal sharing number is greater than $n_1$ (or less than $-n_2$) when the constraint is not considered. Thus, $LSC$ 1 (or $LSC$ 2) must provide its entire inventory to $LSC$ 2 (or $LSC$ 1). Also, in case 2 (or case 4), the optimal sharing number is determined, and it is between 0 and $n_1$ (or between $-n_2$ and 0). Thus, given $p$, the optimal number is $\dot{q}$ (where $\frac{\partial \prod(\dot{q},p)}{\partial q} = 0$) (or $\ddot{q}$, where $\frac{\partial \prod(\ddot{q},p)}{\partial q} = 0$). Case 3 is exceptional because the optimal number $\dot{q}$ or $\ddot{q}$ is less or greater than 0, respectively. Thus, there should not be any empty containers shared between the two $LSCs$, and both $LSCs$ must maintain the status quo.

However, although equation 4.7 provides a way to calculate the system profit, it does not reveal whether the $LSCs$' profit positively or negatively increases during the $ECS$ activity under constant $CT$ impact. For example, if $\prod(q,p)$ is positive, but it is less than the initial system profit $\prod(0,0)$ when there is no $ECS$ activity under no government $CT$ imposed, then the system profit increment is negative, which does not create any benefit for the $LSCs$. Thus, the situation where no $ECS$ activity and no $CT$ imposed by the government $\prod(0,0)$ is the baseline to calculate the system profit increment. The calculation of system profit increment in the centralised decision-making model is introduced in **Lemma 4.2**. (Appendix D)

***Lemma 4.2*** *The system profit increment* $\Delta\prod(q^*, p)$ *in the centralised decision-making model is:*

$$\begin{aligned}\Delta\prod(q^*,p) &= \prod(q^*,p) - \prod(0,0)\\ &= -\alpha_1\int_{n_1-a_1}^{n_1-q^*-a_1+b_1p} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2}^{n_2+q^*-a_2+b_2p} Z_2(d_2)dd_2 \\ &+ (\alpha_2-\alpha_1+h_1-h_2)q^* - c_t\,|q^*| + g_1b_1p + g_2b_2p\end{aligned} \quad \textbf{4.8}$$

$\Delta\prod(q^*,p)$ can be further transformed as (Appendix D)

$$\begin{aligned}\Delta\prod(q^*,p) = &-\alpha_1\left[\int_{n_1-a_1}^{n_1-q^*-a_1+b_1p} Z_1(d_1)dd_1 + q^* - b_1p\right]\\ &-\alpha_2\left[\int_{n_2-a_2}^{n_2+q^*-a_2+b_2p} Z_2(d_2)dd_2 - q^* - b_2p\right] - (\alpha_1-g_1)b_1p - (\alpha_2-g_2)b_2p\\ &+ (h_1-h_2)q^* - c_t\,|q^*|\end{aligned} \quad \textbf{4.9}$$

The system profit increment in the centralised decision-making model can be divided into four parts, which are:

- $-\alpha_1\left[\int_{n_1-a_1}^{n_1-q^*-a_1+b_1p} Z_1(d_1)dd_1 + q^* - b_1p\right]$ and $-\alpha_2\left[\int_{n_2-a_2}^{n_2+q^*-a_2+b_2p} Z_2(d_2)dd_2 - q^* - b_2p\right]$ : the profit obtained by two $LSCs$ from the $ECS$ activity under the constant $CT$ impact.
- $-(\alpha_1-g_1)b_1p$ and $-(\alpha_2-g_2)b_2p$: the $LSCs$' profit loss due to decrease in empty container demand when the government imposes $CT$ on transportation of containers.
- $(h_1-h_2)q^*$: the variation of the total holding cost because of the two $LSCs$' leftover inventory changes during the $ECS$ activity.
- The transport cost $c_t$.

Next, given $p$, the decentralised decision-making model is built by introducing a $BBC$ to explore the two $LSCs$' equilibrium $ECS$ strategy and the system coordination conditions.

### 4.2.2. The decentralised decision-making model

In this subsection, a practical decentralised model is built where two $LSCs$ make decisions independently but are bound to a pre-determined $BBC$. The $BBC$ regulates that a wholesale price $w$ per shared empty container should be paid by $LSC\ i$, when it requests containers from the other. Furthermore, to encourage $LSC\ i$ to lease more empty containers, the other $LSC$ agrees to pay a buy-back price $\eta_i$ to $LSC\ i$ for each unused empty container, and vice versa. In other words, the buy-back price can be treated as a credit offered by one $LSC$ to the other to compensate for the loss of over ordering. Therefore, in summary, the contract is the mechanism to allocate the profit between two $LSCs$. Now, I can obtain the transfer payment $\theta$ between two $LSCs$.

$$\begin{aligned}\theta = wq^+ - \eta_2 \boldsymbol{E}\min\{(n_2 + q + Y_2 - X_2)^+, q^+\} - w(-q)^+ \\ + \eta_1 \boldsymbol{E}\min\{(n_1 - q + Y_1 - X_1)^+, (-q)^+\}\end{aligned} \quad \textbf{4.10}$$

Based on equation 4.10, it is simple to find that a positive $\theta$ means the payment is transferred from $LSC$ 2 to $LSC$ 1, and vice versa. Next, given $p$, combining the transfer payment $\theta$, two $LSCs'$ profit functions in the decentralised decision-making model are formulated. $LSC$ 1's profit function is:

$$\begin{aligned}&\pi_1(q, p, w, \eta_1, \eta_2) \\ &= r_1\, \boldsymbol{E}min[X_1, n_1 - q + Y_1] - h_1 \boldsymbol{E}(n_1 - q + Y_1 - X_1)^+ \\ &- g_1 \boldsymbol{E}(X_1 + q - n_1 - Y_1)^+ - c_t q^+ + \theta\end{aligned} \quad \textbf{4.11}$$

Then, the equation 4.11 is transformed as: (Appendix E)

$$\begin{aligned}&\pi_1(q, p, w, \eta_1, \eta_2) \\ &= (\alpha_1 - \eta_1 \mathbf{1}_{q<0})[\beta_1 - \Phi_1(\beta_1)] + \eta_2 \mathbf{1}_{q>0}[\beta_2 - \Phi_2(\beta_2)] \\ &+ \eta_1 \mathbf{1}_{q<0}[(\beta_1 + q) - \Phi_1(\beta_1 + q)] - \eta_2 \mathbf{1}_{q>0}[(\beta_2 - q) - \Phi_2(\beta_2 - q)] \\ &- r_1 \boldsymbol{E}(-a_1 + b_1 p - Y_1) - \eta_1 \mathbf{1}_{q<0} q - \eta_2 \mathbf{1}_{q>0} q - h_1 \beta_1 - g_1 \boldsymbol{E}(\xi_1 - Y_1) \\ &+ p^+(w - c_t) - w(-q)^+\end{aligned} \quad \textbf{4.12}$$

Similarly, $LSC$ 2's profit function is:

$$\begin{aligned}&\pi_2(q, p, w, \eta_1, \eta_2) \\ &= (\alpha_2 - \eta_2 \mathbf{1}_{q>0})[\beta_2 - \Phi_2(\beta_2)] + \eta_1 \mathbf{1}_{q<0}[\beta_1 - \Phi_1(\beta_1)] \\ &- \eta_1 \mathbf{1}_{q<0}[(\beta_1 + q) - \Phi_1(\beta_1 + q)] + \eta_2 \mathbf{1}_{q>0}[(\beta_2 - q) - \Phi_2(\beta_2 - q)] \\ &- \alpha_2 \boldsymbol{E}(-a_2 + b_2 p - Y_2) + \eta_1 \mathbf{1}_{q<0} q + \eta_2 \mathbf{1}_{q>0} q - h_2 \beta_2 - g_2 \boldsymbol{E}(\xi_2 - Y_2) + (w \\ &- c_t)(-q)^+ - wq^+\end{aligned} \quad \textbf{4.13}$$

Next, **Condition 4.1** is given to ensure that the two $LSCs'$ profit function is strictly concave in $q$. (Appendix F)

***Condition 4.1*** $0 < \eta_i < \alpha_i$

From the perspective of mathematical illustration, **Condition 4.1** indicates that the buy-back price paid by the empty container supplier to the demander for every unmet empty container should not be greater than the all-in-revenue that the demander obtains from every satisfied empty container. Otherwise, the two $LSCs$ profit function cannot be concave in $q$. Practically, in other words, if $\eta_i$ is greater than $\alpha_i$, it indicates that the demander would not tend to use the shared empty container in shipping activity because it can eventually get a buy-back price per unused empty container from the supplier, which is even greater than the unit profit obtained from the shipping activity. It is not a realistic situation for the shipping company.

Next, based on **Condition 4.1**, **Lemma 4.3** is shown to prove that both $\pi_1(q,p,w,\eta_1,\eta_2)$ and $\pi_2(q,p,w,\eta_1,\eta_2)$ are strictly concave in $q$. (Appendix F)

***Lemma 4.3*** *Given* ***Condition 4.1****, $\pi_1(q,p,w,\eta_1,\eta_2)$ and $\pi_2(q,p,w,\eta_1,\eta_2)$ are strictly concave in $q$ where $q_1$ and $q_2$ are two optimal strategies for two carriers, respectively.*

**Lemma 4.3** reveals that both $LSCs'$ have optimal $ECS$ strategies to maximise their profits, respectively, given the impact of the constant government $CT$ imposed. They are denoted as $q_1$ and $q_2$, respectively. The sign of $q_1$ and $q_2$ should be regulated to be consistent with $q^*$ in the centralised decision-making model. $q_1^+$ is denoted as the number of empty containers that $LSC$ 1 provides to $LSC$ 2, and $q_1^-$ means $LSC$ 1 receives the optimal number of empty containers from $LSC$ 2. Similarly, $q_2^+$ refers to the number of empty containers that $LSC$ 2 borrows from $LSC$ 1 while $q_2^-$ is used to represent the number of empty containers that $LSC$ 2 gives $LSC$ 1. Based on **Lemma 4.3**, each $LSC$ tends to selfishly adopt its own optimal strategy while disregarding the other's optimal strategy. Therefore, two the $LSCs'$ optimal $ECS$ strategy tends to move to the Nash equilibrium, which is the Pareto optimal. It means no $LSC$ can get a better payoff without diminishing the other's profit, i.e.,

$$q^e = \min\{q_1^+(p), q_2^+(p)\} - \min\{q_1^-(p), q_2^-(p)\} \tag{4.14}$$

Based on equation 4.14, it is not difficult to find that there are only three potential results of $q^e$, which are $\min\{q_1^+(p), q_2^+(p)\}$, $\min\{q_1^-(p), q_2^-(p)\}$ and 0. Bearing in mind that the $CT$ keeps affecting the two $LSCs'$ profit increment in the decentralised decision-making model, so, to explore the condition for system coordination in the next section, the two $LSCs'$ profit increments should be defined:

$$\begin{aligned} \Delta\pi_1(q^e,p,w,\eta_1,\eta_2) &= \pi_1(q^e,p,w,\eta_1,\eta_2) - \pi_1(0,0,w,\eta_1,\eta_2) \\ \Delta\pi_2(q^e,p,w,\eta_1,\eta_2) &= \pi_2(q^e,p,w,\eta_1,\eta_2) - \pi_2(0,0,w,\eta_1,\eta_2) \end{aligned} \tag{4.15}$$

Equation 4.15 shows the two $LSCs'$ profit increment between the scenarios with and without $ECS$ activity under the constant $CT$ rate impact. Similarly with the equation 4.8, the situation where no $ECS$ activity and no imposed $CT$ [i.e., $\pi_i(0,0,w,\eta_1,\eta_2)$] provides the baseline for the two $LSCs'$ profit increment calculation. Equation 4.16 and 4.17 show $\Delta\pi_1$ and $\Delta\pi_2$, respectively. (Appendix G)

$$\begin{aligned} \Delta\pi_1 = &-\alpha_1 \int_{n_1-a_1}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1 + \eta_1 \mathbf{1}_{q^e<0} \int_{n_1-a_1+b_1p}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1 \\ &- \eta_2 \mathbf{1}_{q^e>0} \int_{n_2-a_2+b_2p}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 + (h_1-\alpha_1+w)q^e + g_1b_1p - c_t(q^e)^+ \end{aligned} \tag{4.16}$$

$$\begin{aligned} \Delta\pi_2 = &-\alpha_2 \int_{n_2-a_2}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 - \eta_1 \mathbf{1}_{q^e<0} \int_{n_1-a_1+b_1p}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1 \\ &+ \eta_2 \mathbf{1}_{q^e>0} \int_{n_2-a_2+b_2p}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 + (\alpha_2-h_2-w)q^e + g_2b_2p - c_t(-q^e)^+ \end{aligned} \tag{4.17}$$

Clearly, $\eta_1$ only exists when $q^e$ is less than 0 (i.e., when empty containers are shared from $LSC$ 2 to $LSC$ 1) and $\eta_2$ is only meaningful when $q^e$ is greater than 0 (i.e., the empty containers are shared from $LSC$ 1 to $LSC$ 2). Next, I will design the system coordination when the conditions are satisfied and analyse how the $ECS$ activity and imposition of the $CT$ affects the two $LSCs$' profit increment.

### 4.3. Mechanism design for system coordination

In the first instance, I define conditions for system coordination. Two conditions should be satisfied, firstly:

***Condition 4.2*** *The optimal number of shared empty containers in the centralised decision-making model $q^e(p)$ equals that in the decentralised decision-making model $q^*(p)$ at the Nash equilibrium.*

By appropriately adjusting the $BBC$ parameters ($\eta_1$, $\eta_1$ and $w$), the optimal number of shared empty containers in the centralised decision-making model equals the equilibrium number of shared empty containers in the decentralised decision-making model. Thus, **Condition 4.2** ensures that the optimal sharing strategy is equivalent to the equilibrium strategy. In addition, it should be noted that the sum of the two $LSCs$' profit increments in the decentralised decision-making model equals the system profit increment in the centralised decision-making model under the system coordination. In other words, under the system coordination, the two $LSCs$' profit increments in the decentralised decision-making model are actually "allocated" from the system profit increment in the centralised decision-making model. Therefore, the two $LSCs$' "profit increment" in the decentralised decision-making model can be replaced by "profit allocation" under the system coordination. i.e.,

$$\Delta\prod(q^e, p) = \Delta\prod(q^*, p) = \Delta\pi_1(q^e, p, w, \eta_1, \eta_2) + \Delta\pi_2(q^e, p, w, \eta_1, \eta_2) \tag{4.18}$$

For consistency, I still use "profit increment" to represent $\Delta\pi_i$ in the rest of the thesis. **Condition 4.2** only guarantees that the two $LSCs$ act to maximise the total system profit. The two $LSCs$ may still be able to obtain a negative profit increment in the decentralised decision-making model, which is not allowed in the coordination, as both $LSCs$ do not sign a contract if they cannot obtain any benefit from it. This is the reason why the terms "system profit increment" and "two $LSCs$' profit increment" were introduced previously. **Condition 4.3** regulates the non-negativity of the two $LSCs'$ profit increment under the system coordination.

***Condition 4.3*** $0 \leq \Delta\pi_1(q^e, p, w, \eta_1, \eta_2) \leq \Delta\prod$ *or* $0 \leq \Delta\pi_2(q^e, p, w, \eta_1, \eta_2) \leq \Delta\prod$

Next, I will design the appropriate $BBC$ to coordinate the system, given the constant $CT$ impact. Three steps will be conducted in sequence.

- The contract parameters (i.e., $w, \eta_1$ and $\eta_2$) will be appropriately decided to meet **Condition 4.2**
- Given **Condition 4.2**, the two $LSCs$' profit increment will be analysed, as well as the way in which the

$ECS$ activity and the imposition of the constant $CT$ rate influences their profit increment.

➢ Based on **Condition 4.3**, the non-negativity of the two $LSCs$' profit increment will be guaranteed to further fulfil the system coordination.

Notice that the coordination conditions will be discussed in five cases shown in **Theorem 4.1**, respectively. Firstly, to meet **Condition 4.2**, **Lemma 4.4** is developed (Appendix H).

***Lemma 4.4*** *Given p, to meet* ***Condition 4.2****, $w, \eta_1$ and $\eta_2$ should be constrained in five cases (Table 4.2).*

**Table 4.2** The $BBC$ parameters' ($w, \eta_1$ and $\eta_2$) constraint if Condition 4.2 is met.

| | |
|---|---|
| $\alpha_1[1 - Z_1(-a_1 + b_1 p)] - h_1 + c_t \le w - \eta_2 Z_2(n_2 + n_1 - a_2 + b_2 p) \le \alpha_2[1 - Z_2(n_2 + n_1 - a_2 + b_2 p)] - h_2$ | Case 1 |
| $\alpha_1[1 - Z_1(n_1 - \dot{q} - a_1 + b_1 p)] - h_1 + c_t = w - \eta_2 Z_2(n_2 + \dot{q} - a_2 + b_2 p) = \alpha_2[1 - Z_2(n_2 + \dot{q} - a_2 + b_2 p)] - h_2$ | Case 2 |
| − | Case 3 |
| $\alpha_2[1 - Z_2(n_2 + \ddot{q} - a_2 + b_2 p)] - h_2 + c_t = w - \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1 p) = \alpha_1[1 - Z_1(n_1 - \ddot{q} - a_1 + b_1 p)] - h_1$ | Case 4 |
| $\alpha_2[1 - Z_2(-a_2 + b_2 p)] - h_2 + c_t \le w - \eta_1 Z_1(n_1 + n_2 - a_1 + b_1 p) \le \alpha_1[1 - Z_1(n_1 + n_2 - a_1 + b_1 p)] - h_1$ | Case 5 |

Given $p$, to let $q^e = q^*$, **Lemma 4.4** shows that $q^e$ should strictly comply with the constraints shown in Table 4.2. For case 1, 3 and 5, **Lemma 4.4** ensures that the monotonicity of $\pi_1$ and $\pi_2$ in $q$ are strictly consistent with that of $\prod$ between $[-n_2, n_1]$. For cases 2 and 4, not only can **Lemma 4.4** ensure the monotonicity consistency, but it also ensures that $q^e(p)$ equals $q^*(p)$ precisely. Figure 4.2 depicts the optimal $ECS$ strategy and the relationship between $\Delta\prod$, $\pi_1$ and $\pi_2$ in five cases when **Condition 4.2** is met.

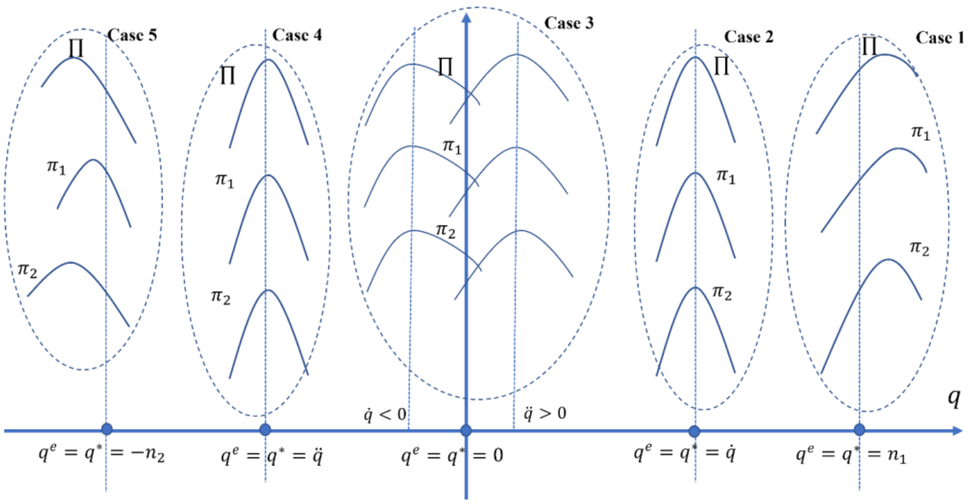


**Figure 4.2** The optimal $ECS$ strategy in five cases when **Condition 4.2** is satisfied.

Overall, **Lemma 4.4** provides the constraints of contract parameters $(w, \eta_i)$ to meet **Condition 4.2**. Next, given **Condition 4.1** and **4.2**, the two $LSCs$' profit increments are decided in **Lemma 4.5**. (Appendix H)

***Lemma 4.5*** the *two $LSCs'$ profit increments are:*

*For cases 1 and 2:*

$$\begin{cases}\Delta\pi_1 = \alpha_1[\Delta S_1^e(q^e,p) + \Delta S_1(0,p)] + \eta_2 \Delta S_2^e(q^e,p) - (\alpha_1 - g_1)b_1 p \\ \Delta\pi_2 = (\alpha_2 - \eta_2)\Delta S_2^e(q^e,p) + \alpha_2 \Delta S_2(0,p) - (\alpha_2 - g_2)b_2 p\end{cases}$$

*For cases 4 and 5:*

$$\begin{cases}\Delta\pi_1 = (\alpha_1 - \eta_1)\Delta S_1^e(q^e,p) + \alpha_1 \Delta S_1(0,p) - (\alpha_1 - g_1)b_1 p \\ \Delta\pi_2 = \alpha_2[\Delta S_2^e(q^e,p) + \Delta S_2(0,p)] + \eta_1 \Delta S_1^e(q^e,p) - (\alpha_2 - g_2)b_2 p\end{cases}$$

*For case 3:* $\Delta\pi_1 = \Delta\pi_2 = 0$

Where $\Delta S_1^e(q^e,p) = -\left[\int_{n_1-a_1+b_1p}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1 + q^e Z_1(n_1 - q^e - a_1 + b_1 p)\right]$ , $\Delta S_1(0,p) = -\left[\int_{n_1-a_1}^{n_1-a_1+b_1p} Z_1(d_1)dd_1 - b_1 p\right]$ , $\Delta S_2^e(q^e,p) = -\left[\int_{n_2-a_2+b_2p}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 - q^e Z_2(n_2 + q^e - a_2 + b_2 p)\right]$ and $\Delta S_2(0,p) = -\left[\int_{n_2-a_2}^{n_2-a_2+b_2p} Z_2(d_2)dd_2 - b_2 p\right]$

Given **Condition 4.1** and **Condition 4.2**, **Lemma 4.5** provides the two $LSCs'$ profit increments. To analyse how $ECS$ activity and the imposition of $CT$ affects the two $LSCs'$ profit increment, the two $LSCs'$ profit increments can be divided into three parts:

- The profit made from the satisfied demand during $ECS$ $\Delta S_i^e(q^e,p)$, given $p$. Note that the empty container demander loses the buy-back price $\eta_{demander}$ paid by the supplier for each satisfied empty container.
- The profit generated from the satisfied demand $\Delta S_i(0,p)$ due to the imposition of the constant $CT$ when $ECS$ is not considered.
- The $LSCs'$ profit loss because of empty container demand decline between the scenario with and without the imposition of the government $CT$, $[-(\alpha_i - g_i)b_i p]$.

Bearing in mind that aside from those three components, the empty container supplier could save the buy-back price from each empty container that the demander satisfies, $\eta_{demander}\Delta S_{demander}^e(q^e,p)$. Next, I will comprehensively analyse each part to investigate how the $ECS$ activity and the government $CT$ affect the two $LSCs'$ profit increment when **Condition 4.1** and **4.2** are satisfied.

- *The profit generated from the satisfied demand during ECS activity $\Delta S_i^e(q^e,p)$, given p.*

Figures 4.3, 4.4, 4.5 and 4.6 are drawn to investigate how the two $LSCs'$ profit increment is affected by the $ECS$ activity. Firstly, Figures 4.3 and 4.4 demonstrate how the $ECS$ strategy affects the demander's ($LSC$ 2) and supplier's ($LSC$ 1) profit increment in cases 1 and 2, respectively. It is not difficult to see that the satisfied demand $\Delta S_i^e(q^e,p)$ of both the supplier and the demander increases as long as the empty containers are shared between the two $LSCs$. More specifically, for the demander ($LSC$ 2, Figure 4.3), with more and more empty containers being shared with $LSC$ 2, the marginal increase of $\Delta S_2^e(q^e,p)$ falls, which means the impact of $ECS$ activity on $LSC$ 2's profit increment is getting weaker. Conversely, for the supplier ($LSC$ 1, Figure 4.4), with more and more empty containers being shared from $LSC$ 1 to $LSC$ 2, $LSC$ 1's marginal increase of $\Delta S_1^e(q^e,p)$ grows, which means the impact of $ECS$ activity on $LSC$ 1's profit increment is becoming more significant.

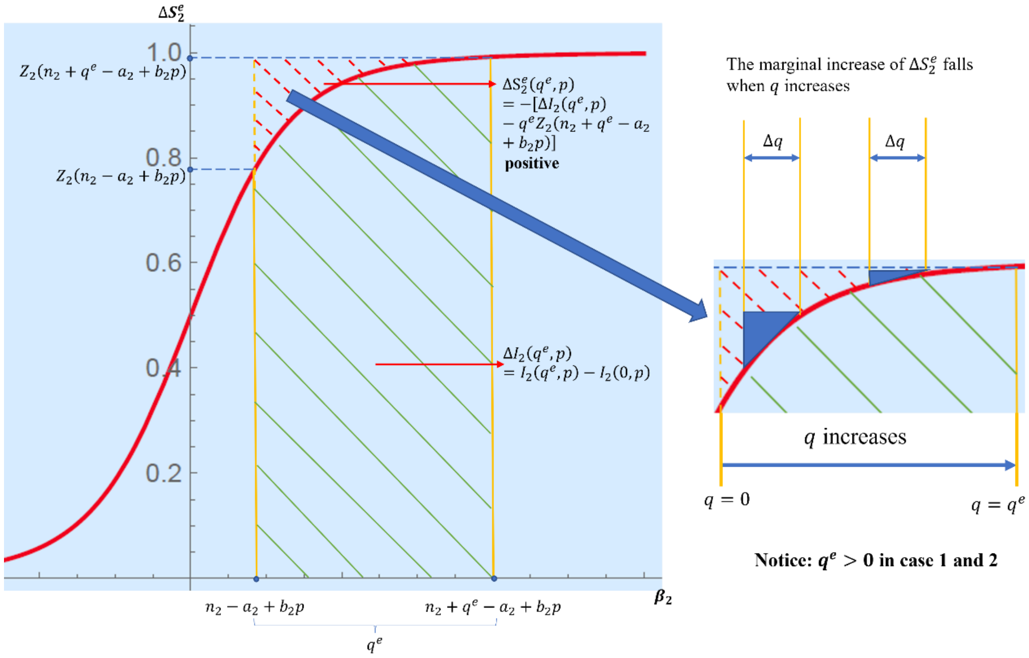


**Figure 4.3** The impact of $ECS$ activity on the demander's satisfied demand change $\Delta S_2(0,p)$ in case 1 and 2.

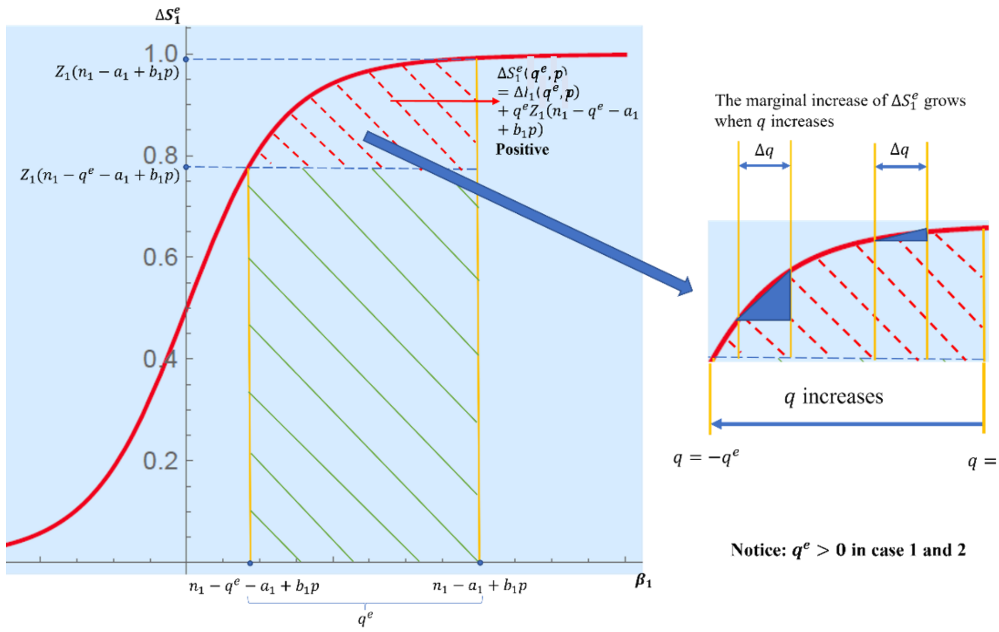


**Figure 4.4** The impact of $ECS$ activity on the supplier's satisfied demand change $\Delta S_1(0,p)$ in cases 1 and 2.

Figure 4.5 and 4.6 illustrate how the $ECS$ activity affects the profit increment of the demander ($LSC$ 1) and supplier ($LSC$ 2) in cases 4 and 5, respectively. In these cases, the two $LSCs$' satisfied demand $\Delta S_i^e(q^e,p)$ also always rise, as long as they share empty containers. For the demander ($LSC$ 1, Figure 4.5), the marginal increase of $\Delta S_1^e(q^e,p)$ declines when more and more empty containers are shared from $LSC$ 2 to $LSC$ 1. It means that the impact of $ECS$ activities on $\Delta S_1^e(q^e,p)$ reduces gradually. For the supplier ($LSC$ 2, Figure 4.6), the marginal increase of $\Delta S_2^e(q^e,p)$ rises when more and more empty containers are shared with $LSC$ 1. This stands for the impact of $ECS$ activities on $\Delta S_2^e(q^e, p)$ gets stronger. The conclusion is made in **Corollary 4.1**.

***Corollary 4.1*** *Given p, the ECS positively affects the two LSCs. They benefit from the activity. Also, when more containers are shared, the impact gets weaker on the demander, but becomes stronger on the supplier.*

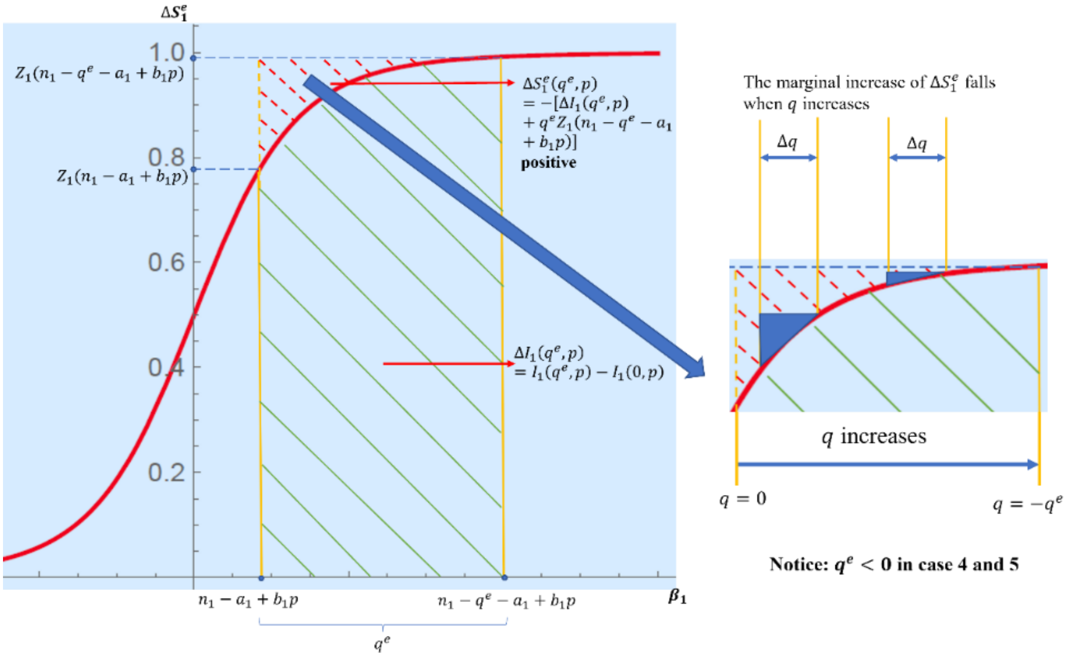


**Figure 4.5** The impact of $ECS$ on the demander's satisfied demand change $\Delta S_1(0, p)$ in case 4 and 5.

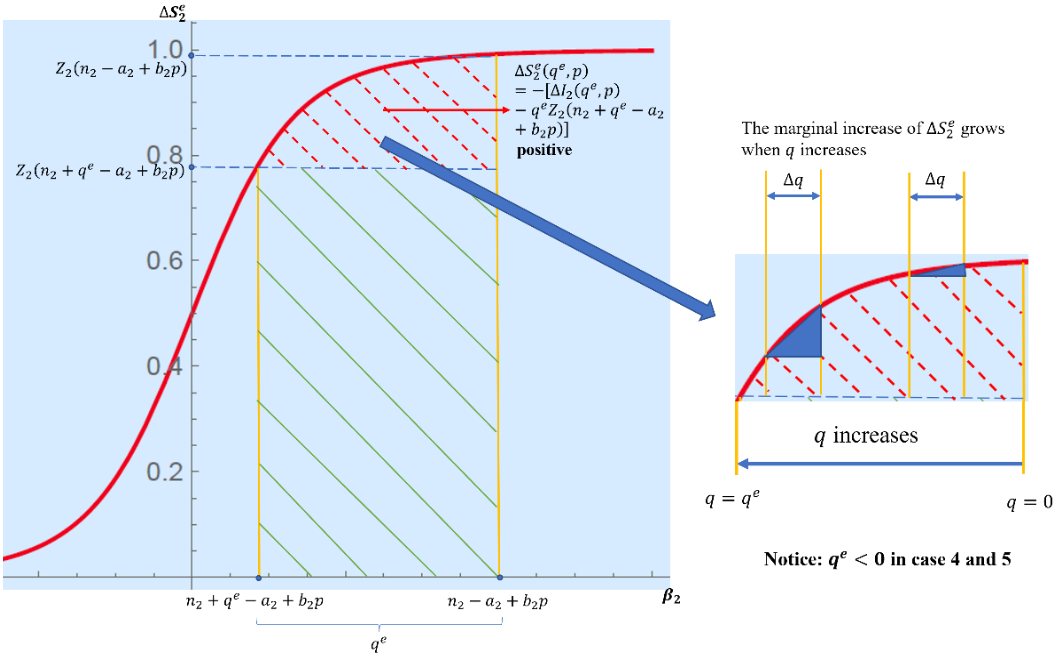


**Figure 4.6** The impact of $ECS$ activity on the supplier's satisfied demand change $\Delta S_2(0, p)$ in case 4 and 5.

Next, the impact of the government $CT$ on the two $LSCs$' profit increment is analysed.

- *The profit generated from the satisfied demand $\Delta S_i(0, p)$ due to the constant imposition of the government $CT$ $p$ when $ECS$ is not considered.*

Figure 4.7 is depicted to illustrate how the imposition of the government $CT$ influences the two $LSCs$'

satisfied demand $\Delta S_i(0,p)$ when the $ECS$ activity is not considered. For both $LSCs$ in cases 1, 2, 4 and 5, when the government imposes the $CT$, then $\Delta S_i(0,p)$ would be positive.

- *The $LSCs$' profit loss because of the decline in empty container demand between the scenario with and without the imposition of the government $CT$, $[-(\alpha_i - g_i)b_i p]$.*

However, the $LSC$ may still suffer the loss because the demand drops when the $CT$ is imposed. It should be noted that there is no actual satisfied demand $\Delta S_i(0,p)$ generated during the imposition of the government $CT$ as the demand fall when the $CT$ is imposed. The profit obtained from satisfied demand is transformed by the goodwill penalty saving. There are two aspects that the imposition of the government $CT$ brings to $LSCs'$ profit change. The first part is the profit loss caused by the decline in demand $-\alpha_i \Delta I_i(0,p^e)$ due to the imposition of $CT$. The other one is the amount of the goodwill penalty saving $+g_i b_i p^e$ due to the demand drop. In other words, the imposition of the $CT$ on $LSCs$ leads to two opposite impacts. So, to be consistent with $\Delta S_i^e(q^e,p)$, I transform $g_i b_i p^e - \alpha_i \Delta I_i(0,p^e)$ to $\alpha_i \Delta S_i(0,p) - (\alpha_i - g_i)b_i p$. The details of transformation can be found in Appendix H. **Corollary 4.2** concludes the impact of the government $CT$ on $\Delta S_i(0,p)$.

**Corollary 4.2** *The imposition of the government CT could make $\Delta S_i(0,p)$ positive, but it causes a loss to the $LSCs$ because the demand declines when CT is imposed.*

Thus, although $\Delta S_i(0,p)$ is positive when $p$ is imposed, the $LSCs$ may still suffer a loss. Overall, the government $CT$ could still potentially negatively affect the profit increment of the two $LSCs$'.

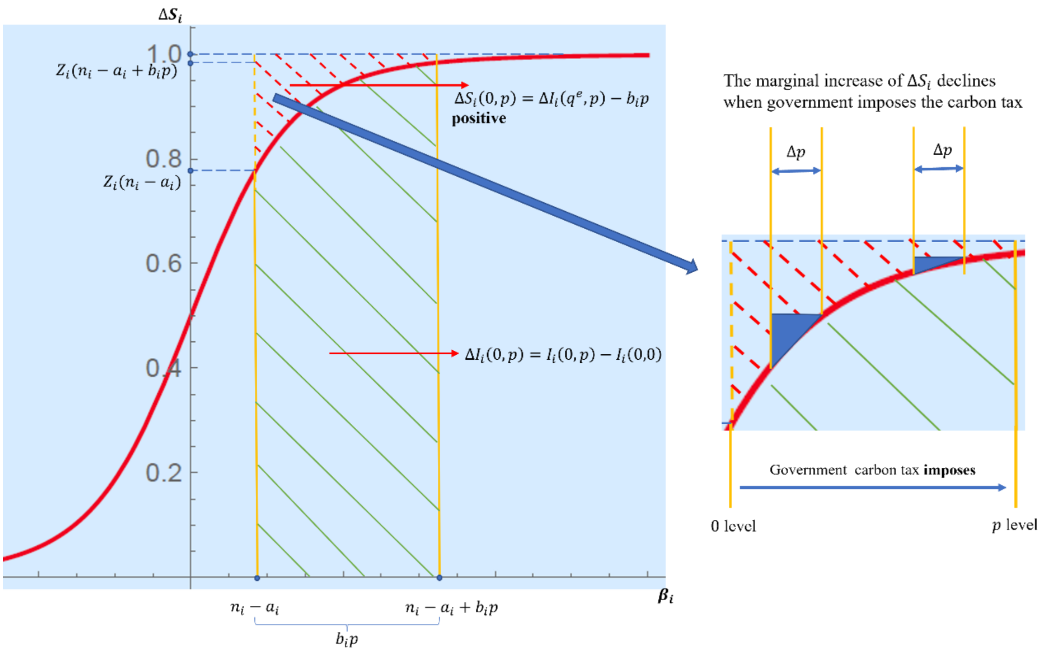


**Figure 4.7** The impact of the government $CT$ rate on the two $LSCs$' satisfied demand change $\Delta S_i(0,p)$.

Case 3 is exceptional because no empty containers are shared between the two $LSCs$, so both $LSCs$' profit increment is 0. Thus, the $CT$ rate does not affect the profit increment of either $LSC$.

In summary, given **Condition 4.1** and **4.2**, **Lemma 4.5** analyses how $ECS$ and the imposition of the government $CT$ affects the profit increment of the two $LSCs$ in the decentralised model. Moreover, according to **Corollary 4.2**, both $LSCs$ might have the chance to be allocated a negative profit under the system coordination because the imposition of the government $CT$ could cause the $LSC$ to suffer a lossTherefore, to meet **Condition 4.3**, **Lemma 4.6** is developed to ensure the profit increment nonnegativity of the two $LSCs$, (Appendix H)

***Lemma 4.6*** *Given* ***Condition 4.1*** *and* ***4.2****, and considering* ***Lemma 4.5****, in order to meet* ***Condition 4.3****, the $BBC$ parameter $\eta_1$ and $\eta_2$ should be constrained within:*

*For cases 1 and 2:*

$$\eta_2' = \frac{(\alpha_1 - g_1)b_1 p - \alpha_1[\Delta S_1^e(q^e, p) + \Delta S_1(0, p)]}{\Delta S_2^e(q^e, p)} \leq \eta_2 \leq \frac{\alpha_2[\Delta S_2^e(q^e, p) + \Delta S_2(0, p)] - (\alpha_2 - g_2)b_2 p}{\Delta S_2^e(q^e, p)} = \eta_2''$$

*For cases 4 and 5:*

$$\eta_1' = \frac{(\alpha_2 - g_2)b_2 p - \alpha_2[\Delta S_2^e(q^e, p) + \Delta S_2(0, p)]}{\Delta S_1^e(q^e, p)} \leq \eta_1 \leq \frac{\alpha_1[\Delta S_1^e(q^e, p) + \Delta S_1(0, p)] - (\alpha_1 - g_1)b_1 p}{\Delta S_1^e(q^e, p)} = \eta_1''$$

To meet **Condition 4.3**, **Lemma 4.6** partly guarantees the nonnegativity of the profit increment of the two $LSCs$ profit increment. It provides a much stricter buy-back price ($\eta_1$ and $\eta_2$) range than that shown in **Condition 4.1** because it further considers the two $LSCs'$ profit increment nonnegativity instead of only considering the two $LSCs'$ profit function concavity in the decentralised decision-making model. $\eta_1'$ and $\eta_1''$, $\eta_2'$ and $\eta_2''$ are denoted as the lower and upper boundaries of $\eta_1$ and $\eta_2$, respectively. However, **Lemma 4.6** cannot perfectly guarantee the nonnegativity of the profit increment of the two $LSCs$. This is because the $LSCs'$ profit increment could still be negative if the loss caused by the imposition of the $CT$ is great enough, even if all profit brought from the sharing activity cannot cover it. Finally, **Theorem 4.2** proves that the system can be conditionally coordinated, given the constant imposition of the government $CT$ $p$ .

***Theorem 4.2*** *Given constant p, the system can be conditionally coordinated only if:*

1) ***Condition 4.1*** *is satisfied.*
2) *For $\eta_1$, $\eta_2$ and w,* ***Lemma 4.4*** *is satisfied so that* ***Condition 4.2*** *is satisfied.*
3) *$\eta_1$ and $\eta_2$ are constrained following in* ***Lemma 4.6*** *so that* ***Condition 4.3*** *is satisfied.*

Given the constant imposition of the government $CT$, **Theorem 4.2** reveals the conditions that ensure the system coordination. **Condition 4.1** guarantees that the two $LSCs'$ profit functions in the decentralised decision-making model are concave in $q$ between $-n_2$ and $n_1$. Then, **Lemma 4.4** makes sure that the number of shared empty containers determined in the centralised decision-making model equals those in the decentralised decision-making model. In doing so, **Condition 4.2** is achieved. Lastly, **Lemma 4.6** provides a feasible range for the buy-back price to ensure that both $LSCs$ can earn the non-negative profit increment, which means **Condition 4.3** is also reached. Overall, the system is coordinated if the three conditions in **Theorem 4.2** are all satisfied. Under the system coordination, given the constant imposition of the government $CT$ , the system profit in the centralised decision-making model and the profits of the two $LSCs$ are all maximised. Also, by appropriately making a $BBC$, the way that the two $LSCs$ cooperate in the decentralised decision-making model, which should be according to the contract, tends to be optimised in such a way that they ideally cooperate in the centralised mode. This means that the sum of the profit that two $LSCs$ get in the decentralised decision-making model equals the maximum system's profit in the centralised decision-making model, and both $LSCs$ get a non-negative profit increment during the $ECS$ activity, given the constant imposition of the $CT$. In the next section, a numerical case is conducted to examine the conclusions and results obtained in this chapter.

### 4.4. Numerical example

Table 4.3 gives the parameters, random and state variables used in the numerical example. I assume that $Y_i$ and $\xi_i$ follow the Normal distribution, and $LSC$ 2 is experiencing more demand pressure for empty containers than $LSC$ 1. Also, I allow that the government $CT$ rate is 100.

**Table 4.3** The assumed parameters, random and state variables in the numerical example

| ***Parameters/variables*** | ***LSC 1*** | ***LSC 2*** |
|---|---|---|
| $\boldsymbol{n_i}$ | 800 | 700 |
| $\boldsymbol{r_i}$ | 900 | 800 |
| $\boldsymbol{h_i}$ | 300 | 300 |
| $\boldsymbol{g_i}$ | 500 | 500 |
| $\boldsymbol{\alpha_i}$ | 1,700 | 1,600 |
| $\boldsymbol{a_i}$ | 850 | 600 |
| $\boldsymbol{b_i}$ | 10 | 10 |
| $\boldsymbol{c_t}$ | 50 | |
| $\boldsymbol{p}$ | 100 | |
| $\boldsymbol{\xi_i}$ | $Normal{\sim}(1{,}400, 600)$ | $Normal{\sim}(2{,}200, 1{,}300)$ |
| $\boldsymbol{Y_i}$ | $Normal{\sim}(350, 200)$ | $Normal{\sim}(400, 250)$ |
| $\boldsymbol{\xi_i - Y_i}$ | $Normal{\sim}(1{,}050, 800)$ | $Normal{\sim}(1{,}800, 1{,}550)$ |

Based on **Lemma 4.1**, given $p = 100$, Figure 4.8 shows that the system profit function in the centralised decision-making model is strictly concave in $q$ as well as the optimal sharing quantities $q^*$ is $+78$. It means that $LSC$ 2 (the empty container demander) should transfer 78 empty containers from $LSC$ 1 (the supplier). Therefore, this situation belongs to case 2 shown in **Theorem 4.1**. Also, the system profit $\prod(q^*, p)$ is $739{,}115$.

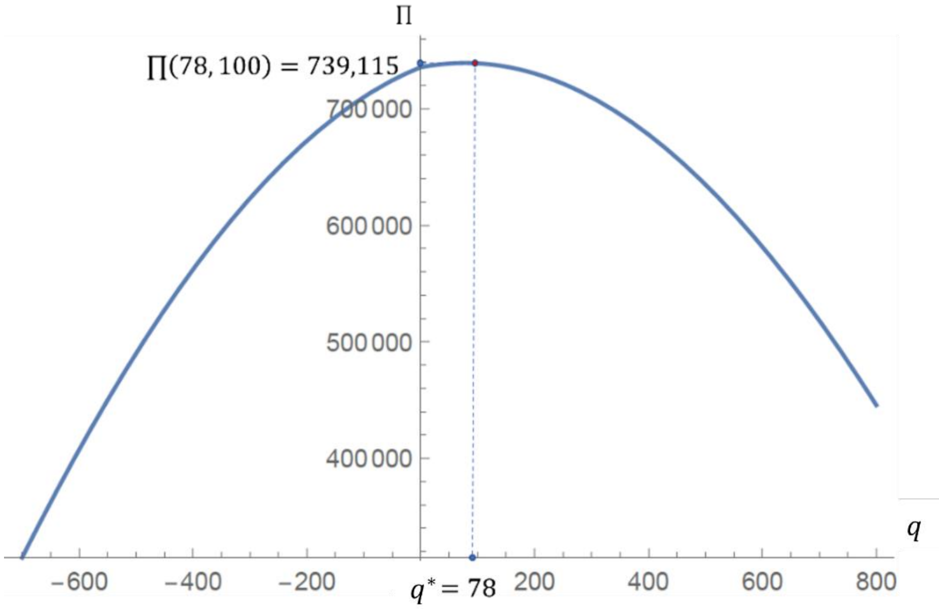


**Figure 4.8** The optimal $ECS$ strategy in the centralised decision-making model

Next, given $p = 100$, when $q^* = 78$, according to the equation 4.9, the system profit increment in the centralised decision-making model is:

$$\Delta\prod(q = 78, p = 100)$$
$$= -1{,}700\left[\int_{800-850}^{800-78-850+10*100} Z_1(d_1)dd_1 + 78 - 10 * 100\right]$$
$$- 1{,}600\left[\int_{700-600}^{700+78-600+10*100} Z_2(d_2)dd_2 - 78 - 10 * 100\right] - 10(1{,}700 - 500) * 100$$
$$- 10(1{,}600 - 500) * 100 + 78(300 - 300) - 50 * \ |78| = 237{,}220$$

Now, to achieve the system coordination, **Condition 4.2** should be met to let $q^e(p = 100) = q^*(p = 100)$. In this case, if buy-back price $\eta_2$ and the wholesale price $w$ are $1142.7$ and $1142$, respectively, then $q^e(p = 100) = q^*(p = 100)$. Figure 4.9 clearly shows the two $LSCs$' maximum profit and the maximum system profit, when $q^e = q^* = 100$.

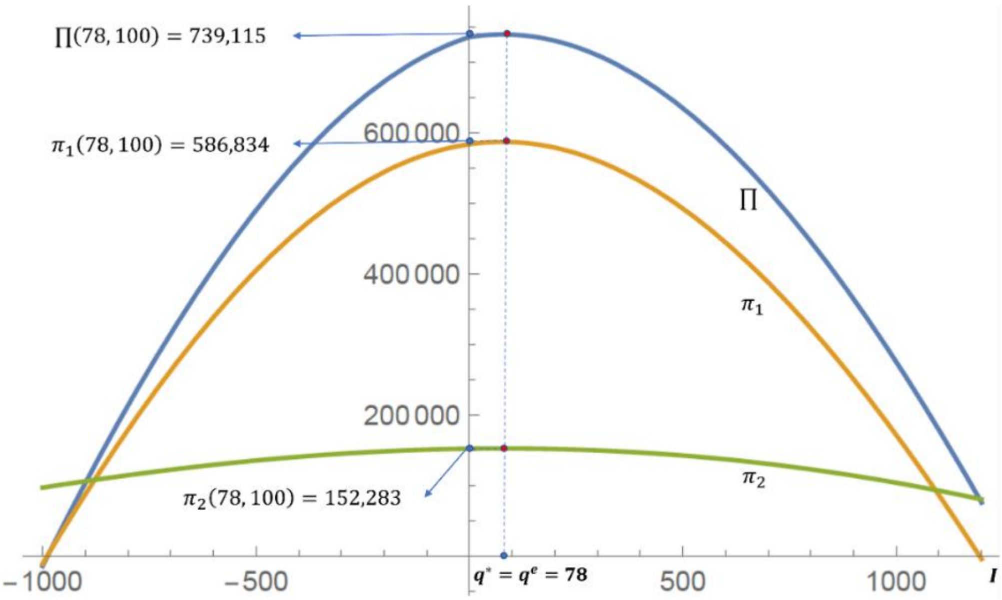


**Figure 4.9** The value of $\prod$, $\pi_1$ and $\pi_2$ when $\eta_2 = 1{,}142.7$ and $w = 1{,}142$

If $\eta_2$ and $w$ are 1142.7 and 1142, then the two $LSCs'$ profit in the decentralised decision-making model and the system profit in the centralised model are maximised. These are $\prod = 739{,}115$; $\pi_1 = 586{,}834$; $\pi_2 = 152{,}283$, respectively (noted that $\prod = \pi_1 + \pi_2$). Consequently, **Condition 4.2** is satisfied. Next, to meet **Condition 4.3,** i.e., examine whether the two $LSCs'$ profit increments are nonnegative. According to **Lemma 4.5**, the impact of the imposition of the $CT$ on the two $LSCs'$ profit increment in the decentralised decision-making model should be determined.

$$
\begin{aligned}
&-\alpha_1 \Delta S_1(0,p) - (\alpha_1 - g_1) b_1 p \\
&\qquad = -1{,}700 * \left[ \int_{800-850}^{800-850+10*100} U_1(d_1) dd_1 - 10 * 100 \right] - (1{,}700 - 500) * 10 * 100 \\
&\qquad = 90{,}884
\end{aligned}
$$

$$
\begin{aligned}
&-\alpha_2 \Delta S_2(0,p) - (\alpha_2 - g_2) b_2 p \\
&\qquad = -1{,}600 * \left[ \int_{700-600}^{700-600+10*100} U_2(d_2) dd_2 - 10 * 100 \right] - (1{,}600 - 500) * 10 * 100 \\
&\qquad = 142{,}674
\end{aligned}
$$

Interestingly, in this case, the government's imposition of the $CT$ brings "profit" to both $LSCs$, which means the two $LSCs$ are able to gain the non-negative profit increment when $\eta_2 = 1142.7$. However, notice that the imposition of the $CT$ does not actually bring "profit" to both $LSCs$. In fact, it means that the goodwill penalty saving is greater than the profit loss caused by the decline in demand because of the imposition of the $CT$ . Finally, I can obtain $\Delta\pi_1$ and $\Delta\pi_2$ are:

$$\Delta\pi_1(q = 78, p = 100, \eta_2 = 1{,}142.7, w = 1{,}142)$$
$$= -1{,}700\left[\int_{800-850+10*100}^{800-78-850+10*100} Z_1(d_1)dd_1 + 78Z_1(800 - 78 - 850 + 10*100)\right]$$
$$- 1{,}142.7\left[\int_{700-600+10*100}^{700+78-600+10*100} Z_2(d_2)dd_2 - 78Z_2(700 + 78 - 600 + 10*100)\right]$$
$$- 1{,}700\left[\int_{800-850}^{800-850+10*100} Z_1(d_1)dd_1 - 10*100\right] - 10(1{,}700 - 500)*100 = 94{,}219$$

$$\Delta\pi_2(q = 78, p = 100, \eta_2 = 1{,}142.7, w = 1{,}142)$$
$$= -(1{,}600 - 1{,}142.7)\left[\int_{700-600+10*100}^{700+78-600+10*100} Z_2(d_2)dd_2 - 78Z_2(700 + 78 - 600 + 10*100)\right]$$
$$- 1{,}600\left[\int_{700-600}^{700-600+10*100} Z_2(d_2)dd_2 - 10*100\right] - 10(1{,}600 - 500)*100$$
$$= 143{,}001$$

Therefore, under the system coordination, the two $LSCs$' profit increments are both non-negative and $\Delta\prod = \Delta\pi_1 + \Delta\pi_2 = 237{,}220$ ($LSC$ 1 and 2's profit increment accounts for 39.72% and 60.28% of the total system profit increment, respectively). Overall, in this case, to coordinate the system, this can entice the two $LSCs$ to choose the $BBC$ where the buy-back price $\eta_2$ is 1,142.7, and the wholesale price $w$ is 1,142 so that the centralised decision-making model can be optimised. Also, under the system coordination, given the constant imposition of the $CT$, the two $LSCs$ behave in such a way that the total system profit is maximised and both $LSCs$ get a non-negative profit increment from the system profit increment. This means that both can accept the contract. In the next section, I will conclude the main findings in this chapter and compare the main results between this chapter and the paper published by Xie et al. (2017).

## 4.5. Conclusion

In this chapter, I mainly investigated how a constant government $CT$ affects two $LSCs$' $ECS$ activity, when the two $LSCs$ apply a $BBC$ as the agreement to decide the number of shared empty containers. Furthermore, I explored how to coordinate the container sharing system when a constant $CT$ is imposed on the two $LSCs$. The conclusion of this chapter can be summarised as follows:

1. In a centralised decision-making mode where two $LSCs$ completely work with each other, and when the government imposes a constant $CT$ on the two $LSCs$, the two $LSCs$' container sharing system can be optimized. This means the whole system profit can be maximised.
2. When a constant $CT$ is imposed, in a decentralised decision-making mode where two $LSCs$ make decisions on $ECS$ strategy independently but bound to a pre-determined $BBC$, this thesis have proved that the

strategies of the two $LSCs$ could reach a Nash equilibrium. In this situation, no $LSC$ can obtain a better payoff without diminishing the other's profit.

3. When the government imposes a constant $CT$ on $LSCs'$ container shipping transportation, the container sharing system can be conditionally coordinated if the two $LSCs$ adopt an appropriate $BBC$ as the contract to decide the number of shared empty containers.

4. Under the system coordination, the optimal number of shared empty containers in the centralised decision-making model equals the equilibrium number of that in the decentralised decision-making model. Also, no $LSC$ can obtain a better payoff without reducing the other's profit, and both $LSCs$ act in a way which can maximise the total system profit.

5. When a constant $CT$ is imposed on the two $LSCs$, the two $LSCs'$profit increment under the system coordination in the decentralised decision-making model includes three parts: (1) the loss due to decline in demand during the imposition of the $CT$; (2) the profit due to $ECS$ activity; (3) the $LSCs'$ goodwill penalty saving due to decrease in demand. In addition to these three parts, the empty container supplier should pay a buy-back price to the demander for every unsatisfied empty container.

6. Given the $CT$ impact, the two $LSCs'$ profit increments during the $ECS$ activity are positive. However, with more and more empty containers being shared, the empty container demander's profit increment gradually falls until the shared number reaches the optimal $ECS$ number. Conversely, the profit increment of the empty container supplier increasingly rises until the shared number reaches the optimal $ECS$ number.

7. When compared to the situation where there is no imposition of $CT$ on the two $LSCs'$, and not taking into consideration any $ECS$ activity, if there is a constant imposition of the $CT$ on the two $LSCs$, their profit may fall because of a leftover inventory increase. However, the two $LSCs'$ may save some of the goodwill penalty due to a decline in the demand for empty container. Overall, the government's imposition of the $CT$ leads to two opposite effects on the profits of the $LSCs$. Therefore, the imposition of the $CT$ could cause the two $LSCs$ to suffer a loss. However, interestingly, the $CT$ could also generate "profit" for the $LSCs$.

This chapter adopts a similar method and analysis to that applied in the research conducted by Xie et al. (2017). However, I introduced a $CT$ as a constant to investigate whether it would affect the $ECS$ system and the system coordination. It has been proved that the system can still be conditionally coordinated even if the $CT$ is introduced, but the two $LSCs'$ profit could be either negatively or positively affected. Table 4.4 clearly shows the comparison in terms of the applied method, analysis and conclusions between this chapter and the paper of Xie et al. (2017).

**Table 4.4** The research difference between Chapter 4 and the paper of Xie et al. (2017).

| ***Applied method, analysis and conclusion*** | ***Xie et al. (2017)*** | ***Chapter 4*** |
|---|---|---|
| Model applied | Intermodal system | Liner shipping system |
| $CT$ introduced? | X | √ |
| $CT$ introduced as? | X | Parameters |
| Government interest involved? | X | X |
| How many players in the model? | One liner firm; one rail firm | Two $LSCs$ |
| Contract applied | $BBC$ | $BBC$ |
| Centralised model involved? | √ | √ |
| Centralised model optimised? | √ | √ |
| Decentralised model involved? | √ | √ |
| $ESC$ strategy's nash equilibrium reached in decentralised model? | √ Pareto optimality | √ Pareto optimality |
| Equilibrium $CT$ reached? | Not applicable | X |
| System coordinated? | √ | √ |
| Two $LSC'$ profit increment analysis? | X | √ |
| The analysis of $CT$ rate impact on system coordination? | X | √ |

Although Chapter 4 developed the research of Xie et al. (2017) by introducing a $CT$ into the two $LSCs'$ $ECS$ system, drawbacks remain. For example, a $CT$ cannot be a constant value, and it is set by government, and fluctuates all the time. Also, as with the paper of Xie et al. (2017), the $BBC$ is still adopted in this chapter, whereas the other contracts have not been considered. So, to fill these research gap, Chapter 5 is developed.

# Chapter 5 How does Carbon Tax Affect Container-sharing and Coordination by Using a Revenue-sharing Contract When Carbon Tax is Considered as Exogenous Variable?

## 5.1. Introduction

In this chapter, I continue to consider the impact of the Carbon Tax ($CT$) policy on Empty Container Sharing ($ECS$) in a shipping alliance, but under a Revenue-sharing Contract ($RSC$) rather than a Buy-back Contract ($BBC$). However, in this chapter, the government $CT$ is considered as a decision variable instead of constant parameter. First, similar to the approach in Chapter 4, I identify the optimal $ECS$ strategy under the perfect collaboration between two Liner Shipping Carriers ($LSCs$) (a centralised decision-making model) taking into account an endogenous $CT$ rate imposed by the government. Secondly, a practical model will be considered where two $LSCs$ make decisions independently but bound to a pre-determined $RSC$ (a decentralised decision-making model). Unlike the result shown in the decentralised decision-making model in Chapter 4, the solution for the decentralised model in this chapter includes the Nash equilibrium $ECS$ strategy between the two $LSCs$ as well as the Nash equilibrium $CT$ rate. Finally, this chapter determines the conditions of the parameters in $RSC$ under which the coordination between the two $LSCs$ can be maintained. It is worth mentioning that this chapter only examines whether, and how an endogenous $CT$ rate affects $ECS$ activity between two the $LSCs$ and the system coordination, instead of helping the government design a carbon taxation scheme. This is because I only focus on a container sharing system and the government's social welfare model is not involved. Thus, the $CT$ rate in this chapter is considered as an endogenous variable, and it can be used as a suggestion to government to make an appropriate $CT$ policy taking into consideration the interests of the $LSCs$. In light of the above statements, this chapter has two primary research objectives: (1) to investigate how $ECS$ activity between two $LSCs$ in a port hinterland may be affected by a $CT$ rate under a $RSC$, and simultaneously explore the conditions of container sharing supply chain coordination; (2) to examine how much profits those $LSCs$ can obtain in a coordinated $ECS$ system under $RSC$ when the equilibrium $CT$ rate is imposed.

The remainder of the chapter is structured as follows. In section 5.2, I formulate the centralised and decentralised decision-making model with the relevant analysis, where a $RSC$ is adopted as the contract binding two $LSCs$ in a decentralised decision-making model. Section 5.3 achieves the system coordination under the impact of the imposition of a $CT$. Section 5.4 presents a numerical case study. Finally, in section 5.5

there will be a conclusion for this chapter.

### 5.2. The model

In this section, I will present both centralised and decentralised decision-making models. Based on the two models, I will firstly investigate the optimisation of the centralised $ECS$ model, when the $CT$ rate is seen as an endogenous variable. Secondly, I will find the Nash equilibrium of $ECS$ sharing strategy between the two $LSCs$ in the decentralised decision-making model, when a $RSC$ is adopted.

#### 5.2.1. The centralised decision-making model

As with the assumption in Chapter 4, in the centralised decision-making model of this chapter, I assume that a virtual planner makes decisions for the two $LSCs$ as well as for the government. Notice that the central planner does not make the $CT$ policy for the government, but rather only calculates a theoretical $CT$ rate which can maximise the total system profit (total profit of the two $LSCs$' profit together). This could be taken as a suggestion to the government regarding the $CT$ decision making from the perspective of international shipping industry.

By solving the centralised decision-making model, I obtain the optimal $ECS$ strategy $q^*$ and optimal $CT$ rate $p^*$ that maximises the system profits. It is worth mentioning that the optimal $CT$ rate $p^*$ is preferred by $LSCs$ and may be different from the government's preferred $CT$ rate. This also is because the government's decision-making target may be maximising social benefits rather than the $LSCs$' profits, and I temporarily do not consider government total social welfare function in this chapter.

Similar to the definition for $S_i$, $I_1$ and $L_i$ made in equation 4.1 to 4.6, I denote $S_i$ as $LSC\ i$ 's ($i = 1, 2$) satisfied demand, $I_i$ as leftover inventory after meeting demands and $L_i$ as unsatisfied demands, their formulations are given as follows, respectively:

$$S_1(q,p) = \boldsymbol{E}\min[X_1, n_1 - q + Y_1] = \boldsymbol{E}\min[a_1 - b_1p + \xi_1, n_1 - q + Y_1] \quad \mathbf{5.1}$$

$$I_1(q,p) = (n_1 - q + Y_1 - X_1)^+ = (n_1 - q + Y_1 - a_1 + b_1p - \xi_1)^+ \quad \mathbf{5.2}$$

$$L_1(q,p) = (X_1 - n_1 + q - Y_1)^+ = (a_1 - b_1p + \xi_1 - n_1 + q - Y_1)^+ \quad \mathbf{5.3}$$

$$S_2(q,p) = \boldsymbol{E}\min[X_2, n_2 + q + Y_2] = \boldsymbol{E}\min[a_2 - b_2p + \xi_2, n_2 + q + Y_2] \quad \mathbf{5.4}$$

$$I_2(q,p) = (n_2 + q + Y_2 - X_2)^+ = (n_2 + q + Y_2 - a_2 + b_2p - \xi_2)^+ \quad \mathbf{5.5}$$

$$L_2(q,p) = (X_2 - n_2 - q - Y_2)^+ = (a_2 - b_2p + \xi_2 - n_2 - q - Y_2)^+ \quad \mathbf{5.6}$$

Since the transportation cost is $c_t|q|$, then $\prod(q,p)$ in the centralised decision-making model is:

$$\prod(q,p) = r_1S_1(q,p) - h_1\boldsymbol{E}I_1(q,p) - g_1\boldsymbol{E}L_1(q,p) + r_2S_2(q,p) - h_2\boldsymbol{E}I_2(q,p) - g_2\boldsymbol{E}L_2(q,p) - c_t|q| \quad \mathbf{5.7}$$

According to Snyder and Shen (2011, Page 281), equation 5.7 can be rewritten as equation 5.8:

$$\prod(q,p) = \alpha_1[\beta_1 - \Phi_1(\beta_1)] + \alpha_2[\beta_2 - \Phi_2(\beta_2)] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) \\ + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p) - c_t|q| \qquad \textbf{5.8}$$

Where $\Phi_i(\cdot)$ is complementary loss function. $z_i(\cdot)$ and $Z_i(\cdot)$ are the $pdf$ and $CDF$ for $\xi_i - Y_i$, respectively, $\left[\frac{d\Phi_i(.)}{d(.)} = Z_i(.); i = 1,2\right]$. For the calculation details, please refer to Appendix A. In the same way as in Chapter 4, I define $\alpha_i = r_i + h_i + g_i$ as two $LSCs'$ all-in-revenue (Xie et al., 2017). I denote $\beta_1 = n_1 - q - a_1 + b_1 p$ and $\beta_2 = n_2 + q - a_2 + b_2 p$, respectively. Next, **Lemma 5.1** is developed to analyse the concavity of the system profit function $\prod(q,p)$ in the centralised decision-making model. (Appendix I)

***Lemma 5.1*** *$\prod(q,p)$ is jointly concave in $p$ and $q$. When $p$ and $q$ are not constrained, the optimal ECS strategy and the optimal CT rate are:*

| | | |
|---|---|---|
| $\boldsymbol{q > 0}$ | $\dot{p} = [Z_1^{-1}(\omega_1) + Z_2^{-1}(\omega_2) + a_1 + a_2 - n_1 - n_2]/(b_1 + b_2)$ <br> $\dot{q} = [b_1 Z_2^{-1}(\omega_2) - b_2 Z_1^{-1}(\omega_1) + b_2(n_1 - a_1) - b_1(n_2 - a_2)]/(b_1 + b_2)$ | **5.9** |
| $\boldsymbol{q < 0}$ | $\ddot{p} = [Z_1^{-1}(\omega_1') + Z_2^{-1}(\omega_2') + a_1 + a_2 - n_1 - n_2]/(b_1 + b_2)$ <br> $\ddot{q} = [b_1 Z_2^{-1}(\omega_2') - b_2 Z_1^{-1}(\omega_1') + b_2(n_1 - a_1) + b_1(a_2 - n_2)]//(b_1 + b_2)$ | **5.10** |

$$\omega_1 = [b_1 g_1 + b_2(r_1 + g_1 - r_2 + c_t)]/[(b_1 + b_2)\alpha_1]; \omega_2 = [b_1(r_2 + g_2 - r_1 - c_t) + b_2 g_2]/[(b_1 + b_2)\alpha_2]$$

$$\omega_1' = [b_1 g_1 + b_2(r_1 + g_1 - r_2 - c_t)]/[(b_1 + b_2)\alpha_1]; \omega_2' = [b_1(r_2 + g_2 - r_1 + c_t) + b_2 g_2]/[(b_1 + b_2)\alpha_2]$$

**Lemma 5.1** indicates that there exists a unique optimal solution for $q$ and $p$ when $p$ and $q$ are unconstrained in the centralised decision-making model. However, in our case, $q \in [-n_2, n_1]$ and $p \in [0, +\infty]$ are applied simultaneously. To incorporate the constraints into the optimal solutions of **Lemma 5.1**, I introduce **Theorem 5.1**. (Appendix J)

***Theorem 5.1*** *Given $n_1$ and $n_2$, and $q \in [-n_2, n_1]$ and $p \in [0, +\infty]$, the optimal ECS quantity $q^*$ and the optimal CT rate $p^*$ in the centralised decision-making model are:*

$$q^*, p^* = \begin{cases} n_1, \bar{p} & 0 < n_1 \le \dot{q};\ \dot{p} \ge 0 & \mathit{Case}\ 1 \\ n_1, 0 & 0 < n_1 < \dot{q};\ \dot{p} < 0 & \mathit{Case}\ 2 \\ \dot{q}, \dot{p} & 0 < \dot{q} < n_1;\ \dot{p} \ge 0 & \mathit{Case}\ 3 \\ \hat{q}, 0 & 0 < \dot{q} < n_1;\ \dot{p} < 0 & \mathit{Case}\ 4 \\ 0, p^0 & \dot{q} < 0; \ddot{q} > 0, \dot{p} \ge 0; \ddot{p} \ge 0 & \mathit{Case}\ 5 \\ 0,0 & \dot{q} < 0; \ddot{q} > 0, \dot{p} \le 0; \ddot{p} \le 0 & \mathit{Case}\ 6 \\ \check{q}, 0 & -n_2 < \ddot{q} < 0, \ddot{p} < 0 & \mathit{Case}\ 7 \\ \ddot{q}, \ddot{p} & -n_2 \le \ddot{q} < 0, \ddot{p} \ge 0 & \mathit{Case}\ 8 \\ -n_2, 0 & \ddot{q} < -n_2, \ddot{p} < 0 & \mathit{Case}\ 9 \\ -n_2, \tilde{p} & \ddot{q} < -n_2, \ddot{p} > 0 & \mathit{Case}\ 10 \end{cases}$$

For the sake of brevity, I name case 1 to 10 according to the different ranges of $\dot{q}, \dot{p}, \ddot{q}, \ddot{p}$ defined in equation 5.9 and 5.10. Given $n_1$ and $n_2$, **Theorem 5.1** reveals that there always exists the optimal $ECS$ strategy between two $LSCs$ in each case when the $CT$ rate is set to maximise the whole system profit in the centralised decision-making model.

For example, in case 1, the optimal $CT$ rate should be $p^* = \bar{p}$ (where $\frac{\partial[\prod(n_1,p)]}{\partial p}|_{p=\bar{p}} = 0$) when the optimal $ECS$ quantity from $LSC$ 1's depot to $LSC$ 2's depot is greater than $LSC$ 1's inventory ($n_1$). However, on the other hand, without taking into consideration the constraint of $p$, the optimal $CT$ rate in the centralised decision-making model might be negative because the model only considers the $LSCs$' profit (e.g., cases 2, 4, 6, 7 and 9). Thus, if the optimal $CT$ rate $p$, without taking into consideration the constraint of $p$ , is less than 0, e.g., in case 4, the optimal $CT$ rate should be 0; furthermore, the optimal $ECS$ quantity should be $\hat{q}$ (where $\frac{\partial[\prod(q,0)]}{\partial q}|_{q=\hat{q}} = 0$). In case 5 where the theoretical optimal $ECS$ quantities from $LSC$ 1 to 2 are less than 0, or the theoretical optimal $ECS$ quantities from $LSC$ 2 to 1 are greater than 0, it means that the two $LSCs$ should keep the status quo and no empty container is shared. Notice that there exists a theoretical initial $CT$ rate $p^0$ (where $\frac{\partial[\prod(0,p)]}{\partial p}|_{p=p^0} = 0$). However, I need to clarify that $p^0$ only exists theoretically in the centralised decision-making model because the $LSCs$ do not have the right to decide the $CT$ rate when there are no $ECS$ activities. This only provides a baseline to calculate the system profit increment when the $CT$ rate fluctuates, which will be shown in section 5.3. In summary, **Theorem 5.1** provides the optimal strategy of $ECS$ quantity $q^*$ and the optimal $CT$ rate $p^*$ for the ten cases according to the constraints of $q$ and $p$ in the centralised decision-making model. Next, I will formulate the decentralised decision-making model where two $LSCs$ make decisions independently but are bound by a $RSC$, when the $CT$ is imposed.

### 5.2.2. Decentralised decision-making model

The above centralised decision-making model provides the optimal $ECS$ strategy when the $CT$ rate is set to maximise the entire system profits under an ideal and perfect collaboration between the two $LSCs$. It also reveals that the $CT$ rate does affect the $LSCs$' profit of container sharing supply chains. In the following section, another practical model will be developed for the decentralised decision-making case in which each $LSC$ makes decisions independently, selfishly aiming to maximise their own profits. In this chapter, I assume that the two $LSCs$ are bound by a $RSC$ that determines the allocation of profits between the two $LSCs$. The decentralised decision-making model reflects how the $LSCs$ may collaborate in shipping practice. Under the $RSC$, $LSC$ 1 charges $LSC$ 2 a wholesale price $w$ if $LSC$ 2 requires the extra empty containers to satisfy its demands. In return, $LSC$ 2 needs to share part of its revenue, which is generated through the satisfied demands through empty containers shared by $LSC$ 1, with $LSC$ 1. I assume that $\emptyset_1$ is the fraction of revenue that $LSC$ 1 keeps, hence $1 - \emptyset_1$ is the fraction of the revenue that $LSC$ 1 shares with $LSC$ 2. Also, $\emptyset_2$ is the fraction of revenue that $LSC$ 2 holds, and $1-\emptyset_2$ is the fraction of revenue that $LSC$ 2 shares with $LSC$ 1. Thus, I can determine the transfer payment $\theta$ between the two $LSCs$.

$$\begin{aligned}&\theta(q,p,w,\emptyset_1,\emptyset_2)\\&\quad= wq^+ + (1-\emptyset_2)r_2\{q^+ - \boldsymbol{E}\,min[(n_2+q+Y_2-X_2)^+,q^+]\} - w(-q)^+\\&\quad- (1-\emptyset_1)r_1\{(-q)^+ - \boldsymbol{E}\,min[(n_1-q+Y_1-X_1)^+,(-q)^+]\}\end{aligned} \tag{5.11}$$

For the sake of convenience, I rewrite the equation 5.11 as equation 5.12 (Appendix K)

$$\begin{aligned}&\theta(q,p,w,\emptyset_1,\emptyset_2)\\&\quad= q^+[w+(1-\emptyset_2)r_2] - (-q)^+[w+(1-\emptyset_1)r_1]\\&\quad- (1-\emptyset_2)r_2\boldsymbol{E}\min[(n_2+q+Y_2-X_2)^+,q^+]\\&\quad+ (1-\emptyset_1)r_1\boldsymbol{E}\min[(n_1-q+Y_1-X_1)^+,(-q)^+]\end{aligned} \tag{5.12}$$

Based on equation 5.12, I find that if $\theta$ is positive, the revenue is shared from $LSC$ 2 to $LSC$ 1; and vice versa for negative transfer payment $\theta$. Therefore, I can formulate $LSC$ 1's profit function as equation 5.13:

$$\begin{aligned}&\pi_1(q,p,w,\emptyset_1,\emptyset_2)\\&\quad= r_1\,\boldsymbol{E}\min[X_1,n_1-q+Y_1] - h_1\boldsymbol{E}(n_1-q+Y_1-X_1)^+\\&\quad- g_1\boldsymbol{E}(X_1+q-n_1-Y_1)^+ - c_tq^+ + \theta\end{aligned} \tag{5.13}$$

According to Snyder and Shen (2011), I further rewrite the equation 5.13 as equation 5.14 (Appendix L), where $\beta_1 = n_1 - q - a_1 + b_1p$ and $\beta_2 = n_2 + q - a_2 + b_2p$.

$$\begin{aligned}&\pi_1(q,p,w,R_1,R_2)\\&\quad= -\alpha_1\Phi_1(\beta_1) + R_2[\Phi_2(\beta_2-q)-\Phi_2(\beta_2)] - R_1[\Phi_1(\beta_1+q)-\Phi_1(\beta_1)]\\&\quad- (-q)^+(R_1+w) + q^+(R_2+w-c_t) + r_1\boldsymbol{E}(Y_1+a_1-b_1p) + (\alpha_1-h_1)\beta_1\\&\quad- g_1\boldsymbol{E}(\xi_1-Y_1)\end{aligned} \tag{5.14}$$

Let $R_1 = (1-\emptyset_1)r_1$ represents the revenue that $LSC$ 1 shares with $LSC$ 2 and $R_2 = (1-\emptyset_2)r_2$ for vice versa. Similarly, $LSC$ 2's profit function is:

$$\begin{aligned}&\pi_2(q,p,w,R_1,R_2)\\&\quad= -\alpha_2\Phi_2(\beta_2) + R_2[\Phi_2(\beta_2)-\Phi_2(\beta_2-q)] - R_1[\Phi_1(\beta_1)-\Phi_1(\beta_1+q)]\\&\quad- q^+(w+R_2) + (-q)^+(w-c_t+R_1) + r_2\boldsymbol{E}(Y_2+a_2-b_2p) + (\alpha_2-h_2)\beta_2\\&\quad- g_2\boldsymbol{E}(\xi_2-Y_2)\end{aligned} \tag{5.15}$$

Before I show that both $LSCs$ have a weakly dominant $ECS$ strategy in the decentralised model, I will introduce **Condition 5.1** to ensure the concavity of the two $LSCs$' profit functions (Appendix M).

**Condition 5.1**

$$\left(\frac{b_1}{b_2}\right)^2\frac{R_1}{R_2} \geq \left[\frac{z_1(n_1-a_1+b_1p)}{z_2(n_2-a_2+b_2p)}\right]^{-1}$$

**Condition 5.1** ensures the concavity of the two $LSCs$' profit functions in the decentralised decision-making model. To be more specific, it can be explained that $z_1(n_1 - a_1 + b_1 p)$ and $z_2(n_2 - a_2 + b_2 p)$ are the initial probability $(pdf)$ of demands that $LSC$ 1 and $LSC$ 2 receive before $ECS$ activity ($q = 0$) under a $CT$ rate $p$. On the other hand, $b_i$ represents $LSC$ $i$'s sensitivity to the $CT$ rate $p$, hence, $\frac{b_1}{b_2}$ implies the degree to which $LSC$ is more sensitive to the $CT$ rate compared to the other. Also, $\frac{R_1}{R_2}$ represents the degree to which $LSC$ is in relatively advantageous positions in the profit allocation in the $RSC$. So, **Condition 5.1** implies that the degree of market share between the two $LSCs$ is negatively proportional to their degree of sensitivity to the $CT$ rate and the degree of dominance in the contract. Next, I will provide **Lemma 5.2** to demonstrate that $\pi_1$ and $\pi_2$ are jointly concave in $p$ and $q$, when **Condition 5.1** is satisfied (Appendix M).

***Lemma 5.2*** *$\pi_1(q, p, w, R_1, R_2)$ and $\pi_2(q, p, w, R_1, R_2)$ are jointly concave in $p$ and $q$.*

**Lemma 5.2** demonstrates that no matter how one $LSC$ changes its $ECS$ strategy, the other $LSC$ has its optimal $ECS$ strategy denoted by $q_i$. Therefore, in the decentralised decision-making model, each $LSC$ can independently decide their $ECS$ quantities, based on their interest. It should be pointed out that $q_1$ and $q_2$ may be imported or exported containers, so, to clarify the direction of empty container movements, I further introduce $q_1^+$ and $q_2^+$ to indicate the number of empty containers that $LSC$ 1 can supply and $LSC$ 2 needs to request when $LSC$ 1 is the supplier and $LSC$ 2 is the demander. Similarly, I introduce $q_1^-$ and $q_2^-$ to stand for the number of empty containers that $LSC$ 1 requests and $LSC$ 2 can supply when $LSC$ 1 is the demander and $LSC$ 2 is the supplier. The directionality of $q_1$ and $q_2$ in this chapter is consistent with that shown in subsection 4.2.2. It is not hard to find that there exists a Nash equilibrium sharing strategy $q^e$ between the two $LSCs$, which can be formulated as:

$$q^e(p^e) = \min\{q_1^+(p^e), q_2^+(p^e)\} - \min\{q_1^-(p^e), q_2^-(p^e)\} \qquad 5.16$$

The sign of $q^e(p^e)$ stands for the direction of empty container flow in the decentralised decision-making model, which is consistent with the sign of the $q^*$ obtained in the centralised decision-making model. Similarly with Chapter 3 (subsection 4.2.2), based on equation 5.16, it is easy to find that $q^e(p^e)$ has only three possible results which are shown in equation 5.17.

$$q^e(p^e) = \begin{cases} \min\{q_1^+(p), q_2^+(p)\} \\ -\min\{q_1^-(p), q_2^-(p)\} \\ 0 \end{cases} \qquad 5.17$$

For example, in the decentralised decision-making model, if $LSC$ 1 can supply 10 empty containers and $LSC$ 2 requires 7 empty containers, then $q^e$ is $\{10,7\} - \{0,0\} = 7$. Hence, $LSC$ 1 should provide $LSC$ 2 with 7 empty containers. Once the optimal $ECS$ quantity $q^e$ is decided, the equilibrium $CT$ rate $p^e$ can be determined subsequently. When $q^e$ and $p^e$ are both reached, this means that no $LSC$ can obtain a better payoff without decreasing the other's profit. This is consistent with the conclusion made in Chapter 4 (subsection 4.2.2). Also, it also means when the equilibrium $CT$ rate is reached, the $ECS$ strategy of the two $LSCs$ can achieve the Pareto optimality simultaneously.

Next, to determine the two $LSCs$' profit increment in the decentralised decision-making models, and explore the system coordination, I let $p^{01}$ and $p^{02}$ denote $LSCs$' Preferred Ideal Carbon Tax Rate ($PICTR$) prior to the implementation of the $RSC$, i.e., $q^e = 0$. Also, similar to $p^0$ in the centralised decision-making model, it is necessary also to clarify that the $PICTRs$, $p^{01}$ and $p^{02}$, are two theoretical rates that the $LSCs$ prefer the government to charge initially, as the two $LSCs$ do not have the right to decide the $CT$ rate. In fact, even in the real world, the government cannot set two different $CT$ rates for the two different shipping companies. The purpose of introducing $PICTRs$, $p^{01}$ and $p^{02}$, is to provide a baseline for calculating the two $LSCs$' profit increment in the decentralised decision-making model relative to the centralised decision-making model, and check whether their profit increment is positive or negative (Appendix N). Also, it should be noted that there is no $PICTR$ in Chapter 4 because the $CT$ rate in Chapter 4 is assumed as a constant parameter.

However, in this chapter, the $CT$ rate is a decision variable, and it is changeable, and it depends on the other factors such as the $ECS$ activity. Therefore, it is fair to examine a theoretical $PICTR$ to be the baseline to calculate $LSCs$' profit increment.

***Theorem 5.2*** *If no empty containers are shared between the two $LSCs$, the two $PICTRs$ are:*

$$p^{01} = \frac{1}{b_1}\left[Z_1^{-1}\left(\frac{g_1}{\alpha_1}\right) - n_1 + a_1\right] \text{ and } p^{02} = \frac{1}{b_2}\left[Z_2^{-1}\left(\frac{g_2}{\alpha_2}\right) - n_2 + a_2\right]$$

**Theorem 5.2** provides a theoretical baseline of the $CT$ rate to calculate the two $LSCs$' profit increments prior to implementing the $RSC$. Next, I still employ a $RSC$ to determine the share of the system's profit increment between the two $LSCs$ and further investigate how an equilibrium $CT$ rate affects the system coordination.

### 5.3. The impact of the Carbon Tax rate and sharing activity on the system coordination

In this section, I will show the optimality conditions of the $RSC$ for system coordination as well as analyse the impact of $CT$ variation on two $LSCs'$ collaboration and discuss how to appropriately make contract to achieve coordination. First, two additional conditions need to be satisfied to coordinate the system under the impact of the $CT$ imposing, where $CT$ rate is an endogenous decision variable.

***Condition 5.2*** $q^e = q^*$; $p^e = p^*$

**Condition 5.2** indicates that the equilibrium $ECS$ quantities $q^e$ in the decentralised decision-making model equals the optimal $ECS$ number $q^*$ in the centralised decision-making model. Also, in this chapter, to achieve coordination, the optimal $CT$ rate $p^*$ in the centralised decision-making model should equal the equilibrium rate of the $CT$ $p^e$ in the decentralised decision-making model.

**Condition 5.3** is the other condition for system coordination.

***Condition 5.3*** $\Delta\pi_i \geq 0$ $(i = 1, 2)$*, where:*

$$\Delta\pi_i(q^e, p^e, w, R_1, R_2) = \pi_i(q^e, p^e, w, R_1, R_2) - \pi_i\left(0, p^{0i}, w, R_1, R_2\right) \quad 5.18$$

$$\Delta\prod(q^e, p^e) = \Delta\pi_1(q^e, p^e, w, R_1, R_2) + \Delta\pi_2(q^e, p^e, w, R_1, R_2) \quad 5.19$$

**Condition 5.3** points out that the share of profit allocation that the two $LSCs$ obtain in the decentralised decision-making model $\Delta\pi_i$ from the system profit increment $\Delta\prod(q^e, p^e)$ relative to the centralised decision-making model should be non-negative. According to equation 5.19, if one $LSC$ is allocated a negative profit, the other $LSC's$ profit will be more than $\Delta\prod$, and vice versa. If this situation happens, the system cannot be coordinated. Otherwise, if **Conditions 5.1**, **5.2** and **5.3** are satisfied, the system can be deemed coordinated.

Next, I will investigate the constraint of different variables and parameters to meet the satisfaction of **Conditions 5.1**, **5.2** and **5.3** and further to achieve system coordination. I first determine the appropriate $RSC$ parameters $R_1$, $R_2$ and $w$ to ensure that **Condition 5.2** is met in different cases shown in **Theorem 5.1**. In other words, by adjusting $R_1$, $R_2$ and $w$ in $RSC$, I will make sure $q^e = q^*$ and $p^e = p^*$. Also, based on **Condition 5.2**, for the sake of clarity and consistency, I replace $q^*$ and $p^*$ with $q^e$ and $p^e$ in the following analysis. **Lemma 5.3** gives the requirements of $R_1$, $R_2$ and $w$ in each case in order to satisfy **Condition 5.2**.

***Lemma 5.3*** *To meet* ***Condition 5.2****,* $R_1$*,* $R_2$ *and* $w$ *in each case should follow Table 5.1*

First, Table 5.2 gives the conditions of $R_1$, $R_2$ and $w$ that meets **Condition 5.2** in each case shown in **Theorem 5.1**. To ensure $q^e = q^*$ and $p^e = p^*$, the monotonicity of $q$ and $p$ in $\pi_1$ and $\pi_2$ should strictly comply with the monotonicity of $q$ in $\prod$ in the centralised model between $-n_2$ and $n_1$ and that of $p$ in $\prod$ between 0 and $+\infty$. Specifically, if $q^e$ is outside the range of two $LSCs'$ inventory $n_i$, then the monotonicity of $q$ in $\pi_1$ and $\pi_2$ should strictly follow the monotonicity of $q$ in $\prod$ between $-n_2$ and $n_1$ (case 1, 2, 9 and 10). Similarly, if $p$ is less than 0, then the monotonicity of $p$ in $\pi_1$ and $\pi_2$ should strictly comply with the monotonicity of $p$ in $\prod$ between 0 and $+\infty$ (case 2, 4, 7 and 9). This is the reason why $w$ and $R_1$, $R_2$ are constrained by an inequation in the above cases, respectively. However, on the one hand, if $q^e$ is just between $-n_2$ and $n_1$, not only should the monotonicity of $q$ in $\pi_1$ and $\pi_2$ be complied with that in $\prod$, but the two $LSCs'$ equilibrium sharing amount of empty containers in the decentralised decision-making model should also equal that in the centralised model, i.e., the partial derivative of $q$ at $q^e$ is 0 for $\pi_1$ and $\pi_2$ (case 3, 4, 7 and 8).

**Table 5.1**. The $R_1$, $R_2$ and $w$ for meeting **Condition 5.2** [where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i); i = 1, 2$], for $\beta_1$ and $\beta_2$, please refer to Table 5.2.

| Case No. | $R_1$; $R_2$ | $w$ |
|---|---|---|
| 1 | $R_2 = \frac{g_1 b_1 - b_1\alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - n_1)]} = \frac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - n_1)}$ | $\alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t \le w + R_2\bar{Z}_2(\beta_2) \le \alpha_2\bar{Z}_2(\beta_2) - h_2$ |
| 2 | $\frac{g_1 b_1 - b_1\alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - n_1)]} \le R_2 \le \frac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - n_1)}$ | $\alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t \le w + R_2\bar{Z}_2(\beta_2) \le \alpha_2\bar{Z}_2(\beta_2) - h_2$ |
| 3 | $R_2 = \frac{g_1 b_1 - b_1\alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)]} = \frac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - q)}$ | $\alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t = w + R_2\bar{Z}_2(\beta_2) = \alpha_2\bar{Z}_2(\beta_2) - h_2$ |
| 4 | $\frac{g_1 b_1 - b_1\alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)]} \le R_2 \le \frac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - q)}$ | $\alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t = w + R_2\bar{Z}_1(\beta_1) = \alpha_2\bar{Z}_2(\beta_2) - h_2$ |
| 5 | X | |
| 6 | | |
| 7 | $\frac{g_2 b_2 - b_2\alpha_2 Z_2(\beta_2)}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]} \le R_1 \le \frac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)}$ | $\alpha_1\bar{Z}_1(\beta_1) - h_1 = w + R_1\bar{Z}_2(\beta_2) = \alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t$ |
| 8 | $R_1 = \frac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)} = \frac{g_2 b_2 - b_2\alpha_2 Z_2(\beta_2)}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]}$ | $\alpha_1\bar{Z}_1(\beta_1) - h_1 = w + R_1\bar{Z}_1(\beta_1) = \alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t$ |
| 9 | $\frac{-b_2\alpha_2 Z_2(\beta_2) + g_2 b_2}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]} \le R_1 \le \frac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)}$ | $\alpha_2\bar{Z}_2(\beta_2) + c_t - h_2 \le w + R_1\bar{Z}_1(\beta_1) \le \alpha_1\bar{Z}_1(\beta_1) - h_1$ |
| 10 | $R_1 = \frac{-b_2\alpha_2 Z_2(\beta_2) + g_2 b_2}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]} = \frac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)}$ | $\alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t \le w + R_1\bar{Z}_1(\beta_1) \le \alpha_1\bar{Z}_1(\beta_1) - h_1$ |

**Table 5.2** The $\beta_1$ and $\beta_2$ in ten cases.

| Case No | $\boldsymbol{\beta_1}$ | $\boldsymbol{\beta_2}$ |
|---|---|---|
| 1 | $-a_1 + b_1\bar{p}$ | $n_2 + n_1 - a_2 + b_2\bar{p}$ |
| 2 | $-a_1$ | $n_2 + n_1 - a_2$ |
| 3 | $n_1 - \dot{q} - a_1 + b_1\dot{p}$ | $n_2 + \dot{q} - a_2 + b_2\dot{p}$ |
| 4 | $n_1 - \hat{q} - a_1$ | $n_2 + \hat{q} - a_2$ |
| 5 | $X$ | |
| 6 | $X$ | |
| 7 | $n_1 - \check{q} - a_1$ | $n_2 + \check{q} - a_2$ |
| 8 | $n_1 - \ddot{q} - a_1 + b_1\ddot{p}$ | $n_2 + \ddot{q} - a_2 + b_2\ddot{p}$ |
| 9 | $n_1 + n_2 - a_1$ | $-a_2$ |
| 10 | $n_1 + n_2 - a_1 + b_1\tilde{p}$ | $-a_2 + b_2\tilde{p}$ |

On the other hand, if $p^e$ is greater than 0, then the optimal $CT$ rate that maximises the whole system profit in the centralised decision-making model should equal the equilibrium government $CT$ rate that maximises the sum of the two $LSCs$' profits in the decentralised decision-making model, i.e., the partial derivative of $p$ at $p^e$ is zero for $\pi_1$ and $\pi_2$ (case 1, 3, 8 and 10). Thus, $w$ and $R_1$, $R_2$ should be limited by an equation for cases 1, 3, 8 and 10. For cases 5 and 6, they are exceptional as there is no empty container shared between the two $LSCs$. Thus, it does not need to determine the range of $R_1$, $R_2$ and $w$. The analysis above is similar with that in **Lemma 4.4** (Figure 4.2) but in a three dimensional.

Next, I will show the two $LSCs$' profit increment from the system profit increment between the centralised and decentralised decision-making models when **Condition 5.2** is met. Then, I will discuss how $ECS$ activity and $CT$ rate variation affects the system coordination. All the calculations in this section can be found in Appendix O.

***Lemma 5.4*** *Given* ***Conditions 5.1*** *and* ***5.2****,* $\Delta\pi_1$ *and* $\Delta\pi_2$ *are shown below:*

*For case 1 to 4:*

$$\begin{cases} \Delta\pi_1 = \alpha_1[\Delta S_1(0,p^e) + \Delta S_1^e(q^e,p^e)] + R_2\Delta S_2^e(q^e,p^e) - (\alpha_1 - g_1)b_1(p^e - p^{01}) \\ \Delta\pi_2 = (\alpha_2 - R_2)\Delta S_2^e(q^e,p^e) + \alpha_2\Delta S_2(0,p^e) - (\alpha_2 - g_2)b_2(p^e - p^{02}) \end{cases}$$

*For case 7 to 10:*

$$\begin{cases} \Delta\pi_1 = (\alpha_1 - R_1)\Delta S_1^e(q^e,p^e) + \alpha_1\Delta S_1(0,p^e) - (\alpha_1 - g_1)b_1(p^e - p^{01}) \\ \Delta\pi_2 = \alpha_2[\Delta S_2(0,p^e) + \Delta S_2^e(q^e,p^e)] + R_1\Delta S_1^e(q^e,p^e) - (\alpha_2 - g_2)b_2(p^e - p^{02}) \end{cases}$$

*For case 5:* $\Delta\pi_1 = \Delta\pi_2 = 0$

*For case 6*: $\Delta\pi_1 = \alpha_1\Delta S_1(0,0) - g_2b_2p^{02}$ and $\Delta\pi_2 = \alpha_2\Delta S_2(0,0) - g_1b_1p^{01}$

Where $\Delta S_1^e(q^e,p^e) = S_1^e(q^e,p^e) - S_1^e(0,p^e)$ and $\Delta S_2^e(q^e,p^e) = S_2^e(q^e,p^e) - S_2^e(0,p^e)$ is the increment of expected satisfied demand between the scenario with and without $ECS$ under the impact of the equilibrium $CT$ rate $p^e$ for $LSC$ 1 and 2, respectively. Similarly, $\Delta S_1(0,p^e) = S_1(0,p^e) - S_1(0,p^{01})$ and $\Delta S_2(0,p^e) = S_2(0,p^e) - S_2(0,p^{02})$ represents the increment of expected satisfied demand between the scenarios with and without the equilibrium $CT$ rate $p^e$ (compared to the $PICTRs$ $p^{0i}$) before the implementation of $ECS$ activity for $LSC$ 1 and 2, respectively (i.e., only considering the impact of the equilibrium $CT$ rate variation from $PICTRs$ $p^{0i}$ to $p^e$ on $LSCs$' profit increment). Also, I further choose the appropriate $RSC$ parameters $R_1$, $R_2$ and $w$ to obtain the maximum and minimum $\Delta\pi_1$ and $\Delta\pi_2$ in ten cases. Below, I state two $LSCs$' theoretical minimum and maximum share of profit increment from the decentralised decision-making model under the system coordination in Table 5.3.

**Lemma 5.4** gives both $LSCs$' share of the system profit increment from the decentralised decision-making model under the equilibrium $CT$ $p^e$. In cases 1 to 4, the empty containers are shared from $LSC$ 1 to $LSC$ 2, and the profit increment of $LSC$ 2 as the empty container demander can be divided into three parts. The first is the profit increment $\Delta S_2^e(q^e, p^e)$ ($q^e = n_1$ in case 1 and 2, $q^e = \dot{q}$ in case 3, $q^e = \hat{q}$ in case 4) that $LSC$ 2 earns from the $ECS$ activity from $LSC$ 1 to $LSC$ 2 under the constant equilibrium $CT$ rate $p^e$. For each unit of satisfied empty container that $LSC$ 1 shares with $LSC$ 2, $LSC$ 2 earns $\alpha_2 - R_2$ since it needs to share a proportion of revenue back with $LSC$ 1 according to the contract. Considering the impact of the equilibrium $CT$ rate $p^e$, it is straightforward to find from Figure 5.1 that $LSC$ 2's increment of expected satisfied demand due to the $RSC$ implementation, $\Delta S_2^e(q^e, p^e)$, is always positive in four cases as long as empty containers are transferred from $LSC$ 1 to $LSC$ 2. It should be noted that the equilibrium $CT$ rate $p^e$ does not affect the positivity of $\Delta S_2^e(q^e, p^e)$ in these cases. Also, according to Figure 5.1, in the initial stage of $ECS$ activity from $LSC$ 1 to $LSC$ 2, $LSC$ 2's marginal increment of expected satisfied demands due to shared empty containers, $\Delta S_2^e(q^e, p^e)$, is high. However, with the number of empty containers shared from $LSC$ 1 to $LSC$ 2 increasing, $LSC$ 2's marginal increment of $\Delta S_2^e(q, p^e)$ decreases. It means that the impact of $ECS$ activity on $LSC$ 2's increment of expected satisfied demands $\Delta S_2^e(q^e, p^e)$ decreases with the number of $ECS$ amount $q$.

On the other hand, for cases 1 to 4, the second part in $LSC$ 2's profit increment is the increment of expected satisfied demands $\Delta S_2(0, p^e)$ due to the impact of $CT$ rate variation from the $PICTR$ $p^{02}$ to the equilibrium $CT$ rate $p^e$ without considering the $ECS$ activity.

$LSC$ 2 can earn a percentage of $\alpha_2$ for every satisfied empty container due to the impact of $CT$ rate variation. The $CT$ rate variation can determine whether $LSC$ 2 can earn a non-negative profit and whether the system can be coordinated. This is the reason why I should investigate the $CT$ rate impact on the system coordination of the $ECS$ supply chain. As shown in Figure 5.2, if equilibrium $CT$ rate $p^e$ is greater than $PICTR$ $p^{02}$, interestingly, the increment of expected satisfied demand $\Delta S_2(0, p^e)$ would be positive. This is consistent with the conclusion made in section 4.3 However, similarly with the statement shown in section 4.3, I should clarify that no actual satisfied demand is generated when the $CT$ imposing because the demand must fall when

**Table 5.3** The maximum and minimum of $\Delta\pi_1$ and $\Delta\pi_2$ under system coordination when $R_1$, $R_2$ and $w$ are appropriately set.

| No | $\boldsymbol{\Delta\pi_1}$ | $\boldsymbol{\Delta\pi_2}$ |
|---|---|---|
| 1 | min: $\alpha_1[\Delta S_1(0,\bar{p}) + \Delta S_1^e(n_1,\bar{p})] + R_2\Delta S_2^e(n_1,\bar{p}) - (\alpha_1 - g_1)b_1(\bar{p} - p^{01})$ | max: $\Delta\prod - \alpha_1[\Delta S_1(0,\bar{p}) + \Delta S_1^e(n_1,\bar{p})] - R_2\Delta S_2^e(n_1,\bar{p}) + (\alpha_1 - g_1)b_1(\bar{p} - p^{01})$ |
| | max: $\Delta\prod - (\alpha_2 - R_2)\Delta S_2^e(n_1,\bar{p}) - \alpha_2\Delta S_2(0,\bar{p}) + (\alpha_2 - g_2)b_2(\bar{p} - p^{02})$ | min: $(\alpha_2 - R_2)\Delta S_2^e(n_1,\bar{p}) + \alpha_2\Delta S_2(0,\bar{p}) - (\alpha_2 - g_2)b_2(\bar{p} - p^{02})$ |
| 2 | min: $\alpha_1[\Delta S_1^e(n_1,0) + \Delta S_1(0{,}0)] + R_{2min}\Delta S_2^e(n_1,0) + (\alpha_1 - g_1)b_1p^{01}$ | max: $\Delta\prod - \alpha_1[\Delta S_1^e(n_1,0) + \Delta S_1(0{,}0)] - R_{2min}\Delta S_2^e(n_1,0) - (\alpha_1 - g_1)b_1p^{01}$ |
| | max: $\Delta\prod - (\alpha_2 - R_{2max})\Delta S_2^e(n_1,0) - \alpha_2\Delta S_2(0{,}0) - (\alpha_2 - g_2)b_2p^{02}$ | min: $(\alpha_2 - R_{2max})\Delta S_2^e(n_1,0) + \alpha_2\Delta S_2(0{,}0) + (\alpha_2 - g_2)b_2p^{02}$ |
| 3 | $\alpha_1[\Delta S_1^e(\dot{q},\dot{p}) + \Delta S_1(0,\dot{p})] + R_2\Delta S_2^e(\dot{q},\dot{p}) - (\alpha_1 - g_1)b_1(\dot{p} - p^{01})$ | $\Delta\prod - \alpha_1[\Delta S_1^e(\dot{q},\dot{p}) + \Delta S_1(0,\dot{p})] - R_2\Delta S_2^e(\dot{q},\dot{p}) + (\alpha_1 - g_1)b_1(\dot{p} - p^{01})$ |
| | $\Delta\prod - (\alpha_2 - R_2)\Delta S_2^e(\dot{q},\dot{p}) - \alpha_2\Delta S_2(0,\dot{p}) + (\alpha_2 - g_2)b_2(\dot{p} - p^{02})$ | $(\alpha_2 - R_2)\Delta S_2^e(\dot{q},\dot{p}) + \alpha_2\Delta S_2(0,\dot{p}) - (\alpha_2 - g_2)b_2(\dot{p} - p^{02})$ |
| 4 | min: $\alpha_1[\Delta S_1^e(\hat{q},0) + \Delta S_1(0{,}0)] + R_{2max}\Delta S_2^e(\hat{q},0) + (\alpha_1 - g_1)b_1p^{01}$ | max: $\Delta\prod - \alpha_1[\Delta S_1^e(\hat{q},0) + \Delta S_1(0{,}0)] - R_{2max}\Delta S_2^e(\hat{q},0) - (\alpha_1 - g_1)b_1p^{01}$ |
| | max: $\Delta\prod - (\alpha_2 - R_{2min})\Delta S_2^e(\hat{q},0) - \alpha_2\Delta S_2(\hat{q},0) - (\alpha_2 - g_2)b_2p^{02}$ | min: $(\alpha_2 - R_{2min})\Delta S_2^e(\hat{q},0) + \alpha_2\Delta S_2(\hat{q},0) + (\alpha_2 - g_2)b_2p^{02}$ |
| 5 | $\Delta\pi_1 = 0; \Delta\pi_2 = 0$ | |
| 6 | $\Delta\pi_1 = \alpha_1\Delta S_1(0{,}0) + (\alpha_1 - g_1)b_1p^{01}; \Delta\pi_2 = \alpha_2\Delta S_2(0{,}0) + (\alpha_2 - g_2)b_2p^{02}$ | |
| 7 | min: $(\alpha_1 - R_{1max})\Delta S_1^e(\check{q},0) + \alpha_1\Delta S_1(0{,}0) + (\alpha_1 - g_1)b_1p^{01}$ | max: $\Delta\prod - (\alpha_1 - R_{1max})\Delta S_1^e(\check{q},0) - \alpha_1\Delta S_1(0{,}0) - (\alpha_1 - g_1)b_1p^{01}$ |
| | max: $\Delta\prod - \alpha_2[\Delta S_2^e(\check{q},0) + \Delta S_2(0{,}0)] - R_{1min}\Delta S_1^e(\check{q},0) - (\alpha_2 - g_2)b_2p^{02}$ | min: $\alpha_2[\Delta S_2^e(\check{q},0) + \Delta S_2(0{,}0)] + R_{1min}\Delta S_1^e(\check{q},0) + (\alpha_2 - g_2)b_2p^{02}$ |
| 8 | $(\alpha_1 - R_1)\Delta S_1^e(\ddot{q},\ddot{p}) + \alpha_1\Delta S_1(0,\ddot{p}) - (\alpha_1 - g_1)b_1(\ddot{p} - p^{01})$ | $\Delta\prod - (\alpha_1 - R_1)\Delta S_1^e(\ddot{q},\ddot{p}) - \alpha_1\Delta S_1^e(0,\ddot{p}) + (\alpha_1 - g_1)b_1(\ddot{p} - p^{01})$ |
| | $\Delta\prod - \alpha_2[\Delta S_2^e(\ddot{q},\ddot{p}) + \Delta S_2(0,\ddot{p})] - R_1\Delta S_1^e(\ddot{q},\ddot{p}) + (\alpha_2 - g_2)b_2(\ddot{p} - p^{02})$ | $\alpha_2[\Delta S_2^e(\ddot{q},\ddot{p}) + \Delta S_2(0,\ddot{p})] + R_1\Delta S_1^e(\ddot{q},\ddot{p}) - (\alpha_2 - g_2)b_2(\ddot{p} - p^{02})$ |
| 9 | min: $(\alpha_1 - R_{1max})\Delta S_1^e(-n_2,0) + \alpha_1\Delta S_1(0{,}0) + (\alpha_1 - g_1)b_1p^{01}$ | max: $\Delta\prod - (\alpha_1 - R_{1max})\Delta S_1^e(-n_2,0) - \alpha_1\Delta S_1(0{,}0) - (\alpha_1 - g_1)b_1p^{01}$ |
| | max: $\Delta\prod - \alpha_2[\Delta S_2^e(-n_2,0) + \Delta S_2(0{,}0)] - R_{1min}\Delta S_1^e(-n_2,0) - (\alpha_2 - g_2)b_2p^{02}$ | min: $\alpha_2[\Delta S_2^e(-n_2,0) + \Delta S_2(0{,}0)] + R_{1min}\Delta S_1^e(-n_2,0) + (\alpha_2 - g_2)b_2p^{02}$ |
| 10 | min: $(\alpha_1 - R_1)\Delta S_1^e(-n_2,\tilde{p}) + \alpha_1\Delta S_1(0,\tilde{p}) - (\alpha_1 - g_1)b_1(\tilde{p} - p^{01})$ | max: $\Delta\prod - (\alpha_1 - R_1)\Delta S_1^e(-n_2,\tilde{p}) - \alpha_1\Delta S_1(0,\tilde{p}) + (\alpha_1 - g_1)b_1(\tilde{p} - p^{01})$ |
| | max: $\Delta\prod - \alpha_2[\Delta S_2(0,\tilde{p}) + \Delta S_2^e(-n_2,\tilde{p})] - R_1\Delta S_1^e(-n_2,\tilde{p}) + (\alpha_2 - g_2)b_2(\tilde{p} - p^{02})$ | min: $\alpha_2[\Delta S_2(0,\tilde{p}) + \Delta S_2^e(-n_2,\tilde{p})] + R_1\Delta S_1^e(-n_2,\tilde{p}) - (\alpha_2 - g_2)b_2(\tilde{p} - p^{02})$ |

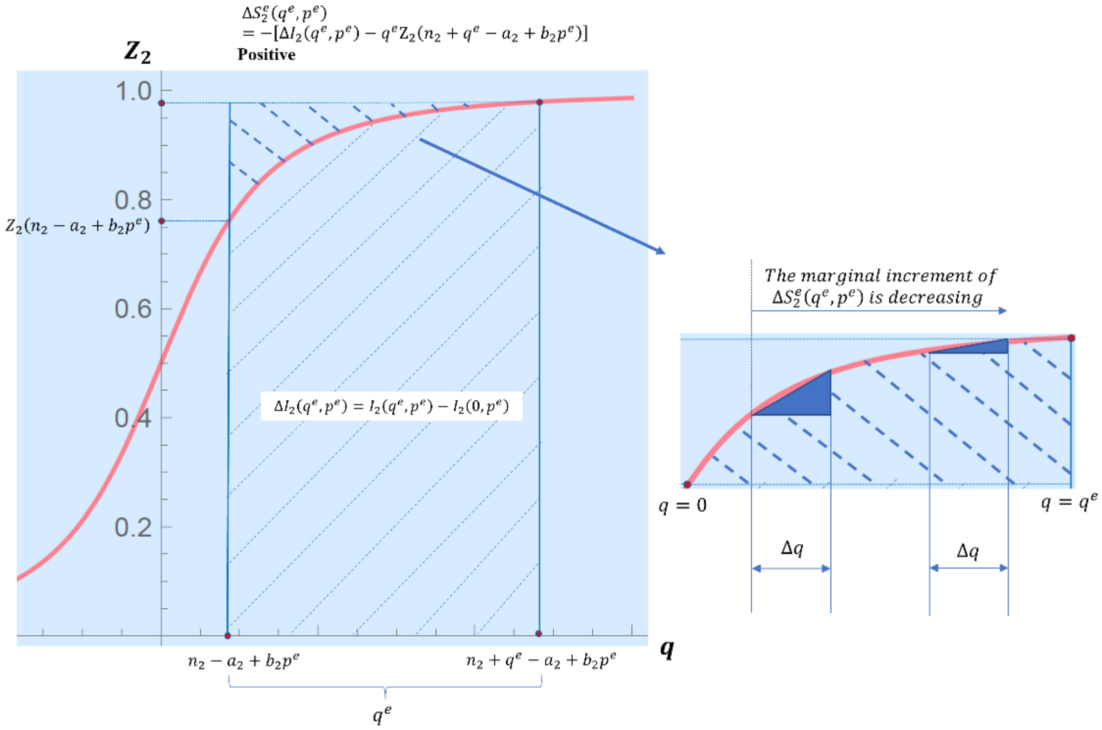


**Figure 5.1.** The impact of $ECS$ activity on the demander's satisfied demand change $\Delta S_2^e(q^e, p^e)$ in case 1-4.

the $CT$ is greater than $PICTR$. The satisfied demand increasing is actually transformed from the goodwill penalty saving $g_i b_i p$ when the demand declines. Conversely, in Figure 5.3, if equilibrium $CT$ rate $p^e$ is less than $PICTR$ $p^{02}$, then the increment of expected satisfied demand $\Delta S_2(0, p^e)$ would be negative because the demand would increase and the goodwill penalty cost would further increase.

More profoundly, based on Figure 5.2, if equilibrium $CT$ rate $p^e$ is greater than $PICTR$ $p^{02}$, the impact of the $CT$ on the marginal increment of expected satisfied demand $\Delta S_2(0, p^e)$ for $LSC$ 2 decreases gradually, although the impact is significant initially, which is also in line with the conclusion made in section 4.3. Conversely, according to Figure 5.3, the impact on the marginal increment of expected satisfied demand $\Delta S_2(0, p^e)$ gradually increases if equilibrium $CT$ rate $p^e$ is less than $PICTR$ $p^{02}$, although the impact is small initially. Therefore, for $LSC$ 2 (the demander), if equilibrium $CT$ rate $p^e$ is greater than $PICTR$ $p^{02}$, the $CT$ variation has a more significant impact on the marginal increment of expected satisfied demand $\Delta S_2(0, p^e)$ in the initial stage, but the impact is gradually diminished. However, if equilibrium $CT$ rate $p^e$ is less than $PICTR$ $p^{02}$, the impact of the $CT$ variation would be insignificant in the initial stage, but the impact gradually rises.

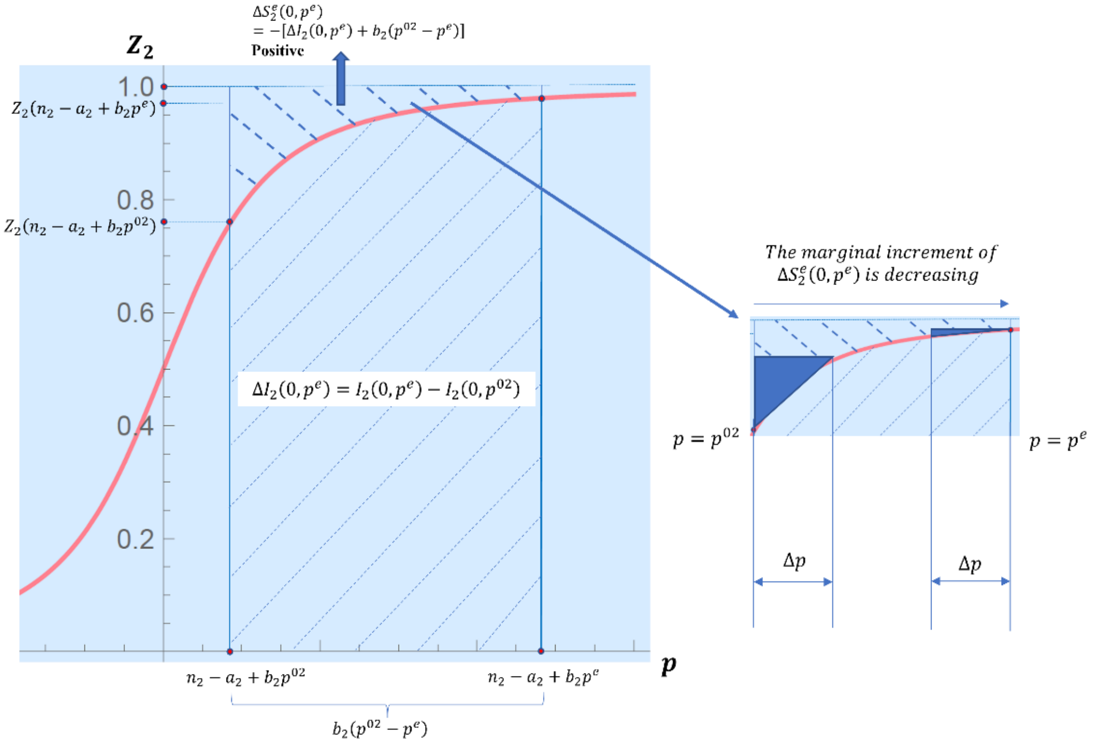


**Figure 5.2**. The impact of the $CT$ on demander's satisfied demand $\Delta S_2(0, p^e)$ when $p^e > p^{02}$ from case 1-4.

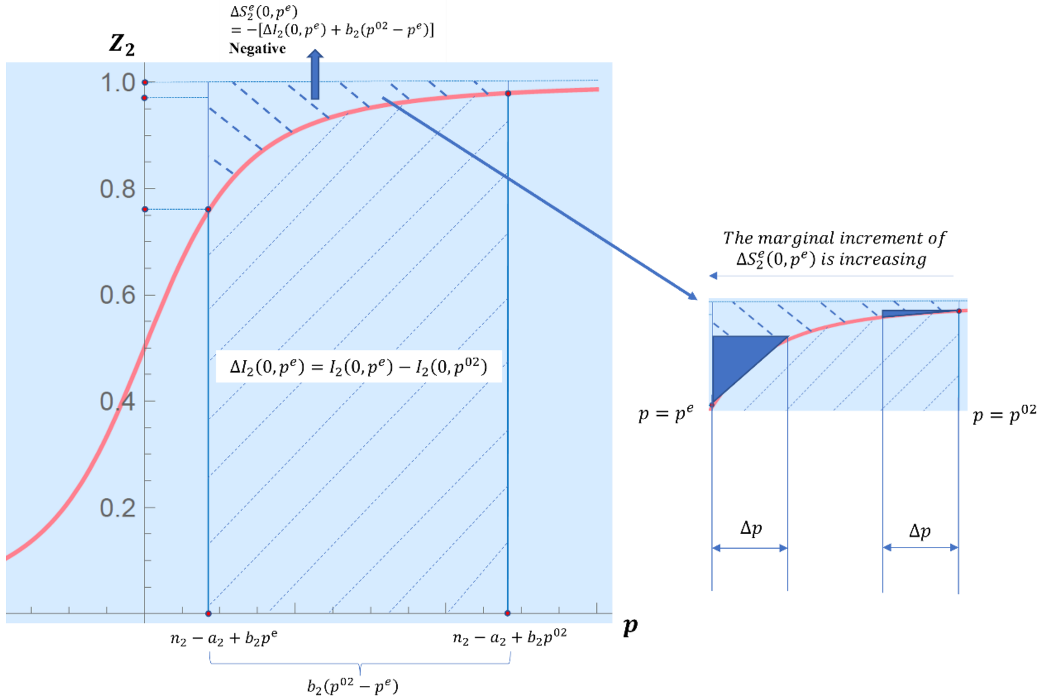


**Figure 5.3**. The impact of the $CT$ on demander's satisfied demand $\Delta S_2(0, p^e)$ when $p^e < p^{02}$ in case 1-4.

Additionally, the third part is $LSC$ 2's loss when the equilibrium $CT$ rate $p^e$ is higher than $PICTR$ $p^{02}$ (only in case 1 and 3) $[-(\alpha_2 - g_2)b_2(p^e - p^{02})]$. Thus, although a $CT$ rate higher than $p^{02}$ makes the satisfied demand $\Delta S_2(0, p^e)$ positive, $LSC$ 2 possibly obtains a negative profit increment as $LSC$ 2's total demand declines. If equilibrium $CT$ rate $p^e$ is lower than $p^{02}$, although $\Delta S_2(0, p^e)$ is negative, $LSC$ 2 may still obtain a non-negative profit increment because it's total demand increases $[(\alpha_2 - g_2)b_2(p^{02} - p^e)]$.

Similarly, in cases 1 to 4, the $LSC$ 1's (the supplier) profit increment consists of four parts. The first one is the profit that $LSC$ 1 gains from the satisfied demand due to the $ECS$ activity $\Delta S_1^e(q^e, p^e)$ from $LSC$ 1 to $LSC$ 2 under the impact of equilibrium $CT$ rate $p^e$. Specifically, as shown in Figure 5.4, $\Delta S_1^e(q^e, p^e)$ is also always positive as long as empty containers are shared from $LSC$ 1 to $LSC$ 2. Moreover, conversely with $LSC$ 2, for $LSC$ 1, with more empty containers being shared to $LSC$ 2, $LSC$ 1's marginal increment of expected satisfied demand $\Delta S_1^e(q^e, p^e)$ rises, which means that the contract implementation has more impact on $\Delta S_1^e(q^e, p^e)$. Interestingly, $LSC$ 1 can receive more revenue $R_2$ shared by $LSC$ 2 according to the contract. Also, based on Figure 5.5, $\Delta S_1(0, p^e)$ is positive if equilibrium $CT$ rate $p^e$ is greater than $PICTR$ $p^{01}$. However, $LSC$ 1 suffers the loss when equilibrium $CT$ rate $p^e$ is greater than $PICTR$ $p^{01}$ because of the total demand decline $[-(\alpha_1 - g_1)b_1(p^e - p^{01})]$. In contrast, based on Figure 5.6, if equilibrium $CT$ rate $p^e$ is less than $PICTR$ $p^{01}$, then $\Delta S_1(0, p^e)$ is negative, but $LSC$ 1 earns the profit when the equilibrium $CT$ rate $p^e$ is lower than $PICTR$ $p^{01}$ as the total demand increases $[(\alpha_1 - g_1)b_1(p^{01} - p^e)]$. Also, according to two Figures (5.5 and 5.6), if equilibrium $CT$ rate $p^e$ is greater than $PICTR$ $p^{01}$, the $CT$ scheme significantly affects $\Delta S_1(0, p^e)$ initially, but the impact is gradually diminished subsequently. In contrast, if equilibrium $CT$ rate $p^e$ is less than $PICTR$ $p^{01}$, $\Delta S_1(0, p^e)$ is slightly influenced in the initial stage, but the influence gradually rises subsequently.

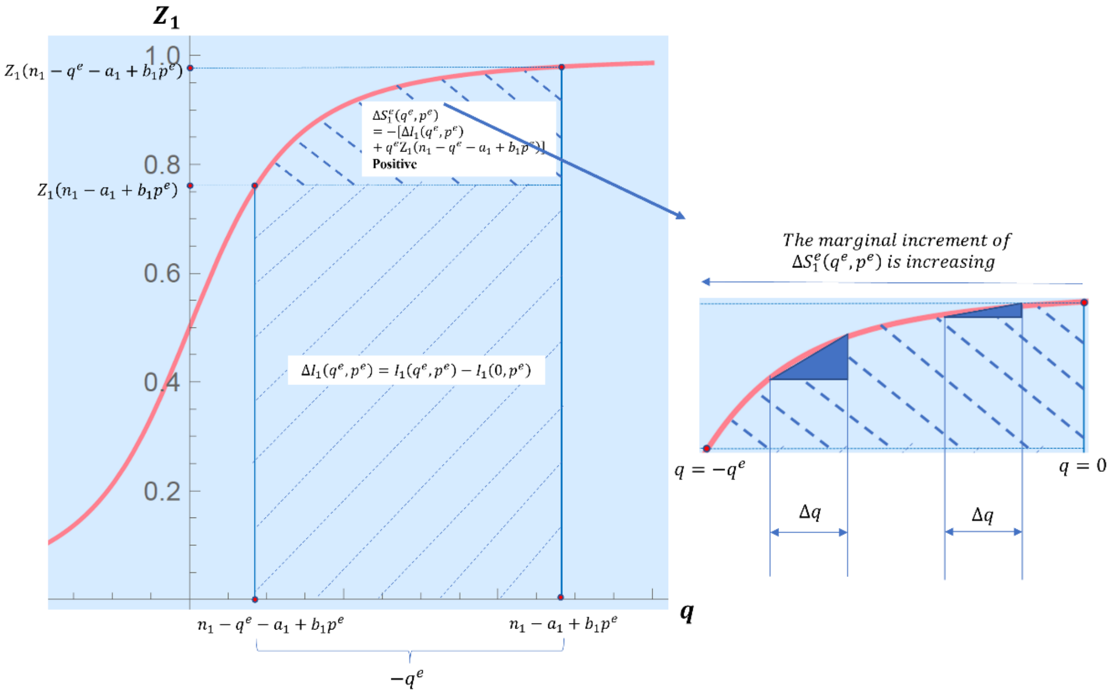


**Figure 5.4**. The impact of $ECS$ activity on the supplier's satisfied demand change $\Delta S_1^e(q^e, p^e)$ in case 1-4.

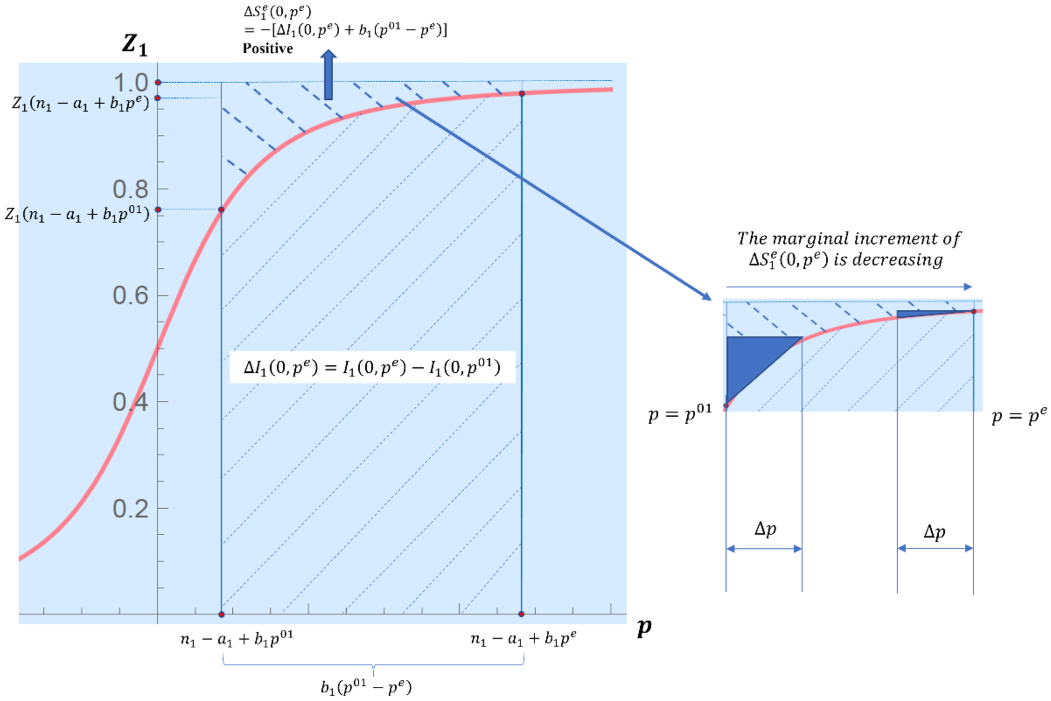


**Figure 5.5**. The impact of the $CT$ on supplier's satisfied demand $\Delta S_1(0, p^e)$ when $p^e > p^{01}$ in case 1-4.

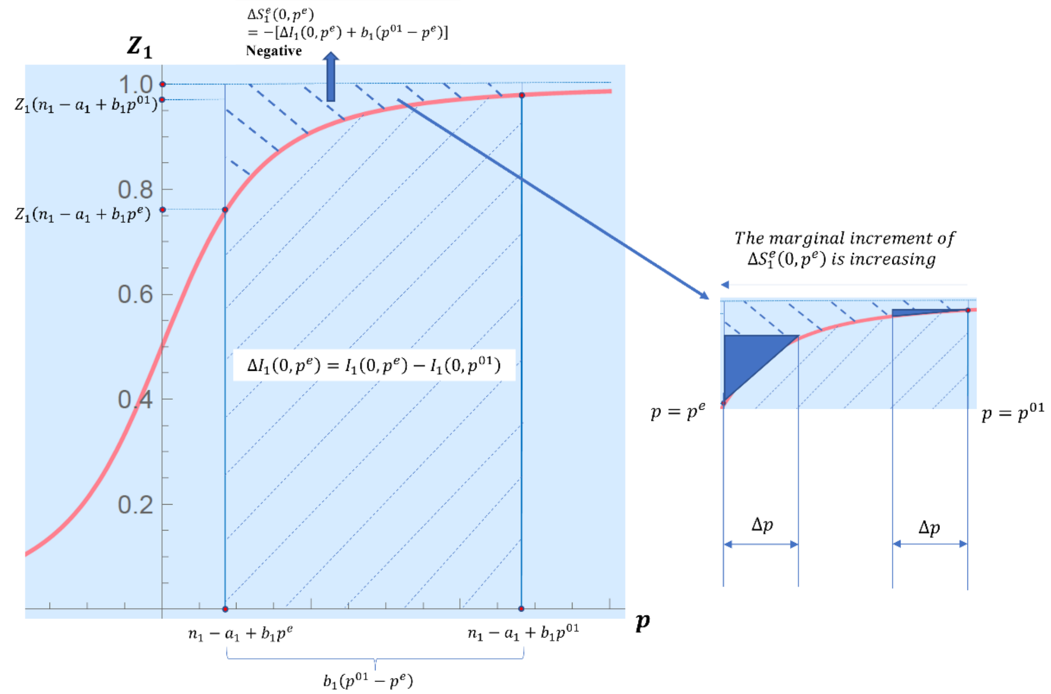


**Figure 5.6**. The impact of the $CT$ on supplier's satisfied demand $\Delta S_1(0, p^e)$ when $p^e < p^{01}$ in case 1-4.

Briefly, in cases 7, 8, 9 and 10 ($q^e = -n_2$ in case 10, $q^e = -n_2$ in case 9, $q^e = \ddot{q}$ in case 8, $q^e = \check{q}$ in case 7), the empty containers are shared from $LSC$ 2 to $LSC$ 1. Due to symmetry between the two cases' groups (see Appendix O), $LSC$ 1's and $LSC$ 2's profit increment in the four cases (i.e., case 7, 8, 9 and 10) have the same structure and conclusions with $LSC$ 2's and $LSC$ 1's profit increment structure and conclusions in cases 4, 3, 2 and 1, respectively. Thus, for convenience, the analysis for these four cases is simplified but the details can be found in Appendix O. In particular, case 5 is a special situation where the two $LSCs$ neither share empty containers nor being affected by the $CT$ rate. So, there is no profit increment for both the $LSCs$ and the system, which means $\Delta\prod = \Delta\pi_1 = \Delta\pi_2 = 0$. Also, in case 6, although no empty containers are shared between the two $LSCs$, there are still profit increment for the two $LSCs$ when $p^e$ is 0 (decreased from $PICTR$). So, the two $LSCs$' profit increment are $\alpha_1\Delta S_1(0,0) + (\alpha_1 - g_1)b_1p^{01}$ and $\alpha_2\Delta S_2(0,0) + (\alpha_2 - g_2)b_2p^{02}$, respectively.

So far, this chapter have investigated how the $ECS$ activity and $CT$ rate affects the two $LSCs'$ profit increment in the decentralised decision-making model. In the following, I will discuss the conditions that the equilibrium $CT$ in the decentralised decision-making model needs to meet to achieve the system coordination.

***Lemma 5.5*** *When **Condition 5.1** and **5.2** are satisfied, two $LSCs$ can obtain non-negative profit allocation only if $p^e$ satisfies the following conditions: (Table 5.4)*

**Table 5.4**. The condition of equilibrium $CT$ rate $p^e$ for meeting Condition 5.3.

| | |
|---|---|
| Cases 1, 2, 3 and 4 | $\lvert p^e - p^{01}\rvert \le \{\alpha_1[\Delta S_1^e(q^e,p^e) + \Delta S_1(0,p^e)] + R_2\Delta S_2^e(q^e,p^e)\}/[(\alpha_1 - g_1)b_1]$<br>$\lvert p^e - p^{02}\rvert \le [(\alpha_2 - R_2)\Delta S_2^e(q^e,p^e) + \alpha_2\Delta S_2(0,p^e)]/[(\alpha_2 - g_2)b_2]$ |
| Cases 7, 8, 9 and 10 | $\lvert p^e - p^{01}\rvert \le [(\alpha_1 - R_1)\Delta S_1^e(q^e,p^e) + \alpha_1\Delta S_1(0,p^e)]/[(\alpha_1 - g_1)b_1]$<br>$\lvert p^e - p^{02}\rvert \le \{\alpha_2[\Delta S_2^e(q^e,p^e) + \Delta S_2(0,p^e)] + R_1\Delta S_1^e(q^e,p^e)\}/[(\alpha_2 - g_2)b_2]$ |
| In case 5: $p^e = p^{01} = p^{02}$; In case 6: $p^{01} \ge -\frac{\alpha_1\Delta S_1(0,0)}{(\alpha_1-g_1)b_1}$; $p^{02} \ge -\frac{\alpha_2\Delta S_2(0,0)}{(\alpha_2-g_2)b_2}$ | |

**Lemma 5.5** reveals that it is possible to achieve the system coordination if the equilibrium $CT$ rate $p^e$ is restricted within a range to ensure the nonnegativity of the two $LSCs'$ profits (**Condition 5.3**). Specifically, in cases 1, 3, 8 and 10, if equilibrium $CT$ rate $p^e$ is less than $PICTRs$ $p^{01}$ or $p^{02}$, **Lemma 5.5** guarantees that the reduction of empty container demander's ($LSC$ 1 in case 8 and 10, $LSC$ 2 in case 1 and 3) profit from satisfied demand $\Delta S_{demander}(0,p^e)$ can be compensated by the profit that demander obtains because of the total demand increase when $CT$ rate decreasing plus the profit that $LSCs$ obtain from $ECS$ activity $\Delta S_{demander}^e(q^e,p^e)$. Similarly, if equilibrium $CT$ rate $p^e$ is greater than $PICTRs$ $p^{01}$ or $p^{02}$, then the empty container demander's loss because of the total demand decreasing must be covered by the profit from the increased satisfied demands $\Delta S_{demander}(0,p^e)$ arising from the $CT$ rate variation plus the increased profit from $ECS$ activity $\Delta S_{demander}^e(q^e,p^e)$. On the other hand, the empty container supplier ($LSC$ 2 in case 8 and 10, $LSC$ 1 in case 1 and 3) can also obtain the empty container demander's revenue sharing for every satisfied empty container based on the $RSC$. In cases 2, 4, 7 and 9, it should be noted that the equilibrium $CT$ rate is 0, so the system can only be coordinated when the $CT$ rate is 0. Thus, the carriers' $PICTRs$ $p^{01}$ and $p^{02}$ are essential for the $LSC's$ non-negative profit increment and for the system coordination.

***Theorem 5.3*** *The system can be coordinated if: **i) Condition 4.1** is satisfied. **ii)** The RSC parameters $w$, $R_i$ follow the conditions in Table 5.1 so that **Condition 5.2** is satisfied. **iii)** The CT rate p should achieve equilibrium level $p^e$ and satisfy the rules shown in **Lemma 5.5** so that **Condition 5.3** is met.*

**Theorem 5.3** shows the system coordination conditions for the two $LSCs$ in the ten cases. These conditions concern the $RSC$ parameters selection and the $CT$ rate level. **Theorem 5.3** indicates that the system can be coordinated under certain conditions when $CT$ rate is an endogenous decision variable. Next, I will conduct a numerical example to verify the results or the conclusions that I gave in this chapter.

### 5.4. Numerical example

In this section, I conduct a numerical experiment to verify how $CT$ scheme affects system coordination. Firstly, I assume that the random variable $\xi_1, \xi_2, Y_1, Y_2$ follow normal distributions. Also, the other parameters such as $n_i, r_i, h_i$ and $g_i$ and the demand parameters $a_i$ and $b_i$ are all given in Table 5.5.

**Table 5.5**. The applied parameters in the numerical example ($i = 1$ and 2)

| | $\boldsymbol{n_i}$ | $\boldsymbol{r_i}$ | $\boldsymbol{h_i}$ | $\boldsymbol{g_i}$ | $\boldsymbol{a_i}$ | $\boldsymbol{b_i}$ | |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{LSC}$ 1 | 900 | 1500 | 200 | 300 | 800 | 15 | $\boldsymbol{c_t = 50}$ |
| $\boldsymbol{LSC}$ 2 | 900 | 1600 | 300 | 400 | 1000 | 25 | |

Based on Table 5.5, the random demand functions for the empty containers received by $LSC$ 1 and 2 are:

$$X_1 = 800 - 15p + \xi_1 \quad X_2 = 1000 - 25p + \xi_2$$

Where $\xi_1$ and $\xi_2$, $Y_1$ and $Y_2$ follow the normal distribution, which are assumed as:

$$\xi_1 \sim N(1650{,}600) \quad \xi_2 \sim N(1600{,}600) \quad Y_1 \sim N(300{,}300) \quad Y_2 \sim N(300{,}300)$$

Then, $\xi_i - Y_i$ ($i = 1$ *and* 2) follows:

$$\xi_1 - Y_1 \sim N(1350{,}900) \quad \xi_2 - Y_2 \sim N(1300{,}900)$$

Next, I show the appropriate $CT$ rate whilst ensuring that the system is coordinated. Firstly, I obtain the optimal $q^*$ and $p^*$ to maximise system profit are 60 and 20.95, in the centralised decision-making model as shown in Figure 5.7. Also, it is understandable that this case belongs to 3 shown in **Theorem 5.1**.

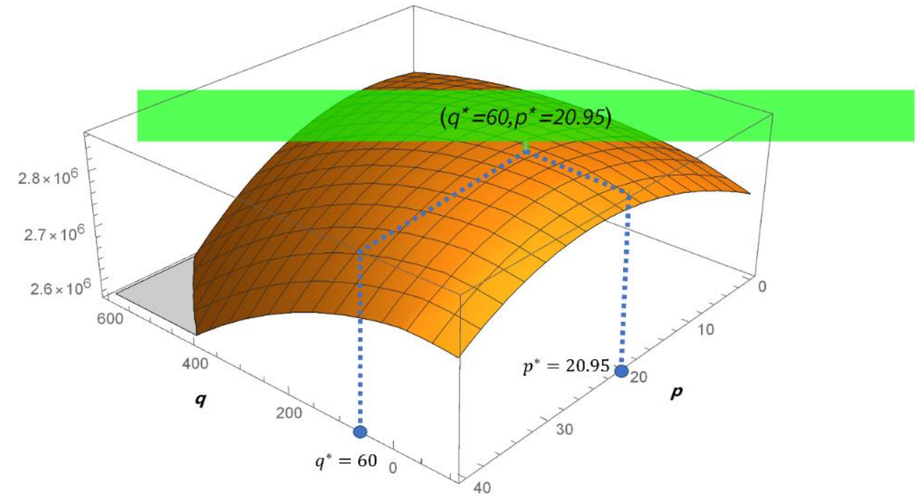


**Figure 5.7**. The optimal strategy of $q^*$ and $p^*$ in the centralised decision-making model.

Then, if the system is coordinated, then $q^* = q^e$ and $p^* = p^e$ must be satisfied and it relies on the $RSC$ parameter selection. According to case 3 in **Lemma 5.3**, $R_2$ and $w$ should be:

$$R_2 = \frac{300 \times 15 - 15 \times 2{,}000 \times Z_1(900 - 60\ - 800 + 15 * 20.95)}{25[Z_2(900 + 60\ - 1{,}000 + 25 * 20.95) - Z_2(900 - 1{,}000 + 25 * 20.95)]}$$
$$= \frac{2{,}300 \times Z_2(900 + 60 - 1{,}000 + 25 * 20.95) - 400}{Z_2(900 + 60\ - 1{,}000 + 25 * 20.95) - Z_2(900 - 1{,}000 + 25 * 20.95)} = 1{,}105$$
$$w = 2{,}000[1 - Z_1(900 - 60\ - 800 + 15 * 20.95)] - 200 + 50$$
$$- 1{,}105[1 - Z_2(900 + 60\ - 1{,}000 + 25 * 20.95)]$$
$$= (2{,}300 - 1{,}105)[1 - Z_2(900 + 60\ - 1{,}000 + 25 * 20.95)] - 300 = 677$$

So, the appropriate revenue $R_2$ that $LSC$ 2 shares with $LSC$ 1, should be 1,105. Then I obtain that $\emptyset_2$ is 0.309 based on $R_2 = (1 - \emptyset_2)r_2$, which means that for each empty container shared, $LSC$ 2 keeps 30.9% of the revenue and return 69% of the revenue to $LSC$ 1. In doing so, $q^* = q^e = 60$ and $p^* = p^e = 20.95$, are satisfied. Figure 5.8 shows the system profit and two $LSCs$' profit when $q^* = 60$ and $p^* = 20.95$, which are $\prod = 2{,}892{,}077$, $\pi_1 = 1{,}434{,}504$ and $\pi_2 = 1{,}457{,}575$. So it is proven that $\prod = \pi_1 + \pi_2$ and the total system profit in the centralised model and two $LSCs$' profit in the decentralised model are maximised. However, the two $LSCs$' profit increments must be non-negative. Otherwise, the $LSC$ would not accept the contract.

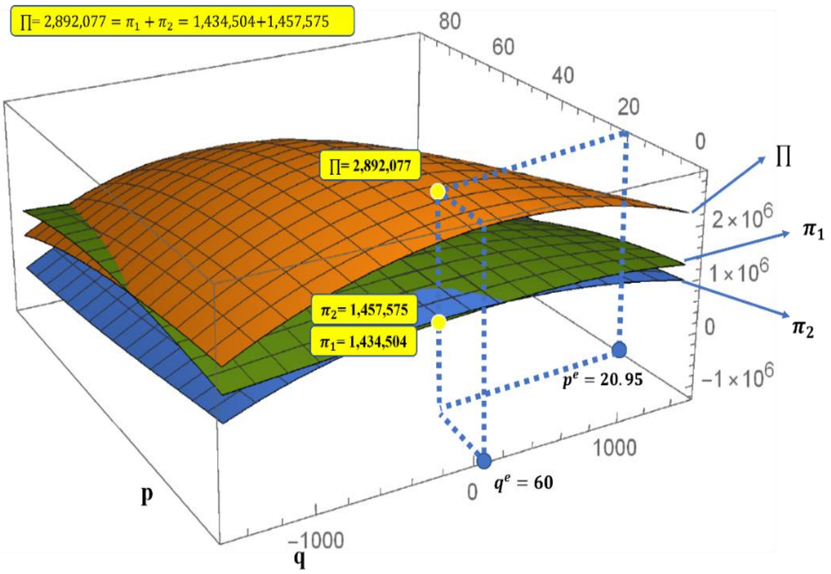


**Figure 5.8**. Given **Condition 5.1** and **5.2** (i.e., $R_2 = 1{,}105$ and $w = 677$), the total system profit in the centralised decision-making model and two $LSCs$' profits in the decentralised decision-making model.

Next, I will determine $PICTRs$ $p^{01} = 21.15$ and $p^{02} = 22.20$, respectively,

$$p^{01} = \frac{1}{15}\left[Z_1^{-1}\left(\frac{300}{2{,}000}\right) - 900 + 800\right] = 21.15;\ p^{02} = \frac{1}{25}\left[Z_2^{-1}\left(\frac{400}{2{,}300}\right) - 900 + 2300\right] = 22.20$$

Finally, I obtain the system profit increment in the centralised decision-making model by setting $q^e = 60$ and $p^e = 20.95$ and $LSC$ 2's profit increment under system coordination:

$$\Delta\prod(60, 20.95) = -2{,}000 \int_{900-800+15*21.15}^{900-60-800+15*20.95} Z_1(d_1)dd_1 - 2300 \int_{900-1{,}000+25*22.20}^{900+60-1{,}000+25*20.95} Z_2(d_2)dd_2$$
$$+ 60(200 - 2{,}000 + 2{,}300 - 300 - 50) + 300 * 15(20.95 - 21.15) + 400$$
$$* 25(20.95 - 22.20) = 1{,}725.38$$

$$
\begin{aligned}
&\Delta\pi_2(60, 20.95, 0, 1105, 677)\\
&\quad = -(2{,}300 - 1{,}105)\left[\int_{900-1{,}000+25*20.95}^{900\,+\,60-1{,}000+25*20.95} Z_2(d_2)dd_2 \right.\\
&\quad \left. - 60Z_2(900 + 60 - 1{,}000 + 25 * 20.95)\right]\\
&\quad - 2{,}300\left[\int_{900-1{,}000+25*22.20}^{900-1{,}000+25*20.95} Z_2(d_2)dd_2 + 25(22.20 - 20.95)\right] + (2{,}300 - 400)\\
&\quad * 25(22.20 - 20.95) = 289.77
\end{aligned}
$$

Thus, $LSC$ 1's profit increment in the decentralised decision-making model is $\Delta\pi_1 = 1{,}725.38 - 289.77 = 1435.61$, and both $LSCs'$ profit increments are positive, which means **Conditions 5.2** and **5.3** are both satisfied. Therefore, the system is coordinated. Moreover, it is easy to find that $LSC$ 1's profit increment accounts for $83.20\%$ of the whole system profit increment; meanwhile, $LSC$ 2's profit increment accounts for $16.80\%$. Overall, in this case, when the $CT$ rate is $20.95$ per empty container, if two $LSCs$ decide the revenue sharing per empty container is $1{,}105$, and the wholesale price is $677$, $LSC$ 1 should share $60$ empty containers with $LSC$ 2, and the system is coordinated. In this situation, the system profit in the centralised decision-making model is maximised, and two $LSCs$ in the decentralised decision-making model accept the optimal $ECS$ strategy as they both get profits from the sharing activity.

### 5.5. Conclusion

This chapter examines how the $CT$ affects the coordination of container sharing supply chains in the container shipping industry. I consider a stylised container shipping system where two $LSCs$ provide international freight transportation services and share empty containers between each other. Their customer demands are uncertain and inversely related to the endogenous change of $CT$ rate. I first consider the case of ideal and perfect collaboration where the two $LSC$ operate under a centralised decision-making model and follow the $ECS$ decisions made by a virtual central planner who has comprehensive information and aims to maximise the overall profit of the entire supply chain. By analysing the centralised decision-making model, I obtain the optimal number of empty containers to be shared subject to the endogenous impact of the $CT$ rate. Secondly, I design a decentralised decision-making model that assumes the two $LSCs$ are bound by a $RSC$ that determines the split of benefits generated from $ECS$ activity. Also, I determine the exact conditions that can make the $ECS$ supply chain coordinated and determine the two $LSCs'$ profit increment under the requirements of system coordination. This chapter provides some vital managerial insights into how the government $CT$ rate affects the container-sharing supply chain and its system coordination, and thereby may help governments design net zero strategies.

The main conclusions of this chapter are made as follows:

1. In the centralised decision-making model where two $LSCs$ perfectly cooperate, when $CT$ as an endogenous decision variable is imposed on two $LSCs$, two $LSCs$' $ECS$ system profit can be maximised. This is a same conclusion with the second conclusion in Chapter 4.

2. When the $CT$ as an endogenous decision variable is imposed on two $LSCs$, by applying a different contract $RSC$, this chapter found that two $LSCs$' individual $ECS$ strategies could also reach Nash equilibrium in a decentralised decision-making model. This is consistent with the conclusion in Chapter 4. However, unlike with Chapter 4, the $CT$ rate in the decentralised model also reaches the equilibrium level.

3. The two $LSCs$' $ECS$ system can be conditionally coordinated using an appropriate $RSC$ under the carbon taxation scheme. When the system is coordinated, the total system profit can be maximised, and both the $LSCs$ can be better off from $ECS$ activity.

4. In the decentralised model, two $LSCs$'profit increments involve three parts under system coordination: (1) the profit increase or decrease due to satisfied demand increase or decrease when equilibrium $CT$ is greater or less than $PICTRs$; (2) the profit from $ECS$ activity; (3) the $LSCs'$ loss or profit obtained because total demand decrease or increase when the equilibrium $CT$ is greater or less than $PICTRs$. Apart from the three parts, the empty container demander should pay a revenue price to the demander for every satisfied empty container.

5. From the two $LSCs$' perspective, the contract that they sign can affect their collaboration. Under the impact of equilibrium $CT$ rate, the number of two $LSCs$' satisfied empty container increases as long as the empty containers are shared between two $LSCs$. Thus, the two $LSCs$ can benefit from their collaboration based on a $RSC$ implementation. This conclusion is in accordance with Chapter 4's sixth conclusion.

6. The effect of $RSC$ on demander's (or borrower) empty containers' satisfied demand is greater in the initial stage of $ECS$ activity, but it drops when more empty containers are shared from the empty container supplier to the demander. Conversely, the impact of the contract implementation on increasing the empty container supplier's satisfied demand is insignificant initially, but it gradually increases when it shares more empty containers to the demander. This conclusion is also aligned with Chapter 4's sixth conclusion.

7. The equilibrium $CT$ rate does affect whether the system can be coordinated. Specifically, if the equilibrium $CT$ rate is greater than the $LSCs$' $PICTRs$, then both $LSCs$' satisfied demand increases due to the $CT$ rate variation. However, the impact of the $CT$ rate variation is significant in the initial stage, then the impact gradually decreases when the $CT$ rate increases until the $CT$ rate reaches the equilibrium level. In the process, the $LSCs$ also suffer the loss simultaneously because the total demand declines when the $CT$ rate is increased. Conversely, when the equilibrium $CT$ rate is less than the $LSC's$ $PICTRs$, both $LSCs$' satisfied demand

decreases because of the $CT$ rate variation impact. Interestingly, the impact is not significant in the initial stage, but the impact gradually rises when the $CT$ rate decreases until it reaches the equilibrium level. Also, $LSCs$ obtain profit as the total demand increases when the $CT$ rate drops. Overall, the equilibrium $CT$ rate leads to two opposite effects on both $LSCs$. This is partly agreeing with the conclusion made in Chapter 4. Table 5.6 clearly illustrates show the comparison of methods, analysis and conclusions among Chapter 4, Chapter 5 and the research conducted by Xie et al. (2017).

**Table 5.6** The comparison in terms of the applied method, analysis and conclusion between Chapter 4, 5 and the paper of Xie et al. (2017).

| ***Applied method, analysis and conclusion*** | ***(Xie et al., 2017)*** | ***Chapter 4*** | ***Chapter 5*** |
|---|---|---|---|
| Model applied | Intermodal system | Liner shipping system | Liner shipping system |
| $CT$ introduced? | X | √ | √ |
| $CT$ introduced as? | X | Constant parameters | Exogenous variable |
| Government interest involved? | X | X | X |
| How many players in the model? | One liner firm; one rail firm | Two $LSCs$ | Two $LSCs$ |
| Contract applied | $BBC$ | $BBC$ | $RSC$ |
| Centralised model involved? | √ | √ | √ |
| Centralised model optimised? | √ | √ | √ |
| Decentralised model involved? | √ | √ | √ |
| $ESC$ strategy's Nash equilibrium reached in decentralised model? | √ Pareto optimality | √ Pareto optimality | √ Pareto optimality |
| Equilibrium $CT$ reached? | Not applicable | X | √ |
| System coordinated? | √ | √ | √ |
| Two $LSCs'$ profit increment analysis? | X | √ | √ |
| The analysis of $CT$ rate impact on system coordination? | X | √ | √ |
| No LSC can be better off without hurting other's profit | √ | √ | √ |

In summary, the research in this chapter is conducted based on the paper of Xie et al. (2017) and Chapter 4. Although it further extended the research scope in the specific dimension, such as introduce the $CT$ as the decision variable rather than a constant parameter and replace the $BBC$ to the new $RSC$, it still includes some fatal flaws. For example, this chapter only proceed the $CT$ rate determination according to optimising the shipping company's business interest, but it should be the government to decide the $CT$ rate based on the target of maximising social welfare. Also, as the policy maker, the government is not included in the $ECS$ model, which is the biggest drawback in this chapter because the government's decision on $CT$ rate must interact with $ECS$ strategy. Unfortunately, this chapter has not discussed this interaction yet. Therefore, in the next section, I will finally try to fill these research gaps.

# Chapter 6 The Impact of Carbon Tax on Empty Container sharing and Coordination in a Stackelberg Game

## 6.1. Introduction

In Chapter 4 and Chapter 5, this research has already investigated how to coordinate a system in an Empty Container Sharing ($ECS$) supply chain between two Liner Shipping Carriers ($LSCs$) when the Carbon Tax ($CT$) scheme is imposed. In Chapter 4, the government $CT$ rate is considered as a constant parameter and the contract that is adopted between two $LSCs$ is the Buy-back Contract ($BBC$). In Chapter 5, the $CT$ rate is included in the model as an endogenous decision variable in the centralised and decentralised decision-making model respectively. Meanwhile, the Revenue-sharing Contract ($RSC$) is used for determining the $ECS$ strategy between the two $LSCs'$. Also, both Chapter 4 and Chapter 5 concentrate on the investigation of the conditions for coordinating the system, including the contract making. Chapter 5 gives the further condition of the $CT$ rate for the system coordination. However, there are some drawbacks existing in both chapters because first, in practice, the $CT$ usually fluctuates rather than being a constant; second, the $CT$ cannot be decided by the $LSCs$ for their business purposes, but is determined by the government, and the government must consider other factors in line with its social responsibility. Therefore, to fill the research gap and make the model more practical, in this chapter, I still mainly explore how the two $LSCs$ improve their profit from the $ECS$ activity given the impact of the government $CT$ rate. However, unlike with the assumptions made in Chapter 4 and Chapter 5, the $CT$ rate in this chapter is dynamic, and it is determined by the government in accordance with the target of achieving maximum total social welfare. In addition, in this chapter, I still adopt the $RSC$ as the agreement which determines the $ECS$ strategy in the decentralised decision-making model. I will also analyse in detail the impact of the $CT$ imposed by the government on the $LSCs$,and make suggestions to the $LSCs$ on how to adjust the $RSC$ to avoid having a negative profit increment when the dynamic $CT$ is imposed.

As the manager and regulator, the government should take social responsibility and maximise total social welfare. On the one hand, government should take action to reduce carbon emission. On the other hand, the government should avoid serious incidents (e.g., serious demand losses or even bankruptcy which the $LSCs$ may suffer because of the imposition of inappropriate $CT$) in shipping industry because they are critical to international transportation and even to the global economy. Thus, the government should formulate an appropriate $CT$ policy taking into consideration, to some extent, the carbon emission reduction and the $LSCs'$ stable operation simultaneously, i.e., achieving maximum social welfare. Then, the $LSCs$ can choose their

$ECS$ strategy, based on the government $CT$ policy, to reach maximum business profit.

Clearly, the government is the leader in the making of the $CT$ policy, and the $LSCs$ are the followers who make their decisions on the based on the $CT$ rate. Thus, both the $LSCs$ and the government form a Stackelberg game. I further develop the research method based on that in Chapter 4 and 5. Firstly, I assume that the two $LSCs$ work harmoniously and play the Stackelberg game with the government with the aim of maximising business profit and social welfare, respectively. This model where the two $LSCs$ completely cooperate is the centralised decision-making model. Secondly, the two $LSCs$ adopt an $RSC$ to decide the $ECS$ strategy in a decentralised decision-making model, given the impact of the government $CT$. Finally, I try to make an appropriate contract to ensure that the optimal $ECS$, which is made in the Stackelberg game in the centralised decision-making model, is equivalent to the strategy made in the decentralised decision-making model when the government $CT$ rate is imposed. In doing so, the system can be coordinated. Overall, this chapter will mainly investigate 1) In a Stackelberg game, how two the $LSCs$ (the follower) share empty containers to improve their profit given the government (the leader) $CT$ impact, and whether the system can be coordinated. 2) how the government $CT$ scheme, as the decision variable, affects the system coordination. 3) how the government $CT$ scheme influences the contract implementation under the system coordination.

This chapter is organised as follows. Section 6.2 develops the centralised decision-making models in the Stackelberg game. In section 6.3, the decentralised decision-making model will be built. In section 6.4, I will explore how the system is coordinated and how the government $CT$ scheme affects $ECS$ activity and the system coordination. I will also analyse how the $LSCs$ adjust the contract to maintain system coordination. Also, a numerical example is set up in section 6.5. The conclusions of this chapter will be made in section 6.6.

## 6.2. The centralised decision-making model and Stackelberg equilibrium

This section will determine how two $LSCs$ make the optimal $ECS$ strategy in practice, when the government is the leader in deciding the $CT$ rate in a Stackelberg game.

### 6.2.1. The centralised decision-making model

In this subsection, the system profit, which consists of the profits of the two $LSCs'$, in the centralised decision-making model and the government social welfare model are explored.

***a. Two $LSCs'$ centralised decision-making model***

In this model, when the government imposes the $CT$, the two $LSCs$ cooperate seamlessly, as if a virtual planner has had all the information and has decided the optimal number of empty containers to share in order to maximise their total combined profit. As this research has comprehensively investigated the centralised decision-making model in Chapter 4 (section 4.2.1), I can immediately give the centralised decision-making model's profit function $\prod_{cen}(q,p)$. For the details of the calculation, please refer to Appendix A.

$$\prod_{cen}(q,p) = \alpha_1[\beta_1 - \Phi_1(\beta_1)] + \alpha_2[\beta_2 - \Phi_2(\beta_2)] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p) - c_t|q| \quad \textbf{6.1}$$

$\Phi_i$ is the complementary loss function of $\xi_i - Y_i$ $(i = 1,2)$. In addition, $\alpha_i = r_i + h_i + g_i$ is defined as the all-in-revenue of the two $LSCs$. Furthermore, for a given tax rate $p$, the number of empty containers on-hand after the $LSCs'$ $ECS$ activity, and the meeting of customer demands, is denoted as $\beta_i = n_i - |q| - a_i + b_i p$. If the empty containers are transferred from $LSC$ 1 to 2, then $q > 0$, while if the empty containers are shared in the opposite direction, then $q < 0$.

***b. The government's social welfare model***

It includes 1) the two $LSCs'$ profit $\pi_1$ and $\pi_2$; 2) the transport cost $c_t$ for each shared empty container between the two $LSCs$; 3) the $CO_2$ treatment cost $C_g$ for each container of satisfied demands.

$$\begin{aligned}\prod_{gov}(q,p) &= \pi_1(q,p) + \pi_2(q,p) - c_t|q| - C_g[\boldsymbol{E}S_1(q,p) + \boldsymbol{E}S_2(q,p)] \\ &= r_1\boldsymbol{E}S_1(q,p) - h_1\boldsymbol{E}I_1(q,p) - g_1\boldsymbol{E}L_1(q,p) + r_2\boldsymbol{E}S_2(q,p) - h_2\boldsymbol{E}I_2(q,p) \\ &\quad - g_2\boldsymbol{E}L_2(q,p) - c_t|q| - C_g[\boldsymbol{E}S_1(q,p) + \boldsymbol{E}S_2(q,p)]\end{aligned} \quad \textbf{6.2}$$

The equation 6.2 can be rewritten as (Appendix P)

$$\prod_{gov}(q,p) = (\alpha_1 - C_g)[\beta_1 - \Phi_1(\beta_1)] + (\alpha_2 - C_g)[\beta_2 - \Phi_2(\beta_2)] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + (r_1 - C_g)\boldsymbol{E}(Y_1 + a_1 - b_1 p) + (r_2 - C_g)\boldsymbol{E}(Y_2 + a_2 - b_2 p) - c_t|q| \quad \textbf{6.3}$$

Both the government and the virtual planner in the above model tend to choose the optimal $CT$ rate and the optimal $ECS$ amount from their own perspective. Their decisions are made sequentially. The government determines the $CT$ rate first, and the virtual planner determines the optimal $ECS$ strategy afterwards based on a tax rate determined by the government. Therefore, the virtual planner and the government's decision-making form a Stackelberg game, where the government is the leader, and the virtual planner is the follower.

### 6.2.2. Stackelberg equilibrium

In this subsection, the Stackelberg equilibrium of the government $CT$ rate $p_S^e$ , and that of the virtual planner's $ECS$ strategy $q_S^e$ will be solved. In the Stackelberg game, the government proposes a $CT$ rate $p$ first; then, the virtual planner decides the optimal sharing strategy $q$ between the two $LSCs$ according to the $CT$ rate $p$ in order to maximise the whole system's profit. Also, to maximise the total social welfare, the government should further adjust the $CT$ rate according to the $LSCs'$ sharing strategy. After a certain number of interactions between the virtual planner and the government, the decision variables $q$ and $p$ in the game will reach the

Stackelberg equilibrium. The process for the game to reach the equilibrium has been illustrated in Table 3.5. For convenience, I outline the process again in Table 6.1.

**Table 6.1** Four steps of the Stackelberg game

| | |
|---|---|
| Step 1. | The government proposes a $CT$ rate $p$ per container. |
| Step 2. | According to the rate $p$, the virtual planner decides the optimal sharing strategy $q^*(p)$ to maximise the system profit in the centralised model. |
| Step 3. | The government adjusts the tax rate $p$ if social welfare can be further improved. Otherwise, the government keeps the rate, and the game reaches the Stackelberg equilibrium. Denoted it as $p_S^e$. |
| Step 4. | Based on $p_S^e$, the virtual planner decides the Stackelberg equilibrium $q_S^e$ to maximise the total $LSCs$' profit. |

Next, **Lemma 6.1** demonstrates how the virtual planner optimally acts to determine the $ECS$ strategy for a given $CT$ rate $p$ (Step 1 and 2). (Appendix Q)

***Lemma 6.1*** *Given* $p$, $\prod_{cen}(q,p)$ *is strictly concave in* $q$, *and the optimal* $ECS$ *strategy is:*

$$q^*(p) = \begin{cases} n_1 & 0 < n_1 < \dot{q}^* & \text{case 1} \\ \dot{q}^* & 0 < \dot{q}^* < n_1 & \text{case 2} \\ 0 & \dot{q}^* < 0 < \ddot{q}^* & \text{case 3} \\ \ddot{q}^* & -n_2 < \ddot{q}^* < 0 & \text{case 4} \\ -n_2 & \ddot{q}^* < -n_2 < 0 & \text{case 5} \end{cases}$$

**Lemma 6.1** shows that no matter how the government changes the $CT$ rate, the virtual planner can always identify the optimal $ECS$ strategy in the centralised decision-making model. Moreover, not only is $q^*(p)$ related to the $CT$ rate, but it is also decided by the two $LSCs$' inventory level $[-n_2, n_1]$. When $q^*(p)$ is greater than $n_1$ or less than $-n_2$ in case 1 and case 5, respectively, the empty container supplier should lend all the empty containers to the other. Then $q^*(p)$ equals $n_1$ or $-n_2$, respectively. Moreover, $q^*(p)$ equals $\dot{q}^*$ when $q^*(p)$ is in $[0, n_1]$ or $\ddot{q}^*$ when $q^*(p)$ is in $[-n_2, 0]$, where $\dot{q}^*$ and $\ddot{q}^*$ satisfy:

$$\frac{\partial \prod_{cen}(\dot{q}^*, p)}{\partial q} = 0;\ 0 < \dot{q}^* < n_1 \ and\ \frac{\partial \prod_{cen}(\ddot{q}^*, p)}{\partial q} = 0;\ -n_2 < \ddot{q}^* < 0$$

However, if $\dot{q}^*$ is less than 0 and $\ddot{q}^*$ is greater than 0, then $q^*(p)$ is 0, indicating that the two $LSCs$ should not share any empty containers. For more clarity, **Corollary 6.1** shows the conditions that $q^*(p)$ satisfies in each case. (Appendix Q)

***Corollary 6.1*** *Given* $p$ *in the centralised decision-making model,* $q^*(p)$ *must satisfy the condition for each case as shown in Table 6.2*

**Table 6.2** The condition of $q^*(p)$ in five cases

| Case | Conditions |
|---|---|
| **1** | $\alpha_1 Z_1(-a_1 + b_1 p) - \alpha_2 Z_2(n_2 + n_1 - a_2 + b_2 p) > (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$ |
| **2** | $\alpha_1 Z_1(n_1 - \dot{q}^* - a_1 + b_1 p) - \alpha_2 Z_2(n_2 + \dot{q}^* - a_2 + b_2 p) = (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$ |
| **3** | $\alpha_1 Z_1(n_1 - \dot{q}^* - a_1 + b_1 p) - \alpha_2 Z_2(n_2 + \dot{q}^* - a_2 + b_2 p) < (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$<br>$\alpha_1 Z_1(n_1 - \ddot{q}^* - a_1 + b_1 p) - \alpha_2 Z_2(n_2 + \ddot{q}^* - a_2 + b_2 p) > (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$ |
| **4** | $\alpha_1 Z_1(n_1 - \ddot{q}^* - a_1 + b_1 p) - \alpha_2 Z_2(n_2 + \ddot{q}^* - a_2 + b_2 p) = (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$ |
| **5** | $\alpha_1 Z_1(n_1 + n_2 - a_1 + b_1 p) - \alpha_2 Z_2(-a_2 + b_2 p) < (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$ |

For the Step 2, given $q^*(p)$, the government can further decide the $CT$ rate to maximise $\prod_{gov}[q^*(p), p]$ in the centralised decision-making model, i.e.,

$$Max \prod_{gov}(q^*, p); \;\; s.t. \;\; q^*(p) = n_1, \dot{q}^*, 0, \ddot{q}^*, -n_2$$

**Lemma 6.2** demonstrates that $\prod_{gov}(q^*, p)$ is strictly concave in $p$. However, **Condition 6.1** should firstly be met to ensure the concavity of $\prod_{gov}[q^*(p), p]$. (Appendix R)

***Condition 6.1*** $\alpha_i \geq C_g$ $(i = 1, 2)$

**Condition 6.1** indicates that the $LSCs$' all-in-revenue $\alpha_i$ obtained from each container of satisfied demand must be no less than the carbon emission treatment cost per empty container. Otherwise, the $LSC$ cannot obtain any profits from the shipping activity because of the high carbon emissions treatment cost. In the following, I will present **Lemma 6.2** for the case where **Condition 6.1** is met. (Appendix R)

***Lemma 6.2*** *Given **Condition 6.1** and $q^*$, $\prod_{gov}(q^*, p)$ is strictly concave in $p$.*

The government choose the optimal $CT$ rate to maximise the total social welfare, which reaches the Stackelberg equilibrium. **Lemma 6.2** demonstrates that no matter how the virtual planner changes the $ECS$ strategy, the government can always make the equilibrium $CT$ rate $p_s^e$ to maximise the total social welfare. Further, once $p_s^e$ is determined, the virtual planner's Stackelberg equilibrium sharing strategy can be decided. For a given $p_s^e$, **Theorem 6.1** gives the virtual planner's Stackelberg equilibrium $ECS$ strategy $q_s^e$ to maximise $\prod_{cen}(q, p_s^e)$. (Appendix S)

***Theorem 6.1*** *Given $p^e$, the $q_s^e(p_s^e)$ in Stackelberg game is:*

$$q_s^e = \begin{cases} n_1 & 0 < n_1 < \dot{q}_s^e & \text{case 1} \\ \dot{q}_s^e & 0 < \dot{q}_s^e < n_1 & \text{case 2} \\ 0 & \dot{q}^e < 0 < \ddot{q}^e & \text{case 3} \\ \ddot{q}_s^e & -n_2 < \ddot{q}_s^e < 0 & \text{case 4} \\ -n_2 & \ddot{q}_s^e < -n_2 < 0 & \text{case 5} \end{cases}$$

Given $p_s^e$, as the follower, the virtual planner can decide the Stackelberg equilibrium $ECS$ strategy $q_s^e$ to maximise the system profit. Therefore, in this situation, both the government (the leader) and the virtual planner (the follower) finally make the equilibrium decision in the Stackelberg game. However, as in Chapter 4 and Chapter 5, the centralised decision-making model reflects an ideal case under perfect collaboration. However, the two $LSCs$ cannot fully cooperate in practice. In the next section, a decentralised decision-making model will be developed to investigate how the $LSCs$ make an optimal $ECS$ strategy under a $RSC$.

### 6.3. Decentralised decision-making model

In this section, a more practical model is developed based on a decentralised decision-making system, where each $LSC$ makes decisions independently but follows a $RSC$ aiming to maximise their own business profits. The assumption is that $p_s^e$ is the $CT$ rate imposed by the government on the two $LSCs$ in the decentralised decision-making model. The two $LSCs$ operate independently in the decentralised decision-making model, and they always seek to maximise their business profit ‘selfishly’. It should be noticed that the meaning of ‘selfishly’ in this context indicates that the two $LSCs$ compete with each other in a mature market system instead of actually acting ‘selfishly’ or diminishing the other one. Therefore, except for the situation where the two $LSCs$’ $ECS$ strategy in the decentralised decision-making model is equivalent to that in the centralised model (i.e., system coordination), the sharing strategy in the decentralised model is always not as optimal as $q_s^e$.

In this chapter, as in Chapter 5, a $RSC$ is adopted in the decentralised decision-making model to incentivise the two $LSCs$ to make decisions that lead to the same performance as the centralised decision-making model. Similarly to Chapter 5, in this chapter, the contract works as follows. $LSC$ 1 charges $LSC$ 2 a wholesale price $w$ per empty container, if $LSC$ 2 requires the extra empty containers from $LSC$ 1. In return, $LSC$ 2 needs to share part of its revenue generated through the shared containers with $LSC$ 1, and vice versa. Let $\emptyset_1$ denote the percentage of revenue that $LSC$ 1 keeps, hence, $1-\emptyset_1$ is the percentage of the revenue that $LSC$ 1 shares with $LSC$ 2. Similarly, $\emptyset_2$ is the percentage of revenue that $LSC$ 2 keeps, and $1-\emptyset_2$ is the percentage of revenue that $LSC$ 2 shares with $LSC$ 1. So, the transfer payment $\theta$ between the two $LSCs$ is:

$$\begin{aligned}\theta(q, p_s^e, w, \emptyset_1, \emptyset_2) \\ &= wq^+ + (1-\emptyset_2)r_2[q^+ - \boldsymbol{E}\min\{(n_2+q+Y_2-X_2)^+, q^+\}] - w(-q)^+ \\ &- (1-\emptyset_1)r_1[(-q)^+ - \boldsymbol{E}\min\{(n_1-q+Y_1-X_1)^+, (-q)^+\}]\end{aligned} \quad \textbf{6.4}$$

Where $R_1 = (1-\emptyset_1)r_1$ means the revenue that $LSC$ 1 shares with $LSC$ 2, and $R_2 = (1-\emptyset_2)r_2$ is the revenue that $LSC$ 2 shares with $LSC$ 1. The equation 6.4 can be further written as equation 6.5: (Appendix K)

$$\begin{aligned}\theta(q, p_s^e, w, R_1, R_2) \\ &= q^+(w+R_2) - (-q)^+(w+R_1) - R_2\boldsymbol{E}\min\{(n_2+q+Y_2-X_2)^+, q^+\} \\ &+ R_1\boldsymbol{E}\min\{(n_1-q+Y_1-X_1)^+, (-q)^+\}\end{aligned} \quad \textbf{6.5}$$

According to equation 6.5, if $\theta$ is positive, the shared revenue is paid from $LSC$ 2 to $LSC$ 1, and vice versa for negative $\theta$. Next, $LSC$ 1's profit function is obtained by:

$$\begin{aligned}\pi_1(q, p_s^e, w, R_1, R_2) \\ &= -\alpha_1\Phi_1(\beta_1) + R_2[\Phi_2(\beta_2 - q) - \Phi_2(\beta_2)] - R_1[\Phi_1(\beta_1 + q) - \Phi_1(\beta_1)] \\ &\quad - (-q)^+(R_1 + w) + q^+(R_2 + w - c_t) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p_s^e) + (\alpha_1 - h_1)\beta_1 \\ &\quad - g_1\boldsymbol{E}(\xi_1 - Y_1)\end{aligned} \tag{6.6}$$

Similarly, $LSC$ 2's profit function is:

$$\begin{aligned}\pi_2(q, p_s^e, w, R_1, R_2) \\ &= -\alpha_2\Phi_2(\beta_2) + R_2[\Phi_2(\beta_2) - \Phi_2(\beta_2 - q)] - R_1[\Phi_1(\beta_1) - \Phi_1(\beta_1 + q)] - q^+(w \\ &\quad + R_2) + (-q)^+(w - c_t + R_1) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p_s^e) + (\alpha_2 - h_2)\beta_2 \\ &\quad - g_2\boldsymbol{E}(\xi_2 - Y_2)\end{aligned} \tag{6.7}$$

Also, to be consistent with the container movement direction of $q_s^e$ in the centralised model, $q_1^+$ and $q_2^+$ are defined as the number of empty containers that $LSC$ 1 could supply, and $LSC$ 2 requires, in the decentralised decision-making model. Similarly, $q_1^-$ and $q_2^-$ stand for the number of empty containers that $LSC$ 2 could offer, and $LSC$ 1 wants to request, respectively. Next, for a given $p_s^e$, to find the Nash equilibrium quantity of $ECS$ activity between the two $LSCs$ in the decentralised model, the two $LSCs'$ optimal $ECS$ strategy should be determined respectively, which requires their profit functions to be concave. So, **Condition 6.2** is introduced to ensure the concavity of the two $LSCs'$ profit functions in the decentralised model (Appendix T).

***Condition 6.2*** $\alpha_i \geq R_i;\ i = 1, 2$

**Condition 6.2** ensures the concavity of $\pi_1$ and $\pi_2$ in the decentralised decision-making model for a given $p_s^e$. It is clear that a specific $LSC's$ all-in-revenue per satisfied empty container should be no more than the revenue that it shares with the other $LSC$. Otherwise, the $LSC$ may suffer a loss for every satisfied empty container in the $ECS$ activity, and the $LSC$ would not accept the contract. Given **Condition 6.2** and $p_s^e$, **Lemma 6.3** will be proposed to show that $\pi_1$ and $\pi_2$ are strictly concave in $q$.

***Lemma 6.3***. *given $p_s^e$, $\pi_1(q, p_s^e, w, R_1, R_2)$ and $\pi_2(q, p_s^e, w, R_1, R_2)$ are strictly concave in $q$.*

**Lemma 6.3** indicates that no matter how one $LSC$ changes its $ECS$ strategy, the other one has its optimal $ECS$ strategy under a $CT$ rate $p_s^e$. In other words, each $LSCs$ tends to selfishly apply its $ECS$ strategy without considering the other's optimal strategy. Therefore, for a given $p_s^e$, the two $LSCs$ in the decentralised decision-making model also form a game and the Nash equilibrium of $q$ is $q_d^e$, which is called Pareto optimal and is formulated as:

$$q_d^e = \min\{q_1^+(p_s^e), q_2^+(p_s^e)\} - \min\{q_1^-(p_s^e), q_2^-(p_s^e)\}$$

Please note that the $q_d^e$ has only three possible results as follows:

$$q_d^e = \begin{cases} \min\{q_1^+(p_s^e), q_2^+(p_s^e)\} \\ -\min\{q_1^-(p_s^e), q_2^-(p_s^e)\} \\ 0 \end{cases}$$

The results of $q_d^e$ further demonstrates that the sign of $q_d^e$ in the decentralised decision-making model complies with the sign of the $q_s^e$ in the centralised decision-making model. **Lemma 6.3** reveals that the two $LSCs$ in the decentralised decision-making model can reach the Pareto optimality in which no $LSC$ can get a better payoff without having a detrimental impact on the other $LSC's$ profit.

### 6.4. Coordination

As set out in Chapter 4 and Chapter 5, in supply chain management, coordination can be achieved when the retailer's order quantity maximises the entire supply chain profit, and simultaneously the supplier also accepts the retailer's order quantity. Similar to the definition shown in Chapter 4 and Chapter 5, in the context of this chapter, the system coordination implies that the $ECS$ strategy (Pareto optimal) $q_d^e$ in the decentralised decision-making model is equivalent to the $ECS$ strategy (Stackelberg equilibrium) $q_s^e$ in the centralised decision-making model, for a given $p_s^e$. Therefore, for a given $p^e$, the system will be coordinated by choosing appropriate $RSC$ parameters (i.e., $R_1, R_2$ and $w$) to make $q_d^e$ equals $q_s^e$. This will ensure the $ECS$ strategy in the practical decentralised decision-making model is equivalent to the ideal $ECS$ strategy in the centralised decision-making model. **Condition 6.3** shows the first condition for the system coordination.

***Condition 6.3*** $\exists\, R_1, R_2$ *and* $w$, *making* $q_s^e = q_d^e$

Apart from **Condition 6.3**, another condition for the system coordination is that each $LSC$ can gain a non-negative profit. Otherwise, the $LSC$ would not sign the contract since it would lose profits during the $ECS$ activity. So, to meet the condition, the two $LSCs$' profit increments in the decentralised decision-making model are defined as the following expressions: (Appendix U)

$$\begin{aligned} \Delta\pi_1(q_d^e, p_s^e, w, R_1, R_2) &= \pi_1(q_d^e, p_s^e, w, R_1, R_2) - \pi_1(0,0,w,R_1,R_2) \\ \Delta\pi_2(q_d^e, p_s^e, w, R_1, R_2) &= \pi_2(q_d^e, p_s^e, w, R_1, R_2) - \pi_2(0,0,w,R_1,R_2) \end{aligned} \quad \textbf{6.8}$$

The profit increment of the two $LSCs$ represents the increased or decreased profits that the two $LSCs$ get between the scenarios, with and without the impact of $p_s^e$ and the $ECS$ activity simultaneously. Thus, the scenario where no $CT$ is imposed and there is no $ECS$ activity provides the baseline for the profit increment calculation. This does not consist with the assumed $PICTR$ (Preferred Ideal Carbon Tax Rate) in Chapter 5 because in this chapter, the $CT$ rate is determined by the government as opposed to being selected by the $LSCs$.

***Condition 6.4*** $\Delta\pi_1(q_d^e, p_s^e, w, R_1, R_2) \geq 0$; $\Delta\pi_2(q_d^e, p_s^e, w, R_1, R_2) \geq 0$

As with the statements of the second condition for the system coordination shown in **Condition 4.3** and **Condition 5.3** in the two previous chapters, **Condition 6.4** is made for ensuring the system coordination in this chapter, and equation 6.9 shows the relationship between $\Delta\prod_{cen}$, $\Delta\pi_1$ and $\Delta\pi_2$.

$$\Delta\prod_{cen}(q_d^e, p_s^e) = \Delta\pi_1(q^e, p_s^e, w, R_1, R_2) + \Delta\pi_2(q^e, p_s^e, w, R_1, R_2) = \prod_{cen}(q_d^e, p^e) - \prod_{cen}(0,0) \qquad \textbf{6.9}$$

If one of the $LSCs$' profit increment is negative, then the other $LSC$ obtains a profit increment more than $\Delta\prod_{cen}(q_d^e, p_s^e)$, and vice versa. If this situation happens, the system is not coordinated. Next, for a given $p_s^e$, to achieve the system coordination, **Condition 6.3** should firstly be satisfied, which requires appropriate contract parameters to be set to make $q_s^e = q_d^e$. Thus, **Lemma 6.4** is presented below (Appendix U).

***Lemma 6.4***. *Given* ***Condition 6.1*** *and* ***6.2*** *and* $p_s^e$*, in order to meet* ***Condition 6.3****,* $R_1$*,* $R_2$ *and* $w$ *should be constrained in each case as follows shown in Table 6.3: where* $\bar{Z}_i(.) = 1 - Z_i(.)$

**Table 6.3** The condition of $R_1$, $R_2$ and $w$ in five cases, given **Condition 6.3**

| Case | Conditions |
|---|---|
| 1 | $\alpha_1\bar{Z}_1(-a_1 + b_1p_s^e) + c_t - h_1 \le w + R_2\bar{Z}_2(n_2 + n_1 - a_2 + b_2p_s^e)$ $\le \alpha_2\bar{Z}_2(n_2 + n_1 - a_2 + b_2p_s^e) - h_2$ |
| 2 | $\alpha_1\bar{Z}_1(n_1 - \dot{q}_s^e - a_1 + b_1p_s^e) + c_t - h_1 = w + R_2\bar{Z}_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e)$ $= \alpha_2\bar{Z}_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e) - h_2$ |
| 3 | $\alpha_2\bar{Z}_2(n_2 - a_2 + b_2p_s^e) - h_2 \le w + R_2\bar{Z}_2(n_2 - a_2 + b_2p_s^e)$ $\le \alpha_1\bar{Z}_1(n_1 - a_1 + b_1p_s^e) - h_1 + c_t$ <br> $\alpha_1\bar{Z}_1(n_1 - a_1 + b_1p_s^e) - h_1 \le w + R_1\bar{Z}_1(n_1 - a_1 + b_1p_s^e) \le \alpha_2\bar{Z}_2(n_2 - a_2 + b_2p_s^e) - h_2 + c_t$ |
| 4 | $\alpha_2\bar{Z}_2(n_2 + \ddot{q}_s^e - a_2 + b_2p_s^e) + c_t - h_2 = w + R_1\bar{Z}_1(n_1 - \ddot{q}_s^e - a_1 + b_1p_s^e)$ $= \alpha_1\bar{Z}_1(n_1 - \ddot{q}_s^e - a_1 + b_1p_s^e) - h_1$ |
| 5 | $\alpha_2\bar{Z}_2(-a_2 + b_2p_s^e) + c_t - h_2 \le w + R_1\bar{Z}_1(n_1 - \ddot{q}^e - a_1 + b_1p_s^e)$ $\le \alpha_1\bar{Z}_1(n_1 - \ddot{q}^e - a_1 + b_1p_s^e) - h_1$ |

Given $p_s^e$, **Lemma 6.4** shows the constraints of $R_1, R_2$ and $w$ in each case, which leads to the satisfaction of **Condition 6.3**. It should be noted that $R_1$ is 0 in cases 1 and 2, and $R_2$ is 0 in cases 4 and 5. To allow $q_s^e = q_d^e$, in cases 1 and 5, the monotonicity of $\pi_1$ and $\pi_2$ in the decentralised decision-making model should be synchronised with $\prod_{cen}$ in the centralised decision-making model between $[-n_2, n_1]$. In both cases, each $LSCs$' should lend all inventories to their partner. In cases 2 and 4, both $\pi_1$ and $\pi_2$ should be strictly concave in $q_d^e$. Therefore, $R_1$, $R_2$ and $w$ are constrained within an equation in the two cases instead of the inequality shown in cases 1 and 5. Case 3 is exceptional as it shows that $LSC$ 1 and $LSC$ 2 should borrow an empty container when $q > 0$ and $q < 0$, respectively, which is supposed to be the opposite of the previous assumption of empty container directionality. Thus, the two $LSCs$ should maintain the status quo, and no empty container should be shared between them. Moreover, given **Condition 6.3**, when the condition shown in

**Lemma 6.4** is strictly followed, the two $LSCs'$ profit increments (i.e., $\Delta\pi_1$ and $\Delta\pi_2$) are obtained in order to further explore the impact of the imposition of $CT$ and $ECS$ activity on $\Delta\pi_1$ and $\Delta\pi_2$, and to analyse the nonnegativity of $\Delta\pi_1$ and $\Delta\pi_2$ (Appendix U).

***Lemma 6.5****. Given* ***Condition 5.1****,* ***5.2*** *and* ***5.3****, given* $p^e$, $\Delta\pi_1$ *and* $\Delta\pi_2$ *are:*

*For cases 1 and 2:*

$$\begin{cases}\Delta\pi_1 = \alpha_1[\Delta S_1^e(q_d^e,p_s^e) + \Delta S_1(0,p_s^e)] + R_2\Delta S_2^e(q_d^e,p_s^e) - (\alpha_1 - g_1)b_1p_s^e \\ \Delta\pi_2 = (\alpha_2 - R_2)\Delta S_2^e(q_d^e,p_s^e) + \alpha_2\Delta S_2(0,p_s^e) - (\alpha_2 - g_2)b_2p_s^e\end{cases}$$

*For cases 4 and 5:*

$$\begin{cases}\Delta\pi_1 = (\alpha_1 - R_1)\Delta S_1^e(q_d^e,p_s^e) + \alpha_1\Delta S_1(0,p_s^e) - (\alpha_1 - g_1)b_1p_s^e \\ \Delta\pi_2 = \alpha_2[\Delta S_2^e(q_d^e,p_s^e) + \Delta S_2(0,p_s^e)] + R_1\Delta S_1^e(q_d^e,p_s^e) - (\alpha_2 - g_2)b_2p_s^e\end{cases}$$

*For case 3:* $\Delta\pi_1 = \alpha_1\Delta S_1(0,p_s^e) + (\alpha_1 - g_1)b_1p_s^e$ ; $\Delta\pi_2 = \alpha_2\Delta S_2(0,p_s^e) + (\alpha_2 - g_2)b_2p_s^e$

In cases 1 and 2, $LSC$ 1 and $LSC$ 2 are the empty container supplier and demander, respectively, while their roles are opposite in cases 4 and 5. $\Delta S_i^e(q_d^e,p_s^e)$ is the increment of expected satisfied demand between the scenarios with and without $ECS$ activity for a given $p_s^e$. $\Delta S_i(0,p_s^e)$ represents the increment of expected satisfied demand between the scenarios with and without $CT$ impact before the $ECS$ activity. Thus, $\Delta\pi_1$ and $\Delta\pi_2$ are affected by both the $ECS$ activity and the government $CT$. Now I examine the $ECS$ activity impact on the profit increment of both $LSCs'$. For the demander, under the impact of tax rate $p_s^e$, the $ECS$ activity affects the demander's satisfied demand $\Delta S_{demander}^e(q_d^e,p_s^e)$. For each shared and satisfied empty container from the supplier to the demander, the demander can earn $\alpha_2 - R_2$ because it has to share a proportion of the revenue with the supplier. Interestingly, as shown in Figure 6.1 (cases 1 and 2) and Figure 6.2 (cases 4 and 5), $\Delta S_{demander}^e(q_d^e,p_s^e)$ is always positive as long as empty containers are shared between the two $LSCs$. Note that the $CT$ impact is not included since $p_s^e$ is constant when $\Delta S_i^e(q_d^e,p_s^e) = S_i^e(q_d^e,p_s^e) - S_i^e(0,p_s^e)$ is examined. Also, as can be seen from Figures 6.1 and 6.2, when the $ECS$ activity just starts, the marginal increment of $\Delta S_{demander}^e(q_d^e,p_s^e)$ is high. However, with more and more empty containers shared from $LSC$ 1 to $LSC$ 2, the marginal increment of $\Delta S_{demander}^e(q_d^e,p_s^e)$ gradually decreases. This means that the impact of $ECS$ activity on the demander's profit increment drops as more and more empty containers are shared.

For the supplier, $\Delta S_{supplier}^e(q_d^e,p_s^e)$ is also positive if the $ECS$ activity is conducted. Although the marginal increment of $\Delta S_{supplier}^e(q_d^e,p_s^e)$ is low initially, it gradually increases as more and more empty containers are shared by the supplier to the demander (Figures 6.3 and 6.4). So, given $p_s^e$, combined with the analysis of $\Delta S_{demander}^e(q_d^e,p_s^e)$, the demander gets profit increments more "quickly" than the supplier at the initial stage of the $ECS$ activity, while the supplier takes the profit increment more "quickly" than the demander at the later stage. It should be noted that the supplier can receive the extra revenue sharing from the demander for every shared and satisfied empty container. All the analysis above is consistent with its counterpart shown in section 5.3 (Figure 5.1 and Figure 5.4) in Chapter 5.

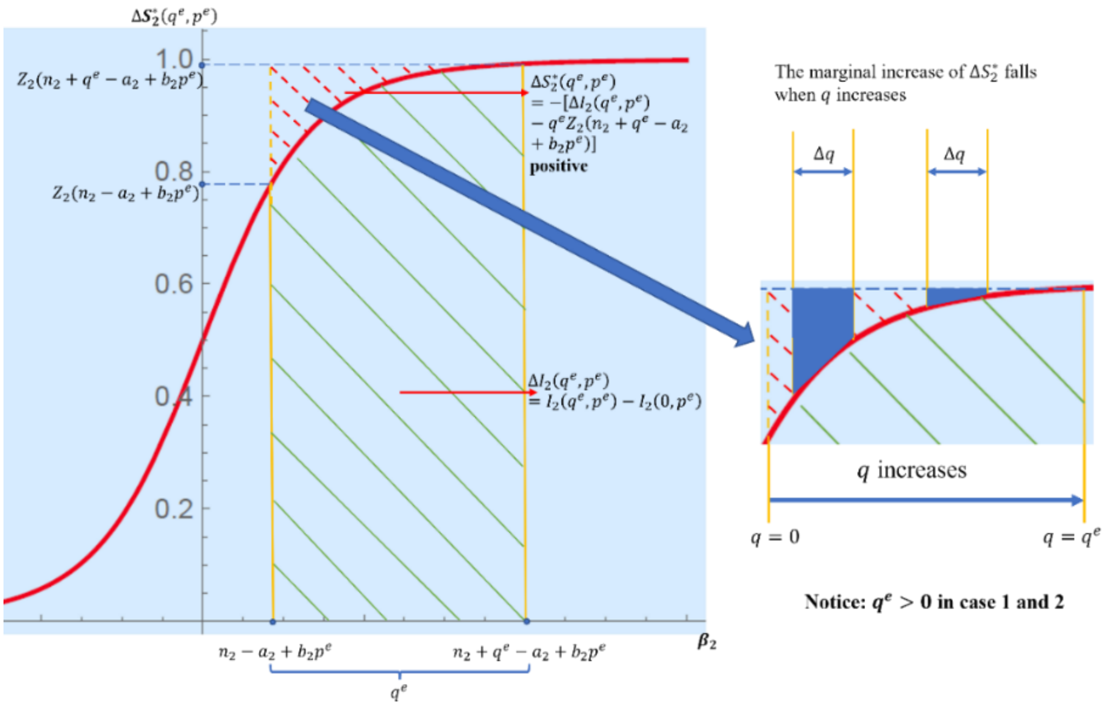


**Figure 6.1.** The number of $LSC$ 2's satisfied demands increment because of the $ECS$ activity in cases 1 and 2, given $p_S^e$.

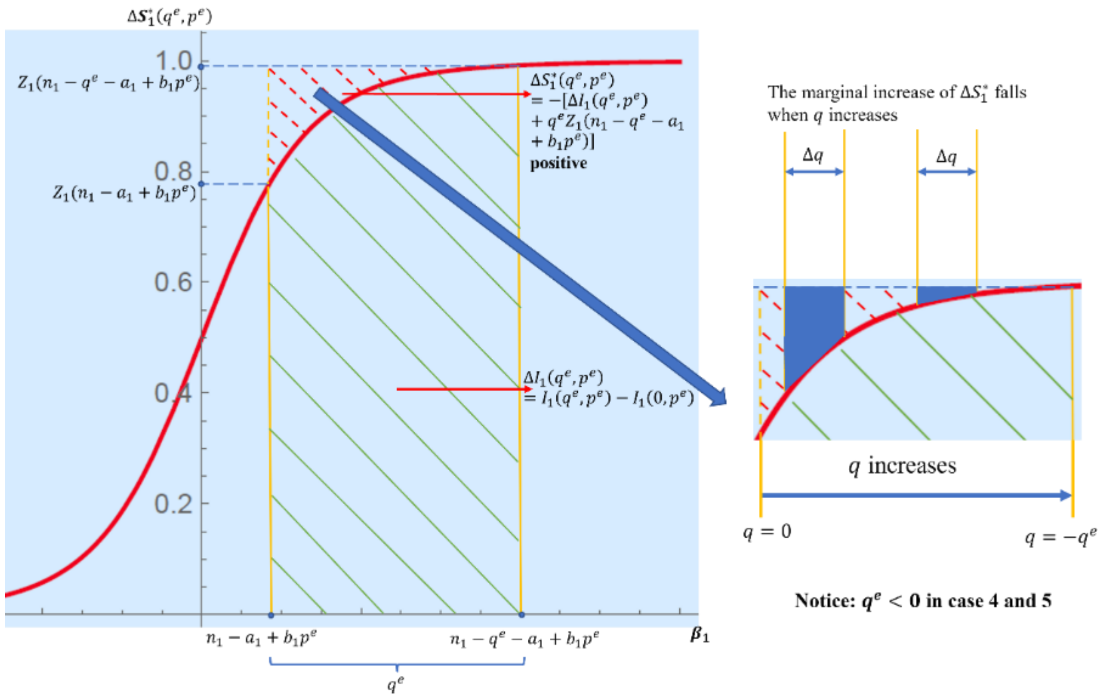


**Figure 6.2.** The number of $LSC$ 1's satisfied demands increment because of the $ECS$ activity in cases 4 and 5, given $p_S^e$.

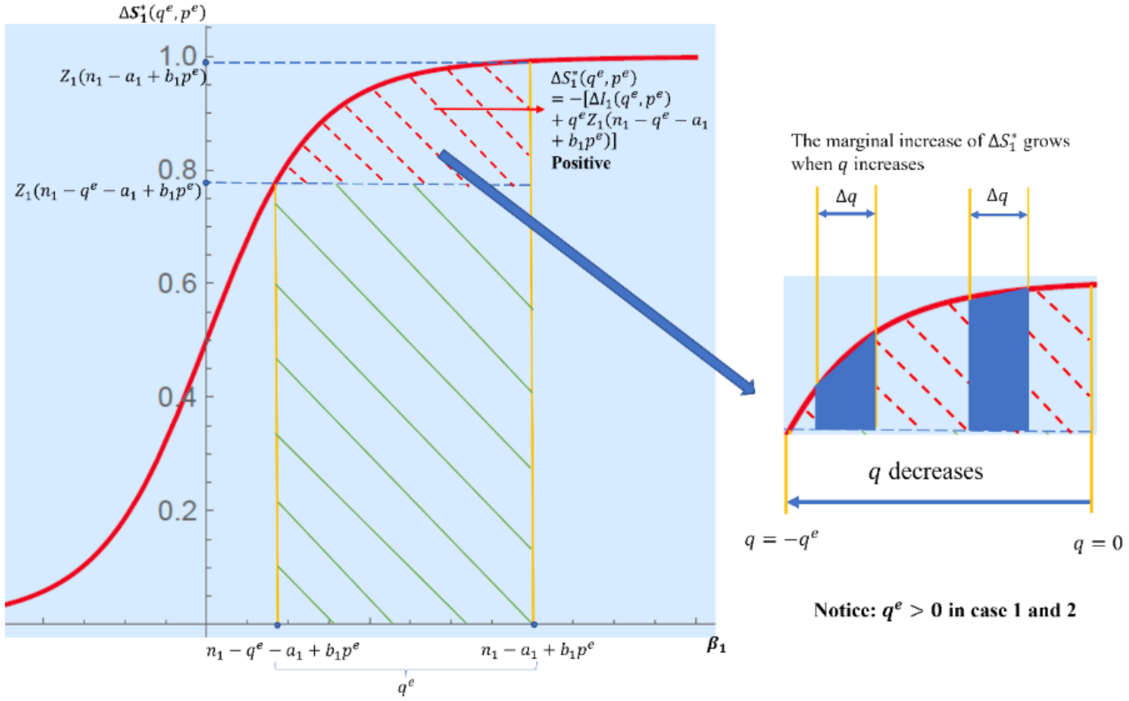


**Figure 6.3.** The number of $LSC$ 1's satisfied demands increment because of the $ECS$ activity in cases 1 and 2, given $p_S^e$.

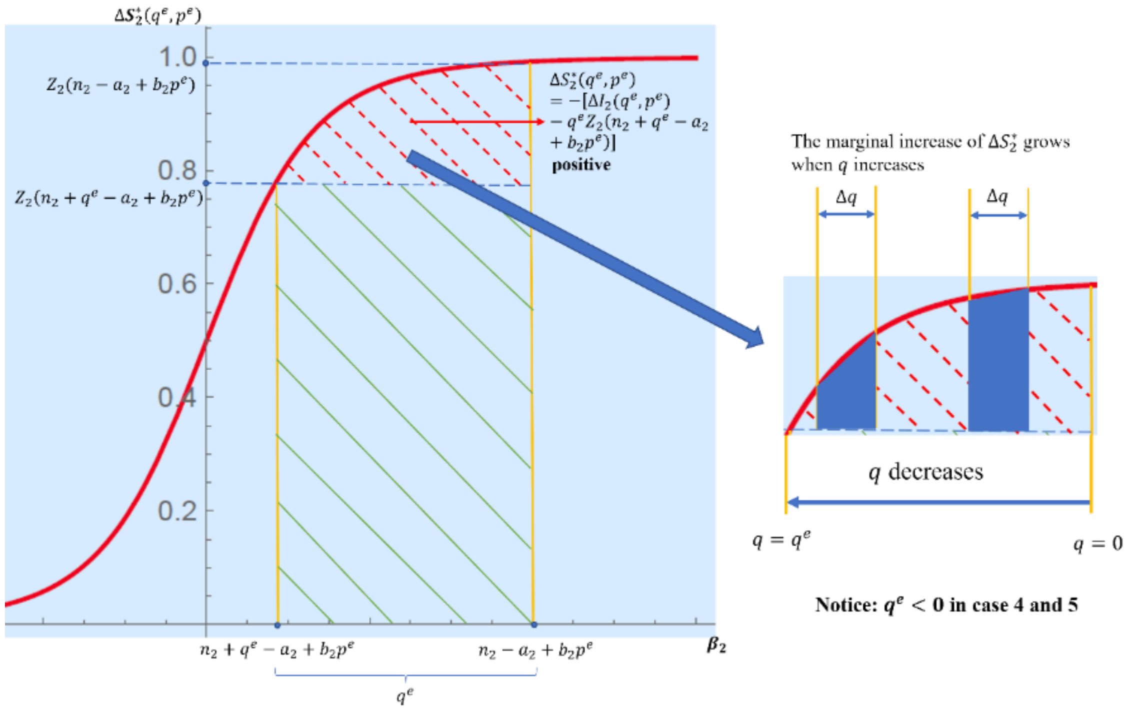


**Figure 6.4.** The number of $LSC$ 2's satisfied demands increments caused by the $ECS$ activity in cases 4 and 5, given $p_S^e$

Meanwhile, the $CT$ rate $p_S^e$ also influences the profit increment of the two $LSCs$. There are two aspects that the imposition of the government $CT$ brings to both $LSCs$' profit change. The first one is the profit loss caused by demand decline $-\alpha_i \Delta I_i(0, p_S^e)$ because of the imposition of the $CT$. The other one is the goodwill penalty amount saving $+g_i b_i p_S^e$ because of the demand decrease. This is the same analysis as its counterpart shown in section 5.3 in Chapter 5. In other words, the imposition of the $CT$ on the $LSCs$ leads to two opposite influences. According to **Lemma 6.5**, **Corollary 6.2** is developed to demonstrate the level of the government's $CT$ rate when it causes the two $LSCs$ generate either profit or loss.

***Corollary 6.2.*** *The imposition of the* $CT$ *leads to a loss for* $LSC$ *if the inequation is satisfied. Otherwise, the* $LSCs$ *would generate a profit.*

$$\frac{g_i}{\alpha_i} < \frac{\Delta I_i(0, p_s^e)}{b_i p_s^e} \tag{6.10}$$

The inequation 6.10 indicates that the $LSCs$ would suffer a loss when the $CT$ is imposed on one container unit if the goodwill penalty saving is less than the loss because of the decrease in demand. Otherwise, the $LSC$ could generate a profit. However, as with the process that I conducted in section 5.3, for the consistency of symbols with $\Delta S_i^e(q_d^e, p_s^e)$, the expression of $-\alpha_i \int_{n_i-a_i}^{n_i-a_i+b_i p_s^e} Z_i(d_i) dd_i + g_i b_i p_s^e$ is transformed as $\alpha_i \Delta S_i(0, p_s^e) - (\alpha_i - g_i) b_i p_s^e$. (See Appendix U).

The imposition of the government's $CT$ affects the increment of expected satisfied demands $\Delta S_i(0, p_s^e)$ of both $LSCs$. The demander and the supplier can earn an all-in-revenue $\alpha_i$ from each shared and satisfied empty container. For $\Delta S_i(0, p_s^e)$, the demander does not need to give the part of the revenue back to the supplier as the $ECS$ activity is not considered in $\Delta S_i(0, p_s^e)$. Moreover, based on Figure 6.5, when the $CT$ is increased from 0 to $p_s^e$, then $\Delta S_i(0, p_s^e)$ would be positive for both supplier and demander, which implies that the $CT$ could benefit both sides. In other words, with the gradual increase in the imposition of the $CT$, its impact on $\Delta S_i(0, p_s^e)$ for both $LSCs$ is significant at the beginning but declines as the $CT$ rate gradually rises until it reaches $p_s^e$. Although the $CT$ generates profit to both $LSCs$, they still lose some profit per each satisfied empty container because of the imposition of the $CT$. Overall, the impact of the $CT$ for both $LSCs$ is divided into two parts: one is the profit increase because of $\Delta S_i(0, p_s^e)$ rise, the other is the profit loss $-(\alpha_i - g_i) b_i p_s^e$ due to the imposition of $CT$. Therefore, the imposition of the government $CT$ could "generate" profit or loss to the $LSCs$. Now considering Case 3 in **Lemma 6.5**. Although no empty container is shared between the two $LSCs$, their profit increments still change due to the impact of the imposition of the $CT$. This means that both $LSCs$ can get benefits from a rising $\Delta S_i(0, p_s^e)$. However, they suffer the loss because of the imposition of the $CT$ $[-(\alpha_i - g_i) b_i p_s^e]$.

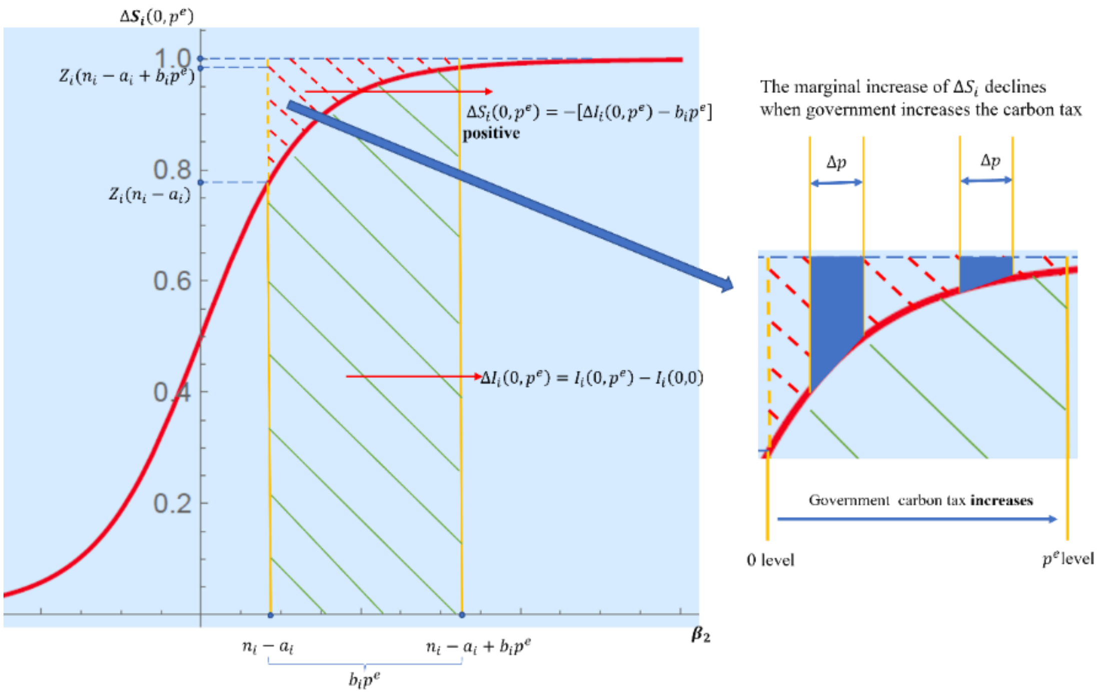


**Figure 6.5.** The number of $\Delta S_i(0, p_s^e)$ increments due to the imposition of the $CT$ without considering the $ECS$ activity

Based on the discussion about the impact of the imposition of the $CT$ the $LSCs$' profit increment, it can be noted that this may still cause a loss of profit to the $LSCs$. Given **Condition 6.3** has been satisfied, **Condition 6.4** is discussed to ensure that both $LSCs$ in the decentralised decision-making model can gain a non-negative profit increment so that the system coordination can finally be achieved. Consequently, according to **Lemma 6.5**, if $\Delta\pi_1$ and $\Delta\pi_2$ are nonnegative, then $R_1$ and $R_2$ should be constrained., These are shown in **Lemma 6.6**.

***Lemma 6.6****. To meet* ***Condition 6.4****,* $R_1$ *and* $R_2$ *should be constrained within:*

$$R_2' = \frac{(\alpha_1 - g_1)b_1 p_s^e - \alpha_1[\Delta S_1^e(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)]}{\Delta S_2^e(q_d^e, p_s^e)} \leq R_2 \leq \frac{\alpha_2[\Delta S_2^e(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] - (\alpha_2 - g_2)b_2 p_s^e}{\Delta S_2^e(q_d^e, p_s^e)} = R_2''$$

$$R_1' = \frac{(\alpha_2 - g_2)b_2 p_s^e - \alpha_2[\Delta S_2^e(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)]}{\Delta S_1^e(q_d^e, p_s^e)} \leq R_1 \leq \frac{\alpha_1[\Delta S_1^e(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] - (\alpha_1 - g_1)b_1 p_s^e}{\Delta S_1^e(q_d^e, p_s^e)} = R_1''$$

**Lemma 6.6** defines the range of $R_1$ and $R_2$ to ensure the nonnegativity of the profit increment of the two $LSCs$. As the imposition of the $CT$ may cause a loss to the $LSC's$ profit increment, the profit made from the $ECS$ activity can be used to cover the loss generated from the $CT$ impact. However, it should be noted that the loss caused by the $CT$ impact could be so huge that the two $LSCs$ might still get a negative profit increment, even if they use all the profit made from the $ECS$ activity to offset the loss. Let us denote the lower and upper boundaries of $R_1$ and $R_2$ as $[R_1', R_1'']$ and $[R_2', R_2'']$, respectively. In particular, to ensure the nonnegativity of $\Delta\pi_1$ and $\Delta\pi_2$, according to **Lemma 6.6,** $R_i''$ decides the upper boundary of the demander's profit increments $\Delta\pi_i$ ($i = 2$ in cases 1 and 2, $i = 1$ in cases 4 and 5), while $R_i'$ determines the lower boundary of the supplier's profit increments $\Delta\pi_i$ ($i = 1$ in cases 1 and 2, $i = 2$ in cases 4 and 5). Unlike **Condition 6.2**, **Lemma 6.6** gives a stricter range of $R_1$ and $R_2$ because it does not only consider the two $LSCs$' profit function concavity in the decentralised decision-making model, but it also considers the two $LSCs$' profit increment nonnegativity if the system is coordinated. So, by combining **Condition 6.2** and **Lemma 6.6**, I develop **Lemma 6.7** to offer a comprehensive range of $R_1$ and $R_2$ that ensures $\Delta\pi_1$ and $\Delta\pi_2$ nonnegativity to meet **Condition 6.4**.

***Lemma 6.7****. Given **Condition 6.1**, **6.2** and **6.3** and* $p_s^e$, *Combining **Lemma 5.6**, the full results of* $R_1$ *and* $R_2$ *are shown in Table 5.4*

**Table 6.4**. The full results of $R_1$ and $R_2$

| *No.* | *Case 1 and 2* | | *No.* | *Case 4 and 5* | |
|---|---|---|---|---|---|
| *A* | $R_2' < 0 = R_2'' < \alpha_2$ | $R_2 = 0$ | *F* | $R_1' < 0 = R_1'' < \alpha_1$ | $R_1 = 0$ |
| *B* | $R_2' \le 0 < R_2'' < \alpha_2$ | $R_2 \in [0, R_2'']$ | *G* | $R_1' \le 0 < R_1'' < \alpha_1$ | $R_1 \in [0, R_1'']$ |
| *C* | $0 < R_2' < R_2'' < \alpha_2$ | $R_2 \in [R_2', R_2'']$ | *H* | $0 < R_1' < R_1'' < \alpha_1$ | $R_1 \in [R_1', R_1'']$ |
| *D* | $0 < R_2' < \alpha_2 \le R_2''$ | $R_2 \in [R_2', \alpha_2]$ | *I* | $0 < R_1' < \alpha_1 \le R_1''$ | $R_1 \in [R_1', \alpha_1]$ |
| *E* | $0 < R_2' = \alpha_2 < R_2''$ | $R_2 = \alpha_2$ | *J* | $0 < R_1' = \alpha_1 < R_1''$ | $R_1 = \alpha_1$ |

**Lemma 6.7** shows the full range of $R_1$ and $R_2$ in different cases to ensure the nonnegativity of $\Delta\pi_1$ and $\Delta\pi_2$ in order that **Condition 6.4** is satisfied. Most importantly, **Lemma 6.7** reveals how the imposition of the $CT$ rate potentially affects the two $LSCs$' profit increment nonnegativity, and further gives the instructions to the $LSCs$ about the feasible range of $R_i$ for coordinating the system in different situations. Below I provide 6 scenarios for **Lemma 6.7.** For the demander, as the $\Delta\pi_{demander}$ ($LSC$ 2 in cases 1 and 2, $LSC$ 1 in cases 4 and 5) is inversely proportional to $R_i$, the lower boundary of $\Delta\pi_{demander}$ depends on the maximum of $R_i$, which can be 0, $R_i''$ or $\alpha_i$.

*i.* If $R_i = R_i'' = 0$, then the imposition of the $CT$ causes the demander to suffer a loss, in which the loss is so huge that the demander has to spend all the revenue generated from the $ECS$ activity to offset the whole loss. Thus, the revenue sharing must be 0, where the demander cannot share any revenue back to the supplier (Cases A and F).

*ii.* If $0 < R_i \le R_i'' < \alpha_i$, the imposition of the $CT$ still causes the demander to suffer a loss. However, in this case the demander only needs to spend part of its profit generated from the $ECS$ activity to offset the loss. In this situation, even if the $R_i$ is maximised to $R_i''$, the demander could still obtain a non-negative profit increment. However, the demander would suffer the loss in profit increment, if $R_i$ is greater than $R_i''$ (Cases B, C, G and H).

*iii.* If $0 < R_i < \alpha_i \leq R_2''$, interestingly, this implies that the imposition of the $CT$ generates profit for the demander. In other words, even if $R_i$ is maximised to $\alpha_i$, which means that if the demander gives all the revenue generated from the $ECS$ activity back to the supplier (notice that $\Delta\pi_{demander}$ decreases in $R_{demadner}$), the demander can still have a non-negative profit increment (Cases D, E, I and J).

All three statements in the different situations above are included in Figure 6.6.

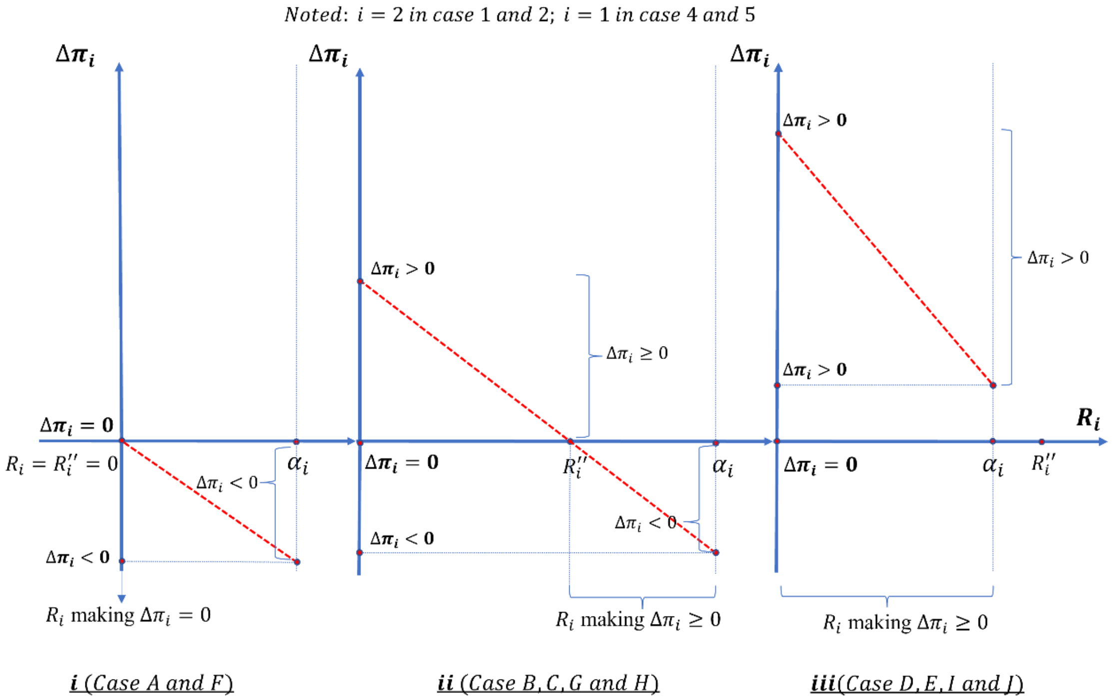


**Figure 6.6**. The impact of government $CT$ rate on $R_i$ selection for demander's profit increment nonnegativity

For the supplier, since $\Delta\pi_{supplier}$ ($LSC$ 1 in cases 1 and 2, $LSC$ 2 in cases 4 and 5) is proportional to $R_i$, the $R_i'$ determines the lower boundary of $\Delta\pi_{supplier}$. The minimum value for $R_i'$ can possibly be 0, $R_i'$ and $\alpha_i$.

*iv.* If $R_i' \leq 0 \leq R_i < \alpha_i$, then the imposition of the $CT$ brings the profit to the supplier, so the supplier does not need to use the profit generated by the $ECS$ activity to offset any loss (note that $\Delta\pi_{supplier}$ increases in $R_{demadner}$) (Cases A, B, F and G). In these cases, the supplier could always obtain a non-negative profit increment.

*v.* If $0 < R_i' \leq R_i \leq \alpha_i$, then it means that the imposition of the $CT$ starts to affect the supplier's profit increment. However, the loss can still be totally offset by the profit generated from the $ECS$ activity (Cases C, D, H and I). In particular, when $R_i$ reaches $R_i'$, the supplier's profit increment is 0.

*vi.* If $R_i = R_i' = \alpha_i$, then it signifies that the imposition of the $CT$ causes such a massive loss that the supplier must use all profit generated from the $ECS$ activity (including the revenue sharing from the demander) to offset the loss. So, in these situations (Cases E and J), the supplier can only get 0 profit increment and the demander gets all the system profit increment.

All the scenarios are depicted in Figure 6.7.

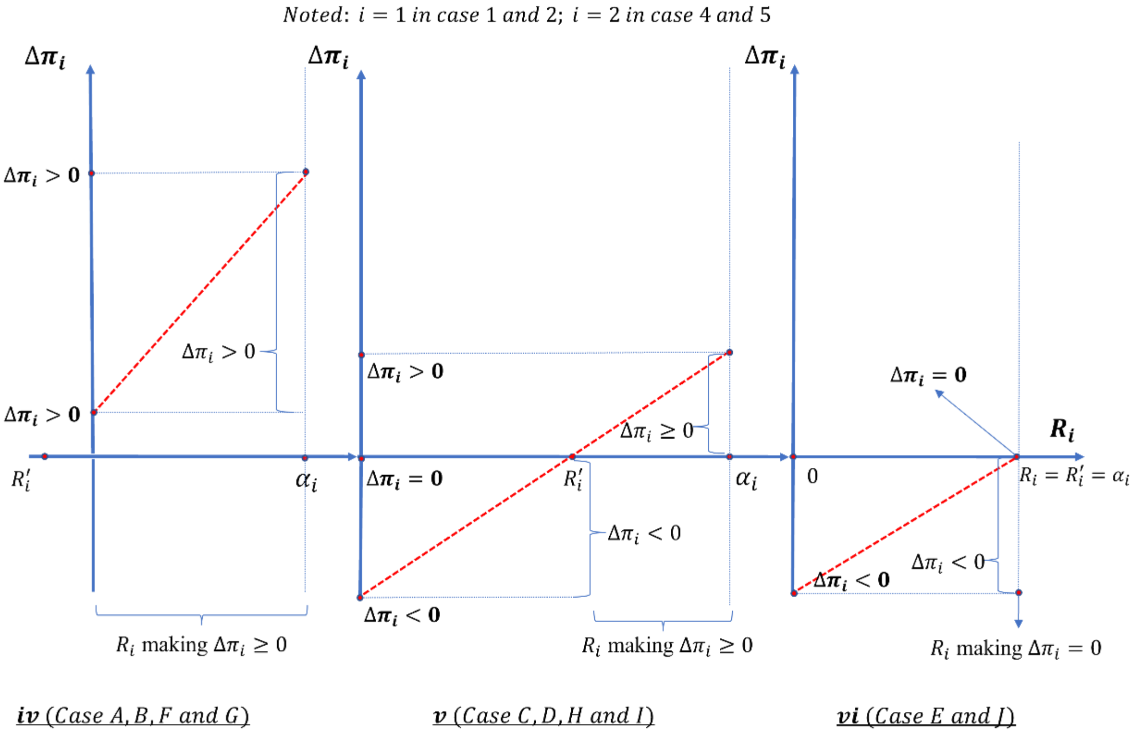


**Figure 6.7**. The impact of the government $CT$ rate on $R_i$ selection for the supplier's profit increment nonnegativity

Overall, to meet **Condition 6.4** and ensure the two $LSCs$' profit increment nonnegativity, the imposition of the $CT$ could affect the range of $R_1$ or $R_2$ in $RSC$. In other words, with the imposition of the $CT$ on the $LSCs$, if the goodwill penalty saving is greater than the profit loss because of demand decline, then the feasible $R_1$ or $R_2$ just needs to follow **Condition 6.2**. Otherwise, $R_1$ or $R_2$ should be strictly constrained within the range shown in **Lemma 6.7**. In doing this, **Condition 6.4** is satisfied. Therefore, finally, **Theorem 6.2** proves that the system can be conditionally coordinated.

***Theorem 6.2***. *Given* $p_s^e$*, the system is coordinated if:*

- **Condition 6.1** and **6.2** are satisfied.
- **Lemma 6.4** is satisfied so that $q_s^e = q_d^e$; thereby, **Condition 6.3** is satisfied.
- $R_1$ or $R_2$ are in the range shown in **Lemma 6.7** so that **Condition 6.4** is satisfied.

Given $p_s^e$, under the system coordination, the two $LSCs$' minimum and maximum profit increment in each case shown in **Lemma 6.7** are demonstrated in Table 6.5. Next, a numerical case is conducted to examine the results and the proof that I established in this chapter.

**Table 6.5.** The maximum and minimum of $\Delta\pi_1$ and $\Delta\pi_2$ under the system coordination.

| *NO* | $\boldsymbol{\Delta\pi_1}$ | $\boldsymbol{\Delta\pi_2}$ |
|---|---|---|
| *A* | min: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] - (\alpha_1 - g_1)b_1 p_s^e$ | min: $\alpha_2 \Delta S_2^*(q_d^e, p_s^e) + \alpha_2 \Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| | max: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2 \Delta S_2^*(q_d^e, p_s^e) + \alpha_2 \Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *B* | min: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] - (\alpha_1 - g_1)b_1 p_s^e$ | min: 0 |
| | max: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] + R_2'' \Delta S_2^*(q_d^e, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2 \Delta S_2^*(q_d^e, p_s^e) + \alpha_2 \Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *C* | min: 0 | min: 0 |
| | max: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] + R_2'' \Delta S_2^*(q_d^e, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $(\alpha_2 - R_2')\Delta S_2^*(q_d^e, p_s^e) + \alpha_2 \Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *D* | min: 0 | min: $\alpha_2 \Delta S_2(0, p^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| | max: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] + \alpha_2 \Delta S_2^*(q_d^e, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $(\alpha_2 - R_2')\Delta S_2^*(q_d^e, p_s^e) + \alpha_2 \Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *E* | min: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] + \alpha_2 \Delta S_2^*(q_d^e, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | min: $\alpha_2 \Delta S_2(0, p^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| | max: $\alpha_1[\Delta S_1^*(q_d^e, p_s^e) + \Delta S_1(0, p_s^e)] + \alpha_2 \Delta S_2^*(q_d^e, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2 \Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *F* | min: $\alpha_1 \Delta S_1^*(q_d^e, p_s^e) + \alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | min: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] - (\alpha_2 - g_2)b_2 p_s^e$ |
| | max: $\alpha_1 \Delta S_1^*(q_d^e, p_s^e) + \alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] - (\alpha_2 - g_2)b_2 p_s^e$ |
| *G* | min: 0 | min: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] - (\alpha_2 - g_2)b_2 p_s^e$ |
| | max: $\alpha_1 \Delta S_1^*(q_d^e, p_s^e) + \alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] + R_1'' \Delta S_1^*(q_d^e, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *H* | min: 0 | min: 0 |
| | max: $(\alpha_1 - R_1')\Delta S_1^*(q_d^e, p_s^e) + \alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] + R_1'' \Delta S_1^*(q_d^e, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *I* | min: $\alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | min: 0 |
| | max: $(\alpha_1 - R_1')\Delta S_1^*(q_d^e, p_s^e) + \alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] + \alpha_1 \Delta S_1^*(q_d^e, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| *J* | min: $\alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | min: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] + \alpha_1 \Delta S_1^*(q_d^e, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |
| | max: $\alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1 p_s^e$ | max: $\alpha_2[\Delta S_2^*(q_d^e, p_s^e) + \Delta S_2(0, p_s^e)] + \alpha_1 \Delta S_1^*(q_d^e, p_s^e) - (\alpha_2 - g_2)b_2 p_s^e$ |

### 6.5. Numerical case

In this section, a numerical case is conducted to verify the imposition of the government $CT$ impact on the two $LSCs'$ $ECS$ and evaluate the system coordination in the decentralised decision-making model. It is assumed that the random variables $\xi_1$, $\xi_2$, $Y_1$, $Y_2$ follow Normal distributions. Table 6.6 shows all the parameters applied in the numerical case.

**Table 6.6.** The parameters value in the numerical case.

| ***Carrier*** | $n_i$ | $r_i$ | $h_i$ | $a_i$ | $a_i$ | $b_i$ | |
|---|---|---|---|---|---|---|---|
| ***LSC* 1** | 800 | 900 | 300 | 500 | 850 | 10 | $c_t = 20$ |
| ***LSC* 2** | 700 | 800 | 300 | 500 | 600 | 10 | $C_g = 250$ |

While $X_1$ and $X_2$ are given by: $X_1 = 850 - 10p + \xi_1$ and $X_2 = 600 - 10p + \xi_2$ ; where $\xi_1 \sim Normal(1400,1000)$ and $\xi_2 \sim (2000,1000)$. Furthermore, $Y_1 \sim Normal(350,200)$ and $Y_2 \sim (400,250)$. So, $\xi_i - Y_i$ can be obtained $(i = 1, 2)$

$$\xi_1 - Y_1 = Normal(1050,1200) \quad \xi_2 - Y_2 = Normal(1600,1250)$$

It can be seen that $LSC$ 2 is facing a higher pressure with regard to satisfying customer demand. Thus, $LSC$ 2 should request empty containers from $LSC$ 1. Firstly, the Stackelberg equilibrium of $q$ and $p$ in the Stackelberg game in the centralised decision-making model is determined. Assuming that an initial $CT$ rate $p = 100$, Figure 6.8 shows that $\prod_{cen}(q,p)$ is strictly concave in $q$, $q^*(100) = 88$, and the maximum $\prod_{cen}(q,p) = 735{,}989$. Therefore, given $p = 100$, $LSC$ 1 should lend 88 empty containers to $LSC$ 2, so that the centralised decision-making model is maximised.

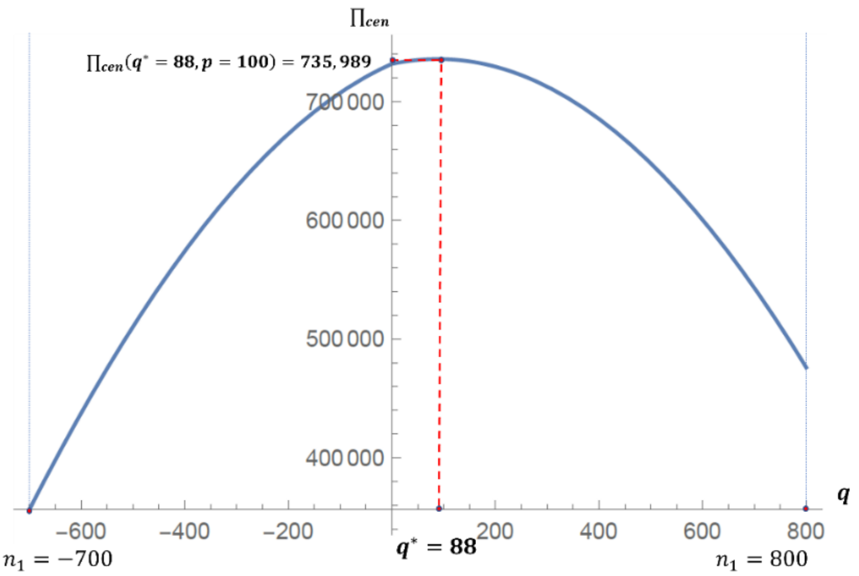


**Figure 6.8**. Given $p = 100$, the optimal $ECS$ strategy in the centralised model.

As the government totally understands the $LSCs'$ $ECS$ strategy, I determine the equilibrium $p_s^e$ in the Stackelberg game by maximising $\prod_{gov}(88,p)$. According to Figure 6.9, given $q^* = 88$, $\prod_{gov}(88,p)$ is strictly concave in $p_s^e = 84.4$, and the total social welfare reaches the maximum at $315{,}100$. Thus, the government should decrease the $CT$ rate by 15.68 per satisfied empty container. Then, to react $p_s^e$, the virtual planner should adjust the optimal $ECS$ strategy by applying $p_s^e = 84.4$ into max $\prod_{cen}(q, p_s^e = 84.4)$, which results in $q_s^e = 76$ (Figure 6.10). Thus, the Stackelberg equilibrium of $q$ and $p$ in the Stackelberg game are $q_s^e = 76, p_s^e = 84.4$, respectively and the centralised decision-making model can be maximised, ($\prod_{cen} = 776{,}175$). Also, I can perceive that this case belongs to case 2 which is shown in **Theorem 6.1**.

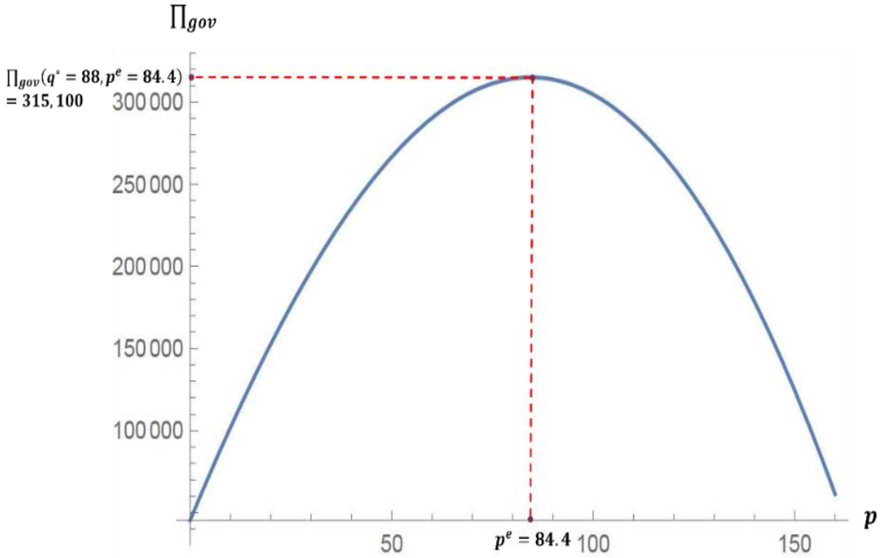


**Figure 6.9**. Given $q^*(100) = 88$, the equilibrium $CT$ rate maximising the social welfare

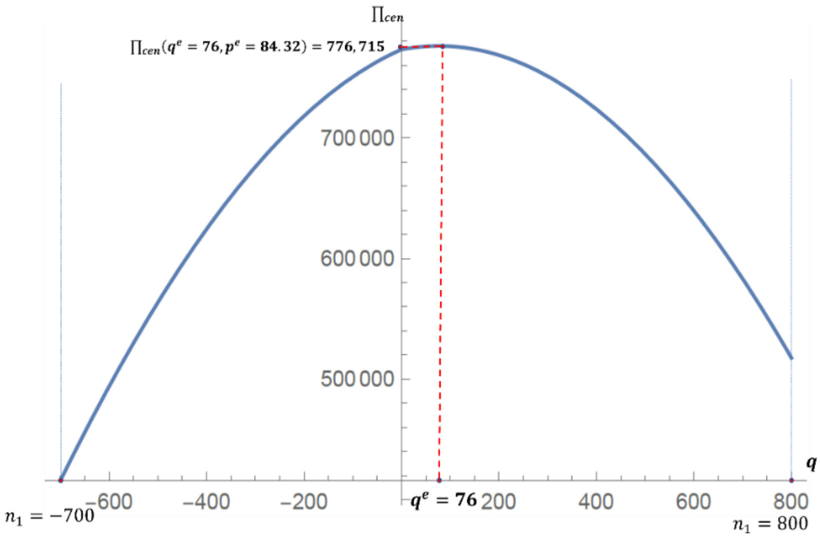


**Figure 6.10**. Given $p_s^e = 84.4$, the Stackelberg equilibrium $ECS$ strategy in the centralised model

Next, given $p_s^e = 84.4$, the contract parameters $R_2$ and $w$ should be appropriately selected to make $q_s^e(p_s^e) = q_d^e(p_s^e) = 76$. Let $w = 491.8$ and $R_2 = 433$. Figure 6.11 clearly shows that the Stackelberg equilibrium shared number between the two $LSCs$ in the centralised decision-making model is 76, which is equal to the decentralised decision-making model's Nash equilibrium shared number. Also, $\prod_{cen} = 776{,}715$, which is equal to the sum of $\pi_1 = 508{,}711$ and $\pi_2 = 267{,}465$.

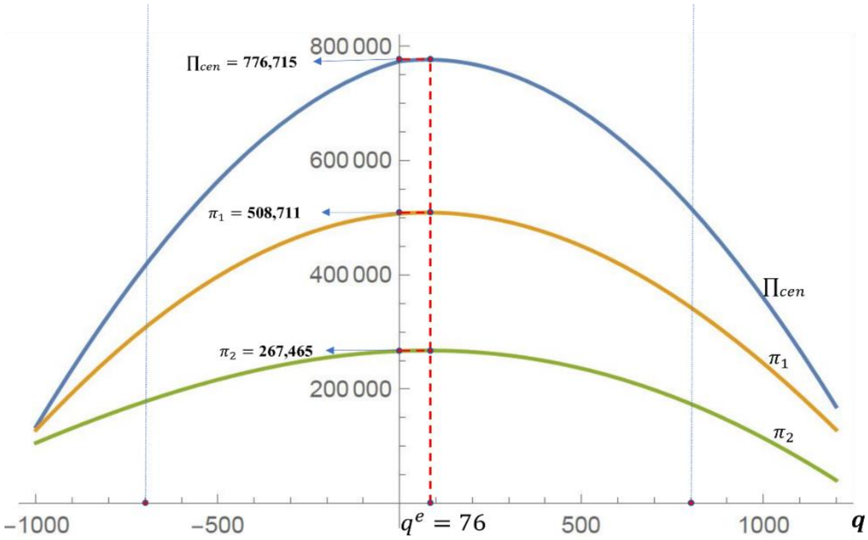


**Figure 6.11**. $\pi_1$, $\pi_2$ and $\prod_{cen}$ when $q_s^e(p_s^e) = q_d^e(p_s^e) = 76$ and $w = 491.8$, $R_2 = 433$

To verify whether the two $LSCs$' profit increments are non-negative and further examine the system coordination, I assess the impact of the imposition of the $CT$ in the Stackelberg game on the two $LSCs$' profit increments in the decentralised model. If the imposition of the $CT$negatively influences the profit of the two $LSCs$, then $R_2 = 433$ should strictly be in the range of $[R_2', R_2'']$. Otherwise, $R_2$ can be in the range of $[0, \alpha_2]$. The following two formulas calculate the profits generated from the impact of the imposition of the government $CT$.

$$-\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p^e} Z_1\,(d_1)dd_1 - b_1p^e\right] - (\alpha_1 - g_1)b_1p^e$$

$$= -1{,}700\left[\int_{800-850}^{800-850+10*84.4} Z_1\,(d_1)dd_1 - 10*84.4\right] - 10(1{,}700-500)*84.4 = 6{,}004$$

$$-\alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p^e} Z_2\,(d_2)dd_2 - b_2p^e\right] - (\alpha_2 - g_2)b_2p^e$$

$$= -1{,}600\left[\int_{700-600}^{700-600+10*84.4} Z_2\,(d_2)dd_2 - 10*84.4\right] - 10(1{,}600-500)*84.4$$
$$= 153{,}700$$

So, as calculated above, when the imposed $CT$ rate $p_s^e = 84.4$, there are +6,004 profits for $LSC$ 1 and 153,700 profits for $LSC$ 2. $R_2$ should just follow the condition shown in **Condition 6.2**, i.e., $[0, \alpha_2]$. Finally, the two $LSCs$ can obtain a non-negative profit allocation, and the system coordination can be achieved. Under the system coordination, the two $LSCs'$ non-negative profit increments are:

$$\Delta\pi_1 = -1{,}700\left[\int_{800-850+10*84.4}^{800-76-850+10*84.4} Z_1\,(d_1)dd_1 + 76Z_1(800-76-850+10*84.4)\right]$$
$$-433\left[\int_{700-600+10*84.4}^{700+76-600+10*84.4} Z_2\,(d_2)dd_2 - 76Z_2(700+76-600+10*84.4)\right]$$
$$-1{,}700\left[\int_{800-850}^{800-850+10*84.4} Z_1\,(d_1)dd_1 - 10*84.4\right] - (1{,}700-500)10*84.4 = 7{,}970$$

$$\Delta\pi_2 = -(1{,}600 - 433)\left[\int_{700-600+10*84.4}^{700+76-600+10*84.4} Z_2\,(d_2)dd_2 - 76Z_2(700 + 76 - 600 + 10 * 84.4)\right] - 1{,}600\left[\int_{700-600}^{700-600+10*84.4} Z_2\,(d_2)dd_2 - 10 * 84.4\right] - (1{,}600 - 500)10 * 84.4 = 154{,}671$$

Also, the system profit increment in the centralised decision-making model is:

$$\Delta\prod_{cen} = -\alpha_1 \int_{800-850}^{800-76-850+10*84.4} Z_1\,(d_1)dd_1 - \alpha_2 \int_{700-600}^{700+76-600+10*84.4} Z_2\,(d_2)dd_2 + 76(1{,}600 - 300 - 1{,}700 + 300 - 50) + 500 * 10 * 76 + 500 * 10 * 76 = 162{,}642$$

Therefore, under the system coordination, the allocated profit to $LSC$ 1 from the system profit increment accounts for 5%, and the $LSC$ 2 profit allocation is 95%. In this case, given the strictest government $CT$ $p^e =$ 84.4, both $LSCs$ prefer to sign the contract as both sides can get a better payoff than the scenario where no contract is agreed. $LSC$ 2 is highly incentivised to sign the contract to obtain most benefits from the sharing activity, while $LSC$ 1 has no reason to reject the contract as it still obtains 7,970 profits from the $ECS$ activity.

### 6.6. Conclusion

In this chapter, I continue to investigate how a dynamic government $CT$ scheme affects $ECS$ activity between different $LSCs$. Based on the research shown in Chapter 4 and Chapter 5, this chapter further developed a Stackelberg game model to explore a similar problem. I found that the $ECS$ supply chain can be coordinated under the government $CT$ impact by applying a $RSC$. Similar to the process in Chapter 4 and Chapter 5, I firstly assumed that the two $LSCs$ fully cooperate in the centralised decision-making model, and the optimal $ECS$ strategy is identified to achieve system optimality. In this centralised model, the two $LSCs$ and the government make decisions following a Stackelberg game, where the government is the leader who decides the $CT$ rate in the first instance, and the two $LSCs$ are the followers who determine the optimal $ECS$ strategy afterwards according to the government's $CT$ rate. This research has solved the optimal $ECS$ strategy and the optimal $CT$ rate at equilibrium in the centralised model. Next, a decentralised decision-making model was developed, in which the two $LSCs$ make decisions independently, but they are bound by an $RSC$ that decides the number of shared empty containers and the split of revenue. Under the Stackelberg equilibrium $CT$ rate, the $ESC$ strategy of the two $LSCs$can reach the Pareto optimality. Lastly, by appropriately selecting the parameters in the contract, the sum of the two $LSCs$' profit obtained in the decentralised decision-making model equals the profit that the virtual planner acquires in the centralised decision-making model, Therefore, the system is eventually coordinated.

The chapter's findings are as follows:

1. In the centralised decision-making model, the government, as the leader, and the virtual planner, as the

follower, form a Stackelberg game, and both decision variables (i.e., $ECS$ number and $CT$ rate) can converge to the Stackelberg equilibrium.

2. Under the Stackelberg equilibrium $CT$ rate, in the decision-making model, the two $LSCs'$ decisions on the $ECS$ strategy reach Pareto optimal, where one $LSC$ cannot gain more benefits without reducing the other's profit. This is consistent with the conclusion shown in Chapter 4 and 5.

3. Under the equilibrium $CT$ rate in the Stackelberg game, the system can be coordinated by appropriately selecting the parameters for the $RSC$. In doing so, the Pareto optimal $ECS$ strategy in the decentralised decision-making model is equivalent to the $ECS$ strategy in the Stackelberg game in the centralised decision-making model.

4. Under the coordination, the two $LSCs$ are better off as long as the $ECS$ activity is conducted. However, the impact of the $ECS$ activity on the empty container demander's profit increment is initially significant but reduces as more empty containers are shared. For the supplier, the situation is the opposite, where the impact is minor initially, but subsequently increases.

5. The imposition of the $CT$ could affect the feasible range of contract parameters selection for achieving the system coordination. If the goodwill penalty saving amount caused by imposition of the $CT$ is greater than the loss due to the decrease in demand, then the $LSCs'$ revenue sharing amount should only be less than the $LSCs'$ all-in-revenue per satisfied container. However, if the goodwill penalty saving is less than the loss due to the decrease in demand, the revenue sharing amount should be strictly constrained within the level which keeps the two $LSCs'$ profit increment nonnegativity. In doing so, the system can remain coordinated.

Overall, unlike with the models I built in Chapter 4 and Chapter 5, in this chapter, I introduced a Stackelberg game to determine the optimal $CT$ rate and $ECS$ strategy based on the interests of both the government' and the shipping carriers. In contrast to the models in Chapter 4 and Chapter 5, the $CT$ rate become a real decision variable decided by the government and it is independent of the interests of the shipping industry. Therefore, this is the perfect and most realistic model when compared with the previous research. Table 6.7 has been created to compare the paper produced by Xie et al. (2017), the research developed in Chapter 4, Chapter 5 and Chapter 6 of this thesis in terms of research methods, analysis and conclusions.

In the next chapter, I will make a comprehensive conclusion for the whole research.

**Table 6.7** The comparison in terms of the method, analysis and conclusion between Chapter 4, 5, 6 and the paper of Xie et al. (2017)

| ***Applied method, analysis and conclusion*** | ***Xie et al. (2017)*** | ***Chapter 4*** | ***Chapter 5*** | ***Chapter 6*** |
|---|---|---|---|---|
| System | Intermodal system | Liner shipping system | Liner shipping system | Liner shipping system |
| Model applied | Inventory sharing model | Inventory sharing model | Inventory sharing model | Stackelberg game |
| $CT$ introduced? | X | √ | √ | √ |
| $CT$ introduced as? | X | Parameters | Decision variable | Decision variable |
| Government interest involved? | X | X | X | √ |
| How many players in the model? | 1 liner firm; 1 rail firm | 2 $LSCs$ | 2 $LSCs$ | 2 $LSCs$; Government |
| Contract applied | $BBC$ | $BBC$ | $RSC$ | $RSC$ |
| Centralised model involved? | √ | √ | √ | √ |
| Centralised model optimised? | √ | √ | √ | √ |
| Decentralised model involved? | √ | √ | √ | √ |
| $ESC$ strategy's Nash equilibrium reached? | √ Pareto optimality | √ Pareto optimality | √ Pareto optimality | √ (Pareto optimality and Stackelberg equilibrium) |
| Equilibrium $CT$ reached? | Not applicable | X | √ | √ (Stackelberg equilibrium) |
| System coordinated? | √ | √ | √ | √ |
| Two $LSCs$' profit increment analysis? | X | √ | √ | √ |
| Analyse $CT$ impact on coordination? | X | √ | √ | √ |

# Chapter 7 Conclusion

## 7.1. Conclusion

This thesis mainly focuses on the problem of how the government Carbon Tax ($CT$) scheme affects Liner Shipping Carriers' ($LSCs'$) Empty Container Sharing ($ECS$) activity. The main idea of this thesis originated from the research of Xie et al. (2017), but I further innovatively develop the research subject. The whole thesis is divided into 8 chapters and appendices. In the first chapter, I introduced the topics and related research background on which this thesis focuses. I presented the current vital issue of global warming and climate change, and the problem of empty container shortage and accumulation in the container shipping industry. Also, in the first chapter, I introduced important management tools, including Game Theory ($GT$), the Newsvendor problem and the Stackelberg game, which are related to this thesis. Overall, this thesis subject is identified based on these research topics and is developed within these research fields. In the second chapter, I comprehensively examined the literature related to these research subjects. For instance, I outlined the existing methods used to solve the container accumulation and shortage, such as saving operational costs, empty container leasing, Empty Container Repositioning ($ECR$) and $ECS$. I comprehensively demonstrated the research on, and application of $ECR$ and $ECS$ in the international shipping industry. In particular, the paper of "*Empty container management and coordination in intermodal transport*" published by Xie et al. (2017) is fully reviewed in the second chapter. In addition, I briefly introduced the application of a Stackelberg game in container management. Lastly, I illustrated the two main existing methods to reduce carbon emissions, the Cap-and-Trade ($CAT$) system and the levying of $CT$, and their impact on the international shipping industry. In addition, based on a comprehensive examination of the literature, I identified and justified three main research gaps between this thesis and the previous research, which are:

- No studies focus on the impact of the imposition of $CT$ on the $ECS$ system when Revenue-sharing Contracts ($RSC$) and Buy-back Contracts ($BBC$) are adopted by the $LSCs$ to determine the $ECS$ strategy.
- In previous studies of general supply chain management and inventory sharing problems, the $CT$ rate was usually set as a fixed constant parameter instead of a decision variable.
- No scholars involved the government as a player in the container sharing game.

Therefore, in accordance with the three gaps, I developed related research in Chapter 4, Chapter 5, and Chapter 6. Before presenting the three main chapters, I pointed out the methodology applied in this thesis including ontology, epistemology and philosophical perspective in Chapter 3. I also presented the research subjects and outputs for Chapter 4, Chapter 5, and Chapter 6.

In Chapter 4, I mainly examined the effects of the constant government $CT$ on the two $LSCs$' $ECS$ strategy and focused on how to coordinate systems when a $BBC$ is used to bind the two $LSCs$. Moreover, in Chapter 5, I continued to investigate the same problem, but I considered that the $CT$ as a decision variable instead of a constant parameter. Nevertheless, the $CT$ rate determination in this chapter is not based on the government's decision, but rather it is dependent on the interests of the $LSCs$, which is not a realistic assumption. Therefore, in Chapter 6, the $CT$ rate was assumed to be dynamic, as well as being determined by the government with the aim of achieving maximum aggregate social welfare. As Chapter 4 was developed based on the paper of Xie et al. (2017), the $BBC$ was still adopted as the agreement between the two $LSCs$. However, in Chapter 5 and 6, a new contract $RSC$ was applied for further investigation. Also, all three chapters successfully coordinate the $ECS$ system in different situations. In Chapter 8, I presented the list of references that have been cited in this thesis. Finally, the appendices which detail the mathematical proof and transformation of Lemma, Corollary and Theorem presented in Chapter 4, 5 and 6. In the next section, I make a comprehensive list of the findings of this thesis.

### 7.2. Findings

In this section, 12 important findings of this thesis are listed:

1. According to the results in Chapter 4, 5 and 6, I confirm that applying either a $RSC$ or a $BBC$ can improve the profits of the $LSCs$ and stimulate efficiency in the utilization of empty containers, even in the situation where the government imposes $CT$ on the $LSCs$.
2. According to the results in Chapter 4, 5 and 6, I learn that regardless of the $CT$ rate status in the model (i.e., as parameters or decision variables), the $LSCs$ can always generate profits from $ECS$ activity by applying either the contract of the $RSC$ or the $BBC$.
3. According to the results in Chapter 4, 5 and 6, the impact of $ECS$ activity on the empty container demander is more significant at the initial stage, while the impact on the supplier is greater at the later stage.
4. According to the results in Chapter 4, 5 and 6, when the $CT$ rate is imposed, regardless of the status of the $CT$ rate (i.e., as parameters or decision variable), the empty container supplier accumulates more advantages in the $ESC$ activity if the $RSC$ is adopted. This could be because the supplier can obtain the revenue back from the demander. Meanwhile, if the $BBC$ is applied between the $LSCs$, the empty container demander can obtain more benefits than the supplier because of the subsidy from the supplier for the unsatisfied empty containers.
5. According to the results in Chapter 4, 5 and 6, I find that during the $ECS$ activity, the imposition of the

$CT$ does affect the $LSCs'$ $ECS$ optimal strategy, and profit increment. This has two opposite consequences. On the one hand, it could decrease the demand for empty containers so the $LSCs$ suffer the loss. On the other hand, it also could save part of the goodwill penalty cost because of the decline in demand.

6. According to the results in Chapter 4 and 6, when compared with the situation where there is no $CT$ imposed on the $LSCs$, the imposition of the $CT$ significantly affects the profit increments of both $LSCs$in the initial stage, while the impact drops as the rate increases until it reaches the equilibrium.

7. According to the results in Chapter 6, when compared with the situation where the Preferred Ideal Carbon Tax Rate ($PICTR$) is imposed on the two $LSCs$, if the equilibrium $CT$ rate is greater than the $PICTR$, then the impact of the imposition of the $CT$ on the $LSCs$ is consistent with the impact shown in point 6. However, if the equilibrium $CT$ rate is less than the $PICTR$, then the impact of the imposition of the $CT$ on the two $LSCs'$ profit increment is not significant in the early stages, but at the later stages it becomes more significant until it reaches the equilibrium.

8. According to the results in Chapter 4, 5 and 6, the system can be conditionally coordinated as long as the contract (both $RSC$ and $BBC$) is made appropriately, where the $CT$ rate is imposed, regardless of the status of $CT$ rate (i.e., as parameters or decision variable).

9. According to the results in Chapter 4, 5 and 6, when the $CT$ rate is imposed, regardless of the status of the $CT$ rate (i.e., as parameters or decision variables), if the system is coordinated, both $LSCs$ are incentivised to accept the contract because they can obtain a better payoff without detriment each other's profit.

10. According to the results in Chapter 6, both $LSCs'$ $ECS$ strategy and the government $CT$ rate could reach equilibrium in a Stackelberg game, when they try to optimise their interests, i.e., maximum business profit and maximum total social welfare.

11. Compared with the research in Chapter 4 and 5, the model that I developed in Chapter 6 is more realistic because it concerns the interests of both the $LSCs$ and the government. The Stackelberg game also is a suitable means of exploring the relationship between the $LSCs$ and the government.

12. According to the results in Chapter 6, the two $LSCs$ could appropriately adjust the contract parameters (for both $RSC$ and $BBC$) to keep the system coordinated. However, the adjustment cannot totally guarantee that both $LSCs$ will be able to gain a nonnegative profit increment, because the loss caused by the imposition of the $CT$ is so huge that even all the profits gained from the $ESC$ activity cannot cover it.

In the next section, based on the conclusions that was made by this research, a short managerial insight will be given for the related industry and policy maker.

## 7.3. The impact of the research in terms of the implications for $LSCs$ and government

Through this research and according to its conclusions, it is valuable to provide some important managerial insights for both international shipping industry and government. On the one hand, for $LSCs$, the most direct and valuable suggestion is that partnering with each other yields greater benefits than working alone. In particular, in light of the introduction of $CT$ scheme, this kind of cooperation becomes even more urgent and necessary. The cooperation, accomplished by signing contract among $LSCs$ to decide the sharing of a number of empty containers and split the revenue, could reduce the $LSCs'$ operation risk and ensure profit when government impose $CT$. Two most popular contracts, $BBC$ and $RSC$, could potentially achieve the target, when $CT$ is implemented. Specifically, it is highly recommended that a $LSC$ sign a $BBC$ with an empty container borrower if it owns surplus empty containers because it will achieve more profit at the end of the contractual term if the buyback price is high. On the contrary, if the $LSC$ only has few extra empty containers and it urgently needs profit through the $ECS$ activity, it is better for it to sign a $RSC$ with the borrower as it could obtain the instant revenue sharing back from the borrower. Additionally, the $CT$ rate could be so high that $LSCs$ are unable to guarantee their profits despite the fact that they are cooperating.

On the other hand, it is imperative that the government be concerned not only with the cost of environmental treatment, but also with the operation of the $LSCs$, which requests government to make $CT$ scheme reasonably. If the $CT$ rate is very high, then the $LSCs$ would suffer loss of profit, unable to lease empty container and further not able to satisfy the customer demand. If the $CT$ rate is optimal and in an appropriate range to not only stimulate $LSCs'$ cooperation on $ECS$ but also the government has sufficient financial support for carbon treatment and process. It would be beneficial for the government to use a portion of $CT$ to invest in innovative carbon treatment methods, which will lower the cost of carbon treatment and therefore reduce the $CT$ over time. As a result, the international shipping industry also will finally benefit. The limitation of this study will be clarified in the next section, along with the potential for future studies in the similar field.

## 7.4. Limitations and future research

Although this research has comprehensively investigated how the $CT$ scheme affects the two $LSCs'$ $ECS$ activity and the system coordination, some limitations have been remained in this thesis. Firstly, two essential contracts, $BBC$ and $RSC$, are applied to investigate the $ECS$ problem when government impose $CT$ on container of cargoes. It is valuable to explore the same topic when other contracts (e.g., cost-sharing contract, $QFC$) are applied. Secondly, the model setting remains relatively simple, the most completed model setting is in Chapter 6 where the government and two $LSCs$ form Stackelberg game and all of them have objective function. However, there are only three players in the game, and it is difficult to apply the model into general problem in practice. Nevertheless, this research still points out that the $LSCs$ cooperation is applicable to reduce the $LSCs'$ operation risk given the gradual $CT$ imposing worldwide. Thirdly, this research only considers two $LSCs$' $ECS$ activity based on two terminals in one port and the internal port transport cost can be ignored if $ECS$ is conducted in the same port. However, if two $LSCs$ are located on two countries, e.g., Shanghai and Los Angeles, the $ECS$ cost could be so high that it can possibly influence the $ECS$ system coordination under the government imposed $CT$. Moreover, if there are multi $LSCs$ in the international shipping network, the problem can be much more difficult, and it might be unsolvable. Therefore, $GT$ may not be an appropriate tool to solve such a complex problem in this case, and it may be needed to employ other more powerful and advanced methods such as dynamic programming to solve the international $ECS$ and system coordination problem, given the $CT$ levied by government. Lastly, due to the global Covid-19 pandemic, the raw data which was to planned to be collected in Mumbai, India (Supported by $UKIERI$, IIT Dehil and Newcastle University) were not conducted so there was no practical data available to be used in the numerical cases in the Chapters 4, 5 and 6. Nevertheless, the data used in the numerical cases is based on the research of Xie et al. (2017), which can be professionally justified.

On the other hand, given the limitations of this research mentioned beforehand, there remains many aspects on which scholars could focus and further develop in the future. For example. firstly, a future study could focus on how the government $CT$ policy affects the shipping carrier's $ECS$ strategy when a different type of contract is applied. For example, the Quantity Flexibility Contract ($QFC$), the Wholesale Price Contract ($WPC$), the Equal Share Contract ($ESC$) and the Fair Share Contract ($FSC$). It would be valuable to confirm whether or not the system can be coordinated when those contracts are applied given the impact of the government $CT$. Also, it would be useful to examine which contract is more efficient and fairer under those same conditions. So far, as stated in the last section, such research could confirm that both $BBC$ and $RSC$ can coordinate the $ECS$ system when government levies $CT$ on each container of cargoes. Also, it can be seen that both contracts work essentially the same, but operate in a different way. For example, $RSC$ focuses on the empty container lender's instant revenue sharing back from the empty container borrower for each shared and satisfied empty container, while $BBC$ concentrates on the empty container borrower's buyback price obtained from the lender for each shared but unsatisfied empty container request at the end of period. Therefore, both contracts encourage all $LSCs$ to share empty container and their status are equal in the contract. However, in some contracts, e.g., $QFC$ and $WPC$, $LSC$ their status is not equal so it is necessary to investigate how and whether the $ECS$ system can be coordinated by using these contracts when $CT$ is levied by government.

Secondly, the coordination could be investigated for multi $LSCs$, given the government $CT$ impact. This would obviously be more realistic if multi shipping carriers worldwide were involved in the model. Also, the intermodal system, including multi $LSCs$ in seaports and rail carriers in dry ports, could also be established to explore the same topic, because the intermodal system is very popular in container cargo transportation. Another way to further investigate the same topic would be to investigate the multi-period $ECS$ decision-making among multi carriers, and the system coordination problem. However, that is a far more challenging and difficult topic to solve.

Lastly, to demonstrate the transferability of this research a more specific quantity analysis for this topic, a quantitative analysis of valid data, including $LSCs'$ empty container demand and government $CT$ scheme etc. from the maritime industry is required. This should have been explored by this thesis but unfortunately was not possible due to global Covid-19 pandemic. Two separate statements were made in the annual reports in 2020 and 2021.

# Chapter 8 Reference


1. Adamopoulos, A. (2019) *Are charterers and consumers to foot the bill for shipping's carbon tax?* Available at: https://lloydslist.maritimeintelligence.informa.com/LL1129917/Are-charterers-and-consumers-to-foot-the-bill-for-shippings-carbon-tax (Accessed: February 25, 2023).
2. Adetunji, O., Yadavalli, S., AlRikabi, R. and Makoena, S. (2020) 'Economic return quantity model for a multi-type empty container management with possible storage constraint and shared cost of shipping', *American Journal of Mathematical and Management Sciences*, 39(4), pp. 345–361. Available at: https://doi.org/10.1080/01966324.2020.1769516.
3. Arikan, E. (2018) 'A Review of the Newsvendor Model', in *Single Period Inventory Control and Pricing: An empirical and analytical study of a generalized model*. Berlin, Germany: Peter Lang International Academic Publishing Group, pp. 22–29.
4. Ashina, S. and Nakata, T. (2008) 'Energy-efficiency strategy for $CO_2$ emissions in a residential sector in Japan', *Applied Energy*, 85(2-3), pp. 101–114. Available at: https://doi.org/10.1016/j.apenergy.2007.06.011.
5. Aumann, R.J. (1989) 'Game Theory', in John Eatwell et al. (eds) *The New Palgrave: Game Theory*. New York, US: W.W. Norton, pp. 1–53.
6. Avinadav, T. (2020) 'The effect of decision rights allocation on a supply chain of perishable products under a revenue-sharing contract', *International Journal of Production Economics*. 225, p. 107587. Available at: https://doi.org/10.1016/j.ijpe.2019.107587.
7. Ayodele, O.F., Ayodele, B.V., Mustapa, S.I. and Fernando, Y. (2021) 'Effect of activation function in modeling the nexus between carbon tax, $CO_2$ emissions, and gas-fired power plant parameters', *Energy Conversion and Management: X*, 12, p. 100111. Available at: https://doi.org/10.1016/j.ecmx.2021.100111.
8. Bandeira, D.L., Becker, J.L. and Borenstein, D. (2009) 'A DSS for integrated distribution of empty and full containers', *Decision Support Systems*, 47(4), pp. 383–397. Available at: https://doi.org/10.1016/j.dss.2009.04.003.
9. Becker-Peth, M. (2019) *Behavioral supply chain contracting decision biases in behavioral operations management*. Wiesbaden, Germany: Springer Gabler.
10. Becker-Peth, M. and Thonemann, U.W. (2016) 'Reference points in revenue-sharing contracts—how to design optimal supply chain contracts', *European Journal of Operational Research*, 249(3), pp. 1033–1049. Available at: https://doi.org/10.1016/j.ejor.2015.06.007.
11. Benjaafar, S., Li, Y. and Daskin, M. (2013) 'Carbon footprint and the management of Supply Chains: Insights from simple models', *IEEE Transactions on Automation Science and Engineering*, 10(1), pp. 99–116. Available at: https://doi.org/10.1109/tase.2012.2203304.

12. Bertsekas, D.P. (1998) ‘A new value iteration method for the average cost dynamic programming problem’, *SIAM Journal on Control and Optimization*, 36(2), pp. 742–759. doi:10.1137/s0363012995291609.
13. Boucher, S.C. (2014) 'What is a philosophical stance? paradigms, policies and perspectives', *Synthese*, 191(10), pp. 2315–2332. Available at: https://doi.org/10.1007/s11229-014-0400-y.
14. Braekers, K., Caris, A. and Janssens, G.K. (2013) 'Optimal shipping routes and vessel size for intermodal barge transport with empty container repositioning', *Computers in Industry*, 64(2), pp. 155–164. Available at: https://doi.org/10.1016/j.compind.2012.06.003.
15. Braekers, K., Janssens, G.K. and Caris, A. (2011) 'Challenges in managing empty container movements at multiple planning levels', *Transport Reviews*, 31(6), pp. 681–708. Available at: https://doi.org/10.1080/01441647.2011.584979.
16. Brock, T. (2020) *Matching pennies: What it means, how it works*, *Investopedia*. Available at: https://www.investopedia.com/terms/m/matching-pennies.asp (Accessed: 20 October 2023).
17. Brouer, B.D., Pisinger, D. and Spoorendonk, S. (2011) 'Liner shipping cargo allocation with repositioning of empty containers', *INFOR: Information Systems and Operational Research*, 49(2), pp. 109–124. Available at: https://doi.org/10.3138/infor.49.2.109.
18. Bryman, A. (2008) *Social Research Methods*. Oxford, UK: Oxford University Press.
19. Burrell, G. and Morgan, G. (2017) *Sociological paradigms and organisational analysis: Elements of the sociology of corporate life*. London, UK: Routledge.
20. Cachon, G.P., 2003. Supply Chain Coordination with Contracts. *In*: *Handbooks in Operations Research and Management Science*. Elsevier B.V, pp. 229–233.
21. Calderón, S., Alvarez, A.C., Loboguerrero, A.M., Arango, S., Calvin, K., Kober, T., Daenzer, K. and Fisher-Vanden, K. (2016) 'Achieving $CO_2$ reductions in Colombia: Effects of carbon taxes and abatement targets', *Energy Economics*, 56, pp. 575–586. Available at: https://doi.org/10.1016/j.eneco.2015.05.010.
22. Cao, F., Zhang, Y. and Zhang, J. (2021) 'Carbon tax, economic uncertainty and tourism: A DSGE approach', *Journal of Hospitality and Tourism Management*, 49, pp. 494–507. Available at: https://doi.org/10.1016/j.jhtm.2021.11.001.
23. Caris, A. and Janssens, G.K. (2010) 'A deterministic annealing algorithm for the pre- and end-haulage of intermodal container terminals', *International Journal of Computer Aided Engineering and Technology*, 2(4), p. 340. Available at: https://doi.org/10.1504/ijcaet.2010.035390.
24. Climate Action Tracker (2022) *The CAT thermometer*. Available at: https://climateactiontracker.org/global/cat-thermometer/ (Accessed: 2 March 2023).
25. Cebr (2017) *The economic contribution of the UK Maritime sector*. A report for Maritime UK. Available at: https://safety4sea.com/wp-content/uploads/2022/06/Cebr-UK-Maritime-Sector-2022_06.pdf (Accessed: 2 March 2023).
26. Chaabane, A., Ramudhin, A. and Paquet, M. (2012) 'Design of sustainable supply chains under the Emission Trading Scheme', *International Journal of Production Economics*, 135(1), pp. 37–49. Available at: https://doi.org/10.1016/j.ijpe.2010.10.025.

27. Chai, Q., Xiao, Z., Lai, K.H. and Zhou, G. (2018) 'Can carbon cap and trade mechanism be beneficial for remanufacturing?', *International Journal of Production Economics*, 203, pp. 311–321. Available at: https://doi.org/10.1016/j.ijpe.2018.07.004.
28. Chang, H., Jula, H., Chassiakos, A. and Ioannou, P. (2008) 'A heuristic solution for the empty container substitution problem', *Transportation Research Part E: Logistics and Transportation Review*, 44(2), pp. 203–216. Available at: https://doi.org/10.1016/j.tre.2007.07.001.
29. Chang, J., Katehakis, M.N., Shi, J. and Yan, Z. (2021) 'Blockchain-empowered newsvendor optimization', *International Journal of Production Economics*, 238, p. 108144. Available at: https://doi.org/10.1016/j.ijpe.2021.108144.
30. Cheang, B. and Lim, A. (2005) 'A network flow based method for the distribution of empty containers', *International Journal of Computer Applications in Technology*, 22(4), p. 198. Available at: https://doi.org/10.1504/ijcat.2005.006957.
31. Chen, D., Zhao, Y., Nelson, P., Li, Y., Wang, X., Zhou, Y., Lang, J. and Guo, X. (2016a) 'Estimating ship emissions based on AIS data for Port of Tianjin, China', *Atmospheric Environment*, 145, pp. 10–18. Available at: https://doi.org/10.1016/j.atmosenv.2016.08.086.
32. Chen, J., Liang, L., Yao, D.Q. and Sun, S. (2017) 'Price and quality decisions in dual-channel supply chains', *European Journal of Operational Research*, 259(3), pp. 935–948. Available at: https://doi.org/10.1016/j.ejor.2016.11.016.
33. Chen, J., Yang, B. and Guan, X. (2012) 'Optimal demand response scheduling with Stackelberg game approach under load uncertainty for smart grid', *2012 IEEE Third International Conference on Smart Grid Communications (SmartGridComm)*. Tainan, 5-8 November. New York City: IEEE, pp. 546–551. Available at: 10.1109/SmartGridComm.2012.648604.
34. Chen, R., Dong, J.X. and Lee, C.Y. (2016b) 'Pricing and competition in a shipping market with waste shipments and empty container repositioning', *Transportation Research Part B: Methodological*, 85, pp. 32–55. Available at: https://doi.org/10.1016/j.trb.2015.12.012.
35. Chen, X. and Hao, G. (2014) 'Sustainable pricing and production policies for two competing firms with carbon emissions tax', *International Journal of Production Research*, 53(21), pp. 6408–6420. Available at: https://doi.org/10.1080/00207543.2014.932928.
36. Chen, X., Benjaafar, S. and Elomri, A. (2013) 'The carbon-constrained EOQ', *Operations Research Letters*, 41(2), pp. 172–179. Available at: https://doi.org/10.1016/j.orl.2012.12.003.
37. Chen, X., Wang, X. and Xia, Y. (2019) 'Production coopetition strategies for competing manufacturers that produce partially substitutable products', *Production and Operations Management*, 28(6), pp. 1446–1464. Available at: https://doi.org/10.1111/poms.12998.
38. Chen, Y., Adams, T.A. and Barton, P.I. (2010) 'Optimal design and operation of Static Energy Polygeneration Systems', *Industrial & Engineering Chemistry Research*, 50(9), pp. 5099–5113. Available at: https://doi.org/10.1021/ie101568v.
39. Chen, Y.H., Wang, C., Nie, P.Y. and Chen, Z.R. (2020) 'A clean innovation comparison between carbon tax and cap-and-trade system', *Energy Strategy Reviews*, 29, p. 100483. Available at: https://doi.org/10.1016/j.esr.2020.100483.
40. Cheung, R.K. and Chen, C.Y. (1998) 'A two-stage stochastic network model and solution methods for the

dynamic empty container allocation problem', *Transportation Science*, 32 (2), pp. 142–162. Available at: https://doi.org/10.1287/trsc.32.2.142.

41. Choi, T.M. (2013) 'Optimal apparel supplier selection with forecast updates under Carbon Emission Taxation Scheme', *Computers & Operations Research*, 40(11), pp. 2646–2655. Available at: https://doi.org/10.1016/j.cor.2013.04.017.
42. Choong, S.T., Cole, M.H. and Kutanoglu, E. (2002) 'Empty Container Management for Intermodal Transportation Networks'. *Transportation Research Part E: Logistics and Transportation Review*, 38(6), pp. 423–438. Available at: https://doi.org/10.1016/s1366-5545(02)00018-2.
43. Chou, C.C., Gou, R.H., Tsai, C.L., Tsou, M.C., Wong, C.P. and Yu, H.L. (2010) 'Application of a mixed fuzzy decision making and optimization programming model to the empty container allocation', *Applied Soft Computing*, 10(4), pp. 1071–1079. Available at: https://doi.org/10.1016/j.asoc.2010.05.008.
44. Climate Watch (2022) *Data explorer*. Available at: https://www.climatewatchdata.org/data-explorer/historical-emissions?historical-emissions-data-sources=cait&historical-emissions-end_year=2018&historical-emissions-gases=All%20Selected&historical-emissions-regions=All%20Selected%2CWORLD&historical-emissions-sectors=All%20Selected&historical-emissions-start_year=2018&page=1 (Accessed: 25 February 2023).
45. COP 27 (2022) *COP27 official*, *COP27*. Available at: https://cop27.eg/#/ (Accessed: 19 October 2023).
46. Cournot, A.A., Bacon, N.T. and Fisher, I. (1897) *Researches into the mathematical principles of the theory of wealth*. New York, NY, US: Macmillan.
47. Crainic, T.G., Gendreau, M. and Dejax, P. (1993a) 'Dynamic and stochastic models for the allocation of empty containers', *Operations Research*, 41(1), pp. 102–126. Available at: https://doi.org/10.1287/opre.41.1.102.
48. Crainic, T.G., Gendreau, M., Soriano, P. and Toulouse, M. (1993b) 'A tabu search procedure for multicommodity location/allocation with balancing requirements', *Annals of Operations Research*, 41(4), pp. 359–383. Available at: https://doi.org/10.1007/bf02023001.
49. Creswell, J.w. (2009) *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA, US: SAGE Publications, Inc.
50. Crotty, M. (1998) *The foundations of social research: Meaning and perspective in the research process*. London, UK: Routledge.
51. Cui, H. and Notteboom, T. (2017) 'Modelling emission control taxes in port areas and port privatization levels in Port Competition and co-operation sub-games', *Transportation Research Part D: Transport and Environment*, 56, pp. 110–128. Available at: https://doi.org/10.1016/j.trd.2017.07.030.
52. Cushing, L., Blaustein-Rejto, D., Wander, M., Pastor, M., Sadd, J., Zhu, A. and Morello-Frosch, R. (2018) 'Carbon trading, co-pollutants, and environmental equity: Evidence from California’s cap-and-trade program (2011–2015)', *PLOS Medicine*, 15(7), pp. 1-20. Available at: https://doi.org/10.1371/journal.pmed.1002604.
53. Dang, Q.V., Nielsen, I.E. and Yun, W.Y. (2013) 'Replenishment policies for empty containers in an inland multi-depot system', *Maritime Economics & Logistics*, 15(1), pp. 120–149. Available at: https://doi.org/10.1057/mel.2012.20.
54. Devi, S. and Gupta, N. (2019) 'Effects of inclusion of delay in the imposition of environmental tax on the

emission of greenhouse gases', *Chaos, Solitons & Fractals*. 125, pp. 41–53. Available at: https://doi.org/10.1016/j.chaos.2019.05.006.

55. Devlin, A.G., Elmaghraby, W. and Hamilton, R.W. (2017) 'Why do suppliers choose wholesale price contracts? end-of-season payments disincentivize retailer marketing effort', *Journal of the Academy of Marketing Science*, 46(2), pp. 212–233. Available at: https://doi.org/10.1007/s11747-017-0550-9.
56. Di Francesco, M., Crainic, T.G. and Zuddas, P. (2009) 'The effect of multi-scenario policies on empty container repositioning', *Transportation Research Part E: Logistics and Transportation Review*, 45(5), pp. 758–770. Available at: https://doi.org/10.1016/j.tre.2009.03.001.
57. Di Francesco, M., Lai, M. and Zuddas, P. (2013) 'Maritime repositioning of empty containers under uncertain Port Disruptions', *Computers & Industrial Engineering*, 64(3), pp. 827–837. Available at: https://doi.org/10.1016/j.cie.2012.12.014.
58. Ding, W., Wang, Y., Dai, L. and Hu, H. (2020) 'Does a carbon tax affect the feasibility of Arctic shipping? ', *Transportation Research Part D: Transport and Environment*, 80, p. 102257. Available at: https://doi.org/10.1016/j.trd.2020.102257.
59. Dong, C., Shen, B., Chow, P.-S., Yang, L. and Ng, C.T. (2014) 'Sustainability investment under Cap-and-trade regulation', *Annals of Operations Research*, 240(2), pp. 509–531. Available at: https://doi.org/10.1007/s10479-013-1514-1.
60. Dong, J.X. and Song, D.P. (2009) 'Container fleet sizing and empty repositioning in liner shipping systems', *Transportation Research Part E: Logistics and Transportation Review*, 45(6), pp. 860–877. Available at: https://doi.org/10.1016/j.tre.2009.05.001.
61. Dong, J.X. and Song, D.P. (2012) 'Lease term optimisation in Container Shipping Systems', *International Journal of Logistics Research and Applications*, 15(2), pp. 87–107. Available at: https://doi.org/10.1080/13675567.2012.683521.
62. Dreyer, J.S. (1998) 'The researcher: Engaged participant or detached observer?', *Journal of Empirical Theology*, 11(2), pp. 5–22. Available at: https://doi.org/10.1163/157092598x00103.
63. Du, Y. and Hall, R. (1997) 'Fleet sizing and empty equipment redistribution for center-terminal transportation networks', *Management Science*, 43(2), pp. 145–157. Available at: https://doi.org/10.1287/mnsc.43.2.145.
64. Dyson, S. and Brown, B. (2006) *Social Theory and Applied Health Research*. Maidenhead, England: Open University Press.
65. Epstein, R., Neely, A., Weintraub, A., Valenzuela, F., Hurtado, S., Gonzalez, G., Beiza, A., Naveas, M., Infante, F., Alarcon, F., Angulo, G., Berner, C., Catalan, J., Gonzalez, C. and Yung, D. (2012) 'A strategic empty container logistics optimization in a major shipping company', *Interfaces*, 42(1), pp. 5–16. Available at: https://doi.org/10.1287/inte.1110.0611.
66. Erera, A.L., Morales, J.C. and Savelsbergh, M. (2005) 'Global Intermodal Tank Container Management for the chemical industry', *Transportation Research Part E: Logistics and Transportation Review*, 41(6), pp. 551–566. Available at: https://doi.org/10.1016/j.tre.2005.06.004.
67. Erera, A.L., Morales, J.C. and Savelsbergh, M. (2009) 'Robust optimization for empty repositioning problems', *Operations Research*, 57(2), pp. 468–483. Available at: https://doi.org/10.1287/opre.1080.0650.

68. Erol, Ö. and Başaran Filik, Ü. (2022) 'A Stackelberg game approach for energy sharing management of a microgrid providing flexibility to entities', *Applied Energy*, 316, p. 118944. Available at: https://doi.org/10.1016/j.apenergy.2022.118944.
69. European Commission (2018) *EUR-Lex*. Available at: https://eur-lex.europa.eu/legal-content/EN/TXT/?qid=1542994250827&uri=COM%3A2018%3A762%3AFIN (Accessed: 26 February 2023).
70. Evely, A.C., Fazey, I., Pinard, M. and Lambin, X. (2008) 'The influence of Philosophical Perspectives in integrative research: A conservation case study in the cairngorms national park', *Ecology and Society*, 13(2), article number 53. Available at: http://www.ecologyandsociety.org/vol13/iss2/art52/.
71. Fahimnia, B., Sarkis, J., Choudhary, A. and Eshragh, A. (2015) 'Tactical Supply Chain Planning under a carbon tax policy scheme: A case study', *International Journal of Production Economics*, 164, pp. 206–215. Available at: https://doi.org/10.1016/j.ijpe.2014.12.015.
72. Fang, G., Tian, L., Fu, M. and Sun, M. (2013) 'The impacts of carbon tax on energy intensity and economic growth – a dynamic evolution analysis on the case of China', *Applied Energy*, 110, pp. 17–28. Available at: https://doi.org/10.1016/j.apenergy.2013.04.041.
73. Farrell, N. (2017) 'What factors drive inequalities in carbon tax incidence? decomposing socioeconomic inequalities in carbon tax incidence in Ireland', *Ecological Economics*. 142, pp. 31–45. Available at: https://doi.org/10.1016/j.ecolecon.2017.04.004.
74. Farrell, N. and Lyons, S. (2016) 'Equity impacts of energy and climate policy: Who is shouldering the burden?', *WIREs Energy and Environment*, 5(5), pp. 492–509. Available at: https://doi.org/10.1002/wene.201.
75. Feng, C.M. and Chang, C.H. (2008) 'Empty Container Reposition Planning for intra-asia liner shipping', *Maritime Policy & Management*, 35(5), pp. 469–489. Available at: https://doi.org/10.1080/03088830802352111.
76. Ford, P. (2013) *China puts up a green wall to us trash*. Available at: https://www.csmonitor.com/World/Asia-Pacific/2013/0619/China-puts-up-a-green-wall-to-US-trash (Accessed: 26 February 2023).
77. Gao, S., Xin, X., Li, C., Liu, Y. and Chen, K. (2022) 'Container Ocean shipping network design considering carbon tax and choice inertia of cargo owners', *Ocean & Coastal Management*, 216, p. 105986. Available at: https://doi.org/10.1016/j.ocecoaman.2021.105986.
78. García-Alvarado, M., Paquet, M., Chaabane, A. and Amodeo, L. (2017) 'Inventory management under Joint Product Recovery and Cap-and-trade constraints', *Journal of Cleaner Production*, 167, pp. 1499–1517. Available at: https://doi.org/10.1016/j.jclepro.2016.10.074.
79. Gaspars-Wieloch, H. (2016) 'Newsvendor problem under complete uncertainty: A case of innovative products', *Central European Journal of Operations Research*, 25(3), pp. 561–585. Available at: https://doi.org/10.1007/s10100-016-0458-3.
80. Giannoccaro, I. and Pontrandolfo, P. (2004) 'Supply Chain Coordination by revenue sharing contracts', *International Journal of Production Economics*', 89(2), pp. 131–139. Available at: https://doi.org/10.1016/s0925-5273(03)00047-1.
81. Gilles, R.P. (2010) *The Cooperative Game Theory of networks and hierarchies*. Heidelberg: Springer.

82. GORI (2022) *Shipping shortages: What's causing the shipping crisis?*. Available at: https://hillebrand.com/media/publication/where-are-all-the-containers-the-global-shortage-explained (Accessed: 26 February 2023).
83. Govindan, K. and Popiuc, M.N. (2014) 'Reverse Supply Chain Coordination by revenue sharing contract: A case for the personal computers industry', *European Journal of Operational Research*, 233(2), pp. 326–336. Available at: https://doi.org/10.1016/j.ejor.2013.03.023.
84. Guba, E.G. (1990) *The paradigm dialog*. Newbury Park, CA, US: Sage.
85. Guba, E.G. and Lincoln, Y.S. (2005) 'Competing Paradigms in Qualitative Research', in N. K. Denzin and Y. S. Lincoln (eds) *The SAGE handbook of qualitative research*. Thousand Oaks, US: Sage Publications, pp. 105–117.
86. Guo, J., Liu, W., Tu, L. and Chen, Y. (2021) 'Forecasting carbon dioxide emissions in BRICS countries by exponential cumulative grey model', *Energy Reports*, 7, pp. 7238–7250. Available at: https://doi.org/10.1016/j.egyr.2021.10.075.
87. Halat, K., Hafezalkotob, A. and Sayadi, M.K. (2021) 'Cooperative inventory games in multi-echelon supply chains under carbon tax policy: Vertical or horizontal? ', *Applied Mathematical Modelling*, 99, pp. 166–203. Available at: https://doi.org/10.1016/j.apm.2021.06.013.
88. Hammami, R., Nouira, I. and Frein, Y. (2015) 'Carbon emissions in a multi-echelon production-inventory model with Lead time constraints', *International Journal of Production Economics*, 164, pp. 292–307. Available at: https://doi.org/10.1016/j.ijpe.2014.12.017.
89. Hammar, H. and Sjöström, M. (2011) 'Accounting for behavioral effects of increases in the carbon dioxide (CO2) tax in revenue estimation in Sweden', *Energy Policy*, 39(10), pp. 6672–6676. Available at: https://doi.org/10.1016/j.enpol.2011.06.014.
90. Hasan, M.R., Roy, T.C., Daryanto, Y. and Wee, H.M. (2021) 'Optimizing inventory level and technology investment under a carbon tax, Cap-and-trade and strict carbon limit regulations', *Sustainable Production and Consumption*, 25, pp. 604–621. Available at: https://doi.org/10.1016/j.spc.2020.12.005.
91. Hayes, A. (2023) *Operations management: Understanding and using it*. Available at: https://www.investopedia.com/terms/o/operations-management.asp (Accessed: 2 March 2023).
92. Heydari, J., Govindan, K., Ebrahimi Nasab, H.R. and Taleizadeh, A.A. (2020) 'Coordination by quantity flexibility contract in a two-echelon supply chain system: Effect of outsourcing decisions', *International Journal of Production Economics*, 225, p. 107586. Available at: https://doi.org/10.1016/j.ijpe.2019.107586.
93. Hoffmann, J., Juan, W., Sirimanne, S., Asariotis, R., Benamara, H., Premti, A., Valentine, V. and Youssef, F., 2016. Review Of Maritime Transport 2016. In: *United Nations Conference on Trade and Development*. New York: United Nations, p.6. Available at: https://unctad.org/system/files/official-document/rmt2016_en.pdf (Accessed: 14 July 2022).
94. Hu, B. and Feng, Y. (2017) 'Optimization and coordination of supply chain with revenue sharing contracts and service requirement under supply and demand uncertainty', *International Journal of Production Economics*, 183, pp. 185–193. Available at: https://doi.org/10.1016/j.ijpe.2016.11.002.
95. Hu, B., Meng, C., Xu, D. and Son, Y.J. (2016) 'Three-echelon supply chain coordination with a loss-averse retailer and revenue sharing contracts', *International Journal of Production Economics*, 179, pp. 192–202.

Available at: https://doi.org/10.1016/j.ijpe.2016.06.001.

96. Hu, H., Du, B. and Bernardo, M. (2020) 'Leasing or repositioning empty containers? determining the time-varying guide leasing prices for decision making', *Maritime Policy & Management*, 48(6), pp. 829–845. Available at: https://doi.org/10.1080/03088839.2020.1803431.
97. Hua, G., Cheng, T.C.E. and Wang, S. (2011) 'Managing carbon footprints in inventory management', *International Journal of Production Economics*, 132(2), pp. 178–185. Available at: https://doi.org/10.1016/j.ijpe.2011.03.024.
98. Huang, D. and Zhao, G. (2019) 'A shared container transportation mode in the Yangtze River', *Sustainability*, 11(10), p. 2886. Available at: https://doi.org/10.3390/su11102886.
99. Ibrahim, A. (2019) 'Profit Comparisons between Cournot and Stackelberg Duopoly with Isoelastic Demand Function', *The 4th Innovation and Analytics Conference & Exhibition*. Shanghai (China), 13th – 14th May, pp. 1–5. Available at: https://doi.org/10.1063/1.5121056
100. IICL (2022) *The Institute of International Container Lessors*. Available at: https://www.iicl.org/ (Accessed 28 February 2023).
101. Imai, A. and Rivera, F. (2001) 'Strategic fleet size planning for maritime refrigerated containers', *Maritime Policy & Management*, 28(4), pp. 361–374. Available at: https://doi.org/10.1080/03088830010020629.
102. Imai, A., Shintani, K. and Papadimitriou, S. (2009) 'Multi-port vs. hub-and-spoke port calls by containerships', *Transportation Research Part E: Logistics and Transportation Review*, 45(5), pp. 740–757. Available at: https://doi.org/10.1016/j.tre.2009.01.002.
103. IMO (2014) *Third IMO GHG study 2014*. Available at: https://www.imo.org/en/OurWork/Environment/Pages/Greenhouse-Gas-Studies-2014.aspx (Accessed: 26 February 2023).
104. IMO (2018) *Initial IMO GHG strategy*. Available at: https://www.imo.org/en/MediaCentre/HotTopics/Pages/Reducing-greenhouse-gas-emissions-from-ships.aspx (Accessed: 26 February 2023).
105. *IPCC* (n.d.) *Understanding Global Warming of 1.5°C*. Available at: https://www.ipcc.ch/sr15/resources/headline-statements/ (Accessed: 26 February 2023).
106. Irshaidat, R. (2022) 'Interpretivism vs. positivism in political marketing research', *Journal of Political Marketing*, 21(2), pp. 126–160. Available at: https://doi.org/10.1080/15377857.2019.1624286.
107. Islam, S. (2017a) 'Empty truck trips problem at Container Terminals', *Business Process Management Journal*, 23(2), pp. 248–274. Available at: https://doi.org/10.1108/bpmj-06-2015-0086.
108. Islam, S. (2017b) 'Simulation of truck arrival process at a seaport: Evaluating truck-sharing benefits for empty trips reduction', *International Journal of Logistics Research and Applications*, 21(1), pp. 94–112. Available at: https://doi.org/10.1080/13675567.2017.1353067.
109. Islam, S. and Olsen, T. (2014) 'Truck-sharing challenges for hinterland trucking companies', *Business Process Management Journal*, 20(2), pp. 290–334. Available at: https://doi.org/10.1108/bpmj-03-2013-0042.
110. Islam, S., Olsen, T. and Daud Ahmed, M. (2013) 'Reengineering the seaport container truck hauling process', *Business Process Management Journal*, 19(5), pp. 752–782. Available at: https://doi.org/10.1108/bpmj-jun-2012-0059.

111. ITF. (2022). Carbon pricing in shipping. *International Transport Forum Policy Papers*, *10*(10). https://doi.org/10.1787/250921ec-en

112. Jaber, M.Y., Glock, C.H. and El Saadany, A.M.A. (2013) 'Supply chain coordination with emissions reduction incentives', *International Journal of Production Research*, 51(1), pp. 69–82. Available at: https://doi.org/10.1080/00207543.2011.651656.

113. Jaržemskis, A. and Vasiliauskas, A.V. (2007) 'Research on dry port concept as intermodal node', *Transport*, 22(3), pp. 207–213. Available at: https://doi.org/10.3846/16484142.2007.9638126.

114. Jeong, Y., Saha, S., Chatterjee, D. and Moon, I. (2018) 'Direct shipping service routes with an empty container management strategy', *Transportation Research Part E: Logistics and Transportation Review*, 118, pp. 123–142. Available at: https://doi.org/10.1016/j.tre.2018.07.009.

115. Johnson, B. and Gray, R. (2010) 'A history of philosophical and theoretical issues for mixed methods research', *SAGE Handbook of Mixed Methods in Social & Behavioral Research*, pp. 69–94. Available at: https://doi.org/10.4135/9781506335193.n3.

116. Jula, H., Chassiakos, A. and Ioannou, P. (2006) 'Port Dynamic Empty Container Reuse', *Transportation Research Part E: Logistics and Transportation Review*, 42(1), pp. 43–60. Available at: https://doi.org/10.1016/j.tre.2004.08.007.

117. Kagan, J. (2023) *What is a carbon tax: Basics, implementation, offsets, Investopedia*. Investopedia. Available at: https://www.investopedia.com/terms/c/carbon-dioxide-tax.asp (Accessed: March 20, 2023).

118. Keat, R. (1979) *Positivism and statistics in social science*. Available at: https://russellkeat.net/admin/papers/60.pdf (Accessed: 2 March 2023).

119. Kelly, A. (2009) 'Introduction', in *Decision making using game theory: An introduction for managers*. Cambridge, UK: Cambridge University Press, pp. 1–3.

120. Kenton, W. (2020) *Cap and trade basics: What it is, how it works, pros & cons*. Available at: https://www.investopedia.com/terms/c/cap-and-trade.asp#:~:text=Cap%20and%20Trade%20Examples,by%20the%20system%20by%202020 (Accessed: 2 March 2023).

121. Kim, G., Wu, K. and Huang, E. (2015) 'Optimal inventory control in a multi-period newsvendor problem with non-stationary demand', *Advanced Engineering Informatics*, 29(1), pp. 139–145. Available at: https://doi.org/10.1016/j.aei.2014.12.002.

122. Kim, Y.D., Han, H.O. and Moon, Y.S. (2011) 'The empirical effects of a gasoline tax on CO2 emissions reductions from transportation sector in Korea', *Energy Policy*, 39(2), pp. 981–989. Available at: https://doi.org/10.1016/j.enpol.2010.11.026.

123. Kolar, P., Schramm, H.J. and Prockl, G. (2018) 'Intermodal transport and repositioning of empty containers in central and Eastern Europe hinterland', *Journal of Transport Geography*, 69, pp. 73–82. Available at: https://doi.org/10.1016/j.jtrangeo.2018.04.014.

124. Kopfer, H. and Sterzik, S. (2012) 'Reduction of Empty Container Repositioning Costs by Container Sharing', in Diethard Klatte et al. (eds) *Operations Research Proceedings 2011: Selected proceedings of the International Conference on Operations Research*. Heidelberg, Germany: Springer, pp, 287–292.

125. Kuehne and Nagel (2023) *Container shortage: Why does the current container shortage happen?*. Available at: https://home.kuehne-nagel.com/-/knowledge/market-insights/container-shortage (Accessed:

26 February 2023).

126. Kuo, T.C., Hong, I.H. and Lin, S.C. (2016) 'Do carbon taxes work? analysis of government policies and enterprise strategies in equilibrium', *Journal of Cleaner Production*, 139, pp. 337–346. Available at: https://doi.org/10.1016/j.jclepro.2016.07.164.
127. Kuzmicz, K.A. and Pesch, E. (2019) 'Approaches to empty container repositioning problems in the context of Eurasian intermodal transportation', *Omega*, 85, pp. 194–213. Available at: https://doi.org/10.1016/j.omega.2018.06.004.
128. Lai, K.K., Lam, K. and Chan, W.K. (1995) 'Shipping Container Logistics and allocation', *The Journal of the Operational Research Society*, 46(6), p. 687. Available at: https://doi.org/10.2307/2584305.
129. Lam, S.W., Lee, L.H. and Tang, L.C. (2007) 'An approximate dynamic programming approach for the empty container allocation problem', *Transportation Research Part C: Emerging Technologies*, 15(4), pp. 265–277. Available at: https://doi.org/10.1016/j.trc.2007.04.005.
130. Lee, C., Realff, M. and Ammons, J. (2011) 'Integration of channel decisions in a decentralized reverse production system with retailer collection under deterministic non-stationary demands', *Advanced Engineering Informatics*, 25(1), pp. 88–102. Available at: https://doi.org/10.1016/j.aei.2010.04.001.
131. Lee, C.H., Rhee, B.D. and Cheng, T.C.E. (2013a) 'Quality uncertainty and quality-compensation contract for Supply Chain Coordination', *European Journal of Operational Research*, 228(3), pp. 582–591. Available at: https://doi.org/10.1016/j.ejor.2013.02.027.
132. Lee, H. and Whang, S. (2002) 'The impact of the secondary market on the supply chain', *Management Science*, 48(6), pp. 719–731. Available at: https://doi.org/10.1287/mnsc.48.6.719.189.
133. Lee, L.H., Chew, E.P. and Luo, Y. (2012) 'Empty Container Management in Mult-port System with Inventory-based Control', *International Journal on Advances in Systems and Measurements*, 5(3 & 4), pp. 164–177. Available at: https://doi.org/http://www.iariajournals.org/systems_and_measurements/.
134. Lee, T.C., Chang, Y.T. and Lee, P.T.W. (2013b) 'Economy-wide impact analysis of a carbon tax on International Container Shipping', *Transportation Research Part A: Policy and Practice*, 58, pp. 87–102. Available at: https://doi.org/10.1016/j.tra.2013.10.002.
135. Legros, B., Bouchery, Y. and Fransoo, J. (2019) 'A Time-based policy for empty container management by consignees', *Production and Operations Management*, 28(6), pp. 1503–1527. Available at: https://doi.org/10.1111/poms.12996.
136. Lewis, S. (2019) *What is operations research and why is it important?*. Available at: https://www.techtarget.com/whatis/definition/operations-research-OR (Accessed: 2 March 2023).
137. Li, J., Luo, X., Wang, Q. and Zhou, W. (2021) 'Supply chain coordination through capacity reservation contract and quantity flexibility contract', *Omega*. 99, p. 102195. Available at: https://doi.org/10.1016/j.omega.2020.102195.
138. Li, J.A., Leung, S.C.H., Wu, Y. and Liu, K. (2007) 'Allocation of empty containers between multi-ports', *European Journal of Operational Research*, 182(1), pp. 400–412. Available at: https://doi.org/10.1016/j.ejor.2006.09.003.
139. Li, J.A., Liu, K., Leung, S.C.H. and Lai, K.K. (2004) 'Empty Container Management in a port with long-run average criterion', *Mathematical and Computer Modelling*, 40(1-2), pp. 85–100. Available at: https://doi.org/10.1016/j.mcm.2003.12.005.

140. Lin, T.Y. and Sarker, B.R. (2017) 'A pull system inventory model with carbon tax policies and imperfect quality items', *Applied Mathematical Modelling*, 50, pp. 450–462. Available at: https://doi.org/10.1016/j.apm.2017.06.001.
141. Lindsey, R. and Dahlman, L.A. (2023) *Climate change: Global temperature*. Available at: https://www.climate.gov/news-features/understanding-climate/climate-change-global-temperature#:~:text=Averaged%20across%20land%20and%20ocean,period%20(1880%2D1900) (Accessed: 2 March 2023).
142. Litman, T. (2009) 'Evaluating carbon taxes as an energy conservation and emission reduction strategy', *Transportation Research Record: Journal of the Transportation Research Board*, 2139(1), pp. 125–132. Available at: https://doi.org/10.3141/2139-15.
143. Liu, L., Chen, C., Zhao, Y. and Zhao, E. (2015) 'China׳s carbon-emissions trading: Overview, challenges and future', *Renewable and Sustainable Energy Reviews*, 49, pp. 254–266. Available at: https://doi.org/10.1016/j.rser.2015.04.076.
144. Liu, Y., Xin, X., Yang, Z., Chen, K. and Li, C. (2021) 'Liner shipping network - transaction mechanism joint design model considering carbon tax and Liner Alliance', *Ocean & Coastal Management*, 212, p. 105817. Available at: https://doi.org/10.1016/j.ocecoaman.2021.105817.
145. Liu, Z., Hua, S. and Zhai, X. (2020) 'Supply chain coordination with risk-averse retailer and option contract: Supplier-led vs. retailer-led', *International Journal of Production Economics*, 223, p. 107518. Available at: https://doi.org/10.1016/j.ijpe.2019.107518.
146. Lloyd's List (2022) *Containers*. Available at: https://lloydslist.maritimeintelligence.informa.com/markets/containers (Accessed: 27 February 2023).
147. Long, Y., Lee, L.H. and Chew, E.P. (2012) 'The sample average approximation method for empty container repositioning with uncertainties', *European Journal of Operational Research*, 222(1), pp. 65–75. Available at: https://doi.org/10.1016/j.ejor.2012.04.018.
148. Loux, M.J. (no date) *Nominalism*. Available at: https://www.rep.routledge.com/articles/thematic/nominalism/v-1 (Accessed: 2 March 2023).
149. Lu, C., Tong, Q. and Liu, X. (2010) 'The impacts of carbon tax and complementary policies on Chinese economy', *Energy Policy*, 38(11), pp. 7278–7285. Available at: https://doi.org/10.1016/j.enpol.2010.07.055.
150. Luo, R., Zhou, L., Song, Y. and Fan, T. (2022) 'Evaluating the impact of carbon tax policy on manufacturing and remanufacturing decisions in a closed-loop supply chain', *International Journal of Production Economics*, 245, p. 108408. Available at: https://doi.org/10.1016/j.ijpe.2022.108408.
151. Luo, T. and Chang, D. (2019) 'Empty container repositioning strategy in intermodal transport with demand switching', *Advanced Engineering Informatics*, 40, pp. 1–13. Available at: https://doi.org/10.1016/j.aei.2019.02.008.
152. Ma, Q., Song, H. and Chen, G. (2014) 'Dynamic Optimal Production Strategies based on the inventory-dependent demand under the cap-and-trade mechanism', *Mathematical Problems in Engineering*, 2014, pp. 1–13. Available at: https://doi.org/10.1155/2014/201708.

153. Maharjan, S., Zhu, Q., Zhang, Y., Gjessing, S. and Basar, T. (2013) 'Dependable demand response management in the Smart Grid: A stackelberg game approach', *IEEE Transactions on Smart Grid*, 4(1), pp. 120–132. Available at: https://doi.org/10.1109/tsg.2012.2223766.
154. Mahdavi, M. and Olsen, T.L. (2017) *Designing effective supply chains in strategic alignment with demand characteristics and market requirements*. Boston, US: now.
155. Matamala, Y. and Feijoo, F. (2021) 'A two-stage stochastic Stackelberg model for microgrid operation with chance constraints for renewable energy generation uncertainty', *Applied Energy*, 303, p. 117608. Available at: https://doi.org/10.1016/j.apenergy.2021.117608.
156. Mcallister, L. (2012) *Cap and trade - researchgate*. Available at: https://www.researchgate.net/publication/256036280_Cap_and_Trade (Accessed: 27 February 2023).
157. Meng, F.L. and Zeng, X.J. (2013) 'A Stackelberg game-theoretic approach to optimal real-time pricing for the smart grid', *Soft Computing*, 17(12), pp. 2365–2380. Available at: https://doi.org/10.1007/s00500-013-1092-9.
158. Meng, Q. and Wang, S. (2011a) 'Liner shipping service network design with empty container repositioning', *Transportation Research Part E: Logistics and Transportation Review*, 47(5), pp. 695–708. Available at: https://doi.org/10.1016/j.tre.2011.02.004.
159. Meng, Q. and Wang, T. (2011b) 'A scenario-based dynamic programming model for multi-period Liner Ship Fleet Planning', *Transportation Research Part E: Logistics and Transportation Review*, 47(4), pp. 401–413. Available at: https://doi.org/10.1016/j.tre.2010.12.005.
160. Meng, X., Yao, Z., Nie, J., Zhao, Y. and Li, Z. (2018) 'Low-carbon product selection with carbon tax and competition: Effects of the power structure', *International Journal of Production Economics*, 200, pp. 224–230. Available at: https://doi.org/10.1016/j.ijpe.2018.03.029.
161. Metcalf, G.E. (2019) 'The distributional impacts of U.S. energy policy', *Energy Policy*, 129, pp. 926–929. Available at: https://doi.org/10.1016/j.enpol.2019.01.076.
162. Metcalf, G.E. (2021) 'Carbon taxes in theory and Practice', *Annual Review of Resource Economics*, 13(1), pp. 245–265. Available at: https://doi.org/10.1146/annurev-resource-102519-113630.
163. Ming, Y., Grabot, B. and Houé, R. (2014) 'A typology of the situations of cooperation in supply chains', *Computers & Industrial Engineering*, 67, pp. 56–71. Available at: https://doi.org/10.1016/j.cie.2013.10.012.
164. Monios, J. and Wilmsmeier, G. (2015) 'Identifying Material, Geographical And Institutional Mobilities In The Global Maritime Trade System', in T Birtchnell et al. (eds) *Cargomobilities: Moving materials in a global age*. Abingdon, UK: Routledge, pp. 125–148.
165. Moon, I. and Hong, H. (2015) 'Repositioning of empty containers using both standard and foldable containers', *Maritime Economics & Logistics*, 18(1), pp. 61–77. Available at: https://doi.org/10.1057/mel.2015.18.
166. Moon, I.K., Ngoc, A.D.D. and Hur, Y.S. (2010) 'Positioning empty containers among multiple ports with leasing and purchasing considerations', *OR Spectrum*, 32(3), pp. 765–786. Available at: https://doi.org/10.1007/s00291-010-0197-0.
167. Moon, K. and Blackman, D. (2014) 'A guide to understanding social science research for natural scientists', *Conservation Biology*, 28(5), pp. 1167–1177. Available at: https://doi.org/10.1111/cobi.12326.

168. Morin, M. (2021) *What are steel shipping containers made of?* Available at: https://www.falconstructures.com/blog/what-are-shipping-containers-made-of (Accessed: 2 March 2023).
169. Morrison, W.M. (2018) *China-U.S. Trade Issues*. Available at: https://sgp.fas.org/crs/row/RL33536.pdf (Accessed: 27 February 2023).
170. Moses, J.W. and Knutsen, T.L. (2019) 'Introduction', in *Ways of knowing: Competing methodologies in social and Political Research*. London, UK: Bloomsbury Academic, pp. 6–10.
171. Nagurney, A., Liu, Z. and Woolley, T. (2006) 'Optimal endogenous carbon taxes for electric power supply chains with power plants', *Mathematical and Computer Modelling*, 44(9-10), pp. 899–916. Available at: https://doi.org/10.1016/j.mcm.2006.02.018.
172. NASA (2023a) *Data.GISS: GISS Surface Temperature Analysis (V4): Analysis graphs and ..., GISS Surface Temperature Analysis (v4)*. Available at: https://data.giss.nasa.gov/gistemp/graphs_v4/ (Accessed: 19 October 2023).
173. NASA (2022) *Carbon dioxide concentration*. Available at: https://climate.nasa.gov/vital-signs/carbon-dioxide/ (Accessed: 25 February 2023).
174. NASA (2023b) *Global temperature*. Available at: https://climate.nasa.gov/vital-signs/global-temperature/ (Accessed: 25 February 2023).
175. Ng, C.T., Song, D.P. and Cheng, T.C. (2012) 'Optimal policy for inventory transfer between two depots with backlogging', *IEEE Transactions on Automatic Control*, 57(12), pp. 3247–3252. Available at: https://doi.org/10.1109/tac.2012.2202055.
176. Nossack, J. and Pesch, E. (2013) 'A truck scheduling problem arising in Intermodal Container Transportation', *European Journal of Operational Research*, 230(3), pp. 666–680. Available at: https://doi.org/10.1016/j.ejor.2013.04.042.
177. Nouri, M., Hosseini-Motlagh, S.M., Nematollahi, M. and Sarker, B.R. (2018) 'Coordinating Manufacturer's innovation and retailer's promotion and replenishment using a compensation-based wholesale price contract', *International Journal of Production Economics*, 198, pp. 11–24. Available at: https://doi.org/10.1016/j.ijpe.2018.01.023.
178. Oertel, C., Matschullat, J., Zurba, K., Zimmermann, F. and Erasmi, S. (2016) 'Greenhouse gas emissions from soils—a review', *Geochemistry*, 76(3), pp. 327–352. Available at: https://doi.org/10.1016/j.chemer.2016.04.002.
179. Oliveira, F.S., Ruiz, C. and Conejo, A.J. (2013) 'Contract design and Supply Chain Coordination in the Electricity Industry', *European Journal of Operational Research*, 227(3), pp. 527–537. Available at: https://doi.org/10.1016/j.ejor.2013.01.003.
180. Olivo, A., Zuddas, P., Di Francesco, M. and Manca, A. (2005) 'An operational model for empty container management', *Maritime Economics & Logistics*, 7(3), pp. 199–222. Available at: https://doi.org/10.1057/palgrave.mel.9100136.
181. Olsson, F. (2010) 'An inventory model with unidirectional lateral transshipments', *European Journal of Operational Research*, 200(3), pp. 725–732. Available at: https://doi.org/10.1016/j.ejor.2009.01.024.
182. Orlov, A. and Grethe, H. (2012) 'Carbon taxation and market structure: A CGE analysis for Russia', *Energy Policy*, 51, pp. 696–707. Available at: https://doi.org/10.1016/j.enpol.2012.09.012.
183. Osborne, M.J. (2000) *An introduction to game theory*. New Delhi, UK: Oxford University Press.

184. Özdemir, Ü. (2018) 'Analysis of empty container accumulation problem of container ports', *Journal of ETA Maritime Science*, 6(4), pp. 319–332. Available at: https://doi.org/10.5505/jems.2018.53325.
185. Parry, I., Heine, D., Kizzier, K. and Smith, T. (2018) 'Carbon taxation for international maritime fuels: Assessing the options', *SSRN Electronic Journal*, 18(203), pp. 3–38. Available at: https://doi.org/10.2139/ssrn.3267230.
186. Pasternack, B.A. (1985) 'Optimal pricing and return policies for perishable commodities', *Marketing Science*, 4(2), pp. 166–176. Available at: https://doi.org/10.1287/mksc.4.2.166.
187. Pasternack, B.A. (1985) 'Optimal pricing and return policies for perishable commodities', *Marketing Science*, 4(2), pp. 166–176. Available at: https://doi.org/10.1287/mksc.4.2.166.
188. Pérez-Rodríguez, N. and Holguín-Veras, J. (2014) 'The accumulation of empty containers in urban areas: Policy implications from a stochastic formulation', *Networks and Spatial Economics*, 14(3-4), pp. 379–408. Available at: https://doi.org/10.1007/s11067-014-9231-0.
189. Petruzzi, N.C. and Dada, M. (1999) 'Pricing and the newsvendor problem: A review with extensions', *Operations Research*, 47(2), pp. 183–194. Available at: https://doi.org/10.1287/opre.47.2.183.
190. Porter, J. (2019) *Shipowners seek to seize initiative as Inevitable Carbon Levy Looms*. Available at: https://lloydslist.maritimeintelligence.informa.com/LL1129791/Shipowners-seek-to-seize-initiative-as-inevitable-carbon-levy-looms (Accessed: 27 February 2023).
191. Powell, T.C. (2001) 'Fallibilism and Organizational Research: The Third Epistemology', *Journal of management research*, 1(4), pp. 201–219.
192. Qin, J., Bai, X. and Xia, L. (2015) 'Sustainable Trade Credit and replenishment policies under the Cap-and-trade and Carbon Tax Regulations', *Sustainability,* 7(12), pp. 16340–16361. Available at: https://doi.org/10.3390/su71215818.
193. Qin, Y., Wang, R., Vakharia, A.J., Chen, Y. and Seref, M.M.H. (2011) 'The newsvendor problem: Review and directions for future research', *European Journal of Operational Research*, 213(2), pp. 361–374. Available at: https://doi.org/10.1016/j.ejor.2010.11.024.
194. Raymond, L. (2019) 'Policy perspective:building political support for carbon pricing—lessons from Cap-and-trade policies', *Energy Policy*. 134, p. 110986. Available at: https://doi.org/10.1016/j.enpol.2019.110986.
195. Rojon, I., Lazarou, N.J., Rehmatulla, N. and Smith, T. (2021) 'The impacts of carbon pricing on maritime transport costs and their implications for developing economies', *Marine Policy*, 132, p. 104653. Available at: https://doi.org/10.1016/j.marpol.2021.104653.
196. Roso, V. and Lumsden, K. (2009) 'The Dry Port Concept: Moving Seaport Activities Inland?', *Transport and Communications Bulletin for Asia and the Pacific – UNESCAP*, 78, pp. 87–102. Available at: https://www.unescap.org/sites/default/d8files/bulletin78_Article-5.pdf.
197. Rudi, N., Kapur, S. and Pyke, D.F. (2001) 'A two-location inventory model with transshipment and local decision making', *Management Science*, 47(12), pp. 1668–1680. Available at: https://doi.org/10.1287/mnsc.47.12.1668.10235.
198. Sáinz Bernat, N., Schulte, F., Voß, S. and Böse, J. (2016) 'Empty Container Management at ports considering pollution, repair options, and street-turns', *Mathematical Problems in Engineering*, 2016, pp. 1–13. Available at: https://doi.org/10.1155/2016/3847163.

199. Salam, M.A. and Noguchi, T. (2005) 'Impact of human activities on carbon dioxide ($CO_2$) emissions: A statistical analysis, *The Environmentalist*, 25(1), pp. 19–30. Available at: https://doi.org/10.1007/s10669-005-3093-4.
200. Schwandt, T.A. (2003) 'Constructivist, interpretivist approaches to human inquiry', in N. K. Denzin and Y. S. Lincoln (eds) *The landscape of qualitative research: Theories and issues*. London, UK: SAGE, pp. 221–259.
201. Shao, J., Krishnan, H. and McCormick, S.T. (2011) 'Incentives for transshipment in a supply chain with decentralized retailers', *Manufacturing & Service Operations Management*, 13(3), pp. 361–372. Available at: https://doi.org/10.1287/msom.1110.0326.
202. Sheffi, Y. (1985) *Urban Transportation Networks: Equilibrium Analysis with mathematical programming methods*. Englewood Cliffs - N.J: Prentice-Hall.
203. Shen, J., Copertaro, B., Zhang, X., Koke, J., Kaufmann, P. and Krause, S. (2019a) 'Exploring the potential of climate-adaptive container building design under future climates scenarios in three different climate zones', *Sustainability*, 12(1), p. 108. Available at: https://doi.org/10.3390/su12010108.
204. Shen, W.S. and Khoong, C.M. (1995) 'A DSS for empty container distribution planning', *Decision Support Systems*, 15(1), pp. 75–82. Available at: https://doi.org/10.1016/0167-9236(94)00037-s.
205. Shen, Y., Shen, K. and Yang, C. (2019b) 'A production inventory model for deteriorating items with collaborative preservation technology investment under Carbon Tax', *Sustainability*, 11(18), p. 5027. Available at: https://doi.org/10.3390/su11185027.
206. Shi, N. and Xu, D. (2011) 'A Markov Decision Process Model for an Online Empty Container Repositioning Problem in a Two-port Fixed Route', *International Journal of Operations Research*, 8(2), pp. 8–17.
207. Shintani, K., Imai, A., Nishimura, E. and Papadimitriou, S. (2007) 'The container shipping network design problem with empty container repositioning', *Transportation Research Part E: Logistics and Transportation Review*, 43(1), pp. 39–59. Available at: https://doi.org/10.1016/j.tre.2005.05.003.
208. Shintani, K., Konings, R. and Imai, A. (2010) 'The impact of foldable containers on container fleet management costs in Hinterland Transport', *Transportation Research Part E: Logistics and Transportation Review*, 46(5), pp. 750–763. Available at: https://doi.org/10.1016/j.tre.2009.12.008.
209. Shoham, Y. and Leyton-Brown, K. (2009) 'Introduction to Noncooperative Game Theory: Games in Normal Form', in *Multiagent Systems: Algorithmic, game-theoretic, and logical foundations*. Cambridge, UK: Cambridge University Press, pp. 47–48.
210. Shu, T., Wu, Q., Chen, S., Wang, S., Lai, K.K. and Yang, H. (2017) 'Manufacturers'/remanufacturers' inventory control strategies with Cap-and-trade regulation', *Journal of Cleaner Production*, 159, pp. 11–25. Available at: https://doi.org/10.1016/j.jclepro.2017.05.021.
211. Silver, E.A., Pyke, D.F. and Peterson, R. (2006) *Inventory management and Production Planning and Scheduling*. New York etc., US: John Wiley & Sons.
212. Snyder, L.V. and Shen, Z.J.M. (2019) *Fundamentals of Supply Chain theory*. Hoboken, NJ, US: John Wiley & Sons, Inc.

213. Song , J. and Leng, M. (2012) 'Analysis of the Single-Period Problem under Carbon Emissions Policies', in T.M. Choi (eds) *Handbook of newsvendor problems: Models, extensions and applications*. New York, US: Springer, pp. 297–314.
214. Song, D. P. and Dong, J. (2023) *Modelling Empty Container Repositioning Logistics*. Cham, Switzerland: Springer.
215. Song, D. P. and Dong, J. X. (2015) 'Empty Container Repositioning', in Chung-Yee Lee and Qiang Meng (eds) *Handbook of Ocean Container Transport Logistics*. New York, US: Springer, pp. 163–208.
216. Song, D., Zhang, J., Carter, J., Field, T., Marshall, J., Polak, J., Schumacher, K., Sinha-Ray, P. and Woods, J. (2005) 'On cost-efficiency of the Global Container Shipping Network', *Maritime Policy & Management*, 32(1), pp. 15–30. Available at: https://doi.org/10.1080/0308883042000176640.
217. Song, D.P. (2007) 'Characterizing optimal empty container reposition policy in periodic-review shuttle service systems', *Journal of the Operational Research Society*, 58(1), pp. 122–133. Available at: https://doi.org/10.1057/palgrave.jors.2602150.
218. Song, D.P. and Carter, J. (2009) 'Empty container repositioning in liner shipping', *Maritime Policy & Management*, 36(4), pp. 291–307. Available at: https://doi.org/10.1080/03088830903056934.
219. Song, D.P. and Dong, J.X. (2008) 'Empty container management in cyclic shipping routes', *Maritime Economics & Logistics*, 10(4), pp. 335–361. Available at: https://doi.org/10.1057/mel.2008.11.
220. Song, D.P. and Dong, J.X. (2011) 'Flow balancing-based empty container repositioning in typical shipping service routes', *Maritime Economics & Logistics*, 13(1), pp. 61–77. Available at: https://doi.org/10.1057/mel.2010.18.
221. Song, D.P. and Dong, J.X. (2012) 'Cargo routing and empty container repositioning in multiple shipping service routes', *Transportation Research Part B: Methodological*. 46(10), pp. 1556–1575. Available at: https://doi.org/10.1016/j.trb.2012.08.003.
222. Song, D.P. and Dong, J.X. (2013) 'Long-haul liner service route design with ship deployment and empty container repositioning', *Transportation Research Part B: Methodological*, 55, pp. 188–211. Available at: https://doi.org/10.1016/j.trb.2013.06.012.
223. Song, D.P. and Zhang, Q. (2010) 'A fluid flow model for empty container repositioning policy with a single port and stochastic demand', *SIAM Journal on Control and Optimization*, 48(5), pp. 3623–3642. Available at: https://doi.org/10.1137/09075785x.
224. Song, D.P., Dinwoodie, J. and Roe, M. (2007) 'Integrated vehicle fleet-sizing, leasing and dispatching policy in a shuttle service system', *International Journal of Logistics Research and Applications*, 10(1), pp. 29–40. Available at: https://doi.org/10.1080/13675560601081482.
225. Soni, S. (2020) *Management and it's functions*. Available at: https://www.thesocialcomment.com/blog/MANAGEMENT-and-ITS-FUNCTIONS?pid=5f4f241a37add53a21ad4e22 (Accessed: 2 March 2023).
226. Sošić, G. (2006) 'Transshipment of inventories among retailers: Myopic vs. farsighted stability', *Management Science*, 52(10), pp. 1493–1508. Available at: https://doi.org/10.1287/mnsc.1060.0558.
227. Spirkin, A.G. (1983) *Dialectical materialism*. Moscow, Russia: Progress Publishers.
228. Stackelberg, H.von (1934) *Marktform und gleichgewicht (In German)*. Düsseldorf, Germany: Verl. Wirtschaft und Finanzen.

229. Statista (2022) *Worldwide Maritime Trade - Transport Volume 2021*. Available at: https://www.statista.com/statistics/264117/tonnage-of-worldwide-maritime-trade-since-1990/ (Accessed: 28 February 2023).
230. Sterzik, S., Kopfer, H. and Yun, W.Y. (2012) 'Reducing hinterland transportation costs through container sharing', *Flexible Services and Manufacturing Journal*, 27(2-3), pp. 382–402. Available at: https://doi.org/10.1007/s10696-012-9167-y.
231. Sumner, J., Bird, L. and Smith, H. (2009) '*Carbon taxes: A review of experience and policy design considerations*'. Available: https://www.nrel.gov/docs/fy10osti/47312.pdf (Accessed: 28 February 2023).
232. Tadelis, S. (2013) *Game theory: An introduction*. Princeton, US: Princeton University Press.
233. Tan, Z., Meng, Q., Wang, F. and Kuang, H. (2018) 'Strategic integration of the inland port and shipping service for the Ocean Carrier', *Transportation Research Part E: Logistics and Transportation Review*, 110, pp. 90–109. Available at: https://doi.org/10.1016/j.tre.2017.12.010.
234. Tang, Y., Chen, S., Feng, Y. and Zhu, X. (2021) 'Optimization of multi- period empty container repositioning and renting in China Railway Express based on container sharing strategy', *European Transport Research Review*, 13(1), pp. 1–12.
235. The World Bank (2014) *Pricing carbon*, *The World Bank*. Available at: https://www.worldbank.org/en/programs/pricing-carbon#:~:text=A%20carbon%20tax%20directly%20sets,but%20the%20carbon%20price%20is. (Accessed: February 3, 2023).
236. The World Bank (2022a) *Map & Data*. Available at: https://carbonpricingdashboard.worldbank.org/map_data (Accessed: 25 February 2023).
237. The World Bank (2022b) *Report: State and trends of carbon pricing*. Available at: https://www.worldbank.org/en/news/press-release/2022/05/24/global-carbon-pricing-generates-record-84-billion-in-revenue (Accessed: 2 March 2023).
238. Tiwari, S., Wee, H.M., Zhou, Y. and Tjoeng, L. (2021) 'Freight consolidation and containerization strategy under business as usual scenario & Carbon Tax Regulation', *Journal of Cleaner Production*, 279, p. 123270. https://doi.org/10.1016/j.jclepro.2020.123270.
239. Toygar, A., Yildirim, U. and İnegöl, G.M. (2022) 'Investigation of empty container shortage based on Swara-aras methods in the COVID-19 ERA', *European Transport Research Review*, 14(8), pp. 1–17. Available at: https://doi.org/10.1186/s12544-022-00531-8.
240. UNCTAD (2011). *Review of Maritime Transport 2011.* Available at: https://unctad.org/system/files/official-document/Chapter_1%20with%20cover%20page.pdf (Accessed: 28 February 2023).
241. UNCTAD (2021) *Review of Maritime Transport 2021*. Available at: https://unctad.org/publication/review-maritime-transport-2021 (Accessed: 28 February 2023).
242. UNFCCC (2015) *The Paris Agreement*. Available at: https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement (Accessed: 2 March 2023).
243. UNFCCC (2021) *Glasgow Climate Pact*. Available at: https://unfccc.int/documents/310475 (Accessed: 28 February 2023).

244. Ungureanu, V. (2018a) 'Introduction', in *Pareto-nash-stackelberg game and control theory intelligent paradigms and applications*. Cham, Switzerland: Springer International Publishing, pp. 1–34.

245. Ungureanu, V. (2018b) 'Stackelberg Equilibrium Sets in Polymatrix Mixed-Strategy Generalized Stackelberg Games', in *Pareto-nash-stackelberg game and control theory intelligent paradigms and applications*. Cham, Switzerland: Springer International Publishing, pp. 129–165.

246. Vafa Arani, H., Rabbani, M. and Rafiei, H. (2016) 'A revenue-sharing option contract toward coordination of Supply Chains', *International Journal of Production Economics*, 178, pp. 42–56. Available at: https://doi.org/10.1016/j.ijpe.2016.05.001.

247. van Dijk, E. (2015) ‘Social Dilemmas, psychology of’, *International Encyclopedia of the Social & Behavioral Sciences*, pp. 245–249. doi:10.1016/b978-0-08-097086-8.24027-0.

248. Vella, D.C. (2021) *Invitation to linear programming and game theory*. Cambridge, UK: Cambridge University Press.

249. Wang, K., Yan, X., Yuan, Y., Jiang, X., Lin, X. and Negenborn, R.R. (2018a) 'Dynamic optimization of ship energy efficiency considering time-varying environmental factors', *Transportation Research Part D: Transport and Environment*, 62, pp. 685–698. Available at: https://doi.org/10.1016/j.trd.2018.04.005.

250. Wang, L., Xu, T. and Qin, L. (2019) 'A study on supply chain emission reduction level based on carbon tax and consumers’ low-carbon preferences under stochastic demand', *Mathematical Problems in Engineering*, 2019, pp. 1–20. Available at: https://doi.org/10.1155/2019/1621395.

251. Wang, S. (2013) 'Essential elements in tactical planning models for Container Liner Shipping', *Transportation Research Part B: Methodological*, 54, pp. 84–99. Available at: https://doi.org/10.1016/j.trb.2013.04.001.

252. Wang, S. and Meng, Q. (2012) 'Liner ship fleet deployment with container transshipment operations', *Transportation Research Part E: Logistics and Transportation Review*, 48 (2), pp. 470–484. Available at: https://doi.org/10.1016/j.tre.2011.10.011.

253. Wang, T., Wang, X. and Meng, Q. (2018b) 'Joint Berth Allocation and Quay Crane assignment under different Carbon Taxation Policies', *Transportation Research Part B: Methodological*, 117, pp. 18–36. Available at: https://doi.org/10.1016/j.trb.2018.08.012.

254. Wang, X., Li, F., Liang, L., Huang, Z. and Ashley, A. (2015) 'Pre-purchasing with option contract and coordination in a relief supply chain', *International Journal of Production Economics*, 167, pp. 170–176. Available at: https://doi.org/10.1016/j.ijpe.2015.05.031.

255. Wang, Y., Zhang, R., Ge, S. and Qian, L. (2018). Investigating the effect of Arctic sea routes on the global maritime container transport system via a generalized Nash equilibrium model. *Polar Research*, 37(1), p.1547042.

256. Wei, Z., Zhao, L., Zhang, X. and Lv, W. (2022) 'Jointly Optimizing Ocean shipping routes and sailing speed while considering involuntary and voluntary speed loss', *Ocean Engineering*, 245, p. 110460. Available at: https://doi.org/10.1016/j.oceaneng.2021.110460.

257. Whitin, T.M. (1955) *Inventory control and price theory*. Cambridge, MA, US: Massachusetts Institute of Technology.

258. Wong, W.K., Qi, J. and Leung, S.Y.S. (2009) 'Coordinating supply chains with sales rebate contracts and vendor-managed inventory', *International Journal of Production Economics*, 120(1), pp. 151–161.

Available at: https://doi.org/10.1016/j.ijpe.2008.07.025.

259. XChange (2019) *Empty container repositioning: Why it happens*. Available at: https://www.container-xchange.com/blog/empty-container-repositioning/ (Accessed: 26 February 2023).

260. XChange (2023) *Empty container repositioning: Why it happens [+how to avoid extra cost]*. Available at: https://www.container-xchange.com/blog/empty-container-repositioning/ (Accessed: 2 March 2023).

261. Xie, Y., Liang, X., Ma, L. and Yan, H. (2017) 'Empty Container Management and coordination in Intermodal Transport', *European Journal of Operational Research*, 257(1), pp. 223–232. Available at: https://doi.org/10.1016/j.ejor.2016.07.053.

262. Xu, L., Wang, C. and Zhao, J. (2018) 'Decision and coordination in the dual-channel supply chain considering Cap-and-trade regulation', *Journal of Cleaner Production*, 197, pp. 551–561. Available at: https://doi.org/10.1016/j.jclepro.2018.06.209.

263. Xu, X., He, P., Xu, H. and Zhang, Q. (2017) 'Supply chain coordination with green technology under CAP-and-trade regulation', *International Journal of Production Economics*, 183, pp. 433–442. Available at: https://doi.org/10.1016/j.ijpe.2016.08.029.

264. Xue, Z., Zhang, C., Yang, P. and Miao, L. (2015) 'A combinatorial benders' cuts algorithm for the local container drayage problem', *Mathematical Problems in Engineering*, 2015, article number 134763. Available at: http://dx.doi.org/10.1155/2015/134763

265. Yang, D., Xue, G., Zhang, J., Richa, A. and Fang, X. (2013) 'Coping with a smart jammer in wireless networks: A stackelberg game approach', *IEEE Transactions on Wireless Communications*, 12(8), pp. 4038–4047. Available at: https://doi.org/10.1109/twc.2013.071913121570.

266. Yang, S., Shi, H., Zhang, J., Liu, Y. and Huang, Y. (2022) 'Leader–follower Stackelberg game oriented adaptive robust constraint-following control design for Fuzzy Exoskeleton Robot Systems', *Information Sciences*, 606, pp. 272–291. Available at: https://doi.org/10.1016/j.ins.2022.05.043.

267. Young Yun, W., Mi Lee, Y. and Seok Choi, Y. (2011) 'Optimal inventory control of empty containers in inland transportation system', *International Journal of Production Economics*, 133(1), pp. 451–457. Available at: https://doi.org/10.1016/j.ijpe.2010.06.016.

268. Yu, C., Qu, Z., Archibald, T.W. and Luan, Z. (2020) 'An inventory model of a deteriorating product considering carbon emissions', *Computers & Industrial Engineering*, 148, p. 106694. Available at: https://doi.org/10.1016/j.cie.2020.106694.

269. Yu, H., Ning, J., Wang, Y., He, J. and Tan, C. (2021) 'Flexible yard management in container terminals for uncertain retrieving sequence', *Ocean & Coastal Management*, 212, p.105794. Available at: https://doi.org/10.1016/j.ocecoaman.2021.105794.

270. Yu, M. and Hong, S.H. (2016) 'A real-time demand-response algorithm for smart grids: A Stackelberg game approach', *IEEE Transactions on Smart Grid*, 7(2), pp. 879–888. Available at: https://doi.org/10.1109/tsg.2015.2413813.

271. Yu, M., Fransoo, J.C. and Lee, C.Y. (2018) 'Detention decisions for empty containers in the hinterland transportation system', *Transportation Research Part B: Methodological*, 110, pp. 188–208. Available at: https://doi.org/10.1016/j.trb.2018.02.007.

272. Yu, W. and Han, R. (2017) 'Coordinating a two-echelon supply chain under Carbon Tax', *Sustainability*, 9(12), p. 2360. Available at: https://doi.org/10.3390/su9122360.

273. Zhang, B. and Xu, L. (2013) 'Multi-item production planning with carbon cap and trade mechanism', *International Journal of Production Economics*, 144(1), pp. 118–127. Available at: https://doi.org/10.1016/j.ijpe.2013.01.024.
274. Zhang, B., Ng, C.T. and Cheng, T.C. (2014) 'Multi-period empty container repositioning with stochastic demand and lost sales', *Journal of the Operational Research Society*, 65(2), pp. 302–319. Available at: https://doi.org/10.1057/jors.2012.187.
275. Zhang, H., Li, P., Zheng, H. and Zhang, Y. (2021) 'Impact of carbon tax on enterprise operation and production strategy for low-carbon products in a co-opetition supply chain', *Journal of Cleaner Production*, 287, p. 125058. Available at: https://doi.org/10.1016/j.jclepro.2020.125058.
276. Zhang, M., Bell, P., Cai, G., and Chen, X. (2010). Optimal Fences and joint price and inventory decisions in distinct markets with demand leakage. *European Journal of Operational Research*, 204(3), pp.589–596. Available at: https://doi.org/10.1016/j.ejor.2009.11.032.
277. Zhang, R., Huang, C. and Feng, X. (2020) 'Empty container repositioning with foldable containers in a river transport network considering the limitations of Bridge Heights', *Transportation Research Part A: Policy and Practice*, 133, pp. 197–213. Available at: https://doi.org/10.1016/j.tra.2020.01.019.
278. Zhang, S., Ruan, X., Xia, Y. and Feng, X. (2017) 'Foldable container in empty container repositioning in intermodal transportation network of belt and road initiative: Strengths and Limitations', *Maritime Policy & Management*, 45(3), pp. 351–369. Available at: https://doi.org/10.1080/03088839.2017.1400699.
279. Zhao, F., Wu, D., Liang, L. and Dolgui, A. (2015) 'Lateral inventory transshipment problem in online-to-offline supply chain', *International Journal of Production Research*, 54(7), pp. 1951–1963. Available at: https://doi.org/10.1080/00207543.2015.1070971.
280. Zhao, H., Deshpande, V. and Ryan, J.K. (2006) 'Emergency transshipment in decentralized dealer networks: When to send and accept transshipment requests', *Naval Research Logistics*, 53(6), pp. 547–567. Available at: https://doi.org/10.1002/nav.20160.
281. Zhao, H., Song, S., Zhang, Y., Liao, Y. and Yue, F. (2020) 'Optimal decisions in supply chains with a call option contract under the Carbon Emissions Tax Regulation', *Journal of Cleaner Production*, 271, p. 122199. Available at: https://doi.org/10.1016/j.jclepro.2020.122199.
282. Zhao, X. and Atkins, D. (2009) 'Transshipment between competing retailers', *IIE Transactions*, 41(8), pp. 665–676. Available at: https://doi.org/10.1080/07408170802702120.
283. Zhao, Y., Choi, T.M., Cheng, T.C.E., Sethi, S.P. and Wang, S. (2014) 'Buyback contracts with price-dependent demands: Effects of demand uncertainty', *European Journal of Operational Research*, 239(3), pp. 663–673. Available at: https://doi.org/10.1016/j.ejor.2014.06.008.
284. Zhao, Y., Ma, L., Xie, G. and Cheng, T.C.E. (2013) 'Coordination of supply chains with bidirectional option contracts', *European Journal of Operational Research*, 229(2), pp. 375–381. Available at: https://doi.org/10.1016/j.ejor.2013.03.020.
285. Zhao, Y., Xue, Q. and Zhang, X. (2018) 'Stochastic empty container repositioning problem with CO2 emission considerations for an Intermodal Transportation System', *Sustainability*, 10(11), p. 4211. Available at: https://doi.org/10.3390/su10114211.
286. Zhao, Y.H. (2011) 'The study of effect of carbon tax on the international competitiveness of energy-intensive industries: An empirical analysis of OECD 21 countries', 1992-2008', *Energy Procedia*, 5, pp.

1291–1302. Available at: https://doi.org/10.1016/j.egypro.2011.03.225.

287. Zheng, J., Sun, Z. and Gao, Z. (2015) 'Empty Container Exchange among liner carriers', *Transportation Research Part E: Logistics and Transportation Review*, 83, pp. 158–169. Available at: https://doi.org/10.1016/j.tre.2015.09.007.
288. Zhou, W.H. and Lee, C.Y. (2009) 'Pricing and competition in a transportation market with empty equipment repositioning', *Transportation Research Part B: Methodological*, 43(6), pp. 677–691. Available at: https://doi.org/10.1016/j.trb.2008.12.001.
289. Zhou, Y. and Kim, K.H. (2019) 'Optimal concession contract between a Port Authority and container-terminal operators by revenue-sharing schemes with Quantity Discount', *Maritime Policy & Management*, 48(7), pp. 1010–1031. Available at: https://doi.org/10.1080/03088839.2019.1707314.
290. Zhou, Y., Fang, W., Li, M. and Liu, W. (2018) 'Exploring the impacts of a low-carbon policy instrument: A case of carbon tax on transportation in China', *Resources*, *Conservation and Recycling*, 139, pp. 307–314. Available at: https://doi.org/10.1016/j.resconrec.2018.08.015.
291. Zou, L., Dresner, M. and Windle, R. (2010) 'A two-location inventory model with transshipments in a competitive environment', *International Journal of Production Economics*, 125(2), pp. 235–250. Available at: https://doi.org/10.1016/j.ijpe.2010.03.001.

# Appendix

## Appendix A

$LSC$ 1's function in the centralised model include: satisfied demands $S_1$ leftover inventory after meeting customer demands $I_1$ and unsatisfied demands $L_1$.

$$\begin{cases} S_1(q,p) = \boldsymbol{E}min[X_1, n_1 - q + Y_1] = \boldsymbol{E}min[a_1 - b_1p + \xi_1, n_1 - q + Y_1] \\ I_1(q,p) = (n_1 - q + Y_1 - X_1)^+ = (n_1 - q + Y_1 - a_1 + b_1p - \xi_1)^+ \\ L_1(q,p) = (X_1 - n_1 + q - Y_1)^+ = (a_1 - b_1p + \xi_1 - n_1 + q - Y_1)^+ \end{cases}$$

Therefore, $LSC$ 1's profit function is:

$$\begin{aligned}\pi_1(q,p) &= r_1\boldsymbol{E}S_1(q,p) - h_1\boldsymbol{E}I_1(q,p) - g_1\boldsymbol{E}L_1(q,p) \\ &= r_1\boldsymbol{E}min[a_1 - b_1p + \xi_1, n_1 - q + Y_1] - h_1\boldsymbol{E}(n_1 - q + Y_1 - a_1 + b_1p - \xi_1)^+ - g_1\boldsymbol{E}(a_1 - b_1p + \xi_1 - n_1 \\ &\quad + q - Y_1)^+ \\ &= (r_1 + h_1 + g_1)\,\boldsymbol{E}min[\xi_1 - Y_1, n_1 - q - a_1 + b_1p] + r_1\boldsymbol{E}(Y_1 + a_1 - b_1p) - h_1(n_1 - q - a_1 + b_1p) \\ &\quad - g_1\boldsymbol{E}(\xi_1 - Y_1)\end{aligned}$$

Similarly, $LSC$ 2's profit function is:

$$\begin{aligned}\pi_2(q,p) &= (r_2 + h_2 + g_2)\,\boldsymbol{E}min[\xi_2 - Y_2, n_2 + q - a_2 + b_2p] + r_2\boldsymbol{E}(Y_2 + a_2 - b_2p) - h_2(n_2 + q - a_2 \\ &\quad + b_2p) - g_2\boldsymbol{E}(\xi_2 - Y_2)\end{aligned}$$

Denote $\alpha_1 = r_1 + h_1 + g_1, \alpha_2 = r_2 + h_2 + g_2, \beta_1 = n_1 - q - a_1 + b_1p, \beta_2 = n_2 + q - a_2 + b_2p$ . Hence, the system profit function in centralised model is:

$$\begin{aligned}\prod(q,p) &= \pi_1(q,p) + \pi_2(q,p) - c_t|q| \\ &= \alpha_1\,\boldsymbol{E}min[\xi_1 - Y_1, \beta_1] + \alpha_2\,\boldsymbol{E}min[\xi_2 - Y_2, \beta_2] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) \\ &\quad - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2p) - c_t|q|\end{aligned}$$

Where $z_i$ is the $pdf$ of $\xi_i - Y_i$; $Z_i$ is the $CDF$ of $\xi_i - Y_i$ and $\Phi_i$ is the complementary loss function $\left[Z_i(.) = \frac{d\Phi_i(.)}{d(.)}\right]$, $i = 1, 2$. $\prod(q,p)$ can be further transformed as below:

$$\begin{aligned}\prod(q,p) &= \alpha_1\boldsymbol{E}\,min[\xi_1 - Y_1, \beta_1] + \alpha_2\boldsymbol{E}min[\xi_2 - Y_2, \beta_2\,] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) \\ &\quad + r_1\boldsymbol{E}(Y_1 + a_1 - b_1p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2p) - c_t|q| \\ &= \alpha_1\left\{\beta_1[1 - Z_1(\beta_1)] + \int_0^{\beta_1}(\xi_1 - Y_1)\,z_1(\xi_1 - Y_1)d(\xi_1 - Y_1)\right\} \\ &\quad + \alpha_2\left\{\beta_2[1 - Z_2(\beta_2)] + \int_0^{\beta_2}(\xi_2 - Y_2)\,z_2(\xi_2 - Y_2)d(\xi_2 - Y_2)\right\} - h_1\beta_1 - h_2\beta_2 \\ &\quad - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2p) - c_t|q| \\ &= \alpha_1[\beta_1 - \Phi_1(\beta_1)] + \alpha_2[\beta_2 - \Phi_2(\beta_2)] - h_1\beta_1 - h_2\beta_2 - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 \\ &\quad - b_1p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2p) - c_t|q|\end{aligned}$$

## Appendix B

$$\frac{\partial\prod(q,p)}{\partial q} = -\alpha_1[1 - Z_1(\beta_1)] + \alpha_2[1 - Z_2(\beta_2)] + h_1 - h_2 - c_t sgn(q)$$

$$\frac{\partial^2\prod(q,p)}{\partial q^2} = -\alpha_1 z_1(\beta_1) - \alpha_2 z_2(\beta_2) < 0$$

$\prod(q,p)$ is not differentiable at $q=0$. For other differentiable points, due to $\frac{\partial^2\prod(q,p)}{\partial q^2}<0$. So, $\prod(q,p)$ is strictly concave in $q$.

**Appendix C**

We denote $q^*=\dot{q}$ when $q>0$ and $q^*=\ddot{q}$ when $q<0$. They satisfy:

$$\frac{\partial\prod(\dot{q},p)}{\partial q}=0 \text{ and } \frac{\partial\prod(\ddot{q},p)}{\partial q}=0$$

Due to $q\in[-n_2,n_1]$, thus, we divide the result of $q$ into five cases.

1. $0<n_1<\dot{q}$; In this case, $q^*=n_1$, and it satisfies:

$$\frac{\partial\prod(n_1,p)}{\partial q}>0\leftrightarrow \alpha_1 Z_1(\beta_1)-\alpha_2 Z_2(\beta_2)>(\alpha_1-h_1)-(\alpha_2-h_2)+T$$

Where $\beta_1=-a_1+b_1p$; $\beta_2=n_2+n_1-a_2+b_2p$. Thus, in this case, the optimal $ECR$ strategy in the centralised model is $n_1$.

2. $0<\dot{q}<n_1$; In this case, $q^*=\dot{q}$ and it satisfies:

$$\frac{\partial\prod(\dot{q},p)}{\partial q}=0\leftrightarrow \alpha_1 Z_1(\beta_1)-\alpha_2 Z_2(\beta_2)=(\alpha_1-h_1)-(\alpha_2-h_2)+T$$

Where $\beta_1=n_1-\dot{q}-a_1+b_1p$; $\beta_2=n_2+\dot{q}-a_2+b_2p$. Thus, in this case, the optimal $ECR$ strategy is $\dot{q}$.

3. $\dot{q}<0<\ddot{q}$; In this case, $q^*=0$ and it satisfies:

$$\frac{\partial\prod(\dot{q},p)}{\partial q}<0\leftrightarrow \alpha_1 Z_1(\beta_1)-\alpha_2 Z_2(\beta_2)<(\alpha_1-h_1)-(\alpha_2-h_2)+T$$

Where $\beta_1=n_1-\dot{q}-a_1+b_1p$; $\beta_2=n_2+\dot{q}-a_2+b_2p$.

$$\frac{\partial\prod(\ddot{q},p)}{\partial q}>0\leftrightarrow \alpha_1 Z_1(\beta_1)-\alpha_2 Z_2(\beta_2)>(\alpha_1-h_1)-(\alpha_2-h_2)+T$$

Where $\beta_1=n_1-\ddot{q}-a_1+b_1p$; $\beta_2=n_2+\ddot{q}-a_2+b_2p$. Thus, in this case, the optimal $ECR$ strategy is 0.

4. $-n_2<\ddot{q}<0$; In this case, $q^*=\ddot{q}$ and it satisfies:

$$\frac{\partial\prod(\ddot{q},p)}{\partial q}=0\leftrightarrow \alpha_1 Z_1(\beta_1)-\alpha_2 Z_2(\beta_2)=(\alpha_1-h_1)-(\alpha_2-h_2)+T$$

Where $\beta_1=n_1-\ddot{q}-a_1+b_1p$; $\beta_2=n_2+\ddot{q}-a_2+b_2p$. Thus, in this case, the optimal $ECR$ strategy is $\ddot{q}$.

5. $\ddot{q}<-n_2<0$; In this case, $q^*=-n_2$ and it satisfies:

$$\frac{\partial\prod(-n_2,p)}{\partial q}<0\leftrightarrow \alpha_1 Z_1(\beta_1)-\alpha_2 Z_2(\beta_2)<(\alpha_1-h_1)-(\alpha_2-h_2)+T$$

Where $\beta_1=n_1+n_2-a_1+b_1p$; $\beta_2=-a_2+b_2p$. Thus, in this case, the optimal $ECR$ strategy is $-n_2$.

**Appendix D**

The system profit increment in the centralised model is:

$\Delta\prod(q^*,p)=\prod(q^*,p)-\prod(0,0)$

$$= \alpha_1 \left\{ n_1 - q^* - a_1 + b_1 p - \int_0^{n_1 - q^* - a_1 + b_1 p} Z_1(d_1) dd_1 - (n_1 - a_1) + \int_0^{n_1 - a_1} Z_1(d_1) dd_1 \right\}$$
$$+ \alpha_2 \left\{ n_2 + q^* - a_2 + b_2 p - \int_0^{n_2 + q^* - a_2 + b_2 p} Z_2(d_2) dd_2 - (n_2 - a_2) + \int_0^{n_2 - a_2} Z_2(d_2) dd_2 \right\}$$
$$- h_1(n_1 - q^* - a_1 + b_1 p) - h_2(n_2 + q^* - a_2 + b_2 p) + h_1(n_1 - a_1) + h_2(n_2 - a_2)$$
$$- g_1 \boldsymbol{E}(\xi_1 - Y_1) - g_2 \boldsymbol{E}(\xi_2 - Y_2) + g_1 \boldsymbol{E}(\xi_1 - Y_1) + \gamma_2 \boldsymbol{E}(\xi_2 - Y_2) + r_1 \boldsymbol{E}(Y_1 + a_1 - b_1 p)$$
$$+ r_1 \boldsymbol{E}(-a_1 + b_1 p - Y_1) + r_2 \boldsymbol{E}(Y_2 + a_2 - b_2 p) + r_2 \boldsymbol{E}(-a_2 + b_2 p - Y_2) - c_t\, |q^*|$$

$$= -\alpha_1 \int_{n_1 - a_1}^{n_1 - q^* - a_1 + b_1 p} U_1(d_1) dd_1 - \alpha_2 \int_{n_2 - a_2}^{n_2 + q^* - a_2 + b_2 p} U_2(d_2) dd_2 + (\alpha_2 - \alpha_1 + h_1 - h_2) q^* - c_t\, |q^*|$$
$$+ g_1 b_1 p + g_2 b_2 p$$

$$= -\alpha_1 \left[ \int_{n_1 - a_1}^{n_1 - q^* - a_1 + b_1 p} Z_1(d_1) dd_1 + q^* - b_1 p \right] - \alpha_2 \left[ \int_{n_2 - a_2}^{n_2 + q^* - a_2 + b_2 p} Z_2(d_2) dd_2 - q^* - b_2 p \right]$$
$$- (\alpha_1 - g_1) b_1 p - (\alpha_2 - g_2) b_2 p + (h_1 - h_2) q^* - c_t\, |q^*|$$

**Appendix E**

$$\pi_1(q, p, w, \eta_1, \eta_2)$$
$$= r_1 \boldsymbol{E} \min[X_1, n_1 - q + Y_1] - \eta_2 \boldsymbol{E} \min[(n_2 + q + Y_2 - X_2)^+, q^+] - h_1 \boldsymbol{E}(n_1 - q + Y_1 - X_1)^+$$
$$+ \eta_1 \boldsymbol{E} \min[(n_1 - q + Y_1 - X_1)^+, (-q)^+] - g_1 \boldsymbol{E}(X_1 + q - n_1 - Y_1)^+ + q^+(w - c_t)$$
$$- w(-q)^+$$
$$= r_1 \{ \boldsymbol{E} \min[a_1 - b_1 p + \xi_1, n_1 - q + Y_1] + \boldsymbol{E} \min[-a_1 + b_1 p - Y_1, -a_1 + b_1 p - Y_1]$$
$$- \boldsymbol{E} \min[-a_1 + b_1 p - Y_1, -a_1 + b_1 p - Y_1] \}$$
$$- \eta_2 \left\{ \boldsymbol{E} \min\left[ \left(\beta_2 - (\xi_2 - Y_2)\right)^+, q^+ \right] + \boldsymbol{E} \min\left[ 0, \left(\beta_2 - q - (\xi_2 - Y_2)\right)^+ \right] \right.$$
$$\left. - \boldsymbol{E} \min\left[ 0, \left(\beta_2 - q - (\xi_2 - Y_2)\right)^+ \right] \right\} - h_1 \boldsymbol{E}[\beta_1 - (\xi_1 - Y_1)]$$
$$+ \eta_1 \left\{ \boldsymbol{E} \min\left[ \left(\beta_1 - (\xi_1 - Y_1)\right)^+, (-q)^+ \right] + \boldsymbol{E} \min\left[ 0, \left(\beta_1 + q - (\xi_1 - Y_1)\right)^+ \right] \right.$$
$$\left. - \boldsymbol{E} \min\left[ 0, \left(\beta_1 + q - (\xi_1 - Y_1)\right)^+ \right] \right\} - g_1 \boldsymbol{E}[(\xi_1 - Y_1) - (n_1 + b_1 p - a_1 - q)^+]$$
$$+ q^+(w - c_t) - w(-q)^+$$
$$= \left(\alpha_1 - \eta_1 \mathbf{1}_{q<0}\right) \boldsymbol{E} \min[\xi_1 - Y_1, \beta_1] + \eta_2 \mathbf{1}_{q>0} \boldsymbol{E} \min[\xi_2 - Y_2, \beta_2] + \eta_1 \mathbf{1}_{q<0} \boldsymbol{E} \min[\xi_1 - Y_1, n_1 - a_1 + b_1 p]$$
$$- \eta_2 \mathbf{1}_{q>0} \boldsymbol{E} \min[\xi_2 - Y_2, n_2 - a_2 + b_2 p] - r_1 \boldsymbol{E}(-a_1 + b_1 p - Y_1) - \eta_1 \mathbf{1}_{q<0} q - \eta_2 \mathbf{1}_{q>0} q$$
$$- h_1 \beta_1 - g_1 \boldsymbol{E}(\xi_1 - Y_1) + q^+(w - c_t) - w(-q)^+$$
$$= \left(\alpha_1 - \eta_1 \mathbf{1}_{q<0}\right) [\beta_1 - \Phi_1(\beta_1)] + \eta_2 \mathbf{1}_{q>0} [\beta_2 - \Phi_1(\beta_2)] + \eta_1 \mathbf{1}_{q<0} [(\beta_1 + q) - \Phi_1(\beta_1 + q)]$$
$$- \eta_2 \mathbf{1}_{q>0} [(\beta_2 - q) - \Phi_2(\beta_2 - q)] - r_1(-a_1 + b_1 p) \boldsymbol{E}(-Y_1) - \eta_1 \mathbf{1}_{q<0} q - \eta_2 \mathbf{1}_{q>0} q$$
$$- h_1 \beta_1 - g_1 \boldsymbol{E}(\xi_1 - Y_1) + q^+(w - c_t) - w(-q)^+$$

**Appendix F**

$$\frac{\partial \pi_1}{\partial q} = -\left(\alpha_1 - \eta_1 \mathbf{1}_{q<0}\right) [1 - Z_1(\beta_1)] + \eta_2 \mathbf{1}_{q>0} [1 - Z_2(\beta_2)] - \eta_1 \mathbf{1}_{q<0} - \eta_2 \mathbf{1}_{q>0} + h_1 + (w - c_t) \mathbf{1}_{q>0}$$
$$- w \mathbf{1}_{q<0}$$

$$\frac{\partial^2\pi_1}{\partial q^2} = -(\alpha_1 - \eta_1\mathbf{1}_{q<0})z_1(\beta_1) - \eta_2\mathbf{1}_{q>0}z_2(\beta_2)$$

To prove $\pi_1(q,p,w,\eta_1,\eta_2)$ is strictly concave in $q$ is equivalent to prove that $\frac{\partial^2\pi_1}{\partial q^2} < 0$. Thus, $\pi_1(q,p,w,\eta_1,\eta_2)$ is strictly concave in $q$ as long as $0 < \eta_1 < \alpha_1$.

$$\frac{\partial\pi_2}{\partial q} = (\alpha_2 - \eta_2\mathbf{1}_{q>0})[1 - Z_2(\beta_2)] - \eta_1\mathbf{1}_{q<0}[1 - Z_1(\beta_1)] + \eta_1\mathbf{1}_{q<0} + \eta_2\mathbf{1}_{q>0} - h_2 + (w - c_t)\mathbf{1}_{q<0} - w\mathbf{1}_{q>0}$$

$$\frac{\partial^2\pi_2}{\partial q^2} = -(\alpha_2 - \eta_2\mathbf{1}_{q>0})z_2(\beta_2) - \eta_1\mathbf{1}_{q<0}z_1(\beta_1)$$

To prove $\pi_2(q,p,w,\eta_1,\eta_2)$ is strictly concave in $q$ is equivalent to prove that $\frac{\partial^2\pi_2}{\partial q^2} < 0$. Thus, $\pi_2(q,p,w,\eta_1,\eta_2)$ is strictly concave in $q$ as long as $0 < \eta_2 < \alpha_2$.

## Appendix G

$$\begin{aligned}
&\Delta\pi_1(q^e,p,w,\eta_1,\eta_2) = \pi_1(q^e,p,w,\eta_1,\eta_2) - \pi_1(0,0,w,\eta_1,\eta_2)\\
&= (\alpha_1 - \eta_1\mathbf{1}_{q^e<0})\left[n_1 - q^e - a_1 + b_1p - \int_0^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1\right]\\
&\quad + \eta_2\mathbf{1}_{q^e>0}\left[n_2 + q^e - a_2 + b_2p - \int_0^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2\right]\\
&\quad + \eta_1\mathbf{1}_{q^e<0}\left[n_1 - a_1 - \int_0^{n_1-a_1} Z_1(d_1)dd_1\right] - \eta_2\mathbf{1}_{q^e>0}\left[n_2 - a_2 - \int_0^{n_2-a_2} Z_2(d_2)dd_2\right]\\
&\quad - (\eta_1\mathbf{1}_{q^e<0} + \eta_2\mathbf{1}_{q^e>0})q^e - h_1(n_1 - q^e - a_1 + b_1p) - g_1\mathbf{E}(\xi_1 - Y_1) + w(q^e)^+\\
&\quad - c_t(q^e)^+ + r_1\mathbf{E}(a_1 - b_1p + Y_1) - \alpha_1\left[n_1 - a_1 - \int_0^{n_1-a_1} Z_1(d_1)dd_1\right] + r_1\mathbf{E}(-a_1 - Y_1)\\
&\quad + h_1(n_1 - a_1) + g_1\mathbf{E}(\xi_1 - Y_1)\\
&= -\alpha_1\int_{n_1-a_1}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1 + \eta_1\mathbf{1}_{q^e<0}\int_{n_1-a_1+b_1p}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1\\
&\quad - \eta_2\mathbf{1}_{q^e>0}\int_{n_2-a_2+b_2p}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 + (h_1 - \alpha_1 + w)q^e + g_1b_1p - c_t(q^e)^+
\end{aligned}$$

$$\begin{aligned}
&\Delta\pi_2(q^e,p,w,\eta_1,\eta_2) = \pi_2(q^e,p,w,\eta_1,\eta_2) - \pi_2(0,0,w,\eta_1,\eta_2)\\
&= (\alpha_2 - \eta_2\mathbf{1}_{q^e>0})\left[n_2 + q^e - a_2 + b_2p - \int_0^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2\right]\\
&\quad + \eta_1\mathbf{1}_{q^e<0}\left[n_1 - q^e - a_1 + b_1p - \int_0^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1\right]\\
&\quad - \eta_1\mathbf{1}_{q^e<0}\left[n_1 - a_1 - \int_0^{n_1-a_1} Z_1(d_1)dd_1\right] + \eta_2\mathbf{1}_{q^e>0}\left[n_2 - a_2 - \int_0^{n_2-a_2} Z_2(d_2)dd_2\right]\\
&\quad + (\eta_1\mathbf{1}_{q^e<0} + \eta_2\mathbf{1}_{q^e>0})q^e - h_2(n_2 + q^e - a_2 + b_2p) - r_2(-a_2 + b_2p - Y_2)\\
&\quad - g_2\mathbf{E}(\xi_2 - Y_2) + (-q^e)^+(w - c_t) - (q^e)^+w - \alpha_2\left[n_2 - a_2 - \int_0^{n_2-a_2} Z_2(d_2)dd_2\right]\\
&\quad + r_2(-a_2 - Y_2) + h_2(n_2 - a_2) + g_2\mathbf{E}(\xi_2 - Y_2)
\end{aligned}$$

$$= -\alpha_2 \int_{n_2-a_2}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 - \eta_1 \mathbf{1}_{q^e<0} \int_{n_1-a_1+b_1p}^{n_1-q^e-a_1+b_1p} Z_1(d_1)dd_1$$

$$+ \eta_2 \mathbf{1}_{q^e>0} \int_{n_2-a_2+b_2p}^{n_2+q^e-a_2+b_2p} Z_2(d_2)dd_2 + (\alpha_2 - h_2 - w)q^e + g_2b_2p - c_t(-q^e)^+$$

**Appendix H**

The marginal profit of $LSC$ 1 and 2 are:

$$\frac{\partial\pi_1(q^e,p,w,\eta_1,\eta_2)}{\partial q} = \begin{cases} -\alpha_1 + \alpha_1 Z_1(n_1 - q^e - a_1 + b_1p) - \eta_2 Z_2(n_2 + q^e - a_2 + b_2p) + h_1 + w - c_t & q > 0 \\ -\alpha_1 + \alpha_1 Z_1(n_1 - q^e - a_1 + b_1p) - \eta_1 Z_1(n_1 - q^e - a_1 + b_1p) + h_1 + w & q < 0 \end{cases}$$

$$\frac{\partial\pi_2(q^e,p,w,\eta_1,\eta_2)}{\partial q} = \begin{cases} \alpha_2 - \alpha_2 Z_2(n_2 + q^e - a_2 + b_2p) + \eta_2 Z_2(n_2 + q^e - a_2 + b_2p) - h_2 - w & q > 0 \\ \alpha_2 - \alpha_2 Z_2(n_2 + q^e - a_2 + b_2p) + \eta_1 Z_1(n_1 - q^e - a_1 + b_1p) - h_2 - w + c_t & q < 0 \end{cases}$$

**Case 1.** $0 < n_1 \le \dot{q}; (q^e = q^* = n_1;\ \eta_1 = 0)$

$$\Delta\prod(n_1,p) = -\alpha_1 \int_{n_1-a_1}^{-a_1+b_1p} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 + (\alpha_2 - \alpha_1 + h_1 - h_2 - c_t)n_1$$

$$+ g_1b_1p + g_2b_2p$$

$$\Delta\pi_1(n_1,p,w,0,\eta_2)$$

$$= -\alpha_1 \int_{n_1-a_1}^{-a_1+b_1p} Z_1(d_1)dd_1 - \eta_2 \int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 + (h_1 - \alpha_1 + w - c_t)n_1$$

$$+ g_1b_1p$$

In this case, the conditions for voluntary compliance of $\eta_2$ and $w$ in two $LSCs'$ profit functions are:

$$\begin{cases} \dfrac{\partial\pi_1(n_1,p,w,0,\eta_2)}{\partial q} \ge 0 \\ \dfrac{\partial\pi_2(n_1,p,w,0,\eta_2)}{\partial q} \ge 0 \end{cases}$$

$$\rightarrow \begin{cases} -\alpha_1 + \alpha_1 Z_1(-a_1 + b_1p) - \eta_2 Z_2(n_2 + n_1 - a_2 + b_2p) + h_1 + w - c_t \ge 0 \\ \alpha_2 - \alpha_2 Z_2(n_2 + n_1 - a_2 + b_2p) + \eta_2 Z_2(n_2 + n_1 - a_2 + b_2p) - h_2 - w \ge 0 \end{cases}$$

$$\rightarrow \alpha_1 - \alpha_1 Z_1(-a_1 + b_1p) - h_1 + c_t \le w - \eta_2 Z_2(n_2 + n_1 - a_2 + b_2p)$$

$$\le \alpha_2 - \alpha_2 Z_2(n_2 + n_1 - a_2 + b_2p) - h_2$$

If $w$ and $\eta_2$ follow the condition, then $q^e = q^* = n_1$. Based on equation 4.18, we know $\Delta\pi_2(n_1,p,w,0,\eta_2) = \Delta\prod(n_1,p) - \Delta\pi_1(n_1,p,w,0,\eta_2)$. So, the system is coordinated if two $LSCs's$ profit increment is non-negative. Let $w_{max} = \alpha_2 - h_2 - (\alpha_2 - \eta_2)Z_2(n_2 + n_1 - a_2 + b_2p)$, then: (Notice that $\Delta\pi_1$ increases in $w$.)

$$\Delta\pi_1(n_1,p,w,0,\eta_2)$$
$$= -\alpha_1 \int_{n_1-a_1}^{-a_1+b_1p} Z_1(d_1)dd_1 - \eta_2 \int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2$$
$$+ [h_1 - \alpha_1 + \alpha_2 - h_2 - (\alpha_2 - \eta_2)Z_2(n_2+n_1-a_2+b_2p) - c_t]n_1 + g_1b_1p$$
$$\Delta\pi_2(n_1,p,w,0,\eta_2) = \Delta\prod(n_1,p) - \Delta\pi_{1\max}(n_1,p,w,0,\eta_2)$$
$$= \eta_2 \int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 - \alpha_2 \int_{n_2-a_2}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2$$
$$+ n_1(\alpha_2 - \eta_2)Z_2(n_2+n_1-a_2+b_2p) + g_2b_2p$$
$$= \eta_2 \int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2$$
$$- \alpha_2 \left[\int_{n_2-a_2+b_2p}^{n_2-a_2} Z_2(d_2)dd_2 + \int_{n_2-a_2}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 - \int_{n_2-a_2+b_2p}^{n_2-a_2} Z_2(d_2)dd_2\right]$$
$$+ n_1(\alpha_2 - \eta_2)Z_2(n_2+n_1-a_2+b_2p) + g_2b_2p$$
$$\Delta\pi_2(n_1,p,w,0,\eta_2)$$
$$= -(\alpha_2 - \eta_2)\left[\int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 - n_1Z_2(n_2+n_1-a_2+b_2p)\right]$$
$$- \alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p} Z_2(d_2)dd_2 - b_2p\right] - (\alpha_2 - g_2)b_2p$$

Based on the definition of $S_2(.)$ shown in Snyder and Shen (2011, Page 281)

$$\Delta S_2(q) = S_2(q_2) - S_2(q_1) = -\left[\int_{q_1}^{q_2} CDFdd + q_1 - q_2\right]$$

Therefore, concerning the government $CT$ rate, we further denote $\Delta S_2^e(n_1,p)$ as:

$$\Delta S_2^e(n_1,p) = -\left[\int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 - n_1Z_2(n_2+n_1-a_2+b_2p)\right]$$

Where $\Delta S_2^e(n_1,p)$ as the expectation of efficient satisfied demand between the scenario with and without $ECS$ under government $CT$ impact. According to the mean value theorem of integrals, we find $\Delta S_2^e(n_1,p)$ is no less than 0. Thus, $\Delta\pi_2$ decreases with $\eta_2$. Also:

$$\Delta S_2(0,p) = \int_{n_2-a_2}^{n_2-a_2+b_2p} Z_2(d_2)dd_2 - b_2p$$

Therefore, $LSC$ 2's profit increment under system coordination is:

$$\Delta\pi_2(n_1,p,w,0,\eta_2) = (\alpha_2 - \eta_2)\Delta S_2^e(n_1,p) + \alpha_2\Delta S_2(0,p) - (\alpha_2 - g_2)b_2p$$

If $\Delta\pi_2(n_1,p,w,0,\eta_2) \geq 0$, then:

$$\eta_2 \leq \frac{\alpha_2[\Delta S_2^e(n_1,p) + \Delta S_2(0,p)] - (\alpha_2 - g_2)b_2p}{\Delta S_2^e(n_1,p)}$$

Moreover, if $w_{min} = \alpha_1 - \alpha_1Z_1(-a_1+b_1p) - h_1 + c_t + \eta_2Z_2(n_2+n_1-a_2+b_2p)$, then $\Delta\pi_1$ is:

$\Delta\pi_1(n_1,p,w,0,\eta_2)$

$$= -\alpha_1\int_{n_1-a_1}^{-n_1+b_1p} Z_1(d_1)dd_1 - \eta_2\int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2$$

$$+ [-\alpha_1 Z_1(-a_1+b_1p) + \eta_2 Z_2(n_2+n_1-a_2+b_2p)]n_1 + g_1b_1p$$

$$= -\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p} Z_1(d_1)dd_1 + \int_{n_1-a_1+b_1p}^{-a_1+b_1p} Z_1(d_1)dd_1\right] - \eta_2\int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2$$

$$+ [-\alpha_1 Z_1(-a_1+b_1p) + \eta_2 Z_2(n_2+n_1-a_2+b_2p)]n_1 + g_1b_1p$$

$$= -\alpha_1\left[\int_{n_1-a_1+b_1p}^{-a_1+b_1p} Z_1(d_1)dd_1 + n_1 Z_1(-a_1+b_1p)\right]$$

$$-\eta_2\left[\int_{n_2-a_2+b_2p}^{n_2+n_1-a_2+b_2p} Z_2(d_2)dd_2 - n_1 Z_2(n_2+n_1-a_2+b_2p)\right]$$

$$-\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p} Z_1(d_1)dd_1 - b_1p\right] - (\alpha_1-g_1)b_1p$$

$\Delta\pi_1(n_1,p,w,0,\eta_2) = \alpha_1[\Delta S_1^e(n_1,p) + \Delta S_1(0,p)] + \eta_2\Delta S_2^e(n_1,p) - (\alpha_1-g_1)b_1p$

If $\Delta\pi_1(n_1,p,w,0,\eta_2) \geq 0$, then

$$\eta_2 \geq \frac{(\alpha_1-g_1)b_1p - \alpha_1[\Delta S_1^e(n_1,p) + \Delta S_1(0,p)]}{\Delta S_2^e(n_1,p)}$$

Therefore, in this case, $\eta_2$ should follow:

$$\frac{(\alpha_1-g_1)b_1p - \alpha_1[\Delta S_1^e(n_1,p) + \Delta S_1(0,p)]}{\Delta S_2^e(n_1,p)} \leq \eta_2 \leq \frac{\alpha_2[\Delta S_2^e(n_1,p) + \Delta S_2(0,p)] - (\alpha_2-g_2)b_2p}{\Delta S_2^e(n_1,p)}$$

**Case 2**. $0 < \dot{q} < n_1(q^* = q^e = \dot{q}; \eta_1 = 0)$

$$\Delta\prod(\dot{q},p) = -\alpha_1\int_{n_1-a_1}^{n_1-\dot{q}-a_1+b_1p} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2}^{n_2+\dot{q}-a_2+b_2p} Z_2(d_2)dd_2 + (\alpha_2-\alpha_1+h_1-h_2-c_t)\dot{q}$$

$$+ g_1b_1p + g_2b_2p$$

$\Delta\pi_1(\dot{q},p,w,0,\eta_2)$

$$= -\alpha_1\int_{n_1-a_1}^{n_1-\dot{q}-a_1+b_1p} Z_1(d_1)dd_1 - \eta_2\int_{n_2-a_2+b_2p}^{n_2+\dot{q}-a_2+b_2p} Z_2(d_2)dd_2 + (h_1-\alpha_1+w-c_t)\dot{q}$$

$$+ g_1b_1p$$

In this case, the conditions for voluntary compliance of $\eta_2$ and $w$ are:

$$\begin{cases} \dfrac{\partial\pi_1(\dot{q},p,w,0,\eta_2)}{\partial q} = 0 \\ \dfrac{\partial\pi_2(\dot{q},p,w,0,\eta_2)}{\partial q} = 0 \end{cases}$$

$$\rightarrow \begin{cases} -\alpha_1 + \alpha_1 Z_1(n_1-\dot{q}-a_1+b_1p) - \eta_2 Z_2(n_2+\dot{q}-a_2+b_2p) + h_1 + w - c_t = 0 \\ \alpha_2 - \alpha_2 Z_2(n_2+\dot{q}-a_2+b_2p) + \eta_2 Z_2(n_2+\dot{q}-a_2+b_2p) - h_2 - w = 0 \end{cases}$$

$$\rightarrow \alpha_1[1 - Z_1(n_1-\dot{q}-a_1+b_1p)] - h_1 + c_t = w - \eta_2 Z_2(n_2+\dot{q}-a_2+b_2p)$$

$$= \alpha_2[1 - Z_2(n_2+\dot{q}-a_2+b_2p)] - h_2$$

If $w = \alpha_2 - h_2 + (\eta_2 - \alpha_2)Z_2(n_2 + \dot{q} - a_2 + b_2 p)$, then:

$\Delta\pi_1(\dot{q}, p, w, 0, \eta_2)$

$$= -\alpha_1 \int_{n_1-a_1}^{n_1-\dot{q}-a_1+b_1 p} Z_1(d_1)dd_1 - \eta_2 \int_{n_2-a_2+b_2 p}^{n_2+\dot{q}-a_2+b_2 p} Z_2(d_2)dd_2$$

$$+ [h_1 - \alpha_1 + \alpha_2 - h_2 + (\eta_2 - \alpha_2)Z_2(n_2 + \dot{q} - a_2 + b_2 p) - c_t]\dot{q} + g_1 b_1 p$$

$\Delta\pi_2(\dot{q}, p, w, 0, \eta_2) = \Delta\prod(\dot{q}, p) - \Delta\pi_1(\dot{q}, p, w, 0, \eta_2)$

$$= -(\alpha_2 - \eta_2)\left[\int_{n_2-a_2+b_2 p}^{n_2+\dot{q}-a_2+b_2 p} Z_2(d_2)dd_2 - \dot{q}Z_2(n_2 + \dot{q} - a_2 + b_2 p)\right]$$

$$-\alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2 p} Z_2(d_2)dd_2 - b_2 p\right] - (\alpha_2 - g_2)b_2 p$$

$$= (\alpha_2 - \eta_2)\Delta S_2^e(\dot{q}, p) + \alpha_2 \Delta S_2(0, p) - (\alpha_2 - g_2)b_2 p$$

$\Delta\pi_2$ decreases with $\eta_2$ because $\int_{n_2-a_2+b_2 p}^{n_2+\dot{q}-a_2+b_2 p} Z_2(d_2)dd_2 - \dot{q}Z_2(n_2 + \dot{q} - a_2 + b_2 p) \leq 0$. If $\Delta\pi_2 \geq 0$, then:

$$\eta_2 \leq \frac{\alpha_2[\Delta S_2^e(\dot{q}, p) + \Delta S_2(0, p)] - (\alpha_2 - g_2)b_2 p}{\Delta S_2^e(\dot{q}, p)}$$

On the other hand, if $w = \eta_2 Z_2(n_2 + \dot{q} - a_2 + b_2 p) + \alpha_1 - \alpha_1 Z_1(n_1 - \dot{q} - a_1 + b_1 p) - h_1 + c_t$, then:

$\Delta\pi_1(\dot{q}, p, w, 0, \eta_2)$

$$= -\alpha_1\left[\int_{n_1-a_1+b_1 p}^{n_1-\dot{q}-a_1+b_1 p} Z_1(d_1)dd_1 + \dot{q}Z_1(n_1 - \dot{q} - a_1 + b_1 p)\right]$$

$$-\eta_2\left[\int_{n_2-a_2+b_2 p}^{n_2+\dot{q}-a_2+b_2 p} Z_2(d_2)dd_2 - \dot{q}Z_2(n_2 + \dot{q} - a_2 + b_2 p)\right]$$

$$-\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1 p} Z_1(d_1)dd_1 - b_1 p\right] - (\alpha_1 - g_1)b_1 p$$

$\Delta\pi_1(\dot{q}, p, w, 0, \eta_2) = \alpha_1[\Delta S_1^e(\dot{q}, p) + \Delta S_1(0, p)] + \eta_2 \Delta S_2^e(\dot{q}, p) - (\alpha_1 - g_1)b_1 p$

$\Delta\pi_1$ rises with $\eta_2$ due to $\int_{n_2-a_2+b_2 p}^{n_2+\dot{q}-a_2+b_2 p} Z_2(d_2)dd_2 - \dot{q}Z_2(n_2 + \dot{q} - a_2 + b_2 p) < 0$, if $\Delta\pi_1 \geq 0$, then

$$\eta_2 \geq \frac{(\alpha_1 - g_1)b_1 p - \alpha_1[\Delta S_1^e(\dot{q}, p) + \Delta S_1(0, p)]}{\Delta S_2^e(\dot{q}, p)}$$

Therefore, in this case, $\eta_2$ should follow:

$$\frac{(\alpha_1 - g_1)b_1 p - \alpha_1[\Delta S_1^e(\dot{q}, p) + \Delta S_1(0, p)]}{\Delta S_2^e(\dot{q}, p)} \leq \eta_2 \leq \frac{\alpha_2[\Delta S_2^e(\dot{q}, p) + \Delta S_2(0, p)] - (\alpha_2 - g_2)b_2 p}{\Delta S_2^e(\dot{q}, p)}$$

**Case 3**. $\dot{q} < 0$ *or* $\ddot{q} > 0 (q^* = q^e = 0;\ \eta_1 = \eta_2 = 0)$

In this case, $w = 0, \eta_1 = \eta_2 = 0$, the conditions for voluntary compliance of $\eta_2$ and $w$ are:

$$\begin{cases} \dfrac{\partial^+ \pi_1(0, p, w, 0, 0)}{\partial p} \leq 0 \\ \dfrac{\partial^+ \pi_2(0, p, w, 0, 0)}{\partial p} \leq 0 \end{cases} \begin{cases} \dfrac{\partial^- \pi_1(0, p, w, 0, 0)}{\partial p} \geq 0 \\ \dfrac{\partial^- \pi_2(0, p, w, 0, 0)}{\partial p} \geq 0 \end{cases}$$

So, there is no $ECS$ between two $LSCs$ and $\Delta\prod(0,p) = \Delta\pi_1(0,p,w,0,0) = \Delta\pi_2(0,p,w,0,0) = 0$. So, the system is coordinated but the profit increment for $LSCs$ is 0.

**Case 4.** $-n_2 < \ddot{q} < 0$; $(q^* = q^e = \ddot{q};\ \eta_2 = 0)$

$$\Delta\prod(\ddot{q},p) = -\alpha_1 \int_{n_1-a_1}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2}^{n_2+\ddot{q}-a_2+b_2p} Z_2(d_2)dd_2 + (\alpha_2 - \alpha_1 + h_1 - h_2 + c_t)\ddot{q} + g_1b_1p + g_2b_2p$$

$$\Delta\pi_1(\ddot{q},p,w,\eta_1,0) = -\alpha_1 \int_{n_1-a_1}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + \eta_1 \int_{n_1-a_1}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + (h_1 - \alpha_1 + w)\ddot{q} + g_1b_1p$$

In this case, the conditions for voluntary compliance of $\eta_2$ and $w$ are:

$$\begin{cases} \dfrac{\partial\pi_1(\ddot{q},p,w,\eta_1,0)}{\partial q} = 0 \\ \dfrac{\partial\pi_2(\ddot{q},p,w,\eta_1,0)}{\partial q} = 0 \end{cases}$$

$$\rightarrow \begin{cases} -\alpha_1 + \alpha_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p) - \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p) + h_1 + w = 0 \\ \alpha_2 - \alpha_2 Z_2(n_2 + \ddot{q} - a_2 + b_2p) + \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p) - h_2 - w + c_t = 0 \end{cases}$$

$$\rightarrow \alpha_2[1 - Z_2(n_2 + \ddot{q} - a_2 + b_2p)] - h_2 + c_t = w - \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p) = \alpha_1[1 - Z_1(n_1 - \ddot{q} - a_1 + b_1p)] - h_1$$

If $w = \alpha_2[1 - Z_2(n_2 + \ddot{q} - a_2 + b_2p)] - h_2 + c_t + \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p)$, then:

$$\Delta\pi_1(\ddot{q},p,w,\eta_1,0) = -\alpha_1 \int_{n_1-a_1}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + \eta_1 \int_{n_1-a_1+b_1p}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + [h_1 - \alpha_1 + \alpha_2 - \alpha_2 Z_2(n_2 + \ddot{q} - a_2 + b_2p) + \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p) - h_2 + c_t]\ddot{q} + g_1b_1p$$

$$\Delta\pi_2(\ddot{q},p,w,\eta_1,0) = \Delta\prod(\ddot{q},p) - \Delta\pi_1(\ddot{q},p,w,\eta_1,0)$$
$$= -\alpha_2\left[\int_{n_2-a_2+b_2p}^{n_2+\ddot{q}-a_2+b_2p} Z_2(d_2)dd_2 - \ddot{q}Z_2(n_2 + \ddot{q} - a_2 + b_2p)\right] - \eta_1\left[\int_{n_1-a_1+b_1p}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + \ddot{q}Z_1(n_1 - \ddot{q} - a_1 + b_1p)\right] - \alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p} Z_2(d_2)dd_2 - b_2p\right] - (\alpha_2 - g_2)b_2p$$

$$\Delta\pi_2(\ddot{q},p,w,\eta_1,0) = \alpha_2[\Delta S_2^e(\ddot{q},p) + \Delta S_2(0,p)] + \eta_1 \Delta S_1^e(\ddot{q},p) - (\alpha_2 - g_2)b_2p$$

So, $\Delta\pi_2$ rises with $\eta_1$ as $\int_{n_1-a_1+b_1p}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + \ddot{q}Z_1(n_1 - \ddot{q} - a_1 + b_1p) \leq 0$. If $\Delta\pi_2 \geq 0$, then:

$$\eta_1 \geq \frac{(\alpha_2 - g_2)b_2p - \alpha_2[\Delta S_2^e(\ddot{q},p) + \Delta S_2(0,p)]}{\Delta S_1^e(\ddot{q},p)}$$

If $w = \alpha_1 - \alpha_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p) - h_1 + \eta_1 Z_1(n_1 - \ddot{q} - a_1 + b_1p)$. Then:

$\Delta\pi_1(\ddot{q},p,w,\eta_1,0)$

$$= -(\alpha_1-\eta_1)\left[\int_{n_1-a_1+b_1p}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + \ddot{q}Z_1(n_1-\ddot{q}-a_1+b_1p)\right]$$

$$-\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p} Z_1(d_1)dd_1 - b_1p\right] - (\alpha_1-g_1)b_1p$$

$\Delta\pi_1(\ddot{q},p,w,\eta_1,0) = (\alpha_1-\eta_1)\Delta S_1^e(\ddot{q},p) + \alpha_1\Delta S_1(0,p) - (\alpha_1-g_1)b_1p$

$\Delta\pi_1$ decrease with $\eta_1$ because $\int_{n_1-a_1+b_1p}^{n_1-\ddot{q}-a_1+b_1p} Z_1(d_1)dd_1 + \ddot{q}Z_1(n_1-\ddot{q}-a_1+b_1p) \le 0$. If $\Delta\pi_1 \ge 0$, then:

$$\eta_1 \le \frac{\alpha_1[\Delta S_1^e(\ddot{q},p)+\Delta S_1(0,p)] - (\alpha_1-g_1)b_1p}{\Delta S_1^e(\ddot{q},p)}$$

Therefore, in this case, $\eta_1$ should follow:

$$\frac{(\alpha_2-g_2)b_2p - \alpha_2[\Delta S_2^e(\ddot{q},p)+\Delta S_2(0,p)]}{\Delta S_1^e(\ddot{q},p)} \le \eta_2 \le \frac{\alpha_1[\Delta S_1^e(\ddot{q},p)+\Delta S_1(0,p)] - (\alpha_1-g_1)b_1p}{\Delta S_1^e(\ddot{q},p)}$$

**Case 5**. $0<\ddot{q}<-n_2, (q^e = q^* = -n_2;\ \eta_2 = 0)$

$$\Delta\prod(-n_2,p) = -\alpha_1\int_{n_1-a_1}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2}^{-a_2+b_2p} Z_2(d_2)dd_2$$

$$+(\alpha_2-\alpha_1+h_1-h_2+c_t)(-n_2) + g_1b_1p + g_2b_2p$$

$\Delta\pi_1(-n_2,p,w,\eta_1,0)$

$$= -\alpha_1\int_{n_1-a_1}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + \eta_1\int_{n_1-a_1+b_1p}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + (h_1-\alpha_1+w)(-n_2)$$

$$+ g_1b_1p$$

In this case, the conditions for voluntary compliance of $\eta_2$ and $w$ are:

$$\begin{cases} \dfrac{\partial\pi_1(-n_2,p,w,\eta_1,0)}{\partial q} = 0 \\ \dfrac{\partial\pi_2(-n_2,p,w,\eta_1,0)}{\partial q} = 0 \end{cases}$$

$$\to \begin{cases} \alpha_1 + \alpha_1Z_1(n_1+n_2-a_1+b_1p) - \eta_1Z_1(n_1+n_2-a_1+b_1p) + h_1 + w \le 0 \\ \alpha_2 - \alpha_2Z_2(-a_2+b_2p) + \eta_1Z_1(n_1+n_2-a_1+b_1p) - h_2 - w + c_t \le 0 \end{cases}$$

$$\to \alpha_2[1-Z_2(-a_2+b_2p)] - h_2 + c_t \le w - \eta_1Z_1(n_1+n_2-a_1+b_1p)$$
$$\le \alpha_1[1-Z_1(n_1+n_2-a_1+b_1p)] - h_1$$

$\Delta\pi_1(-n_2,p,w,\eta_1,0)$ decreases with $w$. If $w_{\max} = \alpha_1 + (\eta_1-\alpha_1)Z_1(n_1+n_2-a_1+b_1p) - h_1$, then:

$\Delta\pi_1(-n_2,p,w,\eta_1,0)$

$$= -(\alpha_1-\eta_1)\left[\int_{n_1-a_1+b_1p}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + (-n_2)Z_1(n_1+n_2-a_1+b_1p)\right]$$

$$-\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p} Z_1(d_1)dd_1 - b_1p\right] - (\alpha_1-g_1)b_1p$$

$\Delta\pi_1(-n_2,p,w,\eta_1,0) = (\alpha_1-\eta_1)\Delta S_1^e(-n_2,p) + \alpha_1\Delta S_1(0,p) - (\alpha_1-g_1)b_1p$

Thus, $\Delta\pi_1$ also decreases with $\eta_1$ because $\int_{n_1-a_1+b_1p}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + (-n_2)Z_1(n_1+n_2-a_1+b_1p) \leq 0$. If $\Delta\pi_1 \geq 0$, then:

$$\eta_1 \leq \frac{\alpha_1[\Delta S_1^e(-n_2,p) + \Delta S_1(0,p)] - (\alpha_1 - g_1)b_1p}{\Delta S_1^e(-n_2,p)}$$

Let $w_{min} = \eta_1 Z_1(n_1+n_2-a_1+b_1p) + \alpha_2 - \alpha_2 Z_2(-a_2+b_2p) - h_2 + c_t$, then:

$$\begin{aligned}
&\Delta\pi_1(-n_2,p,w,\eta_1,0)\\
&= -\alpha_1 \int_{n_1-a_1}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + \eta_1 \int_{n_1-a_1+b_1p}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1\\
&\quad + [h_1 - \alpha_1 + \eta_1 Z_1(n_1+n_2-a_1+b_1p) + \alpha_2 - \alpha_2 Z_2(-a_2+b_2p) - h_2 + c_t](-n_2)\\
&\quad + g_1b_1p
\end{aligned}$$

$$\begin{aligned}
&\Delta\pi_2(-n_2,p,w,\eta_1,0) = \Delta\prod(-n_2,p) - \Delta\pi_1(-n_2,p,w,\eta_1,0)\\
&= -\alpha_2\left[\int_{n_2-a_2+b_2p}^{-a_2+b_2p} Z_2(d_2)dd_2 + b_2 Z_2(-a_2+b_2p)\right]\\
&\quad - \eta_1\left[\int_{n_1-a_1+b_1p}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + (-n_2)Z_1(n_1+n_2-a_1+b_1p)\right]\\
&\quad - \alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p} Z_2(d_2)dd_2 - b_2p\right] - (\alpha_2 - g_2)b_2p
\end{aligned}$$

$$\Delta\pi_2(-n_2,p,w,\eta_1,0) = \alpha_2[\Delta S_2^e(-n_2,p) + \Delta S_2(0,p)] + \eta_1 \Delta S_1^e(-n_1,p) - (\alpha_2 - g_2)b_2p$$

$\Delta\pi_2$ increases with $\eta_1$ because $\int_{n_1-a_1+b_1p}^{n_1+n_2-a_1+b_1p} Z_1(d_1)dd_1 + (-n_2)Z_1(n_1+n_2-a_1+b_1p) \leq 0$, if $\Delta\pi_2 \geq 0$, then:

$$\eta_1 \geq \frac{(\alpha_2 - g_2)b_2p - \alpha_2[\Delta S_2^e(-n_2,p) + \Delta S_2(0,p)]}{\Delta S_1^e(-n_2,p)}$$

Therefore, in this case, $\eta_2$ should follow:

$$\frac{(\alpha_2 - g_2)b_2p - \alpha_2[\Delta S_2^e(-n_2,p) + \Delta S_2(0,p)]}{\Delta S_1^e(-n_2,p)} \leq \eta_1 \leq \frac{\alpha_1[\Delta S_1^e(-n_2,p) + \Delta S_1(0,p)] - (\alpha_1 - g_1)b_1p}{\Delta S_1^e(-n_2,p)}$$

**Appendix I**

$$\begin{aligned}
\frac{\partial\prod(q,p)}{\partial p} &= b_1\alpha_1[1 - Z_1(n_1 - q - a_1 + b_1p)] + b_2\alpha_2[1 - Z_2(n_2 + q - a_2 + b_2p)] - h_1b_1 - h_2b_2 - r_1b_1\\
&\quad - r_2b_2
\end{aligned}$$

$$\frac{\partial\prod(q,p)}{\partial q} = -\alpha_1[1 - Z_1(n_1 - q - a_1 + b_1p)] + \alpha_2[1 - Z_2(n_2 + q - a_2 + b_2p)] + h_1 - h_2 - c_t sgn(q)$$

$$\frac{\partial^2\prod(q,p)}{\partial p^2} = -\alpha_1 b_1^2 z_1(n_1 - q - a_1 + b_1p) - \alpha_2 b_2^2 z_2(n_2 + q - a_2 + b_2p) < 0$$

$$\frac{\partial^2\prod(q,p)}{\partial q^2} = -\alpha_1 z_1(n_1 - q - a_1 + b_1p) - \alpha_2 z_2(n_2 + q - a_2 + b_2p) < 0$$

$$\frac{\partial^2\prod(q,p)}{\partial q\,\partial p} = \frac{\partial^2\prod(q,p)}{\partial p\,\partial q} = b_1\alpha_1 z_1(n_1 - q - a_1 + b_1p) - b_2\alpha_2 z_2(n_2 + q - a_2 + b_2p)$$

Clearly, $\prod(q,p)$ is not differentiable at $q = 0$. For other differentiable segments, $\prod(q,p)$ is jointly concave in $p$ and $q$ when the determinants of Hessian matrix consisting of second-order partial derivatives of $\prod(q,p)$ in terms of $p$ and $q$ is negative semidefinite. We denote $\beta_1 = n_1 - q - a_1 + b_1 p$ and $\beta_2 = n_2 + q - a_2 + b_2 p$.

$$\begin{vmatrix} \frac{\partial^2 \prod(q,p)}{\partial p^2} & \frac{\partial^2 \prod(q,p)}{\partial p\, \partial q} \\ \frac{\partial^2 \prod(q,p)}{\partial q\, \partial p} & \frac{\partial^2 \prod(q,p)}{\partial q^2} \end{vmatrix} = \frac{\partial^2 \prod(q,p)}{\partial p^2} \frac{\partial^2 \prod(q,p)}{\partial q^2} - \frac{\partial^2 \prod(q,p)}{\partial p\, \partial q} \frac{\partial^2 \prod(q,p)}{\partial q\, \partial p}$$

$$= {\alpha_1}^2 b_1^2 z_1^2(\beta_1) + {\alpha_2}^2 b_2^2 z_2^2(\beta_2) - {\alpha_1}^2 b_1^2 z_1^2(\beta_1) - {\alpha_2}^2 b_2^2 z_2^2(\beta_2) + \alpha_1\alpha_2(b_1^2 + b_2^2) z_1(\beta_1) z_2(\beta_2)$$
$$+ 2\alpha_1\alpha_2 b_1 b_2 z_1(\beta_1) z_2(\beta_2) = \alpha_1\alpha_2 z_1(\beta_1) z_2(\beta_2)(b_1 + b_2)^2 \geq 0$$

Hence, $\prod(q,p)$ is jointly concave in $q$ and $p$. We denote the optimal solution as $p^*$ and $q^*$ respectively when the constraints of $p$ and $q$ are not concerned. So, to obtain $p^*$ and $q^*$, we let the first-order condition of $p$ and $q$ equal 0 (i.e., $\frac{\partial \prod(q,p)}{\partial p} = 0$ and $\frac{\partial \prod(q,p)}{\partial q} = 0$), then we have:

$$\begin{cases} Z_1(\beta_1) = \dfrac{b_1 g_1 + b_2(r_1 + g_1 - r_2) + b_2 c_t sgn(q)}{(b_1 + b_2)\alpha_1} \\ Z_2(\beta_2) = \dfrac{b_1(r_2 + g_2 - r_1) + b_2 g_2 - b_1 c_t sgn(q)}{(b_1 + b_2)\alpha_2} \end{cases}$$

When $q > 0$, the value of $Z_1(\beta_1)$ and $Z_2(\beta_2)$ should be:

$$\begin{cases} Z_1(\beta_1) = \dfrac{b_1 g_1 + b_2(r_1 + g_1 - r_2) + b_2 c_t}{(b_1 + b_2)\alpha_1} \\ Z_2(\beta_2) = \dfrac{b_1(r_2 + g_2 - r_1) + b_2 g_2 - b_1 c_t}{(b_1 + b_2)\alpha_2} \end{cases}$$

Let $\omega_1 = \frac{b_1 g_1 + b_2(r_1 + g_1 - r_2) + b_2 c_t}{(b_1 + b_2)\alpha_1}$ and $\omega_2 = \frac{b_1(r_2 + g_2 - r_1) + b_2 g_2 - b_1 c_t}{(b_1 + b_2)\alpha_2}$. Also, due to $\beta_1 = n_1 - q - a_1 + b_1 p$ and $\beta_2 = n_2 + q - a_2 + b_2 p$, then we have $p^*$ and $q^*$:

$$\rightarrow \begin{cases} n_1 - q^* - a_1 + b_1 p^* = Z_1^{-1}[\omega_1] \\ n_2 + q^* - a_2 + b_2 p^* = Z_2^{-1}[\omega_2] \end{cases} \rightarrow \begin{cases} p^* = \dfrac{Z_1^{-1}(\omega_1) + Z_2^{-1}(\omega_2) + a_1 + a_2 - n_1 - n_2}{b_1 + b_2} \\ q^* = \dfrac{b_1 Z_2^{-1}(\omega_2) - b_2 Z_1^{-1}(\omega_1) + b_2(n_1 - a_1) - b_1(n_2 - a_2)}{b_1 + b_2} \end{cases}$$

If $q < 0$, then:

$$\begin{cases} p^* = \dfrac{Z_1^{-1}(\omega_1') + Z_2^{-1}(\omega_2') + a_1 + a_2 - n_1 - n_2}{b_1 + b_2} \\ q^* = \dfrac{b_1 Z_2^{-1}(\omega_2') - b_2 Z_1^{-1}(\omega_1') + b_2(n_1 - a_1) - b_1(n_2 - a_2)}{b_1 + b_2} \end{cases}$$

Where ${\omega_1}' = \frac{b_1 g_1 + b_2(r_1 + g_1 - r_2) - b_2 c_t}{(b_1 + b_2)\alpha_1}$, ${\omega_2}' = \frac{b_1(r_2 + g_2 - r_1) + b_2 g_2 + b_1 c_t}{(b_1 + b_2)\alpha_2}$

**Appendix J**

Due to $q \in [-n_{2,}\, n_1]$ and $p \in [0, +\infty]$, then, the optimal solution of $q$ and $p$ are divided into ten cases. We decide the $q^*$ and $p^*$ in each case based on their constraints. According to *Karush–Kuhn–Tucker* conditions, all constrained minimisation programs are: (Sheffi, 1985)

$$\min z(\boldsymbol{x});\ \boldsymbol{x} = (x_1, x_2 \dots x_i), \text{ s.t. } g_j(\boldsymbol{x}) \geq b_j, \forall\, j \in \boldsymbol{Z}$$

Need to follow:

$$(1).\frac{\partial \mathrm{z}(\boldsymbol{x}^*)}{\partial x_i} = \sum_j \mu_j \frac{\partial g_j(\boldsymbol{x}^*)}{\partial x_i},$$

$$(2).\ \mu_j \geq 0,$$

$$(3).\ \mu_j\left[b_j - g_j(x)\right] = 0,$$

$$(4).\ g_j(\boldsymbol{x}) \geq b_j$$

Where $\mu_j$ is the auxiliary variables which is known as *dual variables* in *Lagrange multipliers*. When $\mu_j = 0$, from $\mu_j\left[b_j - g_j(x)\right] = 0$, there is no binding constraint for $\mathbf{x}^*$. Also, when $\mu_j \geq 0$, then the $j$th constraint is binding at $\mathbf{x}^*$. First, we standard our problem as follow:

$$min -\prod(q,p), \text{ s.t.} \begin{cases} -q \geq -n_1 & (1) \\ q \geq -n_2 & (2) \\ p \geq 0 & (3) \end{cases}$$

We known that $\prod(q,p)$ is jointly concave in $q$ and $p$. Let $\mu_1$, $\mu_1'$ and $\mu_2$ are *dual variables* for three constraints, respectively. Therefore, for $q > 0$, the *Kuhn-Tucker* conditions are:

$$\begin{cases} \dfrac{\partial[-\prod(q,p)]}{\partial q} = \mu_1 \dfrac{\partial(-q)}{\partial q} + \mu_2 \dfrac{\partial(p)}{\partial q} = -\mu_1 & (1) \\ \dfrac{\partial[-\prod(q,p)]}{\partial p} = \mu_1 \dfrac{\partial(-q)}{\partial p} + \mu_2 \dfrac{\partial(p)}{\partial p} = \mu_2 & (2) \\ \mu_1 \geq 0 & (3) \\ \mu_2 \geq 0 & (4) \\ \mu_1[q - n_1] = 0 & (5) \\ \mu_2[-p] = 0 & (6) \\ -q \geq -n_1 & (7) \\ p \geq 0 & (8) \end{cases}$$

For $q < 0$, the *Kuhn-Tucker* conditions should be:

$$\begin{cases} \dfrac{\partial[-\prod(q,p)]}{\partial q} = \mu_1' \dfrac{\partial(q)}{\partial q} + \mu_2 \dfrac{\partial(p)}{\partial q} = \mu_1 & (1) \\ \dfrac{\partial[-\prod(q,p)]}{\partial p} = \mu_1' \dfrac{\partial(q)}{\partial p} + \mu_2 \dfrac{\partial(p)}{\partial p} = \mu_2 & (2) \\ \mu_1' \geq 0 & (3) \\ \mu_2 \geq 0 & (4) \\ \mu_1'[q - n_1] = 0 & (5) \\ \mu_2[-p] = 0 & (6) \\ -q \geq -n_1 & (7) \\ p \geq 0 & (8) \end{cases}$$

**Case 1.** $0 < n_1 \leq \dot{q}, \dot{p} \geq 0$

In this case, according to 5th and 6th $KKT$ condition for $q > 0$ scenario, we know $\mu_1 \geq 0$ and $\mu_2 = 0$ because $\dot{q} \geq n_1$ and $\dot{p} \geq 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial q}$ should be 0 and $q^* = \dot{q}$ if $\mu_1 = 0$ in 1st $KKT$ condition, which is not correct as the $q^*$ cannot be $\dot{q}$ in this case. So, based on all $KKT$ conditions, we obtain $\mu_1 > 0$ and $\mu_2 = 0$, it means 1st constraint is binding on $\dot{q}$ and 3rd constraint is not binding on $p^*$. Also, we know $\prod(q,p)$ is jointly concave in $q$ and $p$. So, we get the optimal $p$ when $q = n_1$ in case 1:

$$-\prod(n_1,p) = -\{\alpha_1[(-a_1+b_1p)-\Phi_1(-a_1+b_1p)] + \alpha_2[(n_2+n_1-a_2+b_2p)-\Phi_2(n_2+n_1-a_2+b_2p)] - h_1(-a_1+b_1p) - h_2(n_2+n_1 - a_2+b_2p) - g_1\boldsymbol{E}(\xi_1-Y_1) - g_2\boldsymbol{E}(\xi_2-Y_2) + r_1\boldsymbol{E}(Y_1+a_1-b_1p) + r_2\boldsymbol{E}(Y_2+a_2-b_2p) - c_t|q|\}$$

$$\frac{\partial[-\prod(n_1,p)]}{\partial p} = -\{\alpha_1b_1[1-Z_1(-a_1+b_1p)] + \alpha_2b_2[1-Z_2(n_2+n_1-a_2+b_2p)] - h_1b_1 - h_2b_2 - r_1b_1 - r_2b_2\}$$

$$\frac{\partial^2[-\prod(n_1,p)]}{\partial p^2} = -\{-\alpha_1b_1^2z_1(-a_1+b_1p) - \alpha_2b_2^2z_2(n_2+n_1-a_2+b_2p)\} > 0$$

So, $\prod(n_1,p)$ is strictly concave in $p$. And $q^* = n_1$, $p^* = \bar{p}$, (where $\frac{\partial[-\prod(n_1,\bar{p})]}{\partial p} = 0$).

**Case 2**. $0 < n_1 \le \dot{q}; \dot{p} < 0$

In this case, according to 5th and 6th $KKT$ condition for $q > 0$ scenario, we know $\mu_1 \ge 0$ and $\mu_2 \ge 0$ because $\dot{q} \ge n_1$ and $\dot{p} < 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial q}$ and $\frac{\partial[-\prod(q,p)]}{\partial p}$ should be 0 and $p^* = \dot{p}$, $q^* = \dot{q}$ if $\mu_1 = 0$ and $\mu_2 = 0$ in 1st and 2nd $KKT$ condition, it is not correct because the $q^*$ and $p^*$ cannot be $\dot{q}$ and $\dot{p}$ in this case. So, we get $\mu_1 > 0$ and $\mu_2 > 0$, which means 1st constraint is binding on $\dot{q}$ and 3rd constraint is also binding on $\dot{p}$. Also, we know $\prod(q,p)$ is jointly concave in $q$ and $p$. Then, we know that $q^* = n_1$, $p^* = 0$.

**Case 3**. $0 < \dot{q} < n_1; \dot{p} \ge 0$

In this case, it is easy to know that $\mu_1 = 0$ and $\mu_2 = 0$, we can obtain:

$$\frac{\partial\prod(\dot{q},\dot{p})}{\partial p} = 0 \text{ and } \frac{\partial\prod(\dot{q},\dot{p})}{\partial q} = 0$$

1st and 3rd constraints are not binding on $\dot{q}$ and $\dot{p}$. So, $q^* = \dot{q}$, $p^* = \dot{p}$ in this case.

**Case 4**. $0 < \dot{q} < n_1, \dot{p} < 0$

In this case, according to 5th and 6th $KKT$ condition for $q > 0$ scenario, we know $\mu_1 = 0$ and $\mu_2 \ge 0$ because $\dot{q} \le n_1$ and $\dot{p} < 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial p}$ should be 0 and $p^* = \dot{p}$ if $\mu_2 = 0$ in 2nd $KKT$ condition, which is not correct as $p^*$ cannot be $\dot{p}$. So, we know $\mu_1 = 0$ and $\mu_2 > 0$, it means 1st constraint is not binding on $\dot{q}$ and 3rd constraint is binding on $p^*$. Also, $\prod(q,p)$ is jointly concave in $q$ and $p$. Then, we get optimal $q$ when $p = 0$ in this case. Thus:

$$-\prod(q,0) = -\{\alpha_1[(n_1-q-a_1)-\Phi_1(n_1-q-a_1)] + \alpha_2[(n_2+q-a_2)-\Phi_2(n_2+q-a_2)] - h_1(n_1-q-a_1) - h_2(n_2+q-a_2) - g_1\boldsymbol{E}(\xi_1-Y_1) - g_2\boldsymbol{E}(\xi_2-Y_2) + r_1\boldsymbol{E}(Y_1+a_1) + r_2\boldsymbol{E}(Y_2+a_2) - c_t|q|\}$$

$$\frac{\partial[-\prod(q,0)]}{\partial q} = -\{\alpha_1[-1+Z_1(n_1-q-a_1)] + \alpha_2[1-Z_2(n_2+q-a_2)] + h_1 - h_2 - c_t sgn[q]\}$$

$$\frac{\partial^2[-\prod(q,0)]}{\partial q^2} = -\{-\alpha_1z_1(n_1-q-a_1) - \alpha_2z_2(n_2+q-a_2)\} > 0$$

So, $\prod(q,0)$ is strictly concave in $q$, $q^* = \hat{q}$ (where $\frac{\partial[-\prod(\hat{q},0)]}{\partial q} = 0$), $p^* = 0$ in this case.

**Case 5**. $\dot{q} < 0,\ \dot{p} \ge 0$ *or* $\ddot{q} > 0, \ddot{p} \ge 0$

In this case, according to the 5th $KKT$ condition for $q > 0$ or $q < 0$ scenarios and 6th $KKT$ condition in both scenarios, we know $\mu_1 \ge 0$ and $\mu_2 = 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial q}$ should be 0 and $q^* = \dot{q}$ if $\mu_1 = 0$ or $\mu_1' = 0$ in

1st $KKT$ condition, it is not satisfied because $q^*$ cannot be $\dot{q}$ or $\ddot{q}$ in this case. So, from all $KKT$ conditions, we know $\mu_1 > 0$ or $\mu_1' > 0$ and $\mu_2 = 0$, it means 1st or 2nd constraint is binding on $q^*$ and 3rd constraint is not binding on $p^*$. Also, $\prod(q,p)$ is concave in $q$ and $p$. So, we get optimal $p$ when $q = 0$ in this case.

$$-\prod(0,p) = -\{\alpha_1[(n_1 - a_1 + b_1 p) - \Phi_1(n_1 - a_1 + b_1 p)] + \alpha_2[(n_2 - a_2 + b_2 p) - \Phi_2(n_2 - a_2 + b_2 p)] - h_1(n_1 - a_1 + b_1 p) - h_2(n_2 - a_2 + b_2 p) - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p)\}$$

$$\frac{\partial[-\prod(0,p)]}{\partial p} = -\{\alpha_1[1 - Z_1(n_1 - a_1 + b_1 p)] + \alpha_2 b_2[1 - Z_2(n_2 - a_2 + b_2 p)] - h_1 b_1 - h_2 b_2 - r_1 b_1 - r_2 b_2\}$$

$$\frac{\partial^2[-\prod(0,p)]}{\partial p^2} = -\{-\alpha_1 b_1^2 z_1(n_1 - a_1 + b_1 p) - \alpha_2 b_2^2 z_2(n_2 - a_2 + b_2 p)\} > 0$$

So $\prod(0,p)$ is strictly concave in $p$, $q^* = 0, p^* = p^0$ (where $\frac{\partial[-\prod(0,p^0)]}{\partial p} = 0$) in this case.

**Case 6.** $\dot{q} < 0, \dot{p} < 0$ or $\ddot{q} > 0, \ddot{p} < 0$

In this case, according to 5th $KKT$ condition for $q > 0$ or $q < 0$ scenario and 6th $KKT$ condition in both scenarios, we know $\mu_1 \geq 0$ or $\mu_1' \geq 0$ and $\mu_2 \geq 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial q}$ should be 0 and $q^* = \dot{q}$ if $\mu_1 = 0$ or $\mu_1' = 0$ in 1st $KKT$ condition and $\frac{\partial[-\prod(q,p)]}{\partial p}$ should also be 0 and $p^* = \dot{p}$ if $\mu_2 = 0$ in 2nd $KKT$ condition in both scenarios, it is not corrrect because the $q^*$ and $p^*$ cannot be $(\dot{q},\dot{p})$ or $(\ddot{q},\ddot{p})$ in this case. Therefore, we can obtain $\mu_1 > 0$ or $\mu_1' > 0$ and $\mu_2 > 0$, it means 1st or 2nd constraint is binding on $q^*$ and 3rd constraint is binding on $p^*$. Moreover, $\prod(q,p)$ is jointly concave in $q$ and $p$. Thus, $q^* = 0, p^* = 0$ in this case.

**Case 7.** $-n_2 < \ddot{q} < 0, \ddot{p} < 0$

In this case, according to 5th and 6th $KKT$ condition for $q < 0$ scenario, we know $\mu_1' = 0$ and $\mu_2 \geq 0$ because $-n_2 \leq \ddot{q} \leq 0$ and $\ddot{p} < 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial p}$ should be 0 and $p^* = \ddot{p}$ if $\mu_2 = 0$ in 2nd $KKT$ condition for $q < 0$ scenario, it is not correct because the optimal $p^*$ cannot be $\ddot{p}$. So, based on all $KKT$ conditions, we know $\mu_1' = 0$ and $\mu_2 > 0$, it means 2nd constraint is not binding on $\ddot{q}$ but 3rd constraint is binding on $p^*$. Also, $\prod(q,p)$ is jointly concave in $q$ and $p$. Then, we get optimal $q$ when $p = 0$ in this case. So:

$$-\prod(q,0) = -\{\alpha_1[(n_1 - q - a_1) - \Phi_1(n_1 - q - a_1)] + \alpha_2[(n_2 + q - a_2) - \Phi_2(n_2 + q - a_2)] - h_1(n_1 - q - a_1) - h_2(n_2 + q - a_2) - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1) + r_2\boldsymbol{E}(Y_2 + a_2) - c_t|q|\}$$

$$\frac{\partial[-\prod(q,0)]}{\partial q} = -\{\alpha_1[-1 + Z_1(n_1 - q - a_1)] + \alpha_2[1 - Z_2(n_2 + q - a_2)] + h_1 - h_2 - c_t sgn[q]\}$$

$$\frac{\partial^2[-\prod(q,0)]}{\partial q^2} = -\{-\alpha_1 z_1(n_1 - q - a_1) - \alpha_2 z_2(n_2 + q - a_2)\} > 0$$

So, $\prod(q,0)$ is strictly concave in $q$, $q^* = \check{q}$ (where $\frac{\partial[-\prod(\check{q},0)]}{\partial q} = 0$), $p^* = 0$ in this case.

**Case 8**. $-n_2 \leq \ddot{q} < 0, \ddot{p} \geq 0$

In this case, we obtain $\mu_1' = 0$ and $\mu_2 = 0$, so:

$$\frac{\partial\prod(\ddot{q},,\ddot{p})}{\partial p} = 0 \text{ and } \frac{\partial\prod(\ddot{q},,\ddot{p})}{\partial q} = 0$$

It means 2nd and 3rd constraint are not binding on $\ddot{q}$ and $\ddot{p}$. So, $q^* = \ddot{q}, p^* = \ddot{p}$ in this case.

**Case 9.** $\ddot{q} < -n_2 < 0, \ddot{p} < 0$

In this case, according to 5th and 6th $KKT$ condition for $q < 0$ scenario, we know $\mu_1' \geq 0$ and $\mu_2 \geq 0$ as $\ddot{q} \geq n_1$ and $\ddot{p} < 0$. But, $\frac{\partial[-\prod(q,p)]}{\partial q}$ and $\frac{\partial[-\prod(q,p)]}{\partial p}$ should be 0 and $q^* = \ddot{q}$ and $p^* = \ddot{p}$ if $\mu_1' = 0$ and $\mu_2 = 0$ in 1st and 2nd $KKT$ condition, it is incorrect as the $q^*$ and $p^*$ cannot be $\ddot{q}$ and $\ddot{p}$. So, based on all $KKT$ conditions, we have $\mu_1' > 0$ and $\mu_2 > 0$, which means 2nd and 3rd constraints are binding on $\ddot{q}$ and $\ddot{p}$. Also, $\prod(q,p)$ is jointly concave in $q$ and $p$. So, $q^* = -n_2, p^* = 0$ in this case.

**Case 10.** $\ddot{q} < -n_2 < 0, \ddot{p} > 0,$

In this case, based on 5th and 6th $KKT$ condition for $q < 0$ scenario, we know $\mu_1' \geq 0$ and $\mu_2 = 0$ because $\ddot{q} \geq n_1$ and $\ddot{p} \geq 0$. However, $\frac{\partial[-\prod(q,p)]}{\partial q}$ should be 0 and $q^* = \ddot{q}$ if $\mu_1' = 0$ in 1st $KKT$ condition, it is incorrect as the $q^*$ cannot be $\ddot{q}$. So, based on all $KKT$ conditions, we get $\mu_1' > 0$ and $\mu_2 = 0$, it means 2nd constraint is binding on $\ddot{q}$ and 3rd constraint is not binding on $\ddot{p}$. Also, $\prod(q,p)$ is jointly concave in $q$ and $p$. So, we get $p^*$ when $q = -n_2$ in this case.

$$
\begin{aligned}
-\prod(-n_2,p) = -\{&\alpha_1[(n_1+n_2-a_1+b_1p) - \Phi_1(n_1+n_2-a_1+b_1p)] \\
&+ \alpha_2[(-a_2+b_2p) - \Phi_2(-a_2+b_2p)] - h_1(n_1+n_2-a_1+b_1p) - h_2(-a_2+b_2p) \\
&- g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2p) - c_t n_2\}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial[-\prod(-n_2,p)]}{\partial p} = -\{&\alpha_1 b_1[1 - Z_1(n_1+n_2-a_1+b_1p)] + \alpha_2 b_2[1 - Z_2(-a_2+b_2p)] - h_1b_1 - h_2b_2 \\
&- r_1b_1 - r_2b_2\}
\end{aligned}
$$

$$
\frac{\partial^2[-\prod(-n_2,p)]}{\partial p^2} = -\{-\alpha_1 b_1^2 z_1(n_1+n_2-a_1+b_1p) - \alpha_2 b_2^2 z_2(-a_2+b_2p)\} > 0
$$

Hence, $\prod(-n_2,p)$ is strictly concave in $p$, $q^* = -n_2$, $p^* = \tilde{p} = \frac{\partial[-\prod(-n_2,p)]}{\partial p}$ in this case.

**Appendix K**

In $RSC$, the transfer payment $\theta(q,p,w,\emptyset_1,\emptyset_2)$ is:

$$
\begin{aligned}
&\theta(q,p,w,\emptyset_1,\emptyset_2) \\
&= wq^+ + (1-\emptyset_2)r_2\{q^+ - \boldsymbol{E}min[(n_2+q+Y_2-X_2)^+, q^+]\} - w(-q)^+ \\
&\qquad - (1-\emptyset_1)r_1\{(-q)^+ - \boldsymbol{E}min[(n_1-q+Y_1-X_1)^+, (-q)^+]\} \\
&= q^+[w + (1-\emptyset_2)r_2] - (-q)^+[w + (1-\emptyset_1)r_1] - (1-\emptyset_2)r_2\,\boldsymbol{E}min[(n_2+q+Y_2-X_2)^+, q^+] \\
&\qquad + (1-\emptyset_1)r_1\,\boldsymbol{E}min[(n_1-q+Y_1-X_1)^+, (-q)^+]
\end{aligned}
$$

**Appendix L**

$$
\begin{aligned}
&\pi_1(q,p,w,\emptyset_1,\emptyset_2) \\
&\qquad = r_1\,\boldsymbol{E}min[X_1, n_1-q+Y_1] - h_1\boldsymbol{E}(n_1-q+Y_1-X_1)^+ - g_1\boldsymbol{E}(X_1+q-n_1-Y_1)^+ - c_tq^+ \\
&\qquad + \boldsymbol{T}
\end{aligned}
$$

According to Snyder and Shen (2011):

$$
\begin{cases}
\boldsymbol{E}[(Quantity - Demand)^+] = Quantity - \boldsymbol{E}\min[Quantity, Demand] \\
\boldsymbol{E}[(Demand - Quantity)^+] = \boldsymbol{E}(Demand) - \boldsymbol{E}\min[Quantity, Demand]
\end{cases}
$$

Therefore, we obtain $\pi_1$:

$\pi_1(q,p,w,\emptyset_1,\emptyset_2)$

$$= (r_1 + h_1 + g_1)\,\boldsymbol{E}min[\xi_1 - Y_1, n_1 - q - a_1 + b_1 p] + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) - h_1(n_1 - q - a_1 + b_1 p\,) - \boldsymbol{E}(\xi_1 - Y_1) - c_t q^+ + q^+[w + (1-\emptyset_2)r_2] - (-q)^+[w + (1-\emptyset_1)r_1] - (1-\emptyset_2)r_2\,\boldsymbol{E}min[(n_2 + q + Y_2 - a_2 + b_2 p - \xi_2)^+, q^+] + (1-\emptyset_1)r_1\boldsymbol{E}\,min[(n_1 - q + Y_1 - a_1 + b_1 p - \xi_1)^+, (-q)^+]$$

We define $R_1 = (1-\emptyset_1)r_1$ as the revenue sharing that $LSC$ 1 gives $LSC$ 2 and $R_2 = (1-\emptyset_2)r_2$ for vice versa. Then, $\pi_1$ is reformulated as:

$\pi_1(q,p,w,R_1,R_2)$

$$= \alpha_1\,\boldsymbol{E}min[\xi_1 - Y_1, n_1 - q - a_1 + b_1 p] - R_2\boldsymbol{E}[(n_2 + q - a_2 + b_2 p) - (\xi_2 - Y_2)]^+ + R_1\boldsymbol{E}[(n_1 - q - a_1 + b_1 p) - (\xi_1 - Y_1)]^+ + R_2\boldsymbol{E}[(n_2 - a_2 + b_2 p) - (\xi_2 - Y_2)]^+ - R_1\boldsymbol{E}[(n_1 - a_1 + b_1 p) - (\xi_1 - Y_1)]^+ + q^+(w - c_t + R_2) - (-q)^+(w + R_1) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) - h_1(n_1 - q - a_1 + b_1 p\,) - \boldsymbol{E}(\xi_1 - Y_1)$$

$$= \alpha_1\,\boldsymbol{E}min[\xi_1 - Y_1, n_1 - q - a_1 + b_1 p] + R_2\{\boldsymbol{E}min[\xi_2 - Y_2, n_2 + q - a_2 + b_2 p] - \boldsymbol{E}min[\xi_2 - Y_2, n_2 - a_2 + b_2 p]\} - R_1\{\boldsymbol{E}min[\xi_1 - Y_1, n_1 - q - a_1 + b_1 p] - \boldsymbol{E}min[\xi_1 - Y_1, n_1 - a_1 + b_1 p]\} - R_1[(-q)^+ + q] + R_2(q^+ - q) + q^+(w - c_t) - (-q)^+ w + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) - h_1(n_1 - q - a_1 + b_1 p) - \boldsymbol{E}(\xi_1 - Y_1)$$

We denote $\beta_1 = n_1 - q - a_1 + b_1 p$ and $\beta_2 = n_2 + q - a_2 + b_2 p$, then $\pi_1$ is determined.

$\pi_1(q,p,w,R_1,R_2)$

$$= \alpha_1\,\boldsymbol{E}min[\xi_1 - Y_1, \beta_1] + R_2\{\boldsymbol{E}min[\xi_2 - Y_2, \beta_2] - \boldsymbol{E}min[\xi_2 - Y_2, \beta_2 - q]\} - R_1\{\boldsymbol{E}min[\xi_1 - Y_1, \beta_1] - \boldsymbol{E}min[\xi_1 - Y_1, \beta_1 + q]\} - R_1[(-q)^+ + q] + R_2(q^+ - q) + q^+(w - c_t) - (-q)^+ w + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) - h_1\beta_1 - \boldsymbol{E}(\xi_1 - Y_1)$$

According to Snyder and Shen (2011), we know:

$$\boldsymbol{E}\min\{Demand, Quantity\} = Quantity - \Phi(Quantity)$$

Where $\Phi()$ is the complementary loss function and it satisfies:

$$\Phi(Quantity) = \int_0^{Quantity} CDF\;\, dd$$

Therefore, we obtain:

$\pi_1(q,p,w,R_1,R_2)$

$$= -\alpha_1\Phi_1(\beta_1) + R_2[\Phi_2(\beta_2 - q) - \Phi_2(\beta_2)] - R_1[\Phi_1(\beta_1 + q) - \Phi_1(\beta_1)] - (-q^+)(R_1 + w) + q^+(R_2 + w - c_t) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) + (\alpha_1 - h_1)\beta_1 - \boldsymbol{E}(\xi_1 - Y_1)$$

**Appendix M**

We firstly get the partial first-order and second-order derivatives of $\pi_1$ in terms of $p$ and $q$.

$$\frac{\partial\pi_1(q,p,w,R_1,R_2)}{\partial p} = -b_1\alpha_1 Z_1(n_1 - q - a_1 + b_1 p) + R_2 b_2[Z_2(n_2 - a_2 + b_2 p) - Z_2(n_2 + q - a_2 + b_2 p)] - R_1 b_1[Z_1(n_1 - a_1 + b_1 p) - Z_1(n_1 - q - a_1 + b_1 p)] + g_1 b_1$$

$$\frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial p^2}$$

$$= -b_1^2\alpha_1 z_1(n_1 - q - a_1 + b_1 p) + R_2 b_2^2[z_2(n_2 - a_2 + b_2 p) - z_2(n_2 + q - a_2 + b_2 p)]$$
$$- R_1 b_1^2[z_1(n_1 - a_1 + b_1 p) - z_1(n_1 - q - a_1 + b_1 p)]$$

$$\frac{\partial \pi_1(q,p,w,R_1,R_2)}{\partial q}$$

$$= \alpha_1 Z_1(n_1 - q - a_1 + b_1 p) - R_2 Z_2(n_2 + q - a_2 + b_2 p) - R_1 Z_1(n_1 - q - a_1 + b_1 p)$$
$$+ (R_2 + w - c_t)|_{q>0} + (R_1 + w)|_{q<0} - (\alpha_1 - h_1)$$

$$\frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial q^2}$$

$$= -\alpha_1 z_1(n_1 - q - a_1 + b_1 p) - R_2 z_2(n_2 + q - a_2 + b_2 p) + R_1 z_1(n_1 - q - a_1 + b_1 p)$$

$$\frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial q\,\partial p} = \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial p\,\partial q}$$

$$= b_1\alpha_1 z_1(n_1 - q - a_1 + b_1 p) - R_2 b_2 z_2(n_2 + q - a_2 + b_2 p)$$
$$- R_1 b_1 z_1(n_1 - q - a_1 + b_1 p)$$

To prove $\pi_1$ is jointly concave in $q$ and $p$ is equivalent to prove that the determinants of Hessian matrix formed by $q$ and $p's$ second-order partial derivatives is negative semidefinite.

$$\begin{vmatrix} \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial p^2} & \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial p\,\partial q} \\ \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial q\,\partial p} & \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial q^2} \end{vmatrix}$$

$$= \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial p^2}\frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial q^2} - \frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial p\,\partial q}\frac{\partial^2 \pi_1(q,p,w,R_1,R_2)}{\partial q\,\partial p}$$

$$= \{-b_1^2\alpha_1 z_1(n_1 - q - a_1 + b_1 p) + R_2 b_2^2[z_2(n_2 - a_2 + b_2 p) - z_2(n_2 + q - a_2 + b_2 p)]$$
$$- R_1 b_1^2[z_1(n_1 - a_1 + b_1 p) - z_1(n_1 - q - a_1 + b_1 p)]\}\,.\,[-\alpha_1 z_1(n_1 - q - a_1 + b_1 p)$$
$$- R_2 z_2(n_2 + q - a_2 + b_2 p) + R_1 z_1(n_1 - q - a_1 + b_1 p)] - [b_1\alpha_1 z_1(n_1 - q - a_1 + b_1 p)$$
$$- R_2 b_2 z_2(n_2 + q - a_2 + b_2 p) - R_1 b_1 z_1(n_1 - q - a_1 + b_1 p)]$$

$$= R_2(b_1 + b_2)^2 z_1(n_1 - q - a_1 + b_1 p) z_2(n_2 + q - a_2 + b_2 p)(\alpha_1 - R_1)$$
$$+ [(\alpha_1 - R_1) z_1(n_1 - q - a_1 + b_1 p) + R_2 z_2(n_2 + q - a_2 + b_2 p)][R_1 b_1^2 z_1(n_1 - a_1 + b_1 p)$$
$$- R_2 b_2^2 z_2(n_2 - a_2 + b_2 p)]$$

We denote $\beta_1 = n_1 - q - a_1 + b_1 p$ and $\beta_2 = n_2 + q - a_2 + b_2 p$, then, the determinants of Hessian matrix is:

$$= R_2(b_1 + b_2)^2 z_1(\beta_1) z_2(\beta_2)(\alpha_1 - R_1) + [(\alpha_1 - R_1) z_1(\beta_1) + R_2 z_2(\beta_2)][R_1 b_1^2 z_1(\beta_1 + q)$$
$$- R_2 b_2^2 z_2(\beta_2 - q)]$$

Clearly, $R_2(b_1 + b_2)^2 z_1(\beta_1) z_2(\beta_2) \geq 0$ and $\alpha_1 - R_1 \geq 0$. So, the determinants of Hessian matrix is greater than 0 as long as:

$$R_1 b_1^2 z_1(\beta_1 + q) - R_2 b_2^2 z_2(\beta_2 - q) \geq 0 \to \left(\frac{b_1}{b_2}\right)^2 \frac{R_1}{R_2} \geq \left[\frac{z_1(n_1 - a_1 + b_1 p)}{z_2(n_2 - a_2 + b_2 p)}\right]^{-1}$$

So, if $\pi_1$ and $\pi_2$ are jointly concave in $p$ and $q$ if $\left(\frac{b_1}{b_2}\right)^2 \frac{R_1}{R_2} \geq \left[\frac{z_1(n_1 - a_1 + b_1 p)}{z_2(n_2 - a_2 + b_2 p)}\right]^{-1}$ is met.

## Appendix N

To obtain $p^{01}$ and $p^{02}$, we formulate two $LSCs'$ profit functions when $q = 0$:

$$\pi_1(0,p,w,R_1,R_2) = -\alpha_1\Phi_1(n_1 - a_1 + b_1 p) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p) + (\alpha_1 - h_1)(n_1 - a_1 + b_1 p) - \boldsymbol{E}(\xi_1 - Y_1)$$

$$\pi_2(0,p,w,R_1,R_2) = -\alpha_2\Phi_2(n_2 - a_2 + b_2 p) + r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p) + (\alpha_2 - h_2)(n_2 - a_2 + b_2 p) - \boldsymbol{E}(\xi_2 - Y_2)$$

$$\frac{\partial \pi_1}{\partial p} = -\alpha_1 b_1 Z_1(n_1 - a_1 + b_1 p^{01}) + g_1 b_1 = 0$$

$$\frac{\partial \pi_2}{\partial p} = -\alpha_2 b_2 Z_2(n_2 - a_2 + b_2 p^{02}) + g_2 b_2 = 0$$

$$\rightarrow p^{01} = \frac{Z_1^{-1}\left(\frac{g_1}{\alpha_1}\right) - n_1 + a_1}{b_1}, p^{02} = \frac{Z_2^{-1}\left(\frac{g_2}{\alpha_2}\right) - n_2 + a_2}{b_2}$$

## Appendix O

The system profit increment $\Delta\prod(q^e,p^e)$ and $LSC$ 1's profit increment $\Delta\pi_1(q^e,p^e,w,R_1,R_2)$ are:

$$\begin{aligned}\Delta\prod(q^e,p^e) &= \prod(q^e,p^e) - \Pi_1(0,p^{01}) - \Pi_2(0,p^{02})\\
&= \alpha_1\left\{n_1 - q^e - a_1 + b_1 p^e - \int_0^{n_1 - q^e - a_1 + b_1 p^e} Z_1(d_1)dd_1 - n_1 + a_1 - b_1 p^{01} + \int_0^{n_1 - a_1 + b_1 p^{01}} Z_1(d_1)dd_1\right\}\\
&+ \alpha_2\left\{n_2 + q^e - a_2 + b_2 p^e - \int_0^{n_2 + q^e - a_2 + b_2 p^e} Z_2(d_2)dd_2 - n_2 + a_2 - b_2 p^{02} + \int_0^{n_2 - a_2 + b_2 p^{02}} Z_2(d_2)dd_2\right\} - h_1(n_1 - q^e - a_1 + b_1 p^e) - h_2(n_2 + q^e - a_2 + b_2 p^e)\\
&+ h_1(n_1 - a_1 + b_1 p^{01}) + h_2(n_2 - a_2 + b_2 p^{02}) - g_1\boldsymbol{E}(\xi_1 - Y_1) - g_2\boldsymbol{E}(\xi_2 - Y_2)\\
&+ g_1\boldsymbol{E}(\xi_1 - Y_1) + g_2\boldsymbol{E}(\xi_2 - Y_2) + r_1\boldsymbol{E}(Y_1 + a_1 - b_1 p^e) + r_1\boldsymbol{E}(-a_1 + b_1 p^{01} - Y_1)\\
&+ r_2\boldsymbol{E}(Y_2 + a_2 - b_2 p^e) + r_2\boldsymbol{E}(-a_2 + b_2 p^{02} - Y_2) - c_t|q^e|\end{aligned}$$

$$\begin{aligned}&= \alpha_1\left[-q^e + b_1(p^e - p^{01}) - \int_{n_1 - a_1 + b_1 p^{01}}^{n_1 - q^e - a_1 + b_1 p^e} Z_1(d_1)dd_1\right]\\
&+ \alpha_2\left[q^e + b_2(p^e - p^{02}) - \int_{n_2 - a_2 + b_2 p^{02}}^{n_2 + q^e - a_2 + b_2 p^e} Z_2(d_2)dd_2\right] + h_1 q^e - h_1 b_1 p^e + h_1 b_1 p^{01}\\
&- h_2 q^e - h_2 b_2 p^e + h_2 b_2 p^{02} - r_1 b_1 p^e + r_1 b_1 p^{01} - r_2 b_2 p^e + r_2 b_2 p^{02} - c_t|q^e|\end{aligned}$$

$$= -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{n_1-q^e-a_1+b_1p^e} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2+b_2p^{02}}^{n_2+q^e-a_2+b_2p^e} Z_2(d_2)dd_2 + (\alpha_2-\alpha_1+h_1-h_2)q^e - c_t|q^e|$$
$$+ g_1b_1(p^e-p^{01}) + g_2b_2(p^e-p^{02})$$

$$\Delta\pi_1(q^e,p^e,w,R_1,R_2) = \pi_1(q^e,p^e,w,R_1,R_2) - \pi_1(0,p^{01},w,R_1,R_2)$$
$$= -\alpha_1\left[\int_0^{n_1-q^e-a_1+b_1p^e} Z_1(d_1)dd_1 - \int_0^{n_1-a_1+b_1p^{01}} Z_1(d_1)dd_1\right]$$
$$+ R_2\left[\int_0^{n_2-a_2+b_2p^e} Z_2(d_2)dd_2 - \int_0^{n_2+q^e-a_2+b_2p^e} Z_2(d_2)dd_2\right]$$
$$- R_1\left[\int_0^{n_1-a_1+b_1p^e} Z_1(d_1)dd_1 - \int_0^{n_1-q^e-a_1+b_1p^e} Z_1(d_1)dd_1\right] + (q^e)^+(w-c_t+R_2)$$
$$- (-q^e)^+(w+R_1) - (\alpha_1-h_1)q^e + g_1b_1(p^e-p^{01})$$
$$= -\alpha_1[\Phi_1(n_1-q^e-a_1+b_1p^e) - \Phi_1(n_1-a_1+b_1p^{01})]$$
$$+ R_2[\Phi_2(n_2-a_2+b_2p^e) - \Phi_2(n_2+q^e-a_2+b_2p^e)]$$
$$- R_1[\Phi_1(n_1-a_1+b_1p^e) - \Phi_1(n_1-q^e-a_1+b_1p^e)] + (q^e)^+(w-c_t+R_2)$$
$$- (-q^e)^+(w+R_1) - (\alpha_1-h_1)q^e + g_1b_1(p^e-p^{01})$$
$$= -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{n_1-q^e-a_1+b_1p^e} Z_1(d_1)dd_1 - R_2 \int_{n_2-a_2+b_2p^e}^{n_2+q^e-a_2+b_2p^e} Z_2(d_2)dd_2 + R_1 \int_{n_1-a_1+b_1p^e}^{n_1-q^e-a_1+b_1p^e} Z_1(d_1)dd_1$$
$$+ (q^e)^+(w-c_t+R_2) - (-q^e)^+(w+R_1) - (\alpha_1-h_1)q^e + g_1b_1(p^e-p^{01})$$

Noticed that $\emptyset_1 = 1$ when $q > 0$ (which means $R_1 = 0$), similarly, $\emptyset_2 = 1$ when $q < 0$ (which means $R_2 = 0$). Then, the marginal profit of $LSC$ 1 and $LSC$ 2 in terms of $q$ and $p$ are:

$$\frac{\partial\pi_1}{\partial p} = \begin{cases} q>0: & -b_1\alpha_1Z_1(\beta_1) - R_2b_2[Z_2(\beta_2) - Z_2(\beta_2-q)] + g_1b_1 \\ q<0: & -b_1\alpha_1Z_1(\beta_1) + R_1b_1[Z_1(\beta_1) - Z_1(\beta_1+q)] + g_1b_1 \end{cases}$$
$$\frac{\partial\pi_2}{\partial p} = \begin{cases} q>0: & -b_2\alpha_2Z_2(\beta_2) + R_2b_2[Z_2(\beta_2) - Z_2(\beta_2-q)] + g_2b_2 \\ q<0: & -b_2\alpha_2Z_2(\beta_2) - R_1b_1[Z_1(\beta_1) - Z_1(\beta_1+q)] + g_2b_2 \end{cases}$$
$$\frac{\partial\pi_1}{\partial q} = \begin{cases} q>0: & \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1-h_1) \\ q<0: & \alpha_1Z_1(\beta_1) - R_1Z_1(\beta_1) + R_1 + w - (\alpha_1-h_1) \end{cases}$$
$$\frac{\partial\pi_2}{\partial q} = \begin{cases} q>0: & -\alpha_2Z_2(\beta_2) + R_2Z_2(\beta_2) - (R_2+w) + \alpha_2 - h_2 \\ q<0: & -\alpha_2Z_2(\beta_2) + R_1Z_1(\beta_1) - R_1 - w + c_t + \alpha_2 - h_2 \end{cases}$$

Where $\beta_1 = n_1 - q^e - a_1 + b_1p^e$; $\beta_2 = n_2 + q^e - a_2 + b_2p^e$. Based on the **Condition 4.2** and **4.3**, we coordinate system in ten cases:

**Case 1**. $0 < n_1 \le \dot{q}$; $\dot{p} \ge 0$; $[q^* = q^e = n_1; p^* = p^e = \bar{p};\ R_1 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases} \dfrac{\partial\pi_1}{\partial p} = 0 \\ \dfrac{\partial\pi_2}{\partial p} = 0 \end{cases} \begin{cases} \dfrac{\partial\pi_1}{\partial q} \ge 0 \\ \dfrac{\partial\pi_2}{\partial q} \ge 0 \end{cases} \rightarrow \begin{cases} -b_1\alpha_1Z_1(\beta_1) - R_2b_2[Z_2(\beta_2) - Z_2(\beta_2-q)] + g_1b_1 = 0 \\ -b_2\alpha_2Z_2(\beta_2) + R_2b_2[Z_2(\beta_2) - Z_2(\beta_2-q)] + g_2b_2 = 0 \\ \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1-h_1) \ge 0 \\ -\alpha_2Z_2(\beta_2) + R_2Z_2(\beta_2) - (R_2+w) + \alpha_2 - h_2 \ge 0 \end{cases} \rightarrow$$
$$\begin{cases} R_2 = \dfrac{g_1b_1 - b_1\alpha_1Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2-n_1)]} = \dfrac{\alpha_2Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2-n_1)} \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t \le w + R_2\bar{Z}_2(\beta_2) \le \alpha_2\bar{Z}_2(\beta_2) - h_2 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = -a_1 + b_1\bar{p}$ and $\beta_2 = n_2 + n_1 - a_2 + b_2\bar{p}$. So, **Condition 4.2** is met as long as $R_2$ and $w$ are constrained shown above. Next, we try to satisfy the **Condition 4.3** (i.e., whether

$\Delta\pi_1 \geq 0$ and $\Delta\pi_2 \geq 0$) and identify whether the system can be coordinated. We know that $\Delta\prod(n_1, \bar{p})$ and $\Delta\pi_1(n_1, \bar{p}, w, 0, R_2)$ are:

$$\Delta\prod(n_1, \bar{p}) = -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{-a_1+b_1\bar{p}} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2+b_2p^{02}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 + n_1(h_1 - \alpha_1 + \alpha_2 - h_2 - c_t)$$
$$+ g_1b_1(\bar{p} - p^{01}) + g_2b_2(\bar{p} - p^{02})$$

$\Delta\pi_1(n_1, \bar{p}, w, 0, R_2)$

$$= -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{-a_1+b_1\bar{p}} Z_1(d_1)dd_1 - R_2 \int_{n_2-a_2+b_2\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2$$
$$+ n_1(w - c_t + R_2 - \alpha_1 + h_1) + g_1b_1(\bar{p} - p^{01})$$

Also, we know $\Delta\prod(n_1, \bar{p}) = \Delta\pi_1(n_1, \bar{p}, w, 0, R_2) + \Delta\pi_2(n_1, \bar{p}, w, 0, R_2)$. So, the system is coordinated only if both $LSCs$' profit increment is no less than 0. Moreover, as $LSC$ 1's profit is increasing in $w$, so if we let $w_{\max} = \alpha_2[1 - Z_2(\beta_2)] - R_2[1 - Z_2(\beta_2)] - h_2$, then:

$\Delta\pi_{2\min}(n_1, \bar{p}, w, 0, R_2) = \Delta\prod(n_1, \bar{p}) - \Delta\pi_{1\max}(n_1, \bar{p}, w, 0, R_2)$

$$= R_2 \int_{n_2-a_2+b_2\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 - \alpha_2 \int_{n_2-a_2+b_2p^{02}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 + g_2b_2(\bar{p} - p^{02}) - n_1R_2$$
$$+ n_1\alpha_2 Z_2(n_2 + n_1 - a_2 + b_2\bar{p}) + n_1R_2[1 - Z_2(n_2 + n_1 - a_2 + b_2\bar{p})]$$
$$= R_2 \int_{n_2-a_2+b_2\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 - \alpha_2 \left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2+b_2\bar{p}} Z_2(d_2)dd_2 + \int_{n_2-a_2+b_2\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2\right]$$
$$+ g_2b_2(\bar{p} - p^{02}) + n_1Z_2(n_2 + n_1 - a_2 + b_2\bar{p})(\alpha_2 - R_2)$$
$$= -(\alpha_2 - R_2)\left[\int_{n_2-a_2+b_2\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 - n_1Z_2(n_2 + n_1 - a_2 + b_2\bar{p})\right]$$
$$- \alpha_2 \left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2+b_2\bar{p}} Z_2(d_2)dd_2 + b_2(p^{02} - \bar{p})\right] - (\alpha_2 - g_2)b_2(\bar{p} - p^{02})$$

According to the introduction of $\Delta S_i(.)$ in Appendix H, we obtain $\Delta\pi_{2\min}$ and $\Delta\pi_{1\max}$.

$\Delta\pi_{2\min} = (\alpha_2 - R_2)\Delta S_2^e(n_1, \bar{p}) + \alpha_2\Delta S_2(0, \bar{p}) - (\alpha_2 - g_2)b_2(\bar{p} - p^{02})$

$\Delta\pi_{1\max} = \Delta\prod - (\alpha_2 - R_2)\Delta S_2^e(n_1, \bar{p}) - \alpha_2\Delta S_2(0, \bar{p}) + (\alpha_2 - g_2)b_2(\bar{p} - p^{02})$

Now, we discuss and analyse the impact of $CT$ rate $\bar{p}$ on $LSC$ 2's profit increment.

If $p^e = \bar{p} \geq p^{02}$, which means equilibrium $CT$ rate is greater than $p^{02}$. Thus, to ensure $\Delta\pi_{2\min}$ is nonnegative, $\bar{p} - p^{02}$ should follow:

$$\bar{p} - p^{02} \leq \frac{(\alpha_2 - R_2)\Delta S_2^e(n_1, \bar{p}) + \alpha_2\Delta S_2(0, \bar{p})}{(\alpha_2 - g_2)b_2}$$

If $p^e = \bar{p} < p^{02}$, which means equilibrium $CT$ rate is less than $p^{02}$. Thus, to ensure $\Delta\pi_{2\min}$ is nonnegative, $p^{02} - \bar{p}$ should follow:

$$p^{02} - \bar{p} \geq -\frac{(\alpha_2 - R_2)\Delta S_2^e(n_1, \bar{p}) + \alpha_2\Delta S_2(0, \bar{p})}{(\alpha_2 - g_2)b_2}$$

If we let $w_{\min} = R_2[Z_2(\beta_2) - 1] + \alpha_1[1 - Z_1(\beta_1)] - h_1 + c_t$, then we have:

$\Delta\pi_{1\min}(n_1, \bar{p}, w, 0, R_2)$

$$= -\alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{-a_1+b_1\bar{p}} Z_1(d_1)dd_1 + \int_{-a_1+b_1\bar{p}}^{n_1-a_1+b_1\bar{p}} Z_1(d_1)dd_1 - \int_{-a_1+b_1\bar{p}}^{n_1-a_1+b_1\bar{p}} Z_1(d_1)dd_1\right]$$

$$- R_2\int_{n_2-a_2+b_2\,\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 + n_1[R_2Z_2(n_2+n_1-a_2+b_2\bar{p}) - \alpha_1Z_1(-a_1+b_1\bar{p})]$$

$$+ g_1b_1(\bar{p} - p^{01})$$

$$= -\alpha_1\left[\int_{n_1-a_1+b_1\bar{p}}^{-a_1+b_1\bar{p}} Z_1(d_1)dd_1 + n_1Z_1(-a_1+b_1\bar{p})\right]$$

$$- R_2\left[\int_{n_2-a_2+b_2\bar{p}}^{n_2+n_1-a_2+b_2\bar{p}} Z_2(d_2)dd_2 - n_1Z_2(n_2+n_1-a_2+b_2\bar{p})\right]$$

$$- \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1+b_1\bar{p}} Z_1(d_1)dd_1 + b_1(p^{01}-\bar{p})\right] - (\alpha_1 - g_1)b_1(\bar{p} - p^{01})$$

$\Delta\pi_{1\min} = \alpha_1[\Delta S_1(0,\bar{p}) + \Delta S_1^e(n_1,\bar{p})] + R_2\Delta S_2^e(n_1,\bar{p}) - (\alpha_1 - g_1)b_1(\bar{p} - p^{01})$

$\Delta\pi_{2\max} = \Delta\prod - \alpha_1[\Delta S_1(0,\bar{p}) + \Delta S_1^e(n_1,\bar{p})] - R_2\Delta S_2^e(n_1,\bar{p}) + (\alpha_1 - g_1)b_1(\bar{p} - p^{01})$

We now discuss the impact of $CT$ rate $\bar{p}$ on $LSC$ 1's profit increment.

If $p^e = \bar{p} \geq p^{01}$, which means the equilibrium $CT$ rate $p^e$ is greater than $p^{01}$. Thus, to ensure $\Delta\pi_{1\min}$ is nonnegative, $\bar{p} - p^{01}$ must follow:

$$\bar{p} - p^{01} \leq \frac{\alpha_1[\Delta S_1(0,\bar{p}) + \Delta S_1^e(n_1,\bar{p})] + R_2\Delta S_2^e(n_1,\bar{p})}{(\alpha_1 - g_1)b_1}$$

If $p^e = \bar{p} \leq p^{01}$, which means the equilibrium $CT$ rate $p$ is less than $p^{01}$. Thus, to ensure $\Delta\pi_{1\min}$ is nonnegative, $p^{01} - \bar{p}$ must follow:

$$p^{01} - \bar{p} \geq -\frac{\alpha_1[\Delta S_1(0,\bar{p}) + \Delta S_1^e(n_1,\bar{p})] + R_2\Delta S_2^e(n_1,\bar{p})}{(\alpha_1 - g_1)b_1}$$

So, if $\bar{p}$ satisfy the inequations above, the system can be coordinated in this case.

**Case 2**. $0 < n_1 \leq \dot{q}, \dot{p} < 0$; $[q^* = q^e = n_1; p^* = p^e = 0;\ R_1 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial p} \leq 0 \\ \dfrac{\partial \pi_2}{\partial p} \leq 0 \end{cases} \begin{cases} \dfrac{\partial \pi_1}{\partial q} \geq 0 \\ \dfrac{\partial \pi_2}{\partial q} \geq 0 \end{cases} \rightarrow \begin{cases} -b_1\alpha_1Z_1(\beta_1) - R_2b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)] + g_1b_1 \leq 0 \\ -b_2\alpha_2Z_2(\beta_2) + R_2b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)] + g_2b_2 \leq 0 \\ \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1 - h_1) \geq 0 \\ -\alpha_2Z_2(\beta_2) + R_2Z_2(\beta_2) - [R_2 + w] + \alpha_2 - h_2 \geq 0 \end{cases} \rightarrow$$

$$\begin{cases} \dfrac{g_1b_1 - b_1\alpha_1Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - n_1)]} \leq R_2 \leq \dfrac{\alpha_2Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - n_1)} \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t \leq w + R_2\bar{Z}_2(\beta_2) \leq \alpha_2\bar{Z}_2(\beta_2) - h_2 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = -a_1$; $\beta_2 = n_2 + n_1 - a_2$ in this case. Therefore, **Condition 4.2** is met as long as $R_2$ and $w$ are constrained shown above. Also, $\Delta\prod(n_1, 0)$ and $\Delta\pi_1(n_1, 0, w, 0, R_2)$ are:

$$\Delta\prod(n_1, 0) = -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{-a_1} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2+b_2\,p^{02}}^{n_2+n_1-a_2} Z_2(d_2)dd_2 + n_1(h_1 - \alpha_1 + \alpha_2 - h_2 - c_t)$$
$$- g_1b_1p^{01} - g_2b_2p^{02}$$

$$\Delta\pi_1(n_1,0,w,0,R_2)$$
$$= -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{-a_1} Z_1(d_1)dd_1 - R_2\left[\int_{n_2-a_2}^{n_2+n_1-a_2} Z_2(d_2)dd_2 - n_1\right] + n_1(w - c_t - \alpha_1 + h_1)$$
$$- g_1b_1p^{01}$$

According to mean value theorem, so we have:

$$\int_{n_2-a_2}^{n_2+n_1-a_2} Z_2(d_2)dd_2 - n_1 = n_1Z_2(.) - n_1 \leq 0$$

As $\Delta\pi_1$ is increasing in $w$ and $R_2$, so if we let $R_{2max} = \frac{\alpha_2 Z_2(\beta_2)-g_2}{Z_2(\beta_2)-Z_2(\beta_2-n_1)}$ and $w_{max} = (\alpha_2 - R_2)[1 - Z_2(\beta_2)] - h_2$, then:

$$\Delta\pi_{2\min}(n_1,0,w,0,R_2) = \Delta\prod(n_1,0) - \Delta\pi_{1\max}(n_1,0,w,0,R_2)$$
$$= -(\alpha_2 - R_{2\max})\left[\int_{n_2-a_2}^{n_2+n_1-a_2} Z_2(d_2)dd_2 - n_1Z_2(n_2+n_1-a_2)\right]$$
$$- \alpha_2\left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2} Z_2(d_2)dd_2 + b_2p^{02}\right] + (\alpha_2 - g_2)b_2p^{02}$$

$$\Delta\pi_{2\min} = (\alpha_2 - R_{2\max})\Delta S_2^e(n_1,0) + \alpha_2\Delta S_2(0,0) + (\alpha_2 - g_2)b_2p^{02}$$
$$\Delta\pi_{1\max} = \Delta\prod - (\alpha_2 - R_{2\max})\Delta S_2^e(n_1,0) - \alpha_2\Delta S_2(0,0) - (\alpha_2 - g_2)b_2p^{02}$$

Now, we discuss and analyse the impact of $p^{02}$ on $LSC$ 2's profit increment.

If $\Delta\pi_{2\min}$ get a non-negative profit increment in this case, then $p^{02}$ must satisfy:

$$p^{02} \geq -\frac{(\alpha_2 - R_{2\max})\Delta S_2^e(n_1,0) + \alpha_2\Delta S_2(0,0)}{(\alpha_2 - g_2)b_2}$$

Let $R_{2\min} = \frac{g_1b_1 - b_1\alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2)-Z_2(\beta_2-n_1)]}$ and $w_{\min} = R_2[Z_2(\beta_2) - 1] + \alpha_1[1 - Z_1(\beta_1)] - h_1 + c_t$. Then:

$$\Delta\pi_{1\min}(n_1,0,w,0,R_2)$$
$$= -\alpha_1\left[\int_{n_1-a_1}^{-a_1} Z_1(d_1)dd_1 + n_1Z_1(-a_1)\right]$$
$$- R_{2min}\left[\int_{n_2-a_2}^{n_2+n_1-a_2} Z_2(d_2)dd_2 - n_1Z_2(n_2+n_1-a_2)\right]$$
$$- \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1} Z_1(d_1)dd_1 + b_1p^{01}\right] + (\alpha_1 - g_1)b_1p^{01}$$

$$\Delta\pi_{1\min} = \alpha_1[\Delta S_1^e(n_1,0) + \Delta S_1(0,0)] + R_{2\min}\Delta S_2^e(n_1,0) + (\alpha_1 - g_1)b_1p^{01}$$
$$\Delta\pi_{2\max} = \Delta\prod - \alpha_1[\Delta S_1^e(n_1,0) + \Delta S_1(0,0)] - R_{2\min}\Delta S_2^e(n_1,0) - (\alpha_1 - g_1)b_1p^{01}$$

Now, we discuss the impact of $p^{01}$ on $LSC$ 1's profit increment.

If $\Delta\pi_{1\min}$ get a non-negative profit allocation in this case, then $p^{01}$ must satisfy:

$$p^{01} \geq -\frac{\alpha_1[\Delta S_1^e(n_1,0) + \Delta S_1(0,0)] + R_{2\min}\Delta S_2^e(n_1,0)}{(\alpha_1 - g_1)b_1}$$

So, if $p^{01}$ and $p^{02}$ satisfy the inequations above, the system is coordinated in this case.

**Case 3.** $0 < \dot{q} < n_1, \dot{p} \geq 0, [q^* = q^e = \dot{q}, p^* = p^e = \dot{p};\ R_1 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases}\dfrac{\partial \pi_1}{\partial p} = 0 \\ \dfrac{\partial \pi_2}{\partial p} = 0\end{cases}\begin{cases}\dfrac{\partial \pi_1}{\partial q} = 0 \\ \dfrac{\partial \pi_2}{\partial q} = 0\end{cases} \rightarrow \begin{cases}-b_1\alpha_1Z_1(\beta_1) - R_2b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)] + g_1b_1 = 0 \\ -b_2\alpha_2Z_2(\beta_2) + R_2b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)] + g_2b_2 = 0 \\ \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1 - h_1) = 0 \\ -\alpha_2Z_2(\beta_2) + R_2Z_2(\beta_2) - [R_2 + w] + \alpha_2 - h_2 = 0\end{cases} \rightarrow$$

$$\begin{cases} R_2 = \dfrac{g_1 b_1 - b_1 \alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)]} = \dfrac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - q)} \\ \alpha_1 \bar{Z}_1(\beta_1) - h_1 + c_t = w + R_2 \bar{Z}_2(\beta_2) = \alpha_2 \bar{Z}_2(\beta_2) - h_2 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 - \dot{q} - a_1 + b_1\dot{p}$ and $\beta_2 = n_2 + \dot{q} - a_2 + b_2\dot{p}$. So, **Condition 4.2** is met as long as $R_2$ and $w$ are constrained shown above. Also, $\Delta\prod(\dot{q},\dot{p})$ and $\Delta\pi_1(\dot{q},\dot{p},w,0,R_2)$ are:

$$\Delta\prod(\dot{q},\dot{p}) = -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{n_1-\dot{q}-a_1+b_1\dot{p}} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2+b_2\,p^{02}}^{n_2+\dot{q}-a_2+b_2\dot{p}} Z_2(d_2)dd_2 + \dot{q}(h_1 - \alpha_1 + \alpha_2 - h_2 - c_t)$$

$$+ g_1 b_1(\dot{p} - p^{01}) + g_2 b_2(\dot{p} - p^{02})$$

$\Delta\pi_1(\dot{q},\dot{p},w,0,R_2)$

$$= -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{n_1-\dot{q}-a_1+b_1\dot{p}} Z_1(d_1)dd_1 - R_2 \int_{n_2-a_2+b_2\,\dot{p}}^{n_2+\dot{q}-a_2+b_2\dot{p}} Z_2(d_2)dd_2$$

$$+ \dot{q}(w - c_t + R_2 - \alpha_1 + h_1) + g_1 b_1(\dot{p} - p^{01})$$

Let $w = (\alpha_2 - R_2)[1 - Z_2(\beta_2)] - h_2$ and take it into $\Delta\pi_1$, then:

$\Delta\pi_2(\dot{q},\dot{p},w,0,R_2) = \Delta\prod(\dot{q},\dot{p}) - \Delta\pi_1(\dot{q},\dot{p},w,0,R_2)$

$$= -(\alpha_2 - R_2)\left[\int_{n_2-a_2+b_2\dot{p}}^{n_2+\dot{q}-a_2+b_2\dot{p}} Z_2(d_2)dd_2 - \dot{q}Z_2(n_2 + \dot{q} - a_2 + b_2\dot{p})\right]$$

$$- \alpha_2\left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2+b_2\dot{p}} Z_2(d_2)dd_2 + b_2(p^{02} - \dot{p})\right] + (\alpha_2 - g_2)b_2(p^{02} - \dot{p})$$

$\Delta\pi_2 = (\alpha_2 - R_2)\Delta S_2^e(\dot{q},\dot{p}) + \alpha_2\Delta S_2(0,\dot{p}) + (\alpha_2 - g_2)b_2(p^{02} - \dot{p})$

$\Delta\pi_1 = \Delta\prod - (\alpha_2 - R_2)\Delta S_2^e(\dot{q},\dot{p}) - \alpha_2\Delta S_2(0,\dot{p}) - (\alpha_2 - g_2)b_2(p^{02} - \dot{p})$

Now, we discuss and analyse the impact of $CT$ rate $\dot{p}$ on $LSC$ 2's profit increment.

If $p^e = \dot{p} \geq p^{02}$, which means equilibrium $CT$ rate is greater than $p^{02}$. Thus, to ensure $\Delta\pi_{2\min}$ is nonnegative, $\dot{p} - p^{02}$ must follow:

$$\dot{p} - p^{02} \leq \frac{(\alpha_2 - R_2)\Delta S_2^e(\dot{q},\dot{p}) + \alpha_2\Delta S_2(0,\dot{p})}{(\alpha_2 - g_2)b_2}$$

If $p^e = \dot{p} < p^{02}$, which means equilibrium $CT$ rate is less than $p^{02}$. Thus, to ensure $\Delta\pi_{2\min}$ is nonnegative, $p^{02} - \dot{p}$ must follow:

$$p^{02} - \dot{p} \geq -\frac{(\alpha_2 - R_2)\Delta S_2^e(\dot{q},\dot{p}) + \alpha_2\Delta S_2(0,\dot{p})}{(\alpha_2 - g_2)b_2}$$

If we let $w = R_2[Z_2(\beta_2) - 1] + \alpha_1[1 - Z_1(\beta_1)] - h_1 + c_t$, then we have:

$\Delta\pi_1(\dot{q},\dot{p},w,0,R_2)$

$$= -\alpha_1\left[\int_{n_1-a_1+b_1\dot{p}}^{n_1-\dot{q}-a_1+b_1\dot{p}} Z_1(d_1)dd_1 + \dot{q}Z_1(n_1 - \dot{q} - a_1 + b_1\dot{p})\right]$$

$$- R_2\left[\int_{n_2-a_2+b_2\dot{p}}^{n_2+\dot{q}-a_2+b_2\dot{p}} Z_2(d_2)dd_2 - \dot{q}Z_2(n_2 + \dot{q} - a_2 + b_2\dot{p})\right]$$

$$- \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1+b_1\dot{p}} Z_1(d_1)dd_1 + b_1(p^{01} - \dot{p})\right] - (\alpha_1 - g_1)b_1(\dot{p} - p^{01})$$

$\Delta\pi_1 = \alpha_1[\Delta S_1^e(\dot{q},\dot{p}) + \Delta S_1(0,\dot{p})] + R_2\Delta S_2^e(\dot{q},\dot{p}) - (\alpha_1 - g_1)b_1(\dot{p} - p^{01})$

$\Delta\pi_2 = \Delta\prod - \alpha_1[\Delta S_1^e(\dot{q},\dot{p}) + \Delta S_1(0,\dot{p})] - R_2\Delta S_2^e(\dot{q},\dot{p}) + (\alpha_1 - g_1)b_1(\dot{p} - p^{01})$

Now, we discuss and analyse the impact of $CT$ rate $\dot{p}$ on $LSC$ 1's profit increment.

If $p^e = \dot{p} \geq p^{01}$, which means the $CT$ rate $p^e$ is greater than $p^{01}$. Thus, to ensure $\Delta\pi_1$ is nonnegative, $\dot{p} - p^{01}$ must follow:

$$\dot{p} - p^{01} \leq \frac{\alpha_1[\Delta S_1^e(\dot{q},\dot{p}) + \Delta S_1(0,\dot{p})] + R_2\Delta S_2^e(\dot{q},\dot{p})}{(\alpha_1 - g_1)b_1}$$

If $p^e = \dot{p} \leq p^{01}$, which means the $CT$ rate $p$ is less than $p^{01}$. Thus, to ensure $\Delta\pi_{1min}$ is nonnegative, $p^{01} - \dot{p}$ must follow:

$$p^{01} - \dot{p} \geq -\frac{\alpha_1[\Delta S_1^e(\dot{q},\dot{p}) + \Delta S_1(0,\dot{p})] + R_2\Delta S_2^e(\dot{q},\dot{p})}{(\alpha_1 - g_1)b_1}$$

So, if $p^e = \dot{p}$ satisfies the inequations above, the system can be coordinated.

**Case 4.** $0 < \dot{q} < n_1;\ \dot{p} < 0,\ [q^* = q^e = \hat{q}; p^* = p^e = 0;\ R_1 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial p} \leq 0 \\ \dfrac{\partial \pi_2}{\partial p} \leq 0 \end{cases} \begin{cases} \dfrac{\partial \pi_1}{\partial q} = 0 \\ \dfrac{\partial \pi_2}{\partial q} = 0 \end{cases} \rightarrow \begin{cases} -b_1\alpha_1 Z_1(\beta_1) - R_2 b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)] + g_1 b_1 \leq 0 \\ -b_2\alpha_2 Z_2(\beta_2) + R_2 b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)] + g_2 b_2 \leq 0 \\ \alpha_1 Z_1(\beta_1) - R_2 Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1 - h_1) = 0 \\ -\alpha_2 Z_2(\beta_2) + R_2 Z_2(\beta_2) - [R_2 + w] + \alpha_2 - h_2 = 0 \end{cases} \rightarrow$$

$$\begin{cases} \dfrac{g_1 b_1 - b_1\alpha_1 Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)]} \leq R_2 \leq \dfrac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - q)} \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t = w + R_2\bar{Z}_1(\beta_1) = \alpha_2\bar{Z}_2(\beta_2) - h_2 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 - \hat{q} - a_1$ and $\beta_2 = n_2 + \hat{q} - a_2$.

Therefore, **Condition 4.2** is met as long as $R_2$ and $w$ are constrained shown above. Also, $\Delta\prod(\hat{q},0)$ and $\Delta\pi_1(\hat{q},0,w,0,R_2)$ are:

$$\Delta\prod(\hat{q},0) = -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{n_1-\hat{q}-a_1} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2+b_2p^{02}}^{n_2+\hat{q}-a_2} Z_2(d_2)dd_2 + \hat{q}(h_1 - \alpha_1 + \alpha_2 - h_2 - c_t) - g_1 b_1 p^{01} - g_2 b_2 p^{02}$$

$\Delta\pi_1(\hat{q},0,w,0,R_2)$

$$= -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{n_1-\hat{q}-a_1} Z_1(d_1)dd_1 - R_2\left[\int_{n_2-a_2}^{n_2+\hat{q}-a_2} Z_2(d_2)dd_2 - \hat{q}\right] + \hat{q}(w - c_t - \alpha_1 + h_1) - g_1 b_1 p^{01}$$

Let $R_{2\max} = \frac{\alpha_2 Z_2(\beta_2) - g_2}{Z_2(\beta_2) - Z_2(\beta_2 - q)}$ and $w = R_2[Z_2(\beta_2) - 1] + \alpha_1[1 - Z_1(\beta_1)] - h_1 + c_t$, So:

$\Delta\pi_{1\min}(\hat{q},0,w,0,R_2)$

$$= -\alpha_1\left[\int_{n_1-a_1}^{n_1-\hat{q}-a_1} Z_1(d_1)dd_1 + \hat{q}Z_1(n_1 - \hat{q} - a_1)\right] - R_{2\max}\left[\int_{n_2-a_2}^{n_2+\hat{q}-a_2} Z_2(d_2)dd_2 - \hat{q}Z_2(n_2 + \hat{q} - a_2)\right] - \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1} Z_1(d_1)dd_1 + b_1p^{01}\right] + \alpha_1 b_1 p^{01} - g_1 b_1 p^{01}$$

$\Delta\pi_{1\min} = \alpha_1[\Delta S_1^e(\hat{q},0) + \Delta S_1(0,0)] + R_{2\max}\Delta S_2^e(\hat{q},0) + (\alpha_1 - g_1)b_1p^{01}$

$\Delta\pi_{2\max} = \Delta\prod - \alpha_1[\Delta S_1^e(\hat{q},0) + \Delta S_1(0,0)] - R_{2\max}\Delta S_2^e(\hat{q},0) - (\alpha_1 - g_1)b_1p^{01}$

Now, we discuss and analyse the impact of $p^{01}$ on $LSC$ 1's profit increment.

If $\Delta\pi_{1\min}$ get a non-negative profit allocation, then $p^{01}$ must satisfy:

$$p^{01} \geq -\frac{\alpha_1[\Delta S_1^e(\hat{q},0) + \Delta S_1(0,0)] + R_{2max}\Delta S_2^e(\hat{q},0)}{(\alpha_1 - g_1)b_1}$$

If we obtain $\Delta\pi_{2\min}$ and $\Delta\pi_{1\max}$, then we let $R_{2\min} = \frac{g_1b_1 - b_1\alpha_1Z_1(\beta_1)}{b_2[Z_2(\beta_2) - Z_2(\beta_2 - q)]}$, $w = \alpha_2 - h_2 - \alpha_2Z_2(\beta_2) - R_2[1 - Z_2(\beta_2)]$. We have:

$\Delta\pi_{2\min}(\hat{q},0,w,R_1,R_2) = \Delta\prod(\hat{q},0) - \Delta\pi_{1\max}(\hat{q},0,w,R_1,R_2)$

$$= -(\alpha_2 - R_{2\min})\left[\int_{n_2-a_2}^{n_2+\hat{q}-a_2} Z_2(d_2)dd_2 - \hat{q}Z_2(n_2 + \hat{q} - a_2)\right]$$

$$- \alpha_2\left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2} Z_2(d_2)dd_2 + b_2p^{02}\right] + \alpha_2b_2p^{02} - g_2b_2p^{02}$$

$\Delta\pi_{2\min} = (\alpha_2 - R_{2\min})\Delta S_2^e(\hat{q},0) + \alpha_2\Delta S_2(\hat{q},0) + (\alpha_2 - g_2)b_2p^{02}$

$\Delta\pi_{1\max} = \Delta\prod - (\alpha_2 - R_{2\min})\Delta S_2^e(\hat{q},0) - \alpha_2\Delta S_2(\hat{q},0) - (\alpha_2 - g_2)b_2p^{02}$

Now, we discuss the impact of $p^{02}$ on $LSC$ 2's profit increment.

If $\Delta\pi_{2min}$ get a non-negative profit increment, then $p^{02}$ must satisfy:

$$p^{02} \geq -\frac{(\alpha_2 - R_{2min})\Delta S_2^e(\hat{q},0) + \alpha_2\Delta S_2(\hat{q},0)}{(\alpha_2 - g_2)b_2}$$

So, if $p^{01}$ and $p^{02}$ satisfy the inequations above, the system can be coordinated.

**Case 5.** $\dot{q} < 0, \dot{p} \geq 0$ *or* $\ddot{q} > 0, \ddot{p} \geq 0; [q^* = q^e = 0;\ p^* = p^e = p^0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases} \dfrac{\partial^+\pi_1}{\partial q} \leq 0 \\ \dfrac{\partial^+\pi_2}{\partial q} \leq 0 \\ \dfrac{\partial^-\pi_1}{\partial q} \geq 0 \end{cases} \begin{cases} \dfrac{\partial^-\pi_2}{\partial q} \geq 0 \\ \dfrac{\partial\pi_1}{\partial p} = 0 \\ \dfrac{\partial\pi_2}{\partial p} = 0 \end{cases} \rightarrow \begin{cases} \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1 - h_1) \leq 0 \\ -\alpha_2Z_2(\beta_1) + R_2Z_2(\beta_2) - (R_2 + w) + \alpha_2 - h_2 \leq 0 \\ \alpha_1Z_1(\beta_1) - R_1Z_1(\beta_1) + R_1 + w - (\alpha_1 - h_1) \geq 0 \\ -\alpha_2Z_2(\beta_2) + R_1Z_1(\beta_1) - R_1 - w + c_t + \alpha_2 - h_2 \geq 0 \\ -b_1\alpha_1Z_1(\beta_1) + g_1b_1 = 0 \\ -b_2\alpha_2Z_2(\beta_2) + g_2b_2 = 0 \end{cases} \rightarrow$$

$$\begin{cases} \alpha_2\bar{Z}_2(\beta_2) - h_2 \leq w + R_2\bar{Z}_2(\beta_2) \leq \alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 \leq w + R_1\bar{Z}_1(\beta_1) \leq \alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t \\ p^0 = p^{01} \\ p^0 = p^{02} \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 - a_1 + b_1p^0$ and $\beta_2 = n_2 - a_2 + b_2p^0$, so, $\Delta\prod = \Delta\pi_1 = \Delta\pi_2 = 0$. So, the system is coordinated; but the profit increment for two $LSCs$ is 0.

**Case 6.** $\dot{q} < 0, \dot{p} < 0$ or $\ddot{q} > 0,\ \ddot{p} < 0$. $[q^* = q^e = 0;\ p^* = p^e = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases} \dfrac{\partial^+\pi_1}{\partial q} \leq 0 \\ \dfrac{\partial^+\pi_2}{\partial q} \leq 0 \\ \dfrac{\partial^-\pi_1}{\partial q} \geq 0 \end{cases} \begin{cases} \dfrac{\partial^-\pi_2}{\partial q} \geq 0 \\ \dfrac{\partial\pi_1}{\partial p} \leq 0 \\ \dfrac{\partial\pi_2}{\partial p} \leq 0 \end{cases} \rightarrow \begin{cases} \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1 - h_1) \leq 0 \\ -\alpha_2Z_2(\beta_2) + R_2Z_2(n_2 - a_2) - (R_2 + w) + \alpha_2 - h_2 \leq 0 \\ \alpha_1Z_1(\beta_1) - R_1Z_1(\beta_1) - (R_1 + w) - (\alpha_1 - h_1) \geq 0 \\ -\alpha_2Z_2(\beta_2) + R_1Z_1(\beta_1) + R_1 + w - c_t + \alpha_2 - h_2 \geq 0 \\ -b_1\alpha_1Z_1(\beta_1) + g_1b_1 = 0 \\ -b_2\alpha_2Z_2(\beta_2) + g_2b_2 = 0 \end{cases}$$

$$\begin{cases} \alpha_2\bar{Z}_2(\beta_2) - h_2 \le w + R_2\bar{Z}_2(\beta_2) \le \alpha_1\bar{Z}_1(\beta_1) - h_1 + c_t \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 \le w + R_1\bar{Z}_1(\beta_1) \le \alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t \\ \alpha_1 Z_1(n_1 - a_1) \le g_1 \\ \alpha_2 Z_2(n_2 - a_2) \le g_2 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 - a_1$ and $\beta_2 = n_2 - a_2$, $\Delta\pi_1$ and $\Delta\pi_2$, are:

$$\Delta\pi_1 = -\alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1} Z_1(d_1)dd_1 + b_1p^{01}\right] + (\alpha_1 - g_1)b_1p^{01} = \alpha_1\Delta S_1(0, p^{01}) + (\alpha_1 - g_1)b_1p^{01}$$

$$\Delta\pi_2 = \Delta\prod - \Delta\pi_1 = \alpha_2\Delta S_2(0, p^{02}) + (\alpha_2 - g_2)b_2p^{02}$$

If $\Delta\pi_1$ and $\Delta\pi_2$ get a non-negative profit increment, then $p^{01}$ and $p^{02}$ must satisfy:

$$p^{01} \ge -\frac{\alpha_1\Delta S_1(0,0)}{(\alpha_1 - g_1)b_1};\ p^{02} \ge -\frac{\alpha_2\Delta S_2(0,0)}{(\alpha_2 - g_2)b_2}$$

So, the system is coordinated when $p^* = 0$ and $p^{01}$ and $p^{02}$ satisfy the inequation above.

**Case 7.** $-n_2 < \ddot{q} < 0;\ \ddot{p} < 0;\ [q^* = q^e = \check{q}, p^* = p^e = 0, R_2 = 0]$

$$\begin{cases} \dfrac{\partial\pi_1}{\partial p} \le 0 \\ \dfrac{\partial\pi_2}{\partial p} \le 0 \end{cases} \begin{cases} \dfrac{\partial\pi_1}{\partial q} = 0 \\ \dfrac{\partial\pi_2}{\partial q} = 0 \end{cases} \rightarrow \begin{cases} -b_1\alpha_1 Z_1(\beta_1) + R_1b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)] + g_1b_1 \le 0 \\ -b_2\alpha_2 Z_2(\beta_2) - R_1b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)] + g_2b_2 \le 0 \\ \alpha_1 Z_1(\beta_1) - R_1Z_1(\beta_1) + R_1 + w - (\alpha_1 - h_1) = 0 \\ -\alpha_2 Z_2(\beta_2) + R_1Z_1(\beta_1) - R_1 - w + c_t + \alpha_2 - h_2 = 0 \end{cases} \rightarrow$$

$$\begin{cases} \dfrac{g_2b_2 - b_2\alpha_2 Z_2(\beta_2)}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]} \le R_1 \le \dfrac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)} \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 = w + R_1\bar{Z}_2(\beta_2) = c_t + \alpha_2\bar{Z}_2(\beta_2) - h_2 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 - \check{q} - a_1$ and $\beta_2 = n_2 + \check{q} - a_2$ in this case. Therefore, **Condition 4.2** is met as long as $R_1$ and $w$ are constrained shown above. Also, $\Delta\prod(\check{q}, 0)$ and $\Delta\pi_1(\check{q}, 0, w, R_1, 0)$ are:

$$\Delta\prod(\check{q}, 0) = -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{n_1-\check{q}-a_1} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2+b_2p^{02}}^{n_2+\check{q}-a_2} Z_2(d_2)dd_2 + \check{q}(h_1 - \alpha_1 + \alpha_2 - h_2 + c_t)$$
$$- g_1b_1p^{01} - g_2b_2p^{02}$$

$$\Delta\pi_1(\check{q}, 0, w, R_1, 0)$$
$$= -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{n_1-\check{q}-a_1} Z_1(d_1)dd_1 + R_1\left[\int_{n_1-a_1}^{n_1-\check{q}-a_1} Z_1(d_1)dd_1 + \check{q}\right] + \check{q}(w - \alpha_1 + h_1)$$
$$- g_1b_1p^{01}$$

Let $R_{1\min} = \frac{g_2b_2 - b_2\alpha_2 Z_2(\beta_2)}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]}$ and $w = R_1[Z_1(\beta_1) - 1] + c_t + \alpha_2[1 - Z_2(\beta_2)] - h_2$, So, we have:

$$\Delta\pi_{2\min}(\check{q}, 0, w, R_1, 0) = \Delta\prod(\check{q}, 0) - \Delta\pi_{1\max}(\check{q}, 0, w, R_1, 0)$$
$$= -\alpha_2\left[\int_{n_2-a_2}^{n_2+\check{q}-a_2} Z_2(d_2)dd_2 - \check{q}Z_2(n_2 + \check{q} - a_2)\right]$$
$$- R_{1\min}\left[\int_{n_1-a_1}^{n_1-\check{q}-a_1} Z_1(d_1)dd_1 + \check{q}Z_1(n_1 - \check{q} - a_1)\right]$$
$$- \alpha_2\left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2} Z_2(d_2)dd_2 + b_2p^{02}\right] + \alpha_2b_2p^{02} - g_2b_2p^{02}$$

$$\Delta\pi_{2\min} = \alpha_2[\Delta S_2^e(\check{q}, 0) + \Delta S_2(0,0)] + R_{1\min}\Delta S_1^e(\check{q}, 0) + (\alpha_2 - g_2)b_2p^{02}$$
$$\Delta\pi_{1\max} = \Delta\prod - \alpha_2[\Delta S_2^e(\check{q}, 0) + \Delta S_2(0,0)] - R_{1\min}\Delta S_1^e(\check{q}, 0) - (\alpha_2 - g_2)b_2p^{02}$$

Now, we discuss the impact of $p^{01}$ on $LSC$ 1's profit increment.

If $\Delta\pi_{2\min}$ get a non-negative profit increment, then $p^{02}$ must satisfy:

$$p^{02} \geq -\frac{\alpha_2[\Delta S_2^e(\check{q},0) + \Delta S_2(0,0)] + R_{1min}\Delta S_1^e(\check{q},0)}{(\alpha_2 - g_2)b_2}$$

If we obtain $\Delta\pi_{1\min}$ and $\Delta\pi_{2\max}$, then we let $R_{1\max} = \frac{\alpha_1 Z_1(\beta_1)-g_1}{Z_1(\beta_1)-Z_1(\beta_1+q)}$ $w = R_1[Z_1(\beta_1) - 1] + \alpha_1[1 - Z_1(\beta_1)] - h_1$. Then, we have:

$\Delta\pi_{1\min}(\check{q}, 0, w, R_1, 0)$

$$= -(\alpha_1 - R_{1\max})\left[\int_{n_1-a_1}^{n_1-\check{q}-a_1} Z_1(d_1)dd_1 + \check{q}Z_1(n_1 - \check{q} - a_1)\right]$$

$$- \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1} Z_1(d_1)dd_1 + b_1p^{01}\right] + \alpha_1 b_1 p^{01} - g_1 b_1 p^{01}$$

$\Delta\pi_{1\min} = (\alpha_1 - R_{1\max})\Delta S_1^e(\check{q},0) + \alpha_1\Delta S_1(0,0) + (\alpha_1 - g_1)b_1p^{01}$

$\Delta\pi_{2\max} = \Delta\prod - (\alpha_1 - R_{1\max})\Delta S_1^e(\check{q},0) - \alpha_1\Delta S_1(0,0) - (\alpha_1 - g_1)b_1p^{01}$

Now, we discuss the impact of $p^{01}$ on $LSC$ 1's profit increment.

If $\Delta\pi_{1\min}$ get a non-negative profit increment, then $p^{01}$ must satisfy:

$$p^{01} \geq -\frac{(\alpha_1 - R_{1max})\Delta S_1^e(\check{q},0) + \alpha_1\Delta S_1(0,0)}{(\alpha_1 - g_1)b_1}$$

So, if $p^{01}$ and $p^{02}$ satisfy the inequations above, the system can be coordinated.

**Case 8.** $-n_2 \leq \ddot{q} < 0;\ \ddot{p} \geq 0,\ [q^* = q^e = \ddot{q}, p^* = p^e = \ddot{p}, R_2 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases}\dfrac{\partial \pi_1}{\partial p} = 0 \\ \dfrac{\partial \pi_2}{\partial p} = 0\end{cases} \begin{cases}\dfrac{\partial \pi_1}{\partial q} = 0 \\ \dfrac{\partial \pi_2}{\partial q} = 0\end{cases} \rightarrow \begin{cases}-b_1\alpha_1 Z_1(\beta_1) + R_1 b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)] + g_1 b_1 = 0 \\ -b_2\alpha_2 Z_2(\beta_2) - R_1 b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)] + g_2 b_2 = 0 \\ \alpha_1 Z_1(\beta_1) - R_1 Z_1(\beta_1) + R_1 + w - (\alpha_1 - h_1) = 0 \\ -\alpha_2 Z_2(\beta_2) + R_1 Z_1(\beta_1) - R_1 - w + c_t + \alpha_2 - h_2 = 0\end{cases} \rightarrow$$

$$\begin{cases}R_1 = \dfrac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)} = \dfrac{g_2 b_2 - b_2\alpha_2 Z_2(\beta_2)}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]} \\ \alpha_1\bar{Z}_1(\beta_1) - h_1 = w + R_1\bar{Z}_1(\beta_1) = \alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t\end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 - \ddot{q} - a_1 + b_1\ddot{p}$ and $\beta_2 = n_2 + \ddot{q} - a_2 + b_2\ddot{p}$. Therefore, **Condition 4.2** is met as long as $R_1$ and $w$ are constrained shown above. Also, $\Delta\prod(\ddot{q},\ddot{p})$ and $\Delta\pi_1(\ddot{q},\ddot{p},w,R_1,0)$ are:

$$\Delta\prod(\ddot{q},\ddot{p}) = -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{n_1-\ddot{q}-a_1+b_1\ddot{p}} Z_1(d_1)dd_1 - \alpha_2\int_{n_2-a_2+b_2\,p^{02}}^{n_2+\ddot{q}-a_2+b_2\ddot{p}} Z_2(d_2)dd_2 + \ddot{q}(h_1 - \alpha_1 + \alpha_2 - h_2 + c_t)$$

$$+ g_1 b_1(\ddot{p} - p^{01}) + g_2 b_2(\ddot{p} - p^{02})$$

$\Delta\pi_1(\ddot{q},\ddot{p},w,R_1,0)$

$$= -\alpha_1\int_{n_1-a_1+b_1p^{01}}^{n_1-\ddot{q}-a_1+b_1\ddot{p}} Z_1(d_1)dd_1 + R_1\int_{n_1-a_1+b_1\ddot{p}}^{n_1-\ddot{q}-a_1+b_1\ddot{p}} Z_1(d_1)dd_1 + \ddot{q}(w + R_1 - \alpha_1 + h_1)$$

$$+ g_1 b_1(\ddot{p} - p^{01})$$

Let $w = \alpha_2\bar{Z}_2(\beta_2) - h_2 + c_t - R_1\bar{Z}_1(\beta_1)$ and take it into $\Delta\pi_1(\ddot{q},\ddot{p},w,R_1,0)$, then:

$\Delta\pi_2(\ddot{q},\ddot{p},w,R_1,0) = \Delta\prod(\ddot{q},\ddot{p}) - \Delta\pi_1(\ddot{q},\ddot{p},w,R_1,0)$

$$= -\alpha_2\left[\int_{n_2-a_2+b_2\ddot{p}}^{n_2+\ddot{q}-a_2+b_2\ddot{p}} Z_2(d_2)dd_2 - \ddot{q}Z_2(n_2+\ddot{q}-a_2+b_2\ddot{p})\right]$$

$$- R_1\left[\int_{n_1-a_1+b_1\ddot{p}}^{n_1-\ddot{q}-a_1+b_1\ddot{p}} Z_1(d_1)dd_1 + \ddot{q}Z_1(n_1-\ddot{q}-a_1+b_1\ddot{p})\right]$$

$$- \alpha_2\left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2+b_2\ddot{p}} Z_2(d_2)dd_2 + b_2(p^{02}-\ddot{p})\right] - (\alpha_2-g_2)b_2(\ddot{p}-p^{02})$$

$\Delta\pi_2 = \alpha_2[\Delta S_2^e(\ddot{q},\ddot{p}) + \Delta S_2(0,\ddot{p})] + R_1\Delta S_1^e(\ddot{q},\ddot{p}) - (\alpha_2-g_2)b_2(\ddot{p}-p^{02})$

$\Delta\pi_1 = \Delta\prod - \alpha_2[\Delta S_2^e(\ddot{q},\ddot{p}) + \Delta S_2(0,\ddot{p})] - R_1\Delta S_1^e(\ddot{q},\ddot{p}) + (\alpha_2-g_2)b_2(\ddot{p}-p^{02})$

Now, we discuss and analyse the impact of $CT$ rate $\dot{p}$ on $LSC$ 2's profit increment.

If $p^e = \ddot{p} \geq p^{02}$, which means equilibrium $CT$ rate is greater than $p^{02}$. Thus, to ensure $\Delta\pi_{2min}$ is nonnegative, $\ddot{p} - p^{02}$ must follow:

$$\ddot{p} - p^{02} \leq \frac{\alpha_2[\Delta S_2^e(\ddot{q},\ddot{p}) + \Delta S_2(0,\ddot{p})] + R_1\Delta S_1^e(\ddot{q},\ddot{p})}{(\alpha_2-g_2)b_2}$$

If $p^e = \ddot{p} < p^{02}$, which means equilibrium $CT$ rate is less than $p^{02}$. Thus, to ensure $\Delta\pi_{2min}$ is nonnegative, $p^{02} - \ddot{p}$ must be followed:

$$p^{02} - \ddot{p} \geq -\frac{\alpha_2[\Delta S_2^e(\ddot{q},\ddot{p}) + \Delta S_2(0,\ddot{p})] + R_1\Delta S_1^e(\ddot{q},\ddot{p})}{(\alpha_2-g_2)b_2}$$

Let $w = R_1[Z_1(\beta_1)-1] + \alpha_1[1-Z_1(\beta_1)] - h_1$, then we have:

$\Delta\pi_1(\ddot{q},\ddot{p},w,R_1,0)$

$$= -(\alpha_1-R_1)\left[\int_{n_1-a_1+b_1\ddot{p}}^{n_1-\ddot{q}-a_1+b_1\ddot{p}} Z_1(d_1)dd_1 + \ddot{q}Z_1(n_1-\ddot{q}-a_1+b_1\ddot{p})\right]$$

$$- \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1+b_1\ddot{p}} Z_1(d_1)dd_1 + b_1(p^{01}-\ddot{p})\right] - (\alpha_1-g_1)b_1(\ddot{p}-p^{01})$$

$\Delta\pi_1 = (\alpha_1-R_1)\Delta S_1^e(\ddot{q},\ddot{p}) + \alpha_1\Delta S_1(0,\ddot{p}) - (\alpha_1-g_1)b_1(\ddot{p}-p^{01})$

$\Delta\pi_2 = \Delta\prod - (\alpha_1-R_1)\Delta S_1^e(\ddot{q},\ddot{p}) - \alpha_1\Delta S_1^e(0,\ddot{p}) + (\alpha_1-g_1)b_1(\ddot{p}-p^{01})$

Now, we discuss and analyse the impact of $CT$ rate $\dot{p}$ on $LSC$ 1's profit increment.

If $p^e = \ddot{p} \geq p^{01}$, which means the equilibrium $CT$ rate $p^e$ is greater than $p^{01}$. Thus, to ensure $\Delta\pi_1$ is non-negative, $\ddot{p} - p^{01}$ must follow:

$$\ddot{p} - p^{01} \leq \frac{(\alpha_1-R_1)\Delta S_1^e(\ddot{q},\ddot{p}) + \alpha_1\Delta S_1(0,\ddot{p})}{(\alpha_1-g_1)b_1}$$

If $p^e = \ddot{p} \leq p^{01}$, which means the equilibrium $CT$ rate $p$ is less than $p^{01}$. Thus, to ensure $\Delta\pi_{1min}$ is non-negative, $p^{01} - \ddot{p}$ must follow:

$$p^{01} - \ddot{p} \geq -\frac{(\alpha_1-R_1)\Delta S_1^e(\ddot{q},\ddot{p}) + \alpha_1\Delta S_1(0,\ddot{p})}{(\alpha_1-g_1)b_1}$$

So, if $p^e = \ddot{p}$ satisfies the inequations above, the system can be coordinated.

**Case 9.** $\ddot{q} < -n_2 < 0, \ddot{p} < 0, [q^* = q^e = -n_2; p^* = p^e = 0;\ R_2 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases}\frac{\partial \pi_1}{\partial p} \le 0 \\ \frac{\partial \pi_2}{\partial p} \le 0\end{cases}\begin{cases}\frac{\partial \pi_1}{\partial q} \le 0 \\ \frac{\partial \pi_2}{\partial q} \le 0\end{cases} \rightarrow \begin{cases}-b_1\alpha_1 Z_1(\beta_1) + R_1 b_1[Z_1(\beta_1) - Z_1(\beta_1+q)] + g_1 b_1 \le 0 \\ -b_2\alpha_2 Z_2(\beta_2) - R_1 b_1[Z_1(\beta_1) - Z_1(\beta_1+q)] + g_2 b_2 \le 0 \\ \alpha_1 Z_1(\beta_1) - R_1 Z_1(\beta_1) + R_1 + w - (\alpha_1 - h_1) \le 0 \\ -\alpha_2 Z_2(\beta_2) + R_1 Z_1(\beta_1) - R_1 - w + c_t + \alpha_2 - h_2 \le 0\end{cases} \rightarrow$$

$$\begin{cases}\frac{-b_2\alpha_2 Z_2(\beta_2) + g_2 b_2}{b_1[Z_1(\beta_1) - Z_1(\beta_1+q)]} \le R_1 \le \frac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1+q)} \\ \alpha_2 \bar{Z}_2(\beta_2) + c_t - h_2 \le w + R_1 \bar{Z}_1(\beta_1) \le \alpha_1 \bar{Z}_1(\beta_1) - h_1\end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 + n_2 - a_1$ and $\beta_2 = -a_2$, So, **Condition 4.2** is met if $R_1$ and $w$ are constrained shown above. Also, $\Delta\prod(\ddot{q}, \ddot{p})$ and $\Delta\pi_1(-n_2, 0, w, R_1, 0)$ are:

$$\Delta\prod(-n_2, 0) = -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{n_1+n_2-a_1} Z_1(d_1)dd_1 + \alpha_2 \int_{-a_2}^{n_2-a_2+b_2p^{02}} Z_2(d_2)dd_2$$
$$+ (-n_2)(h_1 - \alpha_1 + \alpha_2 - h_2 + c_t) - g_1 b_1 p^{01} - g_2 b_2 p^{02}$$

$$\Delta\pi_1(-n_2, 0, w, R_1, 0)$$
$$= -\alpha_1 \int_{n_1-a_1+b_1p^{01}}^{n_1+n_2-a_1} Z_1(d_1)dd_1 + R_1\left[\int_{n_1-a_1}^{n_1+n_2-a_1} Z_1(d_1)dd_1 - n_2\right] + (-n_2)(w - \alpha_1 + h_1)$$
$$- g_1 b_1 p^{01}$$

According to mean value theorem, so we have:

$$\int_{n_1-a_1}^{n_1+n_2-a_1} Z_1(d_1)dd_1 - n_2 \le 0$$

Let $R_{1\min} = \frac{-b_2\alpha_2 Z_2(\beta_2) + g_2 b_2}{b_1[Z_1(\beta_1) - Z_1(\beta_1+q)]}$ and $w_{\min} = \alpha_2[1 - Z_2(\beta_2)] + R_1[Z_1(\beta_1) - 1] + c_t - h_2$. Then:

$$\Delta\pi_{2\min}(-n_2, 0, w, R_1, 0) = \Delta\prod - \Delta\pi_{1\max}(-n_2, 0, w, R_1, 0)$$
$$= -\alpha_2\left[\int_{n_2-a_2}^{-a_2} Z_2(d_2)dd_2 - (-n_2)Z_2(\beta_2)\right]$$
$$- R_{1\min}\left[\int_{n_1-a_1}^{n_1+n_2-a_1} Z_1(d_1)dd_1 + (-n_2)Z_1(\beta_1)\right] - \alpha_2\left[\int_{n_2-a_2+b_2p^{02}}^{n_2-a_2} Z_2(d_2)dd_2 + b_2p^{02}\right]$$
$$+ (\alpha_2 - g_2)b_2p^{02}$$

$$\Delta\pi_{2\min} = \alpha_2[\Delta S_2^e(-n_2, 0) + \Delta S_2(0,0)] + R_{1\min}\Delta S_1^e(-n_2, 0) + (\alpha_2 - g_2)b_2p^{02}$$
$$\Delta\pi_{1\max} = \Delta\prod - \alpha_2[\Delta S_2^e(-n_2, 0) + \Delta S_2(0,0)] - R_{1\min}\Delta S_1^e(-n_2, 0) - (\alpha_2 - g_2)b_2p^{02}$$

Now, we discuss and analyse the impact of $p^{02}$ on $LSC$ 2's profit increment.

If $\Delta\pi_{2min}$ get a non-negative profit increment, then $p^{02}$ must satisfy:

$$p^{02} \ge -\frac{\alpha_2[\Delta S_2^e(-n_2, 0) + \Delta S_2(0,0)] + R_{1\min}\Delta S_1^e(-n_2, 0)}{(\alpha_2 - g_2)b_2}$$

If we obtain $\Delta\pi_{1\min}$ and $\Delta\pi_{2\max}$, we let $R_{1\max} = \frac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1+q)}$ and $w_{\max} = \alpha_1[1 - Z_1(\beta_1)] + R_1[Z_1(\beta_1) - 1] - h_1$. Then:

$$\Delta\pi_{1\min}(-n_2, 0, w, R_1, 0)$$
$$= -(\alpha_1 - R_{1\max})\left[\int_{n_1-a_1}^{n_1+n_2-a_1} Z_1(d_1)dd_1 + (-n_2)Z_1(n_1 + n_2 - a_1)\right]$$
$$- \alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1} Z_1(d_1)dd_1 + b_1p^{01}\right] + (\alpha_1 - g_1)b_1p^{01}$$

$$\Delta\pi_{1\min} = (\alpha_1 - R_{1\max})\Delta S_1^e(-n_2, 0) + \alpha_1\Delta S_1(0,0) + (\alpha_1 - g_1)b_1p^{01}$$
$$\Delta\pi_{2\max} = \Delta\prod - (\alpha_1 - R_{1\max})\Delta S_1^e(-n_2, 0) - \alpha_1\Delta S_1(0,0) - (\alpha_1 - g_1)b_1p^{01}$$

Now, we discuss the impact of $p^{01}$ on $LSC$ 1's profit increment in this case.

If $\Delta\pi_{1\min}$ get a non-negative profit increment, then $p^{01}$ must satisfy:

$$p^{01} \geq -\frac{(\alpha_1 - R_{1\max})\Delta S_1^e(-n_2,0) + \alpha_1 \Delta S_1(0,0)}{(\alpha_1 - g_1)b_1}$$

So, if $p^{01}$ and $p^{02}$ satisfy the inequations above, the system is coordinated in this case.

**Case 10.** $\ddot{q} < -n_2, \ddot{p} > 0, [q^* = q^e = -n_2, p^* = p^e = \tilde{p}, R_2 = 0]$

In this case, the condition for voluntary compliance of the two $LSCs$ are:

$$\begin{cases} \dfrac{\partial \pi_1}{\partial p} = 0 \\ \dfrac{\partial \pi_2}{\partial p} = 0 \end{cases} \begin{cases} \dfrac{\partial \pi_1}{\partial q} \leq 0 \\ \dfrac{\partial \pi_2}{\partial q} \leq 0 \end{cases} \rightarrow \begin{cases} -b_1\alpha_1 Z_1(\beta_1) + R_1 b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)] + g_1 b_1 = 0 \\ -b_2\alpha_2 Z_2(\beta_2) - R_1 b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)] + g_2 b_2 = 0 \\ \alpha_1 Z_1(\beta_1) - R_1 Z_1(\beta_1) + R_1 + w - (\alpha_1 - h_1) \leq 0 \\ -\alpha_2 Z_2(\beta_2) + R_1 Z_1(\beta_1) - R_1 - w + c_t + \alpha_2 - h_2 \leq 0 \end{cases} \rightarrow$$

$$\begin{cases} R_1 = \dfrac{-b_2\alpha_2 Z_2(\beta_2) + g_2 b_2}{b_1[Z_1(\beta_1) - Z_1(\beta_1 + q)]} = \dfrac{\alpha_1 Z_1(\beta_1) - g_1}{Z_1(\beta_1) - Z_1(\beta_1 + q)} \\ \alpha_2 \bar{Z}_2(\beta_2) - h_2 + c_t \leq w + R_1 \bar{Z}_1(\beta_1) \leq \alpha_1 \bar{Z}_1(\beta_1) - h_1 \end{cases}$$

Where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$, $\beta_1 = n_1 + n_2 - a_1 + b_1\tilde{p}$ and $\beta_2 = -a_2 + b_2\tilde{p}$. Also, $\Delta\prod(\ddot{q}, \ddot{p})$ and $\Delta\pi_1(-n_2, 0, w, R_1, 0)$ are:

$$\Delta\prod(-n_2, \tilde{p}) = -\alpha_1 \int_{n_1 - a_1 + b_1 p^{01}}^{n_1 + n_2 - a_1 + b_1\tilde{p}} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2 - a_2 + b_2 p^{02}}^{-a_2 + b_2\tilde{p}} Z_2(d_2)dd_2$$

$$+ (-n_2)(h_1 - \alpha_1 + \alpha_2 - h_2 + c_t) + g_1 b_1(\tilde{p} - p^{01}) + g_2 b_2(\tilde{p} - p^{02})$$

$\Delta\pi_1(-n_2, \tilde{p}, w, R_1, 0)$

$$= -\alpha_1 \int_{n_1 - a_1 + b_1 p^{01}}^{n_1 + n_2 - a_1 + b_1\tilde{p}} Z_1(d_1)dd_1 + R_1 \int_{n_1 - a_1 + b_1\tilde{p}}^{n_1 + n_2 - a_1 + b_1\tilde{p}} Z_1(d_1)dd_1$$

$$+ (-n_2)(w + R_1 - \alpha_1 + h_1) + g_1 b_1(\tilde{p} - p^{01})$$

Let $w_{\min} = R_1[Z_1(\beta_1) - 1] + c_t + \alpha_2[1 - Z_2(\beta_2)] - h_2$, then we have:

$\Delta\pi_{2\min}(-n_2, \tilde{p}, w, R_1, 0) = \Delta\prod - \Delta\pi_{1\max}(-n_2, \tilde{p}, w, R_1, 0)$

$$= -\alpha_2 \left[\int_{n_2 - a_2 + b_2\tilde{p}}^{-a_2 + b_2\tilde{p}} Z_2(d_2)dd_2 - (-n_2)\alpha_2 Z_2(-a_2 + b_2\tilde{p})\right]$$

$$- R_1 \left[\int_{n_1 - a_1 + b_1\tilde{p}}^{n_1 + n_2 - a_1 + b_1\tilde{p}} Z_1(d_1)dd_1 + (-n_2)Z_1(n_1 + n_2 - a_1 + b_1\tilde{p})\right]$$

$$- \alpha_2 \left[\int_{n_2 - a_2 + b_2 p^{02}}^{n_2 - a_2 + b_2\tilde{p}} Z_2(d_2)dd_2 + b_2(p^{02} - \tilde{p})\right] - (\alpha_2 - g_2)b_2(\tilde{p} - p^{02})$$

$\Delta\pi_{2\min} = \alpha_2[\Delta S_2(0, \tilde{p}) + \Delta S_2^e(-n_2, \tilde{p})] + R_1 \Delta S_1^e(-n_2, \tilde{p}) - (\alpha_2 - g_2)b_2(\tilde{p} - p^{02})$

$\Delta\pi_{1\max} = \Delta\prod - \alpha_2[\Delta S_2(0, \tilde{p}) + \Delta S_2^e(-n_2, \tilde{p})] - R_1 \Delta S_1^e(-n_2, \tilde{p}) + (\alpha_2 - g_2)b_2(\tilde{p} - p^{02})$

Now, we discuss and analyse the impact of $CT$ rate $\tilde{p}$ on $LSC$ 2's profit increment.

If $p^e = \tilde{p} \geq p^{02}$, which means equilibrium $CT$ rate is greater than $p^{02}$. Thus, to ensure $\Delta\pi_{2\min}$ is non-negative, $\tilde{p} - p^{02}$ must follow:

$$\tilde{p} - p^{02} \leq \frac{\alpha_2[\Delta S_2(0, \tilde{p}) + \Delta S_2^e(-n_2, \tilde{p})] + R_1 \Delta S_1^e(-n_2, \tilde{p})}{(\alpha_2 - g_2)b_2}$$

If $p^e = \tilde{p} < p^{02}$, which means equilibrium $CT$ rate is less than $p^{02}$. Thus, to ensure $\Delta\pi_{2\min}$ is non-negative, $p^{02}-\tilde{p}$ must be follow:

$$p^{02}-\tilde{p} \geq -\frac{\alpha_2[\Delta S_2(0,\tilde{p})+\Delta S_2^e(-n_2,\tilde{p})]+R_1\Delta S_1^e(-n_2,\tilde{p})}{(\alpha_2-g_2)b_2}$$

Let $w_{\max} = \alpha_1[1-Z_1(\beta_1)]+R_1[Z_1(\beta_1)-1]-h_1$. We have:

$\Delta\pi_{1\min}(-n_2,\tilde{p},w,R_1,0)$

$$= -(\alpha_1-R_1)\left[\int_{n_1-a_1+b_1\tilde{p}}^{n_1+n_2-a_1+b_1\tilde{p}} Z_1(d_1)dd_1+(-n_2)Z_1(n_1+n_2-a_1+b_1\tilde{p})\right]$$

$$-\alpha_1\left[\int_{n_1-a_1+b_1p^{01}}^{n_1-a_1+b_1\tilde{p}} Z_1(d_1)dd_1+b_1(p^{01}-\tilde{p})\right]-(\alpha_1-g_1)b_1(\tilde{p}-p^{01})$$

$\Delta\pi_{1\min}=(\alpha_1-R_1)\Delta S_1^e(-n_2,\tilde{p})+\alpha_1\Delta S_1(0,\tilde{p})-(\alpha_1-g_1)b_1(\tilde{p}-p^{01})$

$\Delta\pi_{2\max}=\Delta\prod-(\alpha_1-R_1)\Delta S_1^e(-n_2,\tilde{p})-\alpha_1\Delta S_1(0,\tilde{p})+(\alpha_1-g_1)b_1(\tilde{p}-p^{01})$

We now discuss the impact of $CT$ rate $\tilde{p}$ on $LSC$ 1's profit increment.

If $p^e=\tilde{p}\geq p^{01}$, which means the equilibrium $CT$ rate $p^e$ is greater than $p^{01}$. Thus, to ensure $\Delta\pi_{1\min}$ is non-negative, $\tilde{p}-p^{01}$ must follow:

$$\tilde{p}-p^{01} \leq \frac{(\alpha_1-R_1)\Delta S_1^e(-n_2,\tilde{p})+\alpha_1\Delta S_1(0,\tilde{p})}{(\alpha_1-g_1)b_1}$$

If $p^e=\tilde{p}\leq p^{01}$, which means the government $CT$ rate $p^e$ is less than $p^{01}$. Thus, to ensure $\Delta\pi_{1\min}$ is non-negative, $p^{01}-\tilde{p}$ must follow:

$$p^{01}-\tilde{p} \geq -\frac{(\alpha_1-R_1)\Delta S_1^e(-n_2,\tilde{p})+\alpha_1\Delta S_1(0,\tilde{p})}{(\alpha_1-g_1)b_1}$$

So, if $\tilde{p}$ satisfy the inequations above, the system can be coordinated in this case.

**Appendix P**

Government's social welfare function are:

$$\begin{aligned}
\prod_{gov}(q,p) &= \pi_1(q,p)+\pi_2(q,p)-c_t|q|-C_g[\boldsymbol{E}S_1(q,p)+\boldsymbol{E}S_2(q,p)]\\
&= r_1\boldsymbol{E}S_1(q,p)-h_1\boldsymbol{E}I_1(q,p)-g_1\boldsymbol{E}L_1(q,p)+r_2\boldsymbol{E}S_2(q,p)-h_2\boldsymbol{E}I_2(q,p)-g_2\boldsymbol{E}L_2(q,p)\\
&\quad -c_t|q|-C_g[\boldsymbol{E}S_1(q,p)+\boldsymbol{E}S_2(q,p)]
\end{aligned}$$

$$\begin{aligned}
&=(\alpha_1-C_g)\boldsymbol{E}min\{\xi_1-Y_1,\beta_1\}+(\alpha_2-C_g)\boldsymbol{E}min\{\xi_2-Y_2,\beta_2\}-h_1\beta_1-h_2\beta_2-g_1\boldsymbol{E}(\xi_1-Y_1)\\
&\quad -g_2\boldsymbol{E}(\xi_2-Y_2)+(r_1-C_g)\boldsymbol{E}(Y_1+a_1-b_1p)+(r_2-C_g)\boldsymbol{E}(Y_2+a_2-b_2p)-c_t|q|\\
&=(\alpha_1-C_g)\left\{\beta_1[1-Z_1(\beta_1)]+\int_0^{\beta_1}(\xi_1-Y_1)\,z_1(\xi_1-Y_1)d(\xi_1-Y_1)\right\}\\
&\quad +(\alpha_2-C_g)\left\{\beta_2[1-Z_2(\beta_2)]+\int_0^{\beta_2}(\xi_2-Y_2)\,z_2(\xi_2-Y_2)d(\xi_2-Y_2)\right\}-h_1\beta_1-h_2\beta_2\\
&\quad -g_1\boldsymbol{E}(\xi_1-Y_1)-g_2\boldsymbol{E}(\xi_2-Y_2)+(r_1-C_g)\boldsymbol{E}(Y_1+a_1-b_1p)+(r_2\\
&\quad -C_g)\boldsymbol{E}(Y_2+a_2-b_2p)-c_t|q|\\
&=(\alpha_1-C_g)\left[\beta_1-\int_0^{\beta_1}Z_1\,(\xi_1-Y_1)d(\xi_1-Y_1)\right]+(\alpha_2-C_g)\left[\beta_2-\int_0^{\beta_2}Z_2\,(\xi_2-Y_2)d(\xi_2-Y_2)\right]-h_1\beta_1\\
&\quad -h_2\beta_2-g_1\boldsymbol{E}(\xi_1-Y_1)-g_2\boldsymbol{E}(\xi_2-Y_2)+(r_1-C_g)\boldsymbol{E}(Y_1+a_1-b_1p)\\
&\quad +(r_2-C_g)\boldsymbol{E}(Y_2+a_2-b_2p)-c_t|q|\\
&=(\alpha_1-C_g)[\beta_1-\Phi_1(\beta_1)]+(\alpha_2-C_g)[\beta_2-\Phi_2(\beta_2)]-h_1\beta_1-h_2\beta_2-g_1\boldsymbol{E}(\xi_1-Y_1)-g_2\boldsymbol{E}(\xi_2-Y_2)\\
&\quad +(r_1-C_g)\boldsymbol{E}(Y_1+a_1-b_1p)+(r_2-C_g)\boldsymbol{E}(Y_2+a_2-b_2p)-c_t|q|
\end{aligned}$$

## Appendix Q

Given $p$, to obtain $q^*(p)$ achieving:

$$Max \prod_{cen}(q^*, p); \ \forall \ p$$

We should prove that $\prod_{cen}(q, p)$ is strictly concave in $q$ first.

$$\frac{\partial \prod_{cen}(q, p)}{\partial q} = -\alpha_1[1 - Z_1(\beta_1)] + \alpha_2[1 - Z_2(\beta_2)] + h_1 - h_2 - c_t sgn(q)$$

$$\frac{\partial^2 \prod_{cen}(q, p)}{\partial q^2} = -\alpha_1 z_1(\beta_1) - \alpha_2 z_2(\beta_2) < 0$$

$\prod_{cen}(q, p)$ is not differentiable at $q_{cen} = 0$. For the other differentiable segments, it is strictly concave in $q$, denote the optimal $ECS$ number is $q^*(p)$, where $\beta_1 = n_1 - q^* - a_1 + b_1 p$; $\beta_2 = n_2 + q^* - a_2 + b_2 p$. Due to $q^*(p) \in [-n_2, n_1]$, the result is divided into five cases. Denote $q^*(p) = \dot{q}^*(p)$ when $0 < q(p) < n_1$ and it satisfies:

$$\frac{\partial \prod_{cen}(\dot{q}^*, p)}{\partial q} = 0$$

Also, denote $q^* = \ddot{q}^*$ when $-n_2 < q(p) < 0$ and it satisfies:

$$\frac{\partial \prod_{cen}(\ddot{q}^*, p)}{\partial q} = 0$$

1. $0 < n_1 < \dot{q}^*(p)$; In this case, $q^*(p) = n_1$, and it satisfies:

$$\frac{\partial \prod_{cen}(n_1, p)}{\partial q} > 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) > (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$$

$\beta_1 = -a_1 + b_1 p$; $\beta_2 = n_2 + n_1 - a_2 + b_2 p$ in this case.

2. $0 < \dot{q}^*(p) < n_1$; In this case, $q^*(p) = \dot{q}^*(p)$

$$\frac{\partial \prod_{cen}(\dot{q}^*, p)}{\partial q} = 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) = (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$$

$\beta_1 = n_1 - \dot{q}^* - a_1 + b_1 p$; $\beta_2 = n_2 + \dot{q}^* - a_2 + b_2 p$ in this case.

3. $\dot{q}^*(p) < 0 < \ddot{q}^*(p)$; In this case, $q^*(p) = 0$

$$\frac{\partial \prod_{cen}(\dot{q}^*, p)}{\partial q} < 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) < (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$$

$\beta_1 = n_1 - \dot{q}^* - a_1 + b_1 p$; $\beta_2 = n_2 + \dot{q}^* - a_2 + b_2 p$ in this case.

$$\frac{\partial \prod_{cen}(\ddot{q}^*, p)}{\partial q} > 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) > (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$$

$\beta_1 = n_1 - \ddot{q}^* - a_1 + b_1 p$; $\beta_2 = n_2 + \ddot{q}^* - a_2 + b_2 p$ in this case.

4. $-n_2 < \ddot{q}^*(p) < 0$; In this case, $q^*(p) = \ddot{q}^*(p)$

$$\frac{\partial \prod_{cen}(q^*, p)}{\partial q} = 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) = (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$$

$\beta_1 = n_1 - \ddot{q}^* - a_1 + b_1 p$; $\beta_2 = n_2 + \ddot{q}^* - a_2 + b_2 p$ in this case.

5. $\ddot{q}^*(p) < -n_2 < 0$; In this case, $q^*(p) = -n_2$

$$\frac{\partial \prod_{cen}(-n_2, p)}{\partial q} < 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) < (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$$

$\beta_1 = n_1 + n_2 - a_1 + b_1 p$; $\beta_2 = -a_2 + b_2 p$ in this case.

## Appendix R

Given $q^*(p)$, to obtain:

$$Max \prod_{gov}(q^*, p)$$

We should prove that $\prod_{gov}(q^*_{cen}, p)$ is strictly concave in $p$, then:

$$\frac{\partial \prod_{gov}(q^*, p)}{\partial p} = b_1(\alpha_1 - C_g)[1 - Z_1(\beta_1)] + b_2(\alpha_2 - C_g)[1 - Z_2(\beta_2)] - h_1 b_1 - h_2 b_2 - (r_1 - C_g)b_1 - (r_2 - C_g)b_2$$

$$\frac{\partial^2 \prod_{gov}(q^*, p)}{\partial p^2} = -(\alpha_1 - C_g)b_1^2 z_1(\beta_1) - (\alpha_2 - C_g)b_2^2 z_2(\beta_2) < 0$$

Where$\beta_1 = n_1 - q^* - a_1 + b_1 p$; $\beta_2 = n_2 + q^* - a_2 + b_2 p$. $\frac{\partial^2 \prod_{gov}(q^*, p)}{\partial p^2}$ is always negative as long as $\alpha_i \geq C_g$ $(i = 1,2)$. So, $\prod_{gov}(q^*, p)$ strictly concave in $p$, given $q^*$ and $p_s^e$ must satisfy:

$$\frac{\partial \prod_{gov}(q^*, p_s^e)}{\partial p} = 0 \rightarrow b_1 g_1 + b_2 g_2 - b_1(\alpha_1 - C_g)Z_1(\beta_1) - b_2(\alpha_2 - C_g)Z_2(\beta_2) = 0 \rightarrow$$

$$\frac{b_1}{b_2} = -\frac{g_2 - (\alpha_2 - C_g)Z_2(n_1 - q^* - a_1 + b_1 p_s^e)}{g_1 - (\alpha_1 - C_g)Z_1(n_2 + q^* - a_2 + b_2 p_s^e)}$$

**Appendix S**

Given $p_s^e$, to obtain $q_s^e$, which means:

$$Max \prod_{cen}(q, p_s^e)$$

Firstly, denote $q_s^e(p_s^e) = \dot{q}_s^e(p_s^e)$ when $q > 0$ and it satisfies:

$$\frac{\partial \prod_{cen}(\dot{q}_s^e, p_s^e)}{\partial q} = 0$$

Also, denote $q_s^e(p_s^e) = \ddot{q}_s^e(p_s^e)$ when $q < 0$ and it satisfies:

$$\frac{\partial \prod_{cen}(\ddot{q}_s^e, p_s^e)}{\partial q} = 0$$

As $q_s^e(p_s^e) \in [-n_2, n_1]$, thus, the $q_s^e(p_s^e)$ should be discussed in five cases.

1. $0 < n_1 < \dot{q}_s^e(p_s^e)$, $q_s^e(p_s^e) = n_1$, and it satisfies:

$$\frac{\partial \prod_{cen}(n_1, p_s^e)}{\partial q} > 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) > (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$$

$\beta_1 = -a_1 + b_1 p_s^e$; $\beta_2 = n_2 + n_1 - a_2 + b_2 p_s^e$.

2. $0 < \dot{q}_s^e(p_s^e) < n_1$, $q_s^e(p_s^e) = \dot{q}_s^e$, and it satisfies:

$$\frac{\partial \prod_{cen}(\dot{q}_s^e, p_s^e)}{\partial q} = 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) = (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$$

$\beta_1 = n_1 - \dot{q}_s^e - a_1 + b_1 p_s^e$; $\beta_2 = n_2 + \dot{q}_s^e - a_2 + b_2 p_s^e$.

3. $\dot{q}_s^e(p_s^e) < 0 < \ddot{q}_s^e(p_s^e)$, $q_s^e(p_s^e) = 0$ and it satisfies:

$$\frac{\partial \prod_{cen}(\dot{q}_s^e, p_s^e)}{\partial q} < 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) < (\alpha_1 - h_1) - (\alpha_2 - h_2) + c_t$$

$\beta_1 = n_1 - \dot{q}_s^e - a_1 + b_1 p_s^e$; $\beta_2 = n_2 + \dot{q}_s^e - a_2 + b_2 p_s^e$.

$$\frac{\partial \prod_{cen}(\ddot{q}_s^e, p_s^e)}{\partial q} > 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) > (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$$

$\beta_1 = n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e$; $\beta_2 = n_2 + \ddot{q}_s^e - a_2 + b_2 p_s^e$.

4. $-n_2 < \ddot{q}_s^e(p_s^e) < 0$, $q_s^e(p_s^e) = \ddot{q}_s^e$, and it satisfies:

$$\frac{\partial \prod_{cen}(\ddot{q}_s^e, p_s^e)}{\partial q} = 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) = (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$$

$$\beta_1 = n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e;\ \beta_2 = n_2 + \ddot{q}_s^e - a_2 + b_2 p_s^e$$

5. $\ddot{q}_s^e(p_s^e) < -n_2 < 0$, $q_s^e(p_s^e) = -n_2$, and it satisfies:

$$\frac{\partial \prod_{cen}(-n_2, p_s^e)}{\partial q} < 0 \leftrightarrow \alpha_1 Z_1(\beta_1) - \alpha_2 Z_2(\beta_2) < (\alpha_1 - h_1) - (\alpha_2 - h_2) - c_t$$

$$\beta_1 = n_1 + n_2 - a_1 + b_1 p_s^e;\ \beta_2 = -a_2 + b_2 p_s^e.$$

**Appendix T**

$$\frac{\partial \pi_1(q, p_s^e, w, R_1, R_2)}{\partial q}$$
$$= \alpha_1 Z_1(n_1 - q - a_1 + b_1 p_s^e) - R_2 Z_2(n_2 + q - a_2 + b_2 p_s^e) - R_1 Z_1(n_1 - q - a_1 + b_1 p_s^e) + (R_2 + w - c_t)|_{q>0} + (R_1 + w)|_{q<0} - (\alpha_1 - h_1)$$

$$\alpha_1 Z_1(n_1 - q - a_1 + b_1 p_s^e) - R_2 Z_2(n_2 + q - a_2 + b_2 p_s^e) + R_2 Z_2(\beta_2) - \alpha_1 Z_1(\beta_1)$$

$$\frac{\partial^2 \pi_1(q, p_s^e, w, R_1, R_2)}{\partial q^2} = -(\alpha_1 - R_1) z_1(n_1 - q - a_1 + b_1 p_s^e) - R_2 z_2(n_2 + q - a_2 + b_2 p_s^e)$$

$$\frac{\partial \pi_2(q, p_s^e, w, R_1, R_2)}{\partial q}$$
$$= -\alpha_2 Z_2(n_2 + q - a_2 + b_2 p_s^e) + R_2 Z_2(n_2 + q - a_2 + b_2 p_s^e) + R_1 Z_1(n_1 - q - a_1 + b_1 p_s^e) - (w + R_2)|_{q>0} + (w - c_t + R_1)|_{q<0} + \alpha_2 - h_2$$

$$\frac{\partial^2 \pi_2(q, p_s^e, w, R_1, R_2)}{\partial q^2} = -(\alpha_2 - R_2) Z_2(n_2 + q - a_2 + b_2 p_s^e) - R_1 Z_1(n_1 - q - a_1 + b_1 p_s^e)$$

Thus, $\frac{\partial^2 \pi_1(q, p_s^e, w, R_1, R_2)}{\partial q^2}$ and $\frac{\partial^2 \pi_2(q, p_s^e, w, R_1, R_2)}{\partial q^2}$ is always negative as long as $\alpha_i \geq R_i$. $\pi_1(q, p_s^e, w, R_1, R_2)$ strictly concave in $q$, given $p_s^e$. Similarly, $\pi_2(q, p_s^e, w, R_1, R_2)$ also strictly concave in $q$, given $p_s^e$.

**Appendix U**

Now, two $LSCs$ adopt the equilibrium $ECS$ number $q_d^e$ and the $p$ reaches $p_s^e$. Next, whether the two $LSCs$ are coordinated by applying the $RSC$ is discussed. Firstly, the centralised model profit increment is obtained.

$$\Delta \prod_{cen}(q_d^e, p_s^e) = \prod_{cen}(q^e, p_s^e) - \prod_{cen}(0,0)$$
$$= \alpha_1 \left[ (n_1 - q_d^e - a_1 + b_1 p_s^e) - \int_0^{n_1 - q_d^e - a_1 + b_1 p_s^e} Z_1(d_1) dd_1 \right]$$
$$+ \alpha_2 \left[ (n_2 + q_d^e - a_2 + b_2 p_s^e) - \int_0^{n_2 + q_d^e - a_2 + b_2 p_s^e} Z_2(d_2) dd_2 \right] - h_1(n_1 - q_d^e - a_1 + b_1 p_s^e)$$
$$- h_2(n_2 + q_d^e - a_2 + b_2 p_s^e) - g_1 \boldsymbol{E}(\xi_1 - Y_1) - g_2 \boldsymbol{E}(\xi_2 - Y_2) + r_1 \boldsymbol{E}(Y_1 + a_1 - b_1 p_s^e)$$
$$+ r_2 \boldsymbol{E}(Y_2 + a_2 - b_2 p_s^e) - c_t |q_d^e|$$
$$- \left\{ \alpha_1 \left[ n_1 - a_1 - \int_0^{n_1 - a_1} Z_1(d_1) dd_1 \right] + \alpha_2 \left[ n_2 - a_2 - \int_0^{n_2 - a_2} Z_2(d_2) dd_2 \right] - h_1(n_1 - a_1) \right.$$
$$\left. - h_2(n_2 - a_2) - g_1 \boldsymbol{E}(\xi_1 - Y_1) - g_2 \boldsymbol{E}(\xi_2 - Y_2) + r_1 \boldsymbol{E}(Y_1 + a_1) + r_2 \boldsymbol{E}(Y_2 + a_2) \right\}$$
$$= -\alpha_1 \int_{n_1 - a_1}^{n_1 - q_d^e - a_1 + b_1 p_s^e} Z_1(d_1) dd_1 - \alpha_2 \int_{n_2 - a_2}^{n_2 + q_d^e - a_2 + b_2 p_s^e} Z_2(d_2) dd_2 + (\alpha_1 - h_1)(-q_d^e)$$
$$+ (\alpha_2 - h_2) q_d^e + g_1 b_1 p_s^e + g_2 b_2 p_s^e - c_t |q_d^e|$$

The $LSC$ 1's profit increment is:

$$\Delta\pi_1(q_d^e, p_s^e, R_1, R_2, w) = \pi_1(q_d^e, p_s^e, R_1, R_2, w) - \pi_1(0,0, R_1, R_2, w)$$

$$= -\alpha_1 \int_0^{n_1-q_d^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 + R_2 \int_{n_2+q_d^e-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e} Z_2(d_2)dd_2$$

$$- R_1 \int_{n_1-q_d^e-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1(d_1)dd_1 - (-q_d^e)^+(R_1+w) + (q_d^e)^+(R_2+w-c_t)$$

$$+ r_1\boldsymbol{E}(Y_1+a_1-b_1p_s^e) + (\alpha_1-h_1)(n_1-q_d^e-a_1+b_1p_s^e) - \boldsymbol{E}(\xi_1-Y_1)$$

$$-\left\{-\alpha_1 \int_0^{n_1-a_1} Z_1(d_1)dd_1 + r_1\boldsymbol{E}(Y_1+a_1) + (\alpha_1-h_1)(n_1-a_1) - \boldsymbol{E}(\xi_1-Y_1)\right\}$$

$$= -\alpha_1 \int_{n_1-a_1}^{n_1-q_d^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 + R_2 \int_{n_2+q_d^e-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e} Z_2(d_2)dd_2$$

$$- R_1 \int_{n_1-q_d^e-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1(d_1)dd_1 - (R_1+w)(-q_d^e)^+ + (R_2+w-c_t)(q_d^e)^+ + g_1b_1p_s^e$$

$$+ (\alpha_1-h_1)(-q_d^e)$$

Notice that $\emptyset_1 = 1$ when $q_d^e > 0$ (which means $R_1 = 0$), similarly, $\emptyset_2 = 1$ when $q_d^e < 0$ (which means $R_2 = 0$). Then, the marginal profit of $LSC$ 1 and $LSC$ 2 are:

$$\frac{\partial\pi_1(q_d^e, p_s^e)}{\partial q} = \begin{cases} \alpha_1Z_1(\beta_1) - R_2Z_2(\beta_2) + R_2 + w - c_t - (\alpha_1-h_1) & q_d^e > 0 \\ \alpha_1Z_1(\beta_1) - R_1Z_1(\beta_1) + R_1 + w - (\alpha_1-h_1) & q_d^e > 0 \end{cases}$$

$$\frac{\partial\pi_2(q_d^e, p_s^e)}{\partial q} \begin{cases} -\alpha_2Z_2(\beta_2) + R_2Z_2(\beta_2) - (w+R_2) + (\alpha_2-h_2) & q_d^e > 0 \\ -\alpha_2Z_2(\beta_2) + R_1Z_1(\beta_1) - (w-c_t+R_1) + (\alpha_2-h_2) & q_d^e > 0 \end{cases}$$

We discuss the system coordination in five cases shown in **Theorem 6.1**.

**Case 1.** $q_d^e = n_1;\ p_s^e;\ R_1 = 0;\ \beta_1 = -a_1 + b_1p_s^e, \beta_2 = n_2 + n_1 - a_2 + b_2p_s^e$

In this case, the system and $LSC$ 1's profit increment in the centralised model are:

$$\Delta\prod_{cen}(n_1, p_s^e) = -\alpha_1 \int_{n_1-a_1}^{-a_1+b_1p_s^e} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2}^{n_2+n_1-a_2+b_2p_s^e} Z_2(d_2)dd_2$$

$$+ (\alpha_2 - h_2 - \alpha_1 + h_1 - c_t)n_1 + g_1b_1p_s^e + g_2b_2p_s^e$$

$$\Delta\pi_1(n_1, p_s^e, 0, R_2, w)$$

$$= -\alpha_1 \int_{n_1-a_1}^{-a_1+b_1p_s^e} Z_1(d_1)dd_1 + R_2 \int_{n_2+n_1-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e} Z_2(d_2)dd_2$$

$$+ (R_2 + w - c_t - \alpha_1 + h_1)n_1 + g_1b_1p_s^e$$

In this case, $\prod_{cen}(q, p_s^e)$ satisfies:

$$\frac{\partial\prod_{cen}(q, p_s^e)}{\partial q} \geq 0$$

So, to coordinate the system, the conditions of two $LSCs$' profit functions should satisfy:

$$\frac{\partial\pi_1}{\partial q} = \alpha_1Z_1(-a_1+b_1p_s^e) - R_2Z_2(n_2+n_1-a_2+b_2p_s^e) + R_2 + w - c_t - (\alpha_1-h_1) \geq 0$$

$$\frac{\partial\pi_2}{\partial q} = R_2Z_2(n_2+n_1-a_2+b_2p_s^e) - \alpha_2Z_2(n_2+n_1-a_2+b_2p_s^e) - (w+R_2) + (\alpha_2-h_2) \geq 0$$

So, we obtain $\alpha_1\bar{Z}_1(\beta_1)+c_t-h_1\le w+\bar{Z}_2(\beta_2)\le\alpha_2\bar{Z}_2(\beta_2)-h_2$, where $\bar{Z}_i(\beta_\mathrm{i})=1-Z_\mathrm{i}(\beta_\mathrm{i})$. As $\Delta\pi_1$ increases with $w$, so, let $w_{max}=(\alpha_2-R_2)\bar{Z}_2(\beta_2)-h_2$, then:

$$\Delta\pi_1(n_1,p_s^e,0,R_2,w)$$

$$=-\alpha_1\int_{n_1-a_1}^{-a_1+b_1p_s^e}Z_1\,(d_1)dd_1$$

$$-R_2\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+n_1-a_2+b_2p_s^e}Z_2\,(d_2)dd_2-n_1Z_2(n_2+n_1-a_2+b_2p_s^e)\right]$$

$$+\{-\alpha_2Z_2(n_2+n_1-a_2+b_2p_s^e)+\alpha_2-h_2-\alpha_1+h_1-c_t\}n_1+g_1b_1p_s^e$$

According to mean value theorem, we know

$$-R_2\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+n_1-a_2+b_2p_s^e}Z_2\,(d_2)dd_2-n_1Z_2(n_2+n_1-a_2+b_2p_s^e)\right]>0$$

So, $\Delta\pi_1$ increases with $R_2$, then:

$$\Delta\pi_1(n_1,p_s^e,0,R_2,w)$$

$$=-\alpha_1\int_{n_1-a_1}^{-a_1+b_1p_s^e}Z_1\,(d_1)dd_1+R_2\int_{n_2+n_1-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e}Z_2\,(d_2)dd_2$$

$$+\{-(\alpha_2-R_2)Z_2(n_2+n_1-a_2+b_2p_s^e)+\alpha_2-h_2-\alpha_1+h_1-c_t\}n_1+g_1b_1p_s^e$$

$$\Delta\pi_2(n_1,p_s^e,0,R_2,w)=\Delta\prod\nolimits_{cen}(n_1,p_s^e)-\Delta\pi_1(n_1,p_s^e,0,R_2,w)$$

$$=-\alpha_2\int_{n_2-a_2}^{n_2+n_1-a_2+b_2p_s^e}Z_2\,(d_2)dd_2+g_2b_2p_s^e$$

$$-\left[R_2\int_{n_2+n_1-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e}Z_2\,(d_2)dd_2\;-n_1(\alpha_2-R_2)Z_2(n_2+n_1-a_2+b_2p_s^e)\right]$$

$$=-(\alpha_2-R_2)\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+n_1-a_2+b_2p_s^e}Z_2\,(d_2)dd_2-n_1Z_2(n_2+n_1-a_2+b_2p_s^e)\right]$$

$$-\alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p_s^e}Z_2\,(d_2)dd_2-b_2p_s^e\right]-(\alpha_2-g_2)b_2p_s^e$$

$$\Delta\pi_2(n_1,p_s^e,0,R_2,w)=(\alpha_2-R_2)\Delta S_2^e(n_1,p_s^e)+\alpha_2\Delta S_2(0,p_s^e)-(\alpha_2-g_2)b_2p_s^e$$

However, if we let $w_{\mathrm{min}}=-R_2\bar{Z}_2(\beta_2)+\alpha_1\bar{Z}_1(\beta_1)+c_t-h_1$, then:

$$\Delta\pi_1(n_1,p_s^e,0,R_2,w)$$

$$=-\alpha_1\int_{n_1-a_1}^{-a_1+b_1p_s^e}Z_1\,(d_1)dd_1+R_2\int_{n_2+n_1-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e}Z_2\,(d_2)dd_2$$

$$+[R_2Z_2(n_2+n_1-a_2+b_2p_s^e)-\alpha_1Z_1(-a_1+b_1p_s^e)]n_1+g_1b_1p_s^e$$

$\Delta\pi_1(n_1, p_s^e, 0, R_2, w)$

$$= -\alpha_1 \left[ \int_{n_1-a_1+b_1p_s^e}^{-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + n_1 Z_1(-a_1 + b_1p_s^e) \right]$$

$$- R_2 \left[ \int_{n_2-a_2+b_2p_s^e}^{n_2+n_1-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - n_1 Z_2(n_2 + n_1 - a_2 + b_2p_s^e) \right]$$

$$- \alpha_1 \left[ \int_{n_1-a_1}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - b_1p_s^e \right] - (\alpha_1 - g_1)b_1p_s^e$$

$\Delta\pi_1(n_1, p_s^e, 0, R_2, w) = \alpha_1[\Delta S_1^e(n_1, p_s^e) + \Delta S_1(0, p_s^e)] + R_2\Delta S_2^e(n_1, p_s^e) - (\alpha_1 - g_1)b_1p_s^e$

**Case 2.** $q_d^e = \dot{q}_s^e;\ p_s^e;\ R_1 = 0;\ \beta_1 = -a_1 + b_1p_s^e, \beta_2 = n_2 + n_1 - a_2 + b_2p_s^e$

In this case, the system and $LSC$ 1's profit increment in the centralised model are:

$$\Delta\prod\nolimits_{cen}(\dot{q}_s^e, p_s^e) = -\alpha_1 \int_{n_1-a_1}^{n_1-\dot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - \alpha_2 \int_{n_2-a_2}^{n_2+\dot{q}_s^e-a_2+b_2p_s^e} Z_2\,(d_2)dd_2$$

$$+ (\alpha_2 - h_2 - \alpha_1 + h_1 - c_t)\dot{q}_s^e + g_1b_1p_s^e + g_2b_2p_s^e$$

$\Delta\pi_1(\dot{q}_s^e, p_s^e, 0, R_2, w)$

$$= -\alpha_1 \int_{n_1-a_1}^{n_1-\dot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + R_2 \int_{n_2+\dot{q}_s^e-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2$$

$$+ (R_2 + w - c_t - \alpha_1 + h_1)\dot{q}_s^e + g_1b_1p_s^e$$

Also, in this case, $\prod_{cen}(q, p_s^e)$ satisfies:

$$\frac{\partial\prod_{cen}(q, p_s^e)}{\partial q} = 0$$

So, to coordinate the system, the conditions of two $LSCs$' profit functions should satisfy:

$$\frac{\partial \pi_1}{\partial q} = \alpha_1 Z_1(n_1 - \dot{q}_s^e - a_1 + b_1p_s^e) - R_2 Z_2\big(n_2 + \dot{q}_s^{e\,e} - a_2 + b_2p_s^e\big) + R_2 + w - c_t - (\alpha_1 - h_1) = 0$$

$$\frac{\partial \pi_2}{\partial q} = -\alpha_2 Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e) + R_2 Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e) - (w + R_2) + (\alpha_2 - h_2) = 0$$

So, we obtain $\alpha_1\bar{Z}_1(\beta_1) + c_t - h_1 = w + R_2\bar{Z}_2(\beta_2) = \alpha_2\bar{Z}_2(\beta_2) - h_2$, where $\bar{Z}_i(\beta_{\mathrm{i}}) = 1 - Z_{\mathrm{i}}(\beta_{\mathrm{i}})$. Llet $w = (\alpha_2 - R_2)\bar{Z}_2(\beta_2) - h_2$, then:

$\Delta\pi_1(\dot{q}_s^e, p_s^e, 0, R_2, w)$

$$= -\alpha_1 \int_{n_1-a_1}^{n_1-\dot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1$$

$$- R_2 \left[ \int_{n_2-a_2+b_2p_s^e}^{n_2+\dot{q}_s^e-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - \dot{q}_s^e Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e) \right]$$

$$+ [-\alpha_2 Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e) + \alpha_2 - h_2 - \alpha_1 + h_1 - c_t]\dot{q}_s^e + g_1b_1p_s^e$$

$$\Delta\pi_2(\dot{q}_s^e, p_s^e, 0, R_2, w) = \Delta\prod_{cen}(\dot{q}_s^e, p_s^e) - \Delta\pi_1(\dot{q}_s^e, p_s^e, 0, R_2, w)$$

$$= -(\alpha_2 - R_2)\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+\dot{q}_s^e-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - \dot{q}_s^e Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e)\right]$$

$$-\,\alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - b_2p_s^e\right] - (\alpha_2 - g_2)b_2p_s^e$$

According to mean value theorem

$$-R_2\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+\dot{q}_s^e-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - \dot{q}^e Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e)\right] > 0$$

So, $\Delta\pi_1$ increases with $R_2$, Finally, we have:

$$\Delta\pi_2(\dot{q}_s^e, p_s^e, 0, R_2, w) = (\alpha_2 - R_2)\Delta S_2^e(\dot{q}_s^e, p_s^e) + \alpha_2\Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2p_s^e$$

Let $w = \alpha_1\bar{Z}_1(\beta_1) - R_2\bar{Z}_2(\beta_2) + c_t - h_1$, then:

$$\Delta\pi_1(\dot{q}_s^e, p_s^e, 0, R_2, w)$$

$$= -\alpha_1\int_{n_1-a_1}^{n_1-\dot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + R_2\int_{n_2+\dot{q}_s^e-a_2+b_2p_s^e}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2$$

$$+\,[R_2Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e) - \alpha_1Z_1(n_1 - \dot{q}_s^e - a_1 + b_1p_s^e)]\dot{q}_s^e + g_1b_1p_s^e$$

$$= -\alpha_1\left[\int_{n_1-a_1+b_1p_s^e}^{n_1-\dot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + \dot{q}_s^e Z_1(n_1 - \dot{q}_s^e - a_1 + b_1p_s^e)\right]$$

$$-\,R_2\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+\dot{q}_s^e-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - \dot{q}_s^e Z_2(n_2 + \dot{q}_s^e - a_2 + b_2p_s^e)\right]$$

$$-\,\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - b_1p_s^e\right] - (\alpha_1 - g_1)b_1p_s^e$$

$$\Delta\pi_1(\dot{q}_s^e, p_s^e, 0, R_2, w) = \alpha_1[\Delta S_1^e(\dot{q}_s^e, p_s^e) + \Delta S_1(0, p_s^e)] + R_2\Delta S_2^e(\dot{q}_s^e, p_s^e) - (\alpha_1 - g_1)b_1p_s^e$$

**Case 3.** $q_d^e = 0$; $p_s^e$; $\beta_1 = n_1 - a_1 + b_1p_s^e, \beta_2 = n_2 - a_2 + b_2p_s^e$

In this case, the system and $LSC$ 1 and 2's profit increment in the centralised model are:

$$\Delta\prod_{cen}(0, p_s^e) = -\alpha_1\int_{n_1-a_1}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - \alpha_2\int_{n_2-a_2}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 + g_1b_1p_s^e + g_2b_2p_s^e$$

$$\Delta\pi_1(0, p_s^e, R_1, R_2, w) = -\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + b_1p_s^e\right] - (\alpha_1 - g_1)b_1p_s^e$$

$$= \alpha_1\Delta S_1(0, p_s^e) - (\alpha_1 - g_1)b_1p_s^e$$

$$\Delta\pi_2(0, p_s^e, R_1, R_2, w) = -\alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 + b_2p_s^e\right] - (\alpha_2 - g_2)b_2p_s^e$$

$$= \alpha_2\Delta S_2(0, p_s^e) - (\alpha_2 - g_2)b_2p_s^e$$

To coordinate the system, the conditions of two $LSCs$' profit functions should satisfy:

$$\frac{\partial \pi_1^+(q,p_s^e)}{\partial q} \le 0 \quad \frac{\partial \pi_1^-(q,p_s^e)}{\partial q} \ge 0 \quad \frac{\partial \pi_2^+(q,p_s^e)}{\partial q} \le 0 \quad \frac{\partial \pi_2^-(q,p_s^e)}{\partial q} \ge 0$$

$$\begin{cases} \alpha_1 Z_1(n_1 - a_1 + b_1 p_s^e) - R_2 Z_2(n_2 - a_2 + b_2 p_s^e) + R_2 + w - c_t - (\alpha_1 - h_1) \le 0 \\ \alpha_1 Z_1(n_1 - a_1 + b_1 p_s^e) - R_1 Z_1(n_2 - a_2 + b_2 p_s^e) + R_1 + w - (\alpha_1 - h_1) \le 0 \\ -\alpha_2 Z_2(n_2 - a_2 + b_2 p_s^e) + R_2 Z_2(n_2 - a_2 + b_2 p_s^e) - (w + R_2) + (\alpha_2 - h_2) \ge 0 \\ -\alpha_2 Z_2(n_2 - a_2 + b_2 p_s^e) + R_1 Z_1(n_1 - a_1 + b_1 p_s^e) - (w - c_t + R_1) + (\alpha_2 - h_2) \ge 0 \end{cases} \rightarrow$$

$$\begin{cases} \alpha_2 \bar{Z}_2(\beta_2) - h_2 \le w + R_2 \bar{Z}_2(\beta_2) \le \alpha_1 \bar{Z}_1(\beta_1) - h_1 + c_t \\ \alpha_1 \bar{Z}_1(\beta_1) - h_1 \le w + R_1 \bar{Z}_1(\beta_1) \le \alpha_2 \bar{Z}_2(\beta_2) - h_2 + c_t \end{cases}$$

where $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$

**Case 4.** $q_d^e = \ddot{q}_s^e;\ p_s^e;\ \beta_1 = n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e;\ \beta_2 = n_2 + \ddot{q}_s^e - a_2 + b_2 p_s^e$

In this case, the system and $LSC$ 1 and 2's profit increment in the centralised model are:

$$\Delta\prod_{cen}(\ddot{q}_s^e, p_s^e) = -\alpha_1 \int_{n_1-a_1}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 - \alpha_2 \int_{n_2-a_2}^{n_2+\ddot{q}_s^e-a_2+b_2p_s^e} Z_2(d_2)dd_2$$
$$+ (\alpha_2 - h_2 - \alpha_1 + h_1 + c_t)\ddot{q}_s^e + g_1 b_1 p_s^e + g_2 b_2 p_s^e$$

$\Delta\pi_1(\ddot{q}_s^e, p_s^e, R_1, 0, w)$

$$= -\alpha_1 \int_{n_1-a_1}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 - R_1 \int_{n_1-\ddot{q}_s^e-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1(d_1)dd_1$$
$$+ \ddot{q}_s^e(R_1 + w - \alpha_1 + h_1) + g_1 b_1 p_s^e$$

In this case, $\prod_{cen}(q, p_s^e)$ satisfies:

$$\frac{\partial \prod_{cen}(q,p_s^e)}{\partial q} = 0$$

So, to coordinate the system, the conditions of two $LSCs$' profit functions should satisfy:

$$\frac{\partial \pi_1}{\partial q} = \alpha_1 Z_1(n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e) - R_1 Z_1(n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e) + R_1 + w - (\alpha_1 - h_1) = 0$$

$$\frac{\partial \pi_2}{\partial q} = -\alpha_2 Z_2(n_2 + \ddot{q}_s^e - a_2 + b_2 p_s^e) + R_1 Z_1(n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e) - (w - c_t + R_1) + (\alpha_2 - h_2) = 0$$

So $\alpha_2 \bar{Z}_2(\beta_2) + c_t - h_2 = w + R_1 \bar{Z}_1(\beta_1) = \alpha_1 \bar{Z}_1(\beta_1) - h_1$ , $\bar{Z}_i(\beta_i) = 1 - Z_i(\beta_i)$ . Let $w = (\alpha_1 - R_1)\bar{Z}_1(\beta_1) - h_1$, then:

$\Delta\pi_1(\ddot{q}_s^e, p_s^e, R_1, 0, w)$

$$= -\alpha_1 \int_{n_1-a_1}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 - R_1 \int_{n_1-\ddot{q}_s^e-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1(d_1)dd_1$$
$$- \ddot{q}_s^e(\alpha_1 - R_1) Z_1(n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e) + g_1 b_1 p_s^e$$
$$= -\alpha_1 \left[\int_{n_1-a_1+b_1p_s^e}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 + \ddot{q}_s^e Z_1(n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e)\right]$$
$$+ R_1 \left[\int_{n_1-a_1+b_1p_s^e}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1(d_1)dd_1 + \ddot{q}_s^e Z_1(n_1 - \ddot{q}_s^e - a_1 + b_1 p_s^e)\right]$$
$$- \alpha_1 \left[\int_{n_1-a_1}^{n_1-a_1+b_1p_s^e} Z_1(d_1)dd_1 - b_1 p_s^e\right] - (\alpha_1 - g_1) b_1 p_s^e$$

$$\Delta\pi_1(\ddot{q}_s^e, p_s^e, R_1, 0, w) = (\alpha_1 - R_1)\Delta S_1^e(\ddot{q}_s^e, p_s^e) + \alpha_1 \Delta S_1(0, p_s^e) - (\alpha_1 - g_1) b_1 p_s^e$$

According to mean value theorem, we know:

$$R_1\left[\int_{n_1-a_1+b_1p_s^e}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + \ddot{q}_s^e Z_1(n_1-\ddot{q}_s^e-a_1+b_1p_s^e)\right] < 0$$

So, $\Delta\pi_1$ decreases with $R_1$. Also, if let $w = \alpha_2\bar{Z}_2(\beta_2) - R_1\bar{Z}_1(\beta_1) - h_2 + c_t$, then:

$\Delta\pi_1(\ddot{q}_s^e, p_s^e, R_1, 0, w)$

$$= -\alpha_1\int_{n_1-a_1}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1$$

$$- R_1\left[\int_{n_1-\ddot{q}_s^e-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - \ddot{q}_s^e Z_1(n_1-\ddot{q}_s^e-a_1+b_1p_s^e)\right]$$

$$+ \ddot{q}_s^e[-\alpha_2 Z_2(n_2+\ddot{q}_s^e-a_2+b_2p_s^e) + \alpha_2 - h_2 + c_t - \alpha_1 + h_1] + g_1b_1p_s^e$$

$\Delta\pi_2(\ddot{q}_s^e, p_s^e, R_1, 0, w) = \prod_{cen}(\ddot{q}_s^e, p_s^e) - \Delta\pi_1(\ddot{q}_s^e, p_s^e, R_1, 0, w)$

$$= -\alpha_2\left[\int_{n_2-a_2+b_2p_s^e}^{n_2+\ddot{q}_s^e-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - \ddot{q}_s^e Z_2(n_2+\ddot{q}_s^e-a_2+b_2p_s^e)\right]$$

$$- R_1\left[\int_{n_1-a_1+b_1p_s^e}^{n_1-\ddot{q}_s^e-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 + \ddot{q}_s^e Z_1(n_1-\ddot{q}_s^e-a_1+b_1p_s^e)\right]$$

$$- \alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2 - b_2p_s^e\right] - (\alpha_2 - g_2)b_2p_s^e$$

$\Delta\pi_2(\ddot{q}_s^e, p_s^e, R_1, 0, w) = \alpha_2[\Delta S_2^e(\ddot{q}_s^e, p_s^e) + \Delta S_2(0, p_s^e)] + R_1\Delta S_2^e(\ddot{q}_s^e, p_s^e) - (\alpha_2 - g_2)b_2p_s^e$

**Case 5.** $q_d^e = -n_2; p_s^e; R_2 = 0;\ \beta_1 = n_1 + n_2 - a_1 + b_1p_s^e, \beta_2 = -a_2 + b_2p_s^e$

In this case, the system and $LSC$ 1 and 2's profit increment in the centralised model are:

$\Delta\prod_{cen}(-n_2, p_s^e)$

$$= -\alpha_1\int_{n_1-a_1}^{n_1+n_2-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - \alpha_2\int_{n_2-a_2}^{-a_2+b_2p_s^e} Z_2\,(d_2)dd_2$$

$$+ (\alpha_2 - h_2 - \alpha_1 + h_1 + c_t)(-n_2) + g_1b_1p_s^e + g_2b_2p_s^e$$

$\Delta\pi_1(-n_2, p_s^e, R_1, 0, w)$

$$= -\alpha_1\int_{n_1-a_1}^{n_1+n_2-a_1+b_1p_s^e} Z_1\,(d_1)dd_1 - R_1\int_{n_1+n_2-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1$$

$$+ (-n_2)(R_1 + w - \alpha_1 + h_1) + g_1b_1p_s^e$$

In this case, $\prod_{cen}(q, p_s^e)$ satisfies:

$$\frac{\partial\prod_{cen}(q, p_s^e)}{\partial q} \le 0$$

So, to coordinate the system, the conditions of two $LSCs$' profit functions should satisfy:

$$\frac{\partial\pi_1}{\partial q} = (\alpha_1 - R_1)Z_1(n_1+n_2-a_1+b_1p_s^e) + R_1 + w - (\alpha_1 - h_1) \le 0$$

$$\frac{\partial\pi_2}{\partial q} = -\alpha_2 Z_2(-a_2+b_2p_s^e) + R_1Z_1(n_1+n_2-a_1+b_1p_s^e) - (w - c_t + R_1) + (\alpha_2 - h_2) \le 0$$

So, $-R_1\bar{Z}_1(\beta_1)+\alpha_2\bar{Z}_2(\beta_2)+c_t-h_2 \le w \le (\alpha_1-R_1)\bar{Z}_1(\beta_1)-h_1$ ; $\bar{Z}_{\mathrm{i}}(\beta_{\mathrm{i}})=1-Z_{\mathrm{i}}(\beta_{\mathrm{i}})$ . $\Delta\pi_1$ decreases with $w$, if $w_{\max}=(\alpha_1-R_1)\bar{Z}_1(\beta_1)-h_1$:

$$\Delta\pi_1(-n_2,p_s^e,R_1,0,w)$$

$$=-(\alpha_1-R_1)\left[\int_{n_1-a_1+b_1p_s^e}^{n_1+n_2-a_1+b_1p_s^e} Z_1\,(d_1)dd_1+(-n_2)Z_1(n_1+n_2-a_1+b_1p_s^e)\right]$$

$$-\alpha_1\left[\int_{n_1-a_1}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1-b_1p_s^e\right]-(\alpha_1-g_1)b_1p_s^e$$

$$\Delta\pi_1(-n_2,p_s^e,R_1,0,w)=(\alpha_1-R_1)\Delta S_1^e(-n_2,p_s^e)+\alpha_1\Delta S_1(0,p_s^e)-(\alpha_1-g_1)b_1p_s^e$$

According to mean value theorem:

$$-R_1\left[\int_{n_1+n_2-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1-(-n_2)Z_1(n_1+n_2-a_1+b_1p_s^e)\right]<0$$

So, $\Delta\pi_1$ decreases with $R_1$. If $w_{\min}=-R_1\bar{Z}_1(\beta_1)+\alpha_2\bar{Z}_2(\beta_2)+c_t-h_2$, then:

$$\Delta\pi_1(-n_2,p_s^e,R_1,0,w)$$

$$=-\alpha_1\int_{n_1-a_1}^{n_1+n_2-a_1+b_1p_s^e} Z_1\,(d_1)dd_1-R_1\int_{n_1+n_2-a_1+b_1p_s^e}^{n_1-a_1+b_1p_s^e} Z_1\,(d_1)dd_1$$

$$+(-n_2)[R_1Z_1(n_1+n_2-a_1+b_1p_s^e)-\alpha_2Z_2(-a_2+b_2p_s^e)+\alpha_2+c_t-\alpha_1+h_1-h_2]$$

$$+g_1b_1p_s^e$$

$$\Delta\pi_2(-n_2,p_s^e,R_1,0,w)=\Delta{\prod}_{cen}(-n_2,p_s^e)-\Delta\pi_1(-n_2,p_s^e,R_1,0,w)$$

$$=-\alpha_2\left[\int_{n_2-a_2+b_2p_s^e}^{-a_2+b_2p_s^e} Z_2\,(d_2)dd_2+n_2Z_2(-a_2+b_2p_s^e)\right]$$

$$-R_{1\min}\left[\int_{n_1-a_1+b_1p_s^e}^{n_1+n_2-a_1+b_1p_s^e} Z_1\,(d_1)dd_1+(-n_2)Z_1(n_1+n_2-a_1+b_1p_s^e)\right]$$

$$-\alpha_2\left[\int_{n_2-a_2}^{n_2-a_2+b_2p_s^e} Z_2\,(d_2)dd_2-b_2p_s^e\right]-(\alpha_2-g_2)b_2p_s^e$$

$$\Delta\pi_2(-n_2,p_s^e,R_1,0,w)=\alpha_2[\Delta S_2^e(-n_2,p_s^e)+\Delta S_2(0,p_s^e)]+R_1\Delta S_2^e(-n_2,p_s^e)-(\alpha_2-g_2)b_2p_s^e$$