Performance Analysis of Spatially-Distributed Cooperative Networks

A thesis submitted for the degree of Doctor of Philosophy

by

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Abstract

Applications of cooperative communications have attracted considerable attention in academia and industry in the past decade for their potential to exploit network densification in meeting the growing demand for data services. However, analytical methods capable of explicitly capturing the impact of the spatial domain on system performance are still rare. The aim of this thesis is to study cooperation between spatially-distributed nodes with the purpose to enhance relevant analytical methods. New approaches to performance analysis of node cooperation and several useful relations are developed in this work in the following three areas.

First part of this thesis investigates broadcasting as an important method for TV and network signalling distribution. Cooperative broadcasting (CB) has been generally studied under the assumptions of asymptotically dense or large networks, which rarely hold in practice. In this work, a method to analyse the latency of CB in finite networks is developed using stochastic geometry. New useful relations and inter-node distance distributions are derived, highlighting interesting network characteristics.

Second part of this thesis studies relay selection (RS), recognised as a way to reduce overheads arising from cooperative communications. In this thesis, a method for analysing RS is developed based on point processes theory. Presented approach is simpler and more intuitive compared to known methods. This has allowed obtaining exact expressions for outage probability of relay-assisted communication. Additionally, analysis of the sources' contention for relays has revealed that relays can be treated as a scarce resource.

Finally, proposed methods are further extended to account for imperfect channel state information (CSI). Practical RS in presence of CSI imperfections remains an active research area, however the aspect of cooperating nodes' spatial distribution remains unexplored. This thesis introduces a novel approach to account for variable levels of CSI accuracy and for the spatial distribution cooperating relays.

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Contents

Nomenclature

Acronyms

3GPP	Third generation partnership project
AF	Amplify and forward
AWGN	Additive white Gaussian noise
BPP	Binomial point process
BS	Base station
CC	Coded cooperation
CDF	Cumulative distribution function
CSI	Channel state information
DF	Decode and forward
i.i.d	Independent and identically distributed
LTE	Long term evolution
MRC	Maximum ratio combining
OR	Opportunistic relaying
PDF	Probability density function
PP	Point process
PPP	Poisson point process
RS	Relay selection
SC	Selection cooperation
SG	Stochastic geometry

Symbols

Φ_d	Set of receivers that have decoded the source transmission correctly, decoding set
α	Path loss exponent
B	Subset of a d-dimensional space
c	Speed of light
c_k	Set of combinations of S_k and T_{k-1} leading to certain value of T_k
ϵ	Channel estimation error
Δf	Channel bandwidth
η_{ij}^2	Variance of the compound channel coefficient g_{ij}
f_c	Carrier frequency
g_{ij}	Compound channel coefficient incorporating path loss and small-scale fading
$\gamma(\cdot)$	Lower incomplete gamma function
$\hat{\eta}_{id}^2$	Variance of the perceived compound channel gain
$\hat{g}_{ij}$	Perceived compound channel gain
h_{ij}	Small-scale fading coefficient for the channel between nodes i and j
K	Number of required retransmission of a symbol
κ	Share of transmission power allocated to the source transmission
$\lambda(w)$	Intensity function of a point process in the location w
$\Lambda(\cdot)$	Intensity measure of a point process
$l\left(r_{ij}\right)$	Path loss factor between nodes i and j
$\nu_d(A)$	Lebesgue measure of subset A in d dimensions
N	Total number of transmitters/receivers in the system
n_w	Additive white Gaussian noise (AWGN)
Φ	Poisson point process
Φ_b	Point process of relays with reliable connections to the destination

Λ_q	Intensity measure of the point process of relays with reliable connections to the source and the destination
Φ_q	Point process of relays with reliable connections to the source and the destination
$P_{\text{out,one}}$	Outage probability for cooperative communication between one source-destination pair
P_{out}	Outage probability
$P_{\text{out,DF}}$	Outage probability for DF relaying
$P_{\text{out,imp}}$	Outage probability for cooperative communication with relay selection based on imperfect instantaneous CSI
$P_{\text{out,mult}}$	Outage probability for cooperative communication between one multiple sources and one destination
$P_{\text{out,stat}}$	Outage probability for cooperative communication with relay selection based on statistical CSI
$P_{\text{tx},r}$	Relay node transmission power
$P_{\mathcal{S}}$	Probability of successful symbol transmission
$P_{\text{tx},s}$	Source node transmission power
P_{tx}	Transmission power
$\mathcal{R}$	Target spectral efficiency
R	Cell radius
$\mathbb{R}^d$	d-dimensional space
r_{ij}	Distance between nodes i and j
σ_ϵ^2	Channel estimation error variance
S_k	Number of nodes that successfully received the source message as a result of k-th retransmission stage
SNR	Signal-to-noise ratio (at the receiver)
σ_w^2	Variance of AWGN
θ	Decoding threshold for the transmitted symbol

T_k	Total number of nodes that successfully received the source message after k retransmission stages
W	Cell coverage area
x_j	Symbol transmitted by node j
y_i	Symbol received by node i

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Chapter 1

Introduction

A wireless cooperative network represents a communication system where network elements collaborate in a coordinated manner to overcome the impairments of a radio channel in order to deliver higher throughput and reliability. Cooperative relays may enhance overall system performance by reducing the probability of communication failure through the introduction of path diversity and the use of shorter hops with lower propagation path loss [1].

First explorations of the relaying channel as a fundamental block of cooperative networks can be dated back to 1970s [2,3], yet the range of practical applications was limited at that time. Interest in cooperative relaying has been then reignited within research and industry after the development of basic relaying protocols in the past decade [4]. Currently cooperative networking is seen as an important component in meeting the growing demand for mobile data services as network operators increase the number of infrastructure elements in order to deliver greater area spectral efficiency. The importance of cooperative networking is also highlighted in the evolving relay node specifications within 3GPP LTE radio access network layer [5,6] and through a number of developments allowing cooperation between mobile devices [7,8].

Challenges associated with the realisation of a cooperative network can be grouped into two broad areas. The first area is related to cooperative relaying protocol design and analysis, i.e. determining an efficient operation method for an individual relay based on the requirements of a specific application. A family of simple relaying protocols has been formulated in [4,9], and has received significant attention from the research community in the past decade.

In particular, the concepts of amplify-and-forward (AF), decode-and-forward (DF) and coded cooperation (CC) have been proposed, each with own strengths and deficiencies. AF relays amplify and retransmit the source signal without decoding, which reduces the complexity of operations at the relays but introduces the problem of error and noise propagation as noisy signals are amplified at the relays. CC improves spectral efficiency of relaying through source coding, though such procedure requires tighter coordination between the source and relay, adding to system complexity. DF relays decode the contents of the source message before re-encoding and forwarding, which demands additional processing power at the relays but opens the opportunities from more sophisticated signal processing. Another weakness of DF is that it requires multiple relays to be available in the system to achieve the diversity order larger than one [10]. Nevertheless, only DF-based relaying is included in the 3GPP LTE-Advanced specification as most network elements are expected to have digital signal processing functionality [1, p.452].

The second major direction of research in the area of cooperative networking is in the design of a coordinated operation strategy for a system of relays. Researchers have proposed a range of strategies with different trade-offs between complexity and performance. For example, transmit beamforming [11] with DF relays is expected to achieve remarkable performance, but it requires instantaneous magnitudes and phases to be available for all relay-destination channels at a centralised controller. Relay selection is another cooperative networking strategy aiming at the reduction of coordination overheads while preserving the performance gains from cooperative relaying.

Research onto the selection criteria to be utilised in a relaying strategy has resulted in a number of proposals, of which selection cooperation (SC) and opportunistic relaying (OR) have gained considerable attention in academia. SC has been proposed in [12] and studied in [13, Sec. III-A-2] under the name of reactive opportunistic DF. Fig. 1.1 schematically illustrates the general communication scenario. Out of n available relays, relay j is chosen based on the selection strategy-specific function of the source-relay or relay-destination channel coefficients. In SC for DF relays, the source first broadcasts a message while all relays listen. Subsequently, all relays that are able to decode the message correctly form the decoding set Φ_d, and one relay with the best channel condition to the destination is selected from Φ_d to retransmit the source message. On the other hand, proposed and studied in [14], OR selects one relay with the best *end-to-end* source-relay-destination path from all available candidates. Performance of SC and OR cooperation strategies have been studied extensively in literature, predicting significant gains in communication performance from the node cooperation [15–19].

A common metric used in the assessment of cooperative relaying protocols is the probability of communication outage, defined as the probability of the event that instantaneous channel capacity will be lower than chosen transmission data rate [10, p.50]. Such probability can be expressed as

$$P_{\text{out}} = \Pr\left(\log_2\left(1 + SNR\right) < \mathcal{R}\right), \tag{1.1}$$

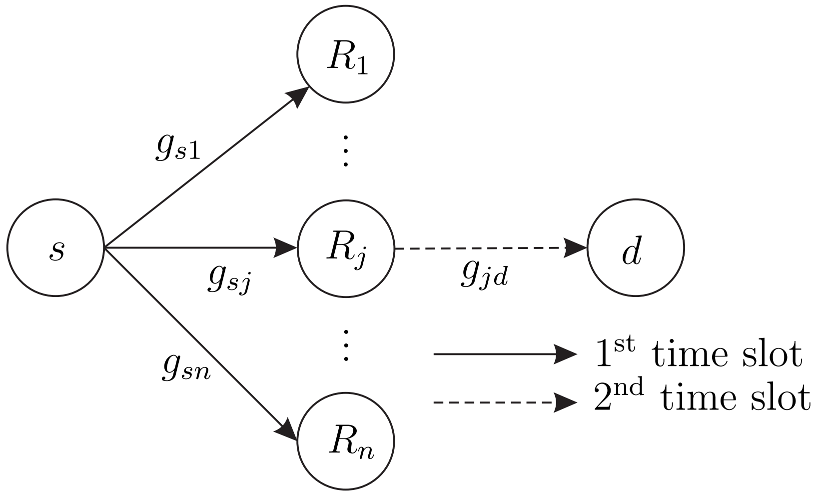

Figure 1.1: Cooperative communication between the source s and destination d via relays $R_j \in [1, n]$ utilising two time slots.

where $\mathcal{R}$ is the spectral efficiency in bits/s/Hz, and SNR is the signal-to-noise ratio at the receiver. Outage probability for DF cooperation using one relay can be expressed as [10, p.70]:

$$P_{\mathrm{out,DF}} = 1 - \exp\left(\frac{2^{2\mathcal{R}} - 1}{P_{\mathrm{s}}\eta_{sr}^2}\sigma_w^2\right) \exp\left(\frac{2^{2\mathcal{R}} - 1}{P_{\mathrm{r}}\eta_{rd}^2}\sigma_w^2\right) \tag{1.2}$$

where $\mathcal{R}$ is the target spectral efficiency, P_{s} and P_{r} are respectively the source and relay transmission power values, η_{sr}^2 is the variance for the source-relay channel coefficient $g_{sr} \sim \mathcal{CN}\left(0, \eta_{sr}^2\right)$ and η_{rd}^2 is the respective quantity for the relay-destination channel; σ_w^2 is the variance of the AWGN at the relay and the destination.

Fig. 1.2 depicts the behaviour of the outage probability $P_{\mathrm{out,DF}}$ as a function of transmission power budget P_{tx} split evenly between the source and relay transmissions. Note that the inter-node distances between the source, selected relay and destination are fixed in order to evaluate (1.2).

1.1 Problem statement

Relation (1.2) provides one example of the fundamental limitation in the approach to performance analysis of cooperative networks used in the majority of literature. This limitation is in that inter-node distances are typically treated as static parameters, as in [10, p.70], or included as the variance of the small-scale fading coefficients [10, p.74]. Such assumptions may be acceptable and convenient in the analysis of point-to-point scenarios or in a lab environment, however cannot be used in the analysis of cooperative systems with a population of spatially-distributed relays [20–22]. Analysis of cooperative systems with multiple relays requires treatment of inter-node distances as random variables because the locations of intermediate cooperating nodes are unknown. They are unknown since whether or not a relay will participate in a coopera-

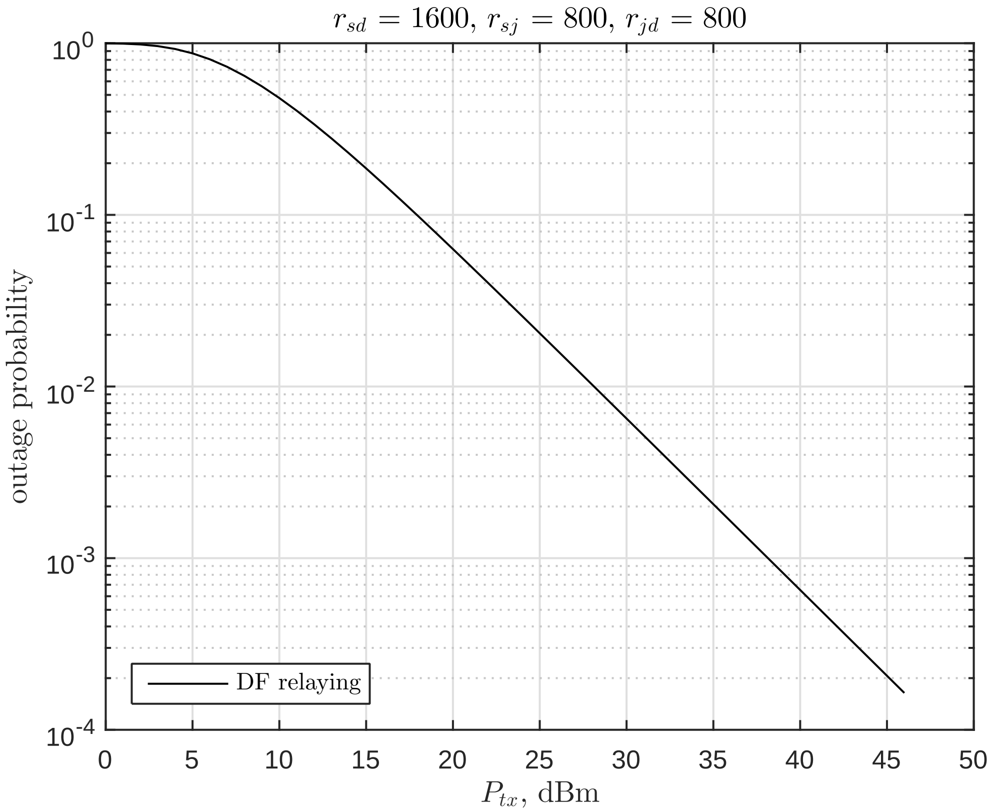

Figure 1.2: Outage probability for cooperative DF relaying with inter-node distances $r_{sj} = r_{jd} = 800$m, free-space propagation path loss at carrier frequency $f_c = 2.4$GHz and subcarrier bandwidth $\Delta f = 15$KHz.

tive transmission depends on the overall channel gain which includes random factors on top of propagation path loss.

Consider for example the network scenario depicted in Fig. 1.3, where a source in the cell centre aims to communicate with a destination through any of the available relays. The location of the relay to be chosen is not known a priori, hence it cannot be modelled as a fixed parameter in general. Intuitively, each individual cooperative relay's location will impose a bias on the statistics of channel gains to the message source and destination. For example, the relays closer to the source are more likely to become a part of the decoding set Φ_d. Quantifying such a bias in statistical modelling of wireless network topologies is therefore of particular importance in performance analysis of cooperative networks.

1.2 Stochastic geometry and alternatives

Stochastic geometry (SG) deals with statistical description of interactions between spatially distributed objects with applications in epidemiology, biology, agriculture, geology and other areas [23]. SG has been recently applied to performance analysis of wireless networks [24], however applications to the analysis of cooperative wireless

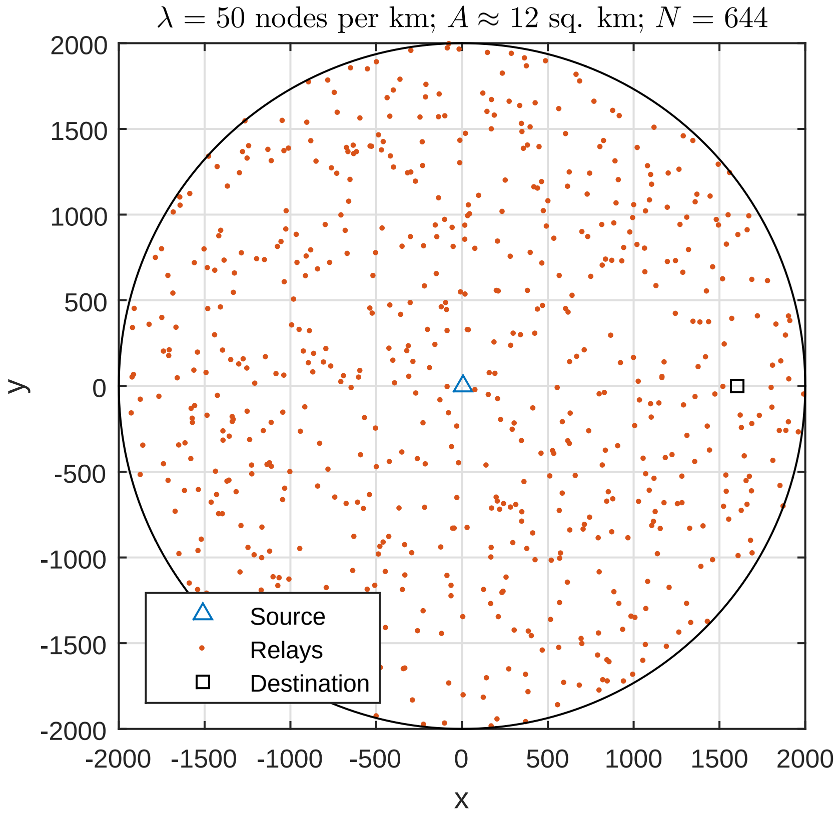

Figure 1.3: Network model based on the Poisson point process with a uniform intensity function $\lambda = 50$ nodes per sq. km.

networks are still limited. This thesis aims to enhance the methods for performance analysis of cooperative networks by investigating elements of cooperative networking with an explicit account for the spatial dimension.

The concept that enables capturing the spatial dimension in performance analysis of wireless networks is the point process. A point process (PP) Φ in a d-dimensional space $\mathbb{R}^d$ is a pattern of points distributed in $\mathbb{R}^d$ with respect to some probabilistic law. Here $d = 1$ corresponds to a PP on line and $d = 2$ – to a PP on Euclidean plane, and some point $x \in \Phi$ can represent a location of a base station (BS) or a user. Such probabilistic law for any PP can be mathematically described using *finite-dimensional (fi-di) distributions* of the form

$$\Pr(\Phi(B_1) = n_1, \dots, \Phi(B_k) = n_k), \tag{1.3}$$

where the notation $\Phi(B_i) = n_i$ denotes that there are exactly n points of some PP Φ in a subset B_i of space $\mathbb{R}^d$. These fi-di distributions, in turn, allow obtaining distributions of random distances between points of the PP, which are of particular importance for cooperative relaying, since they are associated with statistical characterization of inter-node channels.

Different types of point processes have been defined to capture various levels of interactions between network nodes. These interactions could include minimal inter-node distance, node clustering, repelling or attraction conditions, which in practice could translate into minimal inter-BS distance or grouping of users around business and shopping centres. However certain assumptions have to be made for analytical tractability.

The simplest and the most commonly used PPs for modelling nodes in wireless networks are the Poisson point process (PPP) and the Binomial point process (BPP). While PPPs are useful to describe asymptotic scenarios with a very large number of nodes and are more analytically convenient, BPPs more accurately capture properties of finite networks with a fixed number of nodes. In terms of fi-di distributions, the chance of getting exactly n points of a point process in some region B of the plane $\mathbb{R}^2$ for the PPP Φ and BPP Ψ can be found respectively as [23]

$$\Pr(\Phi(B) = n) = e^{-\lambda|B|}\frac{(\lambda|B|)^n}{n!} \tag{1.4}$$

$$\Pr(\Psi(B) = n) = \binom{N}{n}(1-p)^{N-n}p^n, \tag{1.5}$$

where the operator $|\cdot|$ denotes the area of region B, N is the total number of points in the BPP, and p is the probability of getting a point in B. Above relations lead to derivations of distributions of random inter-node distances, distances to ordered neighbours and other important parameters that are used to obtain estimations of network performance metrics, such as mutual information, outage probability, coverage and the level of interference.

1.2.1 Alternative methods

The main objective of the application of SG to analysis of cooperative networks is in an accurate modelling of a cooperative system and in obtaining expressions for performance metrics, such as the probability of communication outage. Alternative methods allowing obtaining the desired performance metrics exist – for example system-level simulations, and the indirect account for inter-node distances discussed in Section 1.1.

System level simulations remain a popular method for assessment of new communication technologies or new network configurations where testing on a live network is undesirable or impossible. Simulations on the system level model the main logical steps taken in a communication system, such as the decisions on correct or erroneous decoding of a received symbol. In order to reduce the computational load in obtaining the statistical network performance metrics, explicit modulation and coding procedures are replaced with the look-up tables, for example utilising established correspondence between SNR and bit error rate. Many communication systems simulators exist, allowing testing different aspects of communication technologies, e.g. [25].

However one of the fundamental limitations of the simulation-based approach is that insights into system performance are obtained indirectly through the interpretation of simulation results for a specific set of system parameters.

Analytical methods, such as SG, operate with general quantities that affect system performance, e.g. transmission power P_{tx} or the number of users N, without necessarily selecting specific values for such quantities. In this way, the general impact of a particular parameter may be obtained. The main drawback of analytical methods is that mathematical complexity can increase when fine details of a specific communication protocol are considered, which requires application of certain assumptions that retain important elements of the system under investigation. Despite this need for simplifying assumptions, analytical description of cooperative communications is indispensable as it helps understand reasons for a specific performance, rather that states numeric results as in the case of simulations.

Overall, mathematical analysis and simulations are expected to complement each other with the ultimate objective of designing efficient communication systems. The focus of this thesis is on the development of analytical methods that help understand the nature of the processes in cooperative networks, and simulations are used to verify obtained results.

Next sections provide an overview of the thesis, highlighting main contributions of presented work, and list the publications generated as a result of this research.

1.3 Thesis plan and contributions

The main areas of novelty of this research are in the development of methods for performance analysis of cooperative communication systems, and in the derivation of fundamental relations describing the performance of specific cooperative communication scenarios. Specific subjects of each chapter and respective contributions are highlighted in the following.

Chapter 2 investigates the effects of node cooperation on the latency of information broadcasting in terms of the number of retransmissions required to reach all nodes in the cell. Practical applications utilising such one-to-many communication pattern include a number of important cases such as television broadcasting and emergency notifications. The hypothesis being tested is that through user cooperation a broadcasted message can reach more nodes in a given time period and thereby improve the performance of relevant applications. In testing this hypothesis, Chapter 2 generalises relations describing inter-node distance distributions in random networks to new forms that allow description of α-th powers of distances. These distributions are important because they open a way to the analysis of cooperative scenarios where distances enter in the α-th powers, e.g. the free-space propagation model with $\alpha = 2$.

These extensions are then applied to the development of a new approach to analysis of cooperative and non-cooperative information broadcasting.

Chapter 3 makes contributions towards a better characterisation of relay selection. Relay selection is applied to improve the efficiency of a cooperative network through involving only those relays that can make the best contribution to communication between a particular source-destination pair. This chapter presents a novel approach to performance analysis of relay selection methods based on thinning operation on point processes. Relations describing the outage probability of relay-assisted communication are derived with an explicit account for the spatial node distribution. These results are then further extended to obtain a lower outage probability bound for a more challenging case of multi-source multi-relay communication, where sources contend for relays in delivering a message to a common destination. Developed analytical methods and relations describing cooperative network performance are significantly simpler and more intuitive compared to existing approaches that typically require multiple levels of numerical integration.

Chapter 4 introduces a new approach to assessment of the impact from imperfections in the information used to make relay selection decisions. Methods and relations that describe communication outage probability are presented for scenarios where the nature of channel state information available at relays ranges from (a) long-term statistics to (b) channel estimates with variable degrees of accuracy due to noisy measurements. An important conclusion from this chapter is that selection decisions made based on false information may lead to a significant performance loss. In this chapter methods are developed to analytically quantify this loss. To the best of author's knowledge there are no comparable published works at the time of writing.

1.4 Publications

Publications arising from this work are listed below [26–29]:

1. **A. Tukmanov**, Z. Ding, S. Boussakta, and A. Jamalipour, "On the broadcast latency in finite cooperative wireless networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1307 –1313, Apr. 2012.

2. **A. Tukmanov**, Z. Ding, S. Boussakta, and A. Jamalipour, "On the impact of network geometric models on multicell cooperative communication systems," *IEEE Wireless Commun. Mag.*, vol. 20, no. 1, pp. 75–81, Feb. 2013.

3. **A. Tukmanov**, S. Boussakta, Z. Ding, and A. Jamalipour, "On the impact of relay-side channel state information on opportunistic relaying," in *IEEE International Conference on Communications (ICC)*, June 2013, pp. 5478–5482.

4. **A. Tukmanov**, S. Boussakta, Z. Ding, and A. Jamalipour, "Outage performance analysis of imperfect-csi-based selection cooperation in random networks," *IEEE Trans. Commun.*, vol. 62, no. 8, pp. 2747–2757, Aug 2014.

Chapter 2

Downlink Broadcast Analysis

2.1 Introduction

In broadcast communication scenarios transmitted information is addressed to multiple destinations [30]. Some of the applications that can be delivered through broadcasting include live sport events, news reports, TV series, emergency signals, network control messaging or software updates since multiple users are simultaneously interested in identical content or data. The benefits from broadcasting arise in the form of a reduction in the amount of duplicate information transmitted over the network compared to unicast, where each destination is addressed individually.

Wireless delivery of common content is especially interesting since radio communications have broadcast nature due to the properties of electro-magnetic energy propagation. Examples of wireless broadcast technologies range from analogue TV to the emerging enhanced Multimedia Broadcast Multicast Service (eMBMS) in LTE [5]. Elements of broadcasting are included in most cellular communications standards in the forms of system information broadcasts. The key challenges associated with wireless broadcast communication differ from application to application, but generally include the limited range of the source transmissions, energy efficiency and the design of multi-hop broadcast algorithms [30].

This chapter explores whether cooperation between receiving nodes offers any benefits in terms of broadcasted message delivery compared to the conventional point-to-multipoint broadcasting model. The hypothesis being tested is that cooperative broadcast may result in a faster information delivery to all nodes, or in a better penetration in the population when the number of transmissions is limited. In testing this hypothesis, novel methods to analyse inter-node communication are developed.

2.1.1 Related works

Cooperation between wireless nodes in general has gained wide attention as it allows trading extra spatial degree of freedom for reduced outage probability, increased capacity or lower power consumption [4,31]. Whether or not node cooperation is beneficial in broadcast scenarios has been studied in several works, for example [32–34].

In particular, [33] studied the effect of message decoding threshold on the number of nodes reached by the cooperative broadcast. Authors report the existence of a critical value of the minimum SNR necessary for message decoding for the case of high-density networks. Decoding threshold set below such a level was reported to trigger a significant increase in the rate of broadcasted message delivery. However, cooperative retransmissions in [33] were assumed to occur simultaneously and arrive at the receiver either with random phases or combined at the receiver constructively using maximum ratio combining (MRC). Simultaneous transmission and MRC may be difficult to implement in practice due to synchronisation and coordination overheads. In this chapter the assumption of random arrival over non-orthogonal channels will be used in a modified form.

A framework to estimate the capacity of cooperative broadcasting protocols was developed in [32] based on the similar assumption of a dense network as in [33]. Reference [34] studied the problem of the optimal transmission schedule and power allocation in a cooperative broadcast scenario. The main conclusion from [34] that in the case of dense networks, optimal transmission schedule is asymptotically approximated by the transmission with minimal power in the order of distance from the source. Another interesting finding is that direct non-cooperative transmission can be more power-efficient than certain sub-optimal cooperative schemes, especially in the case of path loss exponent $\alpha = 2$. While the proposed asymptotically-optimal transmission strategy is simple, implementations may need to be based on some derivative of the distance measurements and account for possible inaccuracies in distance estimation.

2.1.2 Problem statement

One unifying challenge associated with the majority of previous research on cooperative broadcasting is that explicit network geometry and associated path loss (PL) effects on performance of broadcasting scenarios were either ignored, or considered for some asymptotic network settings. For example, [32–34] all assume variations of dense wireless networks, i.e. networks where the density of nodes in the network is large, or networks size is infinite. These assumptions rarely hold in practice, especially in populated areas where cell sizes tend to shrink. In addition, the number of subscribers to a broadcast service may be very low in an otherwise densely-populated area. This chapter focuses specifically on networks of finite sizes and with finite node densities – referred to further as *finite* networks. Joint effects of path loss and fading on system performance have been investigated in [35], however not in the context of cooperative networks. An initial analysis of inter-node distance distributions for finite networks was presented in [36].

The aims of this chapter are to develop methods applicable to the analysis of cooperative broadcast in a finite network, and to compare the performance of cooperative broadcast to conventional non-cooperative protocol in therms of the time required to deliver the source message to all nodes in the network.

2.1.3 Contributions

The main contributions of this chapter are in (a) the developed methodology to analyse the performance of studied broadcasting protocols, and in (b) the derivation of inter-node distance distributions that allow obtaining required system performance metrics. Specific contributions are summarised in the list below:

1. Methodology for derivation of the average number of retransmissions required to deliver a source message to all nodes in the network. This methodology is based on the estimation of probabilities that a specific number of nodes will be reached after a number of retransmission attempts. Such estimation is conducted using new relations developed in this chapter. The value of this contribution is in that it provides a new method to analyse broadcast protocols, which can be re-used and extended to studying related problems.

2. Eq.(2.7) within the Proposition 2.3.1 is a cumulative distribution function (CDF) of the gain in a communication channel between a receiver and the nearest transmitter (or vice-versa). This is an important metric of a cooperative system performance, equivalent to the probability of communication outage. Relation (2.7) is obtained based on rigorous analysis and extensions of the original derivations in [35, 37].

3. Eq.(2.25) within the Proposition 2.4.1 is the probability density function (PDF) of the α-th power of distance to the nearest transmitter from a receiver. This relation is the key to describing the gain in a channel to the destination with an unknown location. Presented result is a generalisation of [36, Theorem 2.1].

Material from this chapter has been published in part in [26].

2.1.4 Chapter organisation

After the description of network and signal models in Section 2.2, Section 2.3 develops the analytical framework for analysis of the latency of the conventional non-cooperative broadcasting. Latency of cooperative broadcasting is the subject of Section 2.4. Numerical results and discussions are included in Section 2.5.

2.2 System model

This section outlines considered broadcast protocols, specifies network and system models, associated assumptions and defines the broadcasting latency metric.

2.2.1 Broadcast protocol models

Broadcast schemes used in [32] will be used as a basis for studied protocols. In particular, in the case of *non-cooperative broadcast*, a single source broadcasts a message while all other nodes listen. This broadcasted message may be repeated by the source in subsequent time slots to increase the population of receivers that are able to decode the message correctly. In *cooperative broadcast* the source broadcasts a message in the first time slot, and continues until at least one receiver decodes it correctly. After that the source remains silent, while all successful receivers cooperate by retransmitting the message to remaining nodes in subsequent time slots until all nodes are able to decode the message.

Following subsections specify associated network and signal models in more detail.

2.2.2 Network model

Consider a cell $W = b_2(0, R)$ with a circular coverage area, and a broadcast source located in the centre of the cell. The notation $b_2(0, R)$ reads as a two-dimensional ball b with the centre at the origin, and radius R. Nodes that are located inside the coverage area $b_2(0, R)$ are assumed to be associated with the transmitter at the centre. Nodes outside the coverage area may or may not be associated with other transmitters. One realisation of the network is shown on Figure 2.1.

Node positions will be modelled as realisations of a point process (PP). As described briefly in Section 1.2 PPs in general provide a mechanism to analyse interactions between spatially distributed objects, such as wireless nodes [23], which allows analytical description of considered system [24]. Among the variety of PPs, of particular importance is the Poisson point process (PPP) since it allows obtaining especially simple expressions for complex network interactions. Specifically, a PPP is characterised by two fundamental properties [23]:

1. the probability of having exactly N nodes in a subset set B of some observation window W is Poisson distributed with parameter $\lambda \cdot \nu_d(B)$. Here λ is the node density function and $\nu_d(B)$ is Lebesgue measure (i.e. length, area or volume of

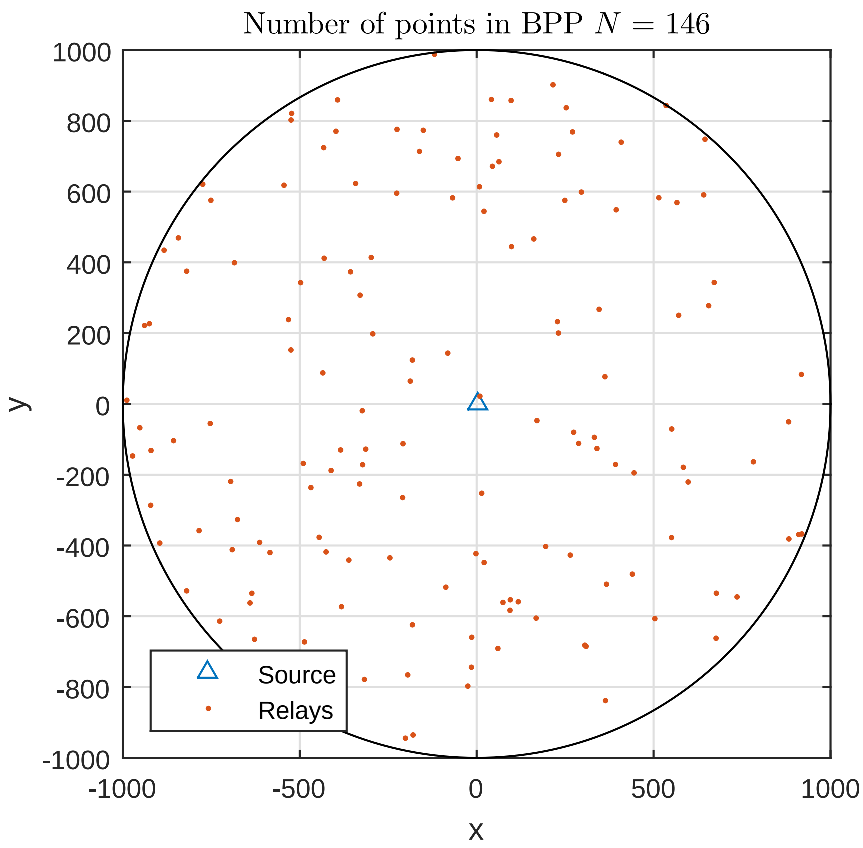

Figure 2.1: Network realisation with $N = 146$ nodes in a circular cell W.

B) and W is understood in this context as a geometrical construction in space where the point process is observed;

2. the counts of points in disjoint subsets of W are independent.

PPPs are very useful in describing large networks where modelled network characteristics are in good agreement with aforementioned properties of the PPPs. Unfortunately the properties of the PPPs do not co-exist well with networks models where network area and node density are limited ("finite networks" [36]). For example, if the number of nodes in the network is known to be N, the PPP model becomes inappropriate since the node counts in different locations become dependent (see [23, p.27] and [36]). Binomial point processes (BPPs) [23] are designed to describe point processes with a fixed number of nodes, hence overcome the limitation of PPPs, however at a cost of analytical complexity.

In this chapter BPPs will be used to analyse the performance of broadcast protocols in finite networks. Following assumptions regarding the network structure and node mobility will be used to facilitate modelling of each broadcasting snapshot as a realisation of a BPP:

A1 high mobility (HM) model [38] will be used, where node positions change randomly and independently in each time slot, A2 all nodes remain within the cell W, i.e. the set of nodes in $b_2(0, R)$ remains constant between snapshots.

In practice, nodes' location changes will follow certain law, and nodes may leave and enter the cell W. Some aspects of such mobility models could be accounted for using specialised point processes (PPs), for example, the hard-core PP [23] could be used to capture the effect of minimal expected node speed. Relaxing assumptions A1 and A2 to account for arbitrary mobility models leads to significantly more complicated analysis and requires a separate study.

2.2.3 Signal model

Let x_j be the symbol transmitted by node j, and h_{ij} be the small-scale fading coefficient between the transmitter j and receiver i. Block fading model is assumed, so that channel remains constant for one symbol duration, and changes with respect to some probability distribution between symbol duration periods. The signal received by node i from transmitter j can be expressed as [10, p.26]

$$y_i = h_{ij}\sqrt{P_{\text{tx}} \cdot l(r_{ij})} \cdot x_j + n_w, \tag{2.1}$$

where P_{tx} is transmission power; n_w is zero-mean AWGN, $l(r_{ij}) = \left(1 + r_{ij}^{\alpha}\right)^{-1}$ is the path loss function [34], r_{ij} is the distance between nodes i and j and α is the path loss exponent. Additive noise component n_w and channel coefficient h_{ij} are assumed to follow respective distributions: $n_w \sim \mathcal{CN}(0; \sigma_w^2)$ and $h_{ij} \sim \mathcal{CN}(0; \sigma_h^2)$.

Symbol x_j is considered to be successfully received if the capacity of communication channel between the transmitter j and receiver i is sufficient for transmission at the chosen spectral efficiency $\mathcal{R}$:

$$\log\left(1 + \frac{|h_{ij}|^2 P_{\text{tx}}}{\left(1 + r_{ij}^{\alpha}\right)\sigma_w^2}\right) \geq \mathcal{R}. \tag{2.2}$$

Probability of such event can be described as probability of success and expressed as

$$P_{\mathcal{S}} = \Pr\left(\frac{|h_{ij}|^2}{1 + r_{ij}^{\alpha}} \geq \theta\right), \tag{2.3}$$

where h_{ij} and r_{ij} are random variables and $\theta = \frac{\sigma_w^2\left(2^{\mathcal{R}}-1\right)}{P_{\text{tx}}}$ is the threshold for successful reception.

Note that non-orthogonal transmissions are assumed, which may lead to performance degradation due to arrivals of the same message from multiple sources. This multipath effect is accounted for through Rayleigh fading in this chapter. A similar setting has been used in [33], where cooperatively transmitted signals were addition-

$$P_k^{nc} = \Pr(K = k|N) = \Pr(T_k = N|N) = \sum_{(T_{k-1},S_k)\in c_k} \Pr(S_k|N - T_{k-1})\Pr(T_{k-1}|N)$$
$$= \sum_{(T_{k-1},S_k)\in c_k} \Bigg\{ \Pr(S_k|N - T_{k-1}) \sum_{(T_{k-2},S_{k-1})\in c_{k-1}} \Bigg\{ \Pr(S_{k-1}|N - T_{k-2}) \cdot \ldots$$
$$\times \sum_{(T_1,S_2)\in c_2} \{\Pr(S_2|N - T_1)\Pr(T_1|N)\} \ldots \Bigg\} \Bigg\}, \tag{2.5}$$

ally coherently combined at destination. Although coherent combining can improve performance, it is difficult to implement due to system overhead.

Each receiver is assumed to be served from the nearest available transmitter as the nearest transmitter's power is expected to dominate the rest.

2.2.4 Latency metric for broadcast transmission

The metric of interest in this chapter is the average number of transmission attempts, or time slots, $\bar{K}$ required to deliver a broadcast message to all N nodes within the observation window W. Such expected number of required transmissions $\bar{K}$ can be found as

$$\bar{K} = \sum_{k=1}^{\infty} k \cdot \Pr(K = k|N) \approx \sum_{k=1}^{K_{\max}} k \cdot \Pr(K = k|N), \tag{2.4}$$

where $\Pr(K = k|N)$ denotes the probability that, conditioned on having N nodes in total, exactly K transmissions will be sufficient to reach all nodes. As will be discussed in Section 2.5, $K_{\max} << \infty$ is a large value used to make calculations feasible.

2.3 Latency analysis for non-cooperative transmission

In order to find the expected number of required transmissions $\bar{K}$ one needs to estimate the probability $\Pr(K = k|N)$, described in (2.4). Let $S_k \in [0, N]$, $k \in [1, K]$, be the random variable, representing the number of successful nodes obtained as a result of k-th transmission stage, and $T_k = \sum_{i=1}^{k} S_i$ be the total number of successful nodes after k transmissions. Then the broadcast process completes when $T_K = \sum_{i=1}^{K} S_i = N$. Therefore, the metric of our interest in (2.4) can be expressed as a sum of probabilities of all possible outcomes of the k transmission stages which lead to the completion of the process in exactly $K = k$ time slots. Relation (2.5) on top of this page describes such a probability.

In (2.5) $\Pr(S_i|N - T_{i-1})$ denotes the probability of getting exactly S_i successful nodes out of $N - T_{i-1}$ as a result of source transmission at the stage i. The conditions of summations $c_i, i \in [1, k]$ in (2.5) can be given as:

$$\begin{aligned} c_k =& \big\{T_{k-1} \in [0, N-1], S_k \in [1, N] : T_{k-1} + S_k = N\big\}; \\ c_{k-1} =& \big\{T_{k-2} \in [0, T_{k-1}], S_{k-1} \in [0, T_{k-1}] : T_{k-2} + S_{k-1} = T_{k-1}\big\}; \end{aligned} \tag{2.6}$$

For example, consider $k = 1$, i.e. the situation where the source broadcast reaches all N nodes in one attempt. Then (2.5) reads as

$$\begin{aligned} P_1^{nc} &= \Pr(K = 1|N) = \Pr(T_1 = N|N) = \sum_{(T_0, S_1) \in c_1} \Pr(S_1|N - T_0) \\ c_1 &= \big\{T_0 = 0, S_1 = N : T_0 + S_1 = N\big\}. \end{aligned}$$

Expressions for arbitrary values of k can be obtain in a similar manner.

Thus to evaluate (2.5), one needs to calculate $\Pr(S_i|N - T_{i-1})$, as demonstrated later in Corollary 2.3.2. To achieve this, the cumulative distribution function of the compound channel state random variable $|g_{ij}|^2 = \frac{|h_{ij}|^2}{1+r_{ij}^{\alpha}}$ for the link between the source and a randomly chosen receiver will be obtained in the Proposition 2.3.1, and then combined with order statistics. The key element in the following analysis is that all receivers are independent and identically distributed (i.i.d.) with respect to fast fading and location.

Proposition 2.3.1 (Joint fading-path loss distribution)**.** *The cumulative distribution function (CDF) of the compound random variable $|g_{ij}|^2 = |h_{ij}|^2 l(r_{ij})$, where $|h_{ij}|^2$ is the amplitude of Rayleigh fading coefficient and $l(r_{ij}) = \left(1 + r_{ij}^{\alpha}\right)^{-1}$ is the path loss function between the source i at origin and a randomly chosen receiver j, is given by*

$$F_{|g_{ij}|^2}(\theta) = 1 - \frac{\delta e^{-\theta}}{R^d \theta^{\delta}} \gamma(\delta, R^{\alpha}\theta), \tag{2.7}$$

with $\gamma(\cdot)$ denoting the lower incomplete Gamma function, d is the number of dimensions and $\delta = d/\alpha$ is used for brevity.

Proof. The CDF of the random variable $|g_{ij}|^2 = \frac{|h_{ij}|^2}{1+r_{ij}^{\alpha}}$ can be expressed as $F_{|g_{ij}|^2}(\theta) = \Pr(|h_{ij}|^2 < \theta\left(1 + r_{ij}^{\alpha}\right))$. Let us consider components of $|g_{ij}|^2$ individually and distinguish three cases: $\theta \in (0, \infty)$, $\theta = 0$ and $\theta = \infty$. Subscripts will be omitted in the rest of the proof for compactness.

The amplitude of Rayleigh fading coefficient is a Chi-square distributed random variable with two degrees of freedom, which is equivalent to the exponential distribution, i.e. $|h|^2 \sim Exp(1)$. The points of the BPP inside W are i.i.d. with a common density function (see [35] and [39])

$$f_{r^\alpha}(y) = \frac{\lambda(y)}{\Lambda(W)} = \frac{\delta y^{\delta-1}}{R^d}, y \in [0, R^\alpha], \tag{2.8}$$

where $\Lambda(W) = \int\limits_W \lambda(w)\mathrm{d}w$ is the intensity measure for the originating PPP, and $\lambda(w)$ is the intensity function of the PPP at the particular location w.

We can now find $F_{|g_{ij}|^2}(\theta)$ for the three regions of θ. For $\theta \in (0, \infty)$:

$$\begin{aligned} F_{|g_{ij}|^2}(\theta) = \Pr\left(|h|^2 < \theta\left(1 + r^\alpha\right)\right) &= \int\limits_{y=0}^{y=R^\alpha} f_{r^\alpha}(y) \int\limits_{x=0}^{x=\theta(1+y)} f_{|h|^2}(x)\mathrm{d}x\mathrm{d}y \\ &= \int\limits_0^{R^\alpha} \frac{\delta y^{\delta-1}}{R^d}\left(1 - e^{-\theta(1+y)}\right)\mathrm{d}y \\ &= \underbrace{\frac{\delta}{R^d}\int\limits_0^{R^\alpha} y^{\delta-1}\mathrm{d}y}_{=1} - \frac{\delta e^{-\theta}}{R^d}\int\limits_0^{R^\alpha} y^{\delta-1}\cdot e^{-\theta y}\mathrm{d}y \\ &= 1 - \frac{\delta e^{-\theta}}{R^d\theta^\delta}\gamma(\delta, R^\alpha\theta). \end{aligned} \tag{2.9}$$

The cases of $\theta = 0$ and $\theta = \infty$ correspond to the events of the path gain being less than zero or less than infinity respectively. Therefore we can write:

$$F_{|g_{ij}|^2}(0) = \Pr(|h|^2 < 0) = 0, \quad F_{|g_{ij}|^2}(\infty) = \Pr(|h|^2 < \infty) = 1.$$

□

Next the above result is used to derive the probability $\Pr(S_i|N - T_{i-1})$ that there are exactly S_i successful nodes out of $N - T_{i-1}$ receivers. Specifically, the distribution of the composite channel gain $|g_{ij}|^2$ between the source and a randomly chosen receiver derived in (2.9) will be used to infer the number of receivers satisfying the given decoding threshold. For simplicity, let $N_r = N - T_{i-1}$ and $Z = |g_{ij}|^2$.

Corollary 2.3.2 (Order statistics)**.** *The conditional probability of having exactly S_i successful nodes out of N_r receivers is*

$$\begin{aligned} Pr(S_i|N_r) = &\binom{N_r}{S_i}\left(1 - \frac{\delta e^{-\theta}}{R^d\theta^\delta}\gamma(\delta, R^\alpha\theta)\right)^{N_r - S_i} \\ &\times \left(\frac{\delta e^{-\theta}}{R^d\theta^\delta}\gamma(\delta, R^\alpha\theta)\right)^{S_i} \end{aligned} \tag{2.10}$$

Proof. Given N_r remaining nodes to be reached, the probability that there are exactly

S_i successful receivers after i-th transmission attempt can be expressed as:

$$\Pr(S_i|N_r) = \Pr(Z_{(N_r-S_i)} < \theta, Z_{(N_r-S_i+1)} \geq \theta), \tag{2.11}$$

where the terms $Z_{(1)} < Z_{(2)} < \ldots < Z_{(N_r-S_i)} < Z_{(N_r-S_i+1)} < \ldots < Z_{(N_r)}$ correspond to ordered realisations of the random variable Z, for which probability distributions are known. Using order statistics and Proposition 2.3.1 we can rewrite (2.11) as

$$\begin{aligned}\Pr(S_i|N_r) &= \int_0^\theta \int_\theta^\infty f_{Z_{(N_r-S_i)},Z_{(N_r-S_i+1)}}(u,v)dvdu \\ &= \binom{N_r}{S_i+1} \int_0^\theta (F_Z(u))^{N_r-S_i-1} \, \mathrm{d}F_Z(u) \\ &\times \int_\theta^\infty (1-F_Z(v))^{S_i-1} \, \mathrm{d}F_Z(v) \\ &= \binom{N_r}{S_i} (F_Z(\theta))^{N_r-S_i} (1-F_Z(\theta))^{S_i},\end{aligned} \tag{2.12}$$

where $F_Z(\theta)$ is defined in (2.7). □

If $\delta = 1$, we get $\gamma(\delta, R^\alpha\theta) = \gamma(1, R^\alpha\theta) = 1 - e^{-R^d\theta}$ and:

$$\begin{aligned}\Pr(S_i|N_r) = &\binom{N_r}{S_i} \frac{1}{(R^d\theta)^{N_r}} \left(e^{-\theta} - e^{-\theta(R^d+1)}\right)^{S_i} \\ &\times \left(R^d\theta - e^{-\theta} + e^{-\theta(R^d+1)}\right)^{N_r-S_i}.\end{aligned} \tag{2.13}$$

Substitution of (2.10) or (2.13) into (2.5) gives the desired expected number of required transmissions required to reach all N nodes in the cell W using non-cooperative broadcast. Next section presents analysis of a cooperative broadcast protocol in terms of the same metric.

2.4 Latency analysis for cooperative transmission

The aim of this section is to estimate the latency of cooperative broadcast in terms of the average number of retransmissions required for a message to reach all nodes.

2.4.1 General setting

Following the same line of reasoning as for (2.5) in the previous section, the calculations to follow are based on the estimation of probabilities for all possible combinations of outcomes of the k transmission stages leading to $T_k = N$ in order to obtain (2.14)

$$P_k^c = \Pr(T_k = N|N) = \sum_{(T_{k-1},S_k)\in c_k} \Pr(S_k|N - T_{k-1})\Pr(T_{k-1}|N)$$
$$\times \sum_{(T_{k-1},S_k)\in c_k} \Bigg\{ \Pr(S_k|N - T_{k-1}) \sum_{(T_{k-2},S_{k-1})\in c_{k-1}} \Bigg\{ \Pr(S_{k-1}|N - T_{k-2}) \cdot \ldots \tag{2.14}$$
$$\times \sum_{(T_1,S_2)\in c_2} \{\Pr(S_2|N - T_1) \cdot \Pr(T_1|N)\} \ldots \Bigg\}\Bigg\},$$

---

on top of the next page. Relation (2.14) is the probability that exactly k stages of cooperative broadcast protocol will be sufficient to deliver the source message to all destinations. The summation conditions c_i are identical to (2.6).

Different from (2.5), in (2.14) the probability $\Pr(S_i|N-T_{i-1})$ denotes the chance of getting exactly S_i successful nodes as a result of i-th stage of cooperative broadcasting given $N - T_{i-1}$ remaining receivers. The case of $i = 1$ corresponds to non-cooperative transmission by the source, which has been analysed in previous section.[1] The probabilities $\Pr(S_i|N - T_{i-1})$ will be estimated in the following subsection.

2.4.2 Estimation of $\Pr(S_i|N - T_{i-1})$ for cooperative broadcasting

At the i-th stage of cooperative broadcasting, the source message is retransmitted by all successful receivers, originated in $(i - 1)$ previous transmission stages. We are interested in the event when exactly S_i of the receivers successfully receive the message while $(N - T_{i-1} - S_i)$ do not.

Following the assumptions in Section 2.2.2, any receiver can be treated as a reference point of a BPP of transmitters, containing T_{i-1} nodes. As each receiver is restricted to processing signals only from the nearest transmitter, we would like to find corresponding distribution of SNR under the joint effect of fading and path loss to the nearest transmitter. Associated difficulty is that the BPP process of the transmitters becomes anisotropic once the observation point is shifted from the origin of a circular cell. However, the focus in this section will be on the isotropic scenario with the reference point located at the origin, which will give an approximation of performance, keeping derivations feasible. In this way, for a given broadcasting stage i, the distributions of distances from any receiver to respective nearest transmitter can be assumed to be i.i.d. Then the probability $\Pr(S_i|N - T_{i-1})$ of getting exactly

[1] It has to be mentioned that there is a non-zero probability that a number of source transmissions will not reach any of the receivers, meaning that cooperative stage cannot start. Equation (2.14) accounts for such events, with the probability of throttle transmissions $\Pr(S_i|N - 0)$ equivalent to the probability of getting zero successful nodes in a non-cooperative scenario $\Pr(S_1 = 0|N)$.

S_i successful nodes given $N - T_{i-1}$ remaining receivers can be expressed as

$$\Pr(S_i|N - T_{i-1}) = \binom{N - T_{i-1}}{S_i} P_{\mathcal{S}}^{S_i} \left(1 - P_{\mathcal{S}}\right)^{N - T_{i-1} - S_i}, \tag{2.15}$$

where $P_{\mathcal{S}} = \Pr\left(\frac{|h|^2}{1+r^\alpha} \geq \theta\right)$ is the probability of successful communication for a transmitter-receiver pair. An estimate for $P_{\mathcal{S}}$ can be obtained as follows.

Proposition 2.4.1 (Probability of success)**.** *Under an isotropic BPP assumption, the probability of successful communication between a receiver and its nearest transmitter under the joint effect of Rayleigh fading and path loss of* $l(r) = (1 + r^\alpha)^{-1}$ *for* $\delta = \frac{d}{\alpha} = 1$ *is*

$$\begin{aligned} P_{\mathcal{S}} &= \Pr\left(\frac{|h|^2}{1 + r^\alpha} \geq \theta\right) \\ &= e^{-\theta} T! \left(\sum_{i=0}^{T-1} \frac{(-1)^i}{(\theta R^d)^{i+1} (T - 1 - i)!} - \frac{(-1)^{T-1} e^{-\theta R^d}}{(\theta R^d)^T} \right). \end{aligned} \tag{2.16}$$

Proof. The proof has two main logical steps. First, the result, originally reported in [36] for the distribution of distances to points of a BPP, is extended to cover the description of the probability density function of α-th powers of distances. Next, the latter is used to derive the probability of success of communication of a node with the nearest transmitter, taking into account both Rayleigh fading and path loss via a compound random variable.

General distribution of distances and α-th powers of distances

We start with a general BPP, i.e. with the reference point located arbitrarily (eg. Figure 1 in [36]). Under the reference point we understand a receiver, and the points of the BPP are the T_{i-1} transmitters. Let us denote T_{i-1} as T for brevity.

Let r_n denote the random distance from a reference point x to n-th nearest neighbor, then, conditioned on having exactly T nodes, the complementary cumulative distribution function (CCDF) of r_n is [36]

$$\bar{F}_{r_n}(r) = \sum_{i=0}^{n-1} \binom{T}{i} p^i (1 - p)^{T-i}, \tag{2.17}$$

where p is the probability that a node falls into a subset B of the observation window W. In case $B = b_d(x, r)$ is a d-dimensional ball with radius r and centered at x, p can be expressed in general in terms of counting measures as [39, p.24]

$$p = p(x, r) = \frac{\Lambda(B)}{\Lambda(W)} = \int\limits_{b_d(x,r) \cap W} f(r) \mathrm{d}r, \tag{2.18}$$

where $f(r)$ is the common probability density function of i.i.d. nodes in W, which can be found as

$$f(r) = \frac{\lambda(r)}{\Lambda(W)} = \frac{dr^{d-1}}{R^d}, r \in [0, R]. \tag{2.19}$$

In case of the nearest neighbor, the CCDF and probability density function (PDF) of r_1 can be found from (2.17) as

$$\bar{F}_{r_1}(r) = (1-p)^T, \tag{2.20}$$
$$f_{r_1}(r) = T(1-p)^{T-1}\frac{\mathrm{d}p}{\mathrm{d}r}. \tag{2.21}$$

With the assumption of an isotropic BPP and the observation point located in the origin o, the intersection of $b_d(o, r)$ and W coincides with the area of $b_d(o, r)$. Therefore p can be expressed as

$$p = p(0, r) = \frac{\Lambda(b_d(o, r))}{\Lambda(W)} = \left(\frac{r}{R}\right)^d \tag{2.22}$$

Substituting this into (2.20) yields

$$\bar{F}_{r_1}(r) = (1 - \left(\frac{r}{R}\right)^d)^T = \frac{1}{R^{dT}}\left(R^d - r^d\right)^T, \tag{2.23}$$
$$\begin{aligned} f_{r_1}(r) &= T(1-p)^{T-1}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{r}{R}\right)^d \\ &= \frac{T}{R^{dT}}\left(R^d - r^d\right)^{T-1} dr^{d-1}. \end{aligned} \tag{2.24}$$

Using derived distributions property [40, p.208] PDF of r_1^α can be expressed as

$$f_{r_1^\alpha}(y) = \frac{\delta T}{R^{dT}}\left(R^d - y^\delta\right)^{T-1} y^{\delta-1}, y \in [0, R^\alpha]. \tag{2.25}$$

The joint distribution

The cumulative distribution function of the compound RV $\frac{|h|^2}{1+r_1^\alpha}$ can now be obtained since each individual distribution is known:

$$\begin{aligned} \Pr\left(\frac{|h|^2}{1+r_1^\alpha} \geq \theta\right) &= \int_0^{R^\alpha} f_{r_1^\alpha}(y) \int_{\theta(1+y)}^{\infty} f_{|h|^2}(x)\mathrm{d}x\mathrm{d}y \\ &= \frac{\delta T e^{-\theta}}{R^{dN}} \int_0^{R^\alpha} \left(R^d - y^\delta\right)^{T-1} y^{\delta-1} e^{-\theta y}\mathrm{d}y. \end{aligned} \tag{2.26}$$

For the special case of $\delta = 1$, we get $p = y/R^d$ and

$$\Pr\left(\frac{|h|^2}{1+r_1^\alpha} \geq \theta\right) = \frac{Te^{-\theta}}{R^{dT}} \int_0^{R^\alpha = R^d} \left(R^d - y\right)^{T-1} e^{-\theta y} \mathrm{d}y. \tag{2.27}$$

Substituting $R^d - r = x$ we obtain

$$\begin{aligned}\Pr\left(\frac{|h|^2}{1+r_1^\alpha} \geq \theta\right) &= \frac{Te^{-\theta}}{R^{dT}} \int_{x=R^d-0}^{x=R^d-R^d} x^{T-1} e^{-\theta(R^d - x)} (-\mathrm{d}x) \\ &= \frac{Te^{-\theta} e^{-\theta R^d}}{R^{dT}} \int_0^{R^d} x^{T-1} e^{\theta x} \mathrm{d}x,\end{aligned} \tag{2.28}$$

Using [41, p.176 5.1.2.1.6] the solution of the integral can be found as

$$\begin{aligned}&\Pr\left(\frac{|h|^2}{1+r_1^\alpha} \geq \theta\right) \\ &= \frac{Te^{-\theta}}{R^{dT}} \left(\frac{e^{-\theta(R^d-x)}}{\theta} \sum_{i=0}^{T-1} \frac{(-1)^i}{\theta^i} \frac{(T-1)!}{(T-1-i)!} x^{T-1-i}\right)\Bigg|_{x=0}^{x=R^d} \\ &= e^{-\theta} T! \left(\sum_{i=0}^{T-1} \frac{(-1)^i}{(\theta R^d)^{i+1} (T-1-i)!} - \frac{(-1)^{T-1} e^{-\theta R^d}}{(\theta R^d)^T}\right).\end{aligned} \tag{2.29}$$

□

Combination of (2.16), (2.15) and (2.14) gives the expected number of transmissions required to reach all nodes using cooperative broadcast protocol.

2.5 Numerical results and discussion

The main goals of this chapter were to develop methods applicable to performance analysis of cooperative and non-cooperative broadcast scenarios, and to attempt understanding whether node cooperation offers any benefits to broadcasted message delivery. Utilising results developed in previous sections and simulations, this section quantifies the performance of cooperative and non-cooperative broadcast protocols in terms of the number of retransmissions required to deliver a message to all nodes in the coverage area.

2.5.1 Numerical results

In order to make the numerical study specific, the broadcast message is assumed to be delivered over a single subcarrier in an LTE system with a 20 MHz channel bandwidth.

In particular, a base station or an access point is assumed to be transmitting with the total power P_{tx}. Corresponding signal and noise powers can be found as

$$P_{\text{tx},s} = P_{\text{tx}} - 10\log_{10}(N_{\text{sc}}) = P_{\text{tx}} - 30.8,$$
$$P_{\text{n}} = -174 + 10\log_{10}(\Delta f) = -132.24,$$

where the units are dBm[2], $N_{\text{sc}} = 1200$ is the number of subcarriers in the 20 MHz LTE channel, and $\Delta f = 15000$ Hz is the bandwidth occupied by a single subcarrier.

As discussed in Section 2.2, small scale Rayleigh fading and path loss are assumed to affect the transmission. For simulations in this section free space path loss model is assumed, with path loss $l(r_{ij})$ expressed as

$$l(r_{ij}) = kr_{ij}^2, \; k = \left(\frac{4\pi f_c}{c}\right)^2$$

where f_c is the carrier frequency, c is the speed of light and r_{ij} is the length of the path between the transmitter i and receiver j. With the carrier frequency set as $f_c = 2.6$ GHz , the coefficient $k = 1.18 \cdot 10^4 \approx 10^4$. Converting above parameters into linear scale, and assuming the target spectral efficiency $\mathcal{R}$ is 1 bit/s/Hz, the decoding threshold θ can be obtained as

$$\theta = \frac{2^{\mathcal{R}} - 1}{P_{\text{tx,s}}/\kappa P_{\text{n}}}, \tag{2.30}$$

so that the outcome of the transmission depends on the realisation of the Rayleigh fading process through the channel coefficient $|h_{ij}|^2$, weighted by the distance r_{ij} between the transmitter and receiver.

Figure 2.2 illustrates the expected number of required transmissions to complete a message broadcast in the cases of cooperative and non-cooperative transmission protocols for node density $\lambda \approx 100$ nodes per sq. km. Simulation results for non-cooperative broadcast are shown to match tightly the analytical calculations, verifying the accuracy of the developed model. For cooperative broadcast, analytical results report lower expected system delay compared to simulations. This mismatch is understood to be caused by the assumption used in calculation of parameter p in (2.22) that the process of transmitters is isotropic with respect to any receiver. In reality, receivers closer to cell boundary have larger expected distance to the nearest transmitter, which would translate into a larger outage probability, especially for lower transmission power values. Precise account for such edge effects is a long-standing problem, and will be discussed in more detail in Section 4.3.

Figure 2.2 shows that cooperative broadcast achieves lower transmission delay

[2]Decibel to milliwatt, $P_{\text{W}} = 10^{0.1(P_{\text{dBm}}-30)}$

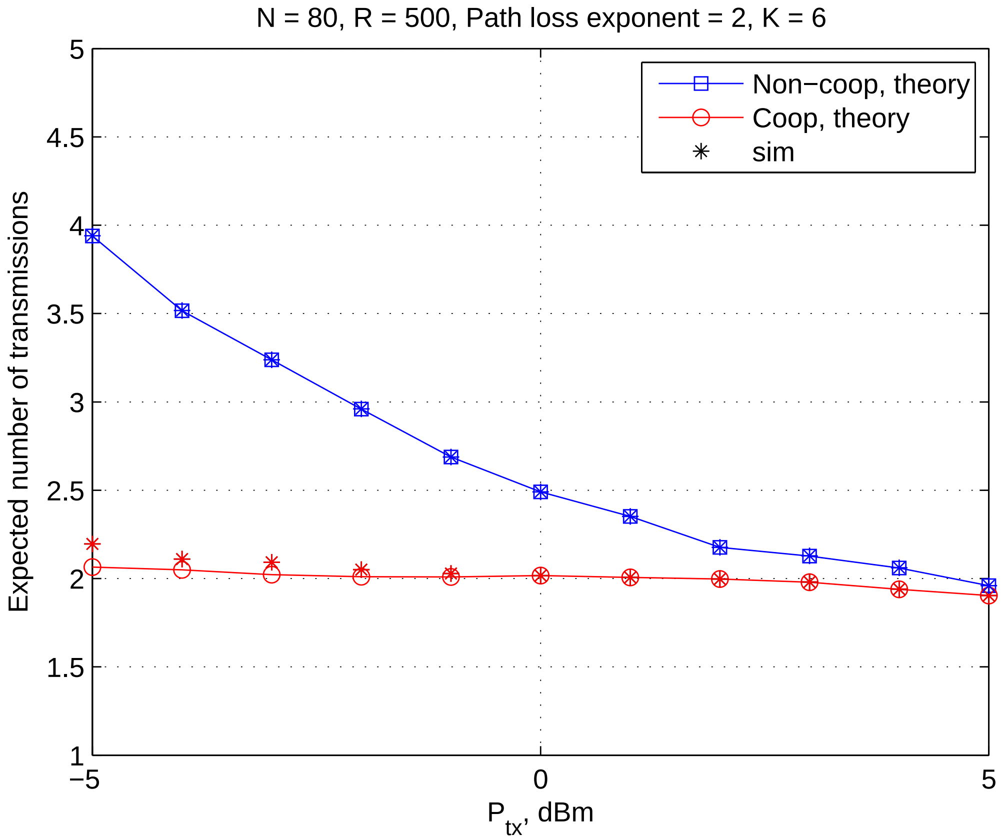

Figure 2.2: Theoretical and simulated system latency.

compared to the non-cooperative scenario, especially for lower transmission powers. Specifically, for the selected parameter set, the broadcasted message is delivered in up to 15% fewer attempts compared to the non-cooperative scenario.

Figures 2.3 and 2.4 illustrate the dependency of broadcast latency on network properties for a wider range of system parameters. Transmitter power level was set to 10 dBm and the number of nodes N in the system was calculated as $N = \lceil \lambda \pi R^2 \rceil$ for different combinations of the node density $\lambda \in [100, 1500]$ nodes per sq. km, and the cell radius $R \in [100, 1000]$ m.

It is evident from Figure 2.3 that performance of the non-cooperative scheme is primarily affected by the cell size, and the node density has a relatively weaker effect. In contrast, results for the cooperative scenario in Fig. 2.4 indicate that the impact of network size on system latency diminishes with the increase of node density. Specifically, subject to sufficiently high node density, approximately two iterations of cooperative broadcasting become sufficient to reach all nodes in the network for a wide range of cell sizes. These results suggest that cooperative schemes can be particularly effective in delivering the broadcast messages in geometrically large networks with high node densities.

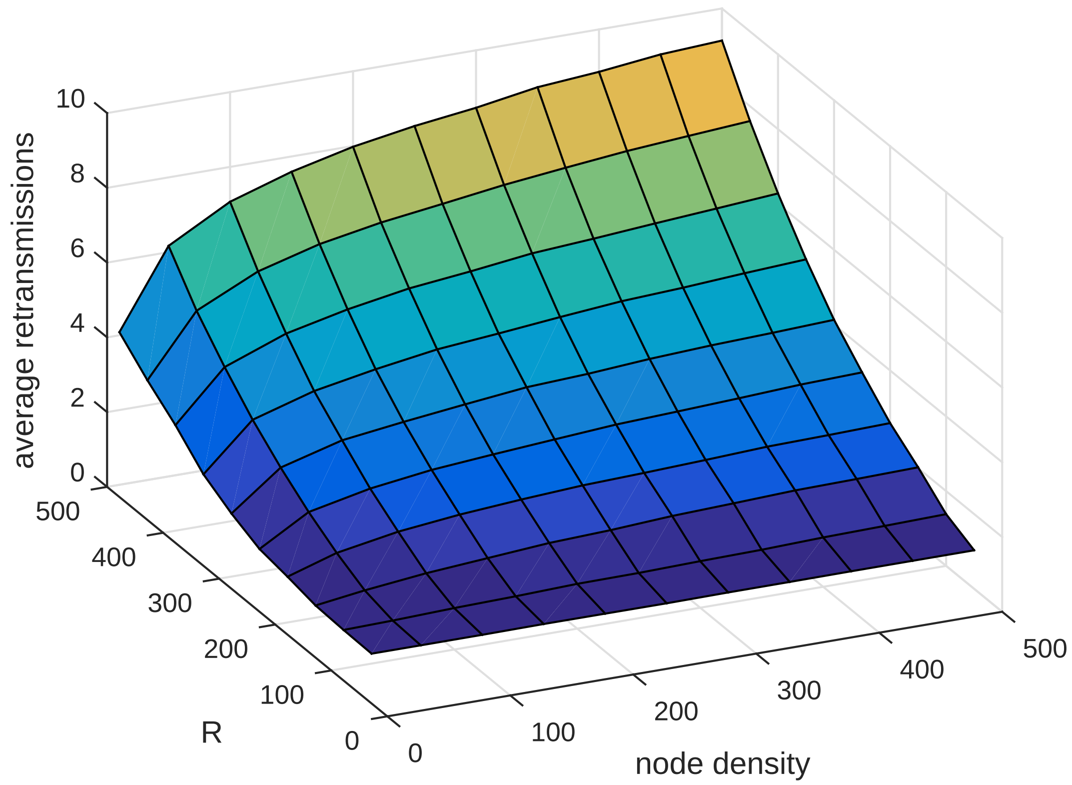

Figure 2.3: Average number of transmissions required to reach all nodes in the cell utilising non-cooperative broadcasting. The number of required transmissions is directly proportional to the node density and cell size.

2.5.2 Complexity analysis

Presented analytical results are not in closed form and require multiple iterations of computations. This subsection provides an estimate of computational complexity required to find the expected number of transmissions $\bar{K}$ using (2.4). Note that complexities for cooperative and non-cooperative schemes differ only in calculation of the exact number of successful nodes after a transmission attempt using (2.10) or (2.15). Therefore, the complexity order is estimated based on the non-cooperative case.

First, to obtain $\bar{K}$, summation of infinite number of terms is required in (2.4). However, the number of terms, contributing significantly to $\bar{K}$ can be limited by some threshold $K_{\max}$, as in (2.4). In the following, the number of operations X_1 required to evaluate (2.4) as a function of the number of nodes N and the threshold $K_{\max}$ will be estimated.

Expression (2.5) is a summation of all possible outcomes of $k \in [1, K_{\max}]$ transmissions leading to delivery of broadcasted message to all N nodes. Using methodology in [42, p.43], one can find that the number of such combinations is $C(N,k) = \frac{(N+k-2)!}{(N-1)!(k-1)!}$. Each summation term in (2.5) consists of a product of k probabilities, calculated us-

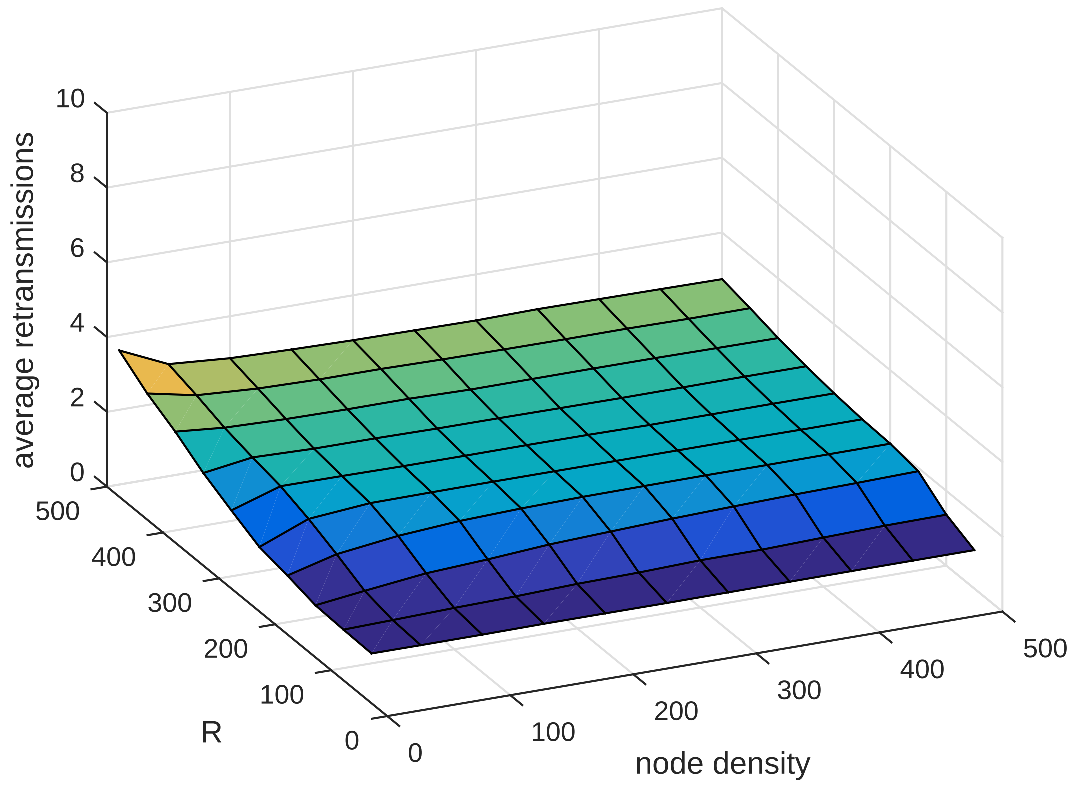

Figure 2.4: Average number of transmissions required to reach all nodes in the cell utilising cooperative broadcasting. The number of required transmissions decreases to approx. 2 with increasing node density.

ing (2.10). Let X denote the number of operations (i.e. additions or multiplications) required to evaluate (2.10). Then, each term of summation in (2.5) would need X^k operations, and evaluation of (2.5) would take $X^k \cdot C(N,k)$ such operations. Finally, the number of operations required to evaluate (2.4) can be found as

$$X_1 = \sum_{k=1}^{K_{\max}} X^k \cdot C(N,k) + \epsilon, \tag{2.31}$$

where ϵ is the total number of intermediate multiplications and additions involved in (2.4) and (2.5). Since order of complexity is determined by the highest-order term [43] we can ignore ϵ and express the order of complexity as

$$\mathcal{O}\left(X^{K_{\max}} C(N, K_{\max})\right). \tag{2.32}$$

Such complexity order limits the application of this type of analysis to large values of system parameters, however for finite networks with fixed and small number of nodes, calculations can be done in realistic times.

2.6 Conclusion

The aim of this chapter was to analyse the impact of node cooperation on information broadcasting with an explicit account for the spatial distribution of cooperating nodes. Results of the analysis and simulations indicate that the number of required retransmissions decreases with increasing node density for cooperative broadcast. This effect is opposite for non-cooperative broadcasting, which is also significantly affected by the network physical dimensions. This allows to conclude that cooperative broadcast can help reduce the time required to deliver a broadcasted message in scenarios where the node density is relatively large.

The main contributions of this chapter were in the development of required analysis methodology, and in the derivation of new relations that are applicable to the analysis of a wider range of problems involving interactions between spatially-distributed nodes. Results have demonstrated that developed relations precisely describe the latency of non-cooperative broadcast and provide a lower bound for latency of cooperative broadcast.

While broadcasting remains an important component of many communication systems, point-to-point information delivery is required to distribute personalised services, such as bi-directional communications or file downloads. The next chapter investigates a technique that utilises the broadcasting nature of wireless transmission, yet optimises the cooperative retransmission stage that follows the source node's message transmission.

Chapter 3

Relay selection

In cooperative wireless networks broadcasting and unicasting differ with the desired network operation following the source transmission because cooperation with relays located between the source and the intended destination is likely to result in a better system performance than cooperation with an arbitrarily chosen relay. Therefore, unlike the point-to-multipoint scenario studied in the previous chapter, different relays have different value in the case of point-to-point information delivery. Performance analysis of a technique that exploits such a difference through relay selection is the subject of this chapter.

3.1 Introduction

Relays have been utilised in communications in different forms and technologies, with the general goal to connect a source to a destination that is unreachable with a direct link for a variety of technical or economical reasons [44]. The concept of relays in wireless communications has gained a particular attention in the past decade within the area of cooperative communications [4]. The basic principle of cooperative relaying is based on the fact that a transmitted signal may be overheard by many nodes on its way to destination. Then the chances of a successful retransmission from a relay node closer to the destination may be higher when compared to the retransmission from the distant original source. In addition, efficient coding and processing at the relay(s) and destination can be employed to extract additional benefits from the redundant transmissions.

The concept of relays is included in the 3GPP Long Term Evolution specification, although focussing on dedicated relays only [6]. Opportunistic relaying, where the relays can also be recruited from a general population of transceivers, is widely studied in academic literature and feature in a number of recent patent applications, e.g. [45–47].

Achieving the gains made available by cooperative relaying is associated with significant challenges, such as the design of efficient coordination and communication protocols [1]. One of the principal questions is on the selection of relays that will assist the source in transmitting a message to the destination. This problem is motivated by the fact that although many relay candidates may be available in the network, only some of them can assist efficiently due to random inter-node connectivity conditions, capabilities of relays or route-cost related factors. Therefore, relay selection is required to minimise the use of resources on coordinating and utilising relays that do not improve system performance.

Significant volume of research has been dedicated to the development and analysis of efficient relay selection algorithms, e.g. [12–14, 16]. However a common characteristic of the majority of research on this subject is that path loss is not explicitly accounted for in the analysis.

The importance of inclusion of spatial node distribution in performance analysis of wireless networks in general has been advocated in [35], and specialised to relay selection (RS) in [20–22, 48–50]. The main reason for such an explicit account for spatial properties of a network in scenarios involving cooperative relaying is that inter-node distances contribute to selection decisions, i.e. affect the outcome of implemented relay selection process. Specifically, these distances are subject to being random, as is small-scale fading, because RS is conducted based on the set of existing nodes with arbitrary locations.

The aim of this chapter is in the development of effective methods for assessment of performance limits of relay selection techniques. In particular, the focus of this chapter is on two important cases where the set of candidate relays is either (a) available exclusively to a single source-destination pair, or (b) shared between multiple sources communicating with a common destination.

3.1.1 Related works

The problem of RS with an explicit account for inter-node distances has been considered in a number of works recently [20–22, 48–50].

In [20] uplink communication between a source and a BS was considered in presence of spatially-random decode-and-forward (DF) relays. Mark theory was used to derive the distribution of distance from qualified relays to the BS, which allowed numerical evaluation of communication outage probability. One limitation of the methodology and results presented in [20] is that expressions for outage probability are complicated and require multiple stages of numerical integration. As will be highlighted in Section 3.2.2 of this chapter, and as authors acknowledge in [20, Sec.IV], there is an alternative approach to analysis of this problem, providing simpler results.

Downlink scenario with DF relays was considered in [21], where thinning of point processes was used to obtain properties of the relays that are connected to the BS. However, presented outage probability expressions [21, Eqs.(37,44)] are still complicated, requiring a number of numerical integrations over surfaces. Developed in this chapter approach to performance analysis is also based on thinning operation, as in [21], yet it provides significantly more simple results that reduce to closed-forms in certain cases.

Point process theory has been used in [49] to obtain outage probability expressions have been derived using geometrical constructions relating channel gains to relay positions relative to the source and the destination (biangular coordinate system). Presented in [49] approach resulted in a more intuitive overall outage probability expression compared to [20, 21], however the key component in the main result [49, Eq.(14a)] bears high complexity.

Performance of AF relays has been studied in [22, 50]. In [22] energy-fair relay selection in sensor networks for the case of the destination located in the far-field of the source and cooperating AF relays. Such an assumption allowed authors to treat distances from the relays to the destination as approximately equal, however the relative locations of communicating nodes may not always satisfy this assumption. Reference [50] applies point process theory to the analysis of outage probability in a scenario with AF relays. One general challenge associated with AF relays is that simplification in hardware is unlikely to provide significant overall gains since most relays in practice are likely to be DF with digital signal processing capabilities allowing the decoding and subsequent processing of the received signals [1].

Different from previous works on relay selection in multi-source environment, including [16,17,51], methods presented in this chapter explicitly consider the impact of both small-scale fading and network topology on the outcome of the selection process.

3.1.2 Chapter overview and contributions

This chapter consists of two parts. The focus of the first part is on the case where a single relay-destination pair has an exclusive access to the pool of relays utilising the SC strategy in the network. This scenario can describe downlink, uplink or an ad-hoc communication. The key contribution of the first part of this chapter is in the development of new methodology that enables simple and intuitive analysis of communication scenarios involving selection of spatially distributed relays. Other contributions include derived relations that characterise performance of relay selection.

The second part of this chapter investigates contention between sources for relays. In particular, the case where multiple sources share a common pool of relays to reach a single destination is investigated. Practical examples corresponding to this scenario include uplink in a cellular system or in a sensor network. Main contribution of this part is in characterising the performance limits of relay selection in such a shared environment.

Specific contributions of this chapter can be summarised as follows:

1. Developed methodology for performance analysis of SC is based on the estimation of properties of custom point processes, such as the probability of the process being empty. Such an approach has been shown to result in intuitive results and simple derivations.

2. Derived relations (3.11), (3.15) and (3.16) allow calculation of the average number of qualified relays in the network for general, special-case and asymptotic sets of system parameters, respectively. One important characteristic of presented expressions is that required precision of calculations can be chosen from exact to approximate without changing the overall outage probability expression.

3. Communication outage probability in the case of contention for relays is obtained in Section 3.3. Interesting features of the shared relay pool are highlighted, e.g. that the set of available relays from the BS decoding set is a valuable and limited resource.

Results in this section have been published in part in [29].

3.2 Single source-destination pair

The focus of this section is on the scenario where a single source-destination pair has an exclusive access to the pool of DF relays. Following subsections define network and signal models, develop analytical methodology to obtain outage probability for the considered scenario.

3.2.1 Network and signal models

Consider a scenario where a single source s communicates with a destination d with assistance from a set of idle users acting as relays (Fig. 3.1). Relays are assumed to form a Poisson point process (PPP) $\Phi(W)$ with a uniform intensity function λ in a circular region W with radius R. The region W can represent a single cell in a cellular system or coverage area of an access point (femto, pico or other type of a cell). For compactness the references to cell W will be suppressed in the rest of the chapter.

Interference-free scenario and half-duplex communication are assumed, so that all nodes communicate over orthogonal channels, and nodes cannot transmit and receive at the same time. The direct source-destination link is assumed to be unavailable.

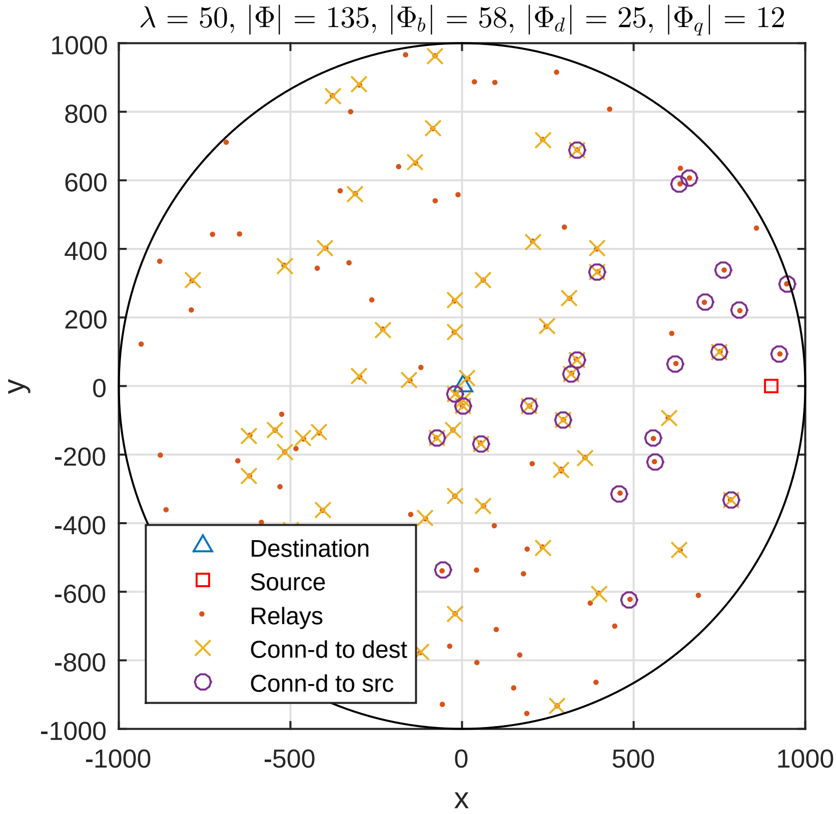

Figure 3.1: Network model: source aims to communicate with destination in presence of a realization of the process Φ of candidate relays. Candidate relay density $\lambda = 50$ nodes per sq. km, with $|\Phi| = 135$ nodes in this realisation, of which $|\Phi_d| = 25$ are connected to the source, $|\Phi_b| = 58$ – to the destination, $|\Phi_q| = 12$ – both to the source and destination.

Cooperative transmission is conducted over two time slots as in [4]. In the first time slot the source broadcasts a message x_s, and each candidate relay $j \in \Phi$ receives

$$y_j = \sqrt{\kappa P_{\text{tx}}} g_{sj} x_s + n_w, \tag{3.1}$$

where P_{tx} is the total available power, $\kappa \in (0, 1)$ is the share of the power P_{tx} allocated for source transmission, n_w is the AWGN with variance σ_n^2, coefficient $g_{sj} = \frac{h_{sj}}{\sqrt{1+r_{sj}^{\alpha}}}$ is the channel coefficient incorporating the small scale Rayleigh fading through $h_{sj} \sim \mathcal{CN}(0, \sigma_h^2)$ and path loss effects through the bounded path loss model $l(r_{sj}) = 1 + r_{sj}^{\alpha}$ with r_{sj} standing for the source-relay distance, and $\alpha \in [2, 6]$ is the path loss exponent. Note that for a given relay j, $g_{sj} \sim \mathcal{CN}\left(0, \eta_{sj}^2\right)$ and has variance $\eta_{sj}^2 = \frac{\sigma_h^2}{1+r_{sj}^{\alpha}}$ [40, p.154].

The relays that receive the source message x_s correctly, form a realization of the decoding set Φ_d:

$$\Phi_d = \left\{ j \in \Phi : |g_{sj}|^2 \geq \frac{\theta}{\kappa} \right\}, \tag{3.2}$$

where $\theta = \frac{2^{2\mathcal{R}-1}}{P_{\text{tx}}/\sigma_w^2}$ and $\mathcal{R}$ is the target spectral efficiency.

Selection cooperation (SC) strategy [12] is considered in this chapter. According to SC, from the set Φ_d of relays that decode the source transmission successfully in the first time slot, one relay J with the best channel gain for the relay-destination channel is selected to forward the message. The selected relay J satisfies the condition

$$J = \arg\max_{j \in \Phi_d} |g_{jd}|^2, \tag{3.3}$$

where $|g_{jd}|^2$ is the channel gain between the relay j and the destination.

The signal model for the transmission from the selected relay J can be written similarly to (3.1) as

$$y_{Jd} = \sqrt{(1-\kappa)P} g_{Jd} x_s + n_w, \tag{3.4}$$

where $1-\kappa$ is the power share allocated to the relay transmission. Relays in the cell W with reliable links both to the source and to the destination form a set of qualified relays:

$$\Phi_q = \left\{ j \in \Phi : |g_{sj}|^2 \geq \frac{\theta}{\kappa}, |g_{jd}|^2 \geq \frac{\theta}{1-\kappa} \right\}, \tag{3.5}$$

where the first condition implies that all qualified relays have to be connected to the source, and the second condition requires the qualified relays to be connected to the destination as well, i.e. $\Phi_q \subseteq \Phi_d$.

In general, outage probability for relay-assisted communication is defined as the probability that the end-to-end SNR γ_J for the source-relay-destination path falls below some predefined threshold θ, i.e. $\Pr(\gamma_J < \theta)$, were J is the index of relay selected for forwarding the source message to the destination [13, 52]. However, a different formulation is possible using point process terminology, as described below.

Clearly, when the relays posses perfect local channel state information (CSI), communication outage is only possible when there are no relays with reliable links both to the source and to the destination, which corresponds to the situation when the point process of qualified relays is empty $\Phi_q = \emptyset$. Therefore, communication outage probability can be related to the properties of the point process of qualified relays. Consequently, for the case of single source-destination pair, the communication outage probability can be defined as

$$P_{\text{out,one}} = \Pr(|\Phi_q| = 0) = \exp(-\Lambda_q), \tag{3.6}$$

where Λ_q is the intensity measure of the process Φ_q of qualified relays, defined in Section 3.2.3. Therefore, in order to quantify outage performance of SC, one needs to be able to characterise the properties of the process of relays that are qualified to assist the source and destination. Following subsection describes thinning operation on point processes – a procedure that will be used for a simple and intuitive derivation of the metrics of interest.

3.2.2 Thinning operation

In its simplest form, thinning is realised by associating each point of a point process with a probability of retention p that is independent of the point's location and of the respective locations of other points in the point process [23]. For example, each point of the parent process Φ could be deleted in a random way with probability $1-p$. However, we are interested in a more advanced type of thinning, termed as $p(r)$-*thinning*, where the probability of retention of a point depends on the location r of this point. It is important to note, that p- and $p(r)$-thinnings of a PPP produce point processes that are still Poisson [23, 53], although such processes may not retain stationarity and/or isotropy properties.

For example, in order to obtain the set of nodes (points) in (3.2) from the original PPP of candidate relays Φ, one can apply a location-dependent thinning $p_d(r_{sj})$ that will select candidate relays from Φ with respect to the connectivity of each relay $j \in \Phi$ to the source s. In particular, relay j located at distance r_{sj} from the source will be retained with probability

$$p_d(r_{sj}) = \Pr\left(\frac{|h_{sj}|^2}{1+r_{sj}^{\alpha}} \geq \frac{\theta}{\kappa}\right) = \exp\left(-\left(1+r_{sj}^{\alpha}\right)\frac{\theta}{\kappa}\right). \tag{3.7}$$

Using this probability of retention, the intensity measure Λ_d of such new thinned process Φ_d can be found from the intensity measure Λ of the original point process Φ as [23]:

$$\Lambda_d = \lambda \int_W p_d(w)\Lambda(\mathrm{d}w), \tag{3.8}$$

where $p_d(w)$ is equivalent to $p_d(r_{sj})$ since the unique location w in the region W can be defined in polar coordinates through the distance r_{sj} from the source to relay j and the angle φ between some reference direction and the line connecting s and j. Connectivity between relays and the source is independent of orientation φ, hence the angle φ is omitted from (3.7). Element area $\mathrm{d}w$ of the region W can be represented as $\mathrm{d}w = r_{sj}\mathrm{d}r_{sj}\mathrm{d}\varphi$. The intensity measure Λ_d can be treated as an average number of relays satisfying the condition in (3.7) within certain area W. This metric is not to be confused with the intensity function $\lambda_d(w)$, which denotes the average number of points of the process Φ_d per unit area (length or volume) at location w.

On the other hand, location-dependent thinning $p_b(r_{jd})$ that will retain relays from Φ with respect to the connectivity to the destination can be applied additionally to obtain the PPP Φ_q of relays connected both to the source and to the destination:

$$p_b\left(r_{jd}\right) = \exp\left(-\left(1+r_{jd}^{\alpha}\right)\frac{\theta}{1-\kappa}\right), \tag{3.9}$$

where

$$r_{jd}^2 = r_{sj}^2 + r_{sd}^2 - 2r_{sj}r_{sd}\cos(\varphi). \tag{3.10}$$

Note that the two thinning stages are conditionally independent for any given relay j, and can be applied in an arbitrary order on the original PPP of candidate relays Φ. In fact the same methodology can be applied to uplink or downlink communication.

3.2.3 Outage probability analysis

The expression for outage probability $P_{\text{out,one}}$ is given in (3.6). To estimate $P_{\text{out,one}}$ the intensity measure Λ_q must be found. Following proposition provides a general expression for Λ_q that can be evaluated numerically. Closed form results for special cases are presented and discussed afterwards.

Proposition 3.2.1 (Mean number of qualified relays)**.** *The intensity measure Λ_q of the PPP Φ_q of relays with reliable links both to the source s and destination d can be expressed as*

$$\begin{aligned}\Lambda_q = \lambda e^{-\frac{\theta}{\kappa(1-\kappa)}} \int_0^R \int_0^{2\pi} r_{sj} \exp\left(-\frac{\theta r_{sj}^\alpha}{\kappa}\right) \\ \times \exp\left(-\frac{\theta\left(r_{sj}^2 + r_{sd}^2 - 2r_{sj}r_{sd}\cos(\varphi)\right)^{\frac{\alpha}{2}}}{1-\kappa}\right) \mathrm{d}r_{sj}\mathrm{d}\varphi,\end{aligned} \tag{3.11}$$

where λ is the intensity function of the process Φ of all candidate relays, $\theta = \frac{2^{2\mathcal{R}}-1}{P_{tx}/\sigma_w^2}$, P_{tx} is the total transmission power, κ and $1-\kappa$ are respectively the source and relay power shares, $\varphi \in [0, 2\pi)$ is the angle between the relay j and the destination, r_{sj} and r_{sd} are distances from the source to the relay j and the destination respectively, and R is the cell radius.

Proof. Following the logic of SC strategy, we will first apply the first thinning stage with retention probability defined in (3.7) to obtain the process of relays connected to the source (the decoding set Φ_d), and then apply the second stage described in (3.9). Then the mean number of relays that are retained after these two thinning stages can be found as

$$\begin{aligned}\Lambda_q &= \int_W p_q(w)\Lambda_d(\mathrm{d}w) \\ &\overset{a}{=} \lambda \int_0^R \int_0^{2\pi} r_{sj} p_q(r_{sj},\varphi) p_d(r_{sj}) \mathrm{d}r_{sj}\mathrm{d}\varphi,\end{aligned} \tag{3.12}$$

where $\Lambda_d(\mathrm{d}w)$ is the mean number of relays in the decoding set in a small region $\mathrm{d}w$ of the cell W, and in step (a) $\Lambda_d(\mathrm{d}w) = \lambda_d(w)\cdot \mathrm{d}w = \lambda p_d(w)\mathrm{d}w$ was used [23].

Note that the probability of retention $p_b(r_{sj},\varphi)$ in (3.12) is equivalent to $p_b(r_{jd})$ in (3.9) with the distance r_{jd} represented as in (3.10). Then by substituting (3.7) and

(3.9) into (3.12) we obtain the result in (3.11). Outage probability $P_{\text{out,one}}$ is then obtained from (3.6). □

Numerical evaluation of (3.11) can be easily accomplished in Matlab. Following corollaries provide closed-form solutions for special cases of system parameters[1].

Corollary 3.2.2 (Special case of $\alpha = 2$)**.** *For the special case of the path loss exponent $\alpha = 2$ the intensity measure of the process Φ_q can be expressed as*

$$\begin{aligned}\Lambda_q(\lambda, 2, R) = &\frac{\kappa(1-\kappa)\pi\lambda}{\theta}\exp\left(-\frac{\theta}{\kappa(1-\kappa)} - \theta r_{sd}^2\right)\\ &\times\left(1 - Q_1\left(r_{sd}\sqrt{\frac{2\kappa\theta}{1-\kappa}}, R\sqrt{\frac{2\theta}{\kappa(1-\kappa)}}\right)\right),\end{aligned} \tag{3.13}$$

where $Q_1(\cdot,\cdot)$ is the first-order Marcum-Q function [54, 55].

Proof. For the case of $\alpha = 2$ the relation (3.11) can be rewritten as

$$\begin{aligned}\Lambda_q(\lambda, 2, R) = &2\pi\lambda e^{-\frac{\theta\left(\kappa^{-1} - r_{sd}^2\right)}{1-\kappa}}\\ &\times\int_0^R r_{sj} e^{-\frac{\theta r_{sj}^2}{\kappa(1-\kappa)}} I_0\left(\frac{2\theta r_{sj} r_{sd}}{1-\kappa}\right) \mathrm{d}r_{sj},\end{aligned} \tag{3.14}$$

where I_0 is the zeroth order Bessel function and [56, 3.364.2] was applied. Introducing a change of variables $r_{sj} = t\sqrt{\frac{\kappa(1-\kappa)}{2\theta}}$ in the last integral and after some algebra (3.13) can be obtained. □

Corollary 3.2.3 (High total power P_{tx})**.** *For the special case of high total power P_{tx} and path loss exponent $\alpha = 2$, the intensity measure of the process Φ_q can be approximated as*

$$\begin{aligned}\Lambda_q(\lambda, 2, R) \approx &\left(\frac{\kappa(1-\kappa)\pi\lambda}{\theta} - \pi\lambda\left(1 + \kappa(1-\kappa)r_{sd}^2\right)\right)\\ &\times\left(1 - e^{-\frac{\theta R^2}{\kappa(1-\kappa)}}\right),\end{aligned} \tag{3.15}$$

Proof. In the case of high transmission power, the decoding threshold $\theta \to 0$, so that the argument in the exponent in (3.13) and (3.16) becomes small. Hence using Taylor expansion and relation $Q_1(0, x) = e^{-\frac{x^2}{2}}$ [55] (3.13) can further approximated as (3.16). □

Corollary 3.2.4 (Special case of $\alpha = 2$ and $R \to \infty$)**.** *For the special cases of path*

[1]The notation $\Lambda(\lambda, \alpha, R)$ is used further to specify the special case parameter chioces

loss exponent $\alpha = 2$ and $R \to \infty$ the intensity measure of the process Φ_q simplifies to

$$\Lambda_q(\lambda, 2, \infty) = \frac{\kappa(1-\kappa)\pi\lambda}{\theta}\exp\left(-\frac{\theta}{\kappa(1-\kappa)} - \theta r_{sd}^2\right). \tag{3.16}$$

Proof. The proof is obtained similarly to Corollary 3.2.2 using [56, 6.614.1]. □

Note that the effect of limited cell size is captured in (3.13) through the Macrum-Q function, which is not involved in (3.16) where the cell size and supply of candidate relays are unlimited.

3.2.4 Illustration of intensity measures

This subsection provides a discussion on the behaviour of intensity measures considered in this section. Fig. 3.2 illustrates the intensity measures Λ_d and Λ_q of respective PPs as functions of the total transmission power P_{tx} for a cell with radius $R = 2000$m and the destination positioned at distance $r_{sd} = 1600$m from the source node. Power allocation coefficient $\kappa = 0.5$, i.e. source and relay transmissions are conducted with equal power. The impact of other possible choices of the power allocation coefficient κ is illustrated and discussed in Section 4.6. Clearly, the mean number of relays connected to the source Λ_d is no less than the mean number of relays that are also connected to the destination Λ_q across all P_{tx} values. This follows the intuition that additional conditions imposed on qualification of a relay for Φ_q will result in a lower number nodes in the point process, compared to the number of elements in Φ_d with milder qualification conditions.

One interesting observation is that although $\Lambda_d \geq \Lambda_q$ on Fig. 3.2 for smaller transmission power values, the ratio Λ_d/Λ_q tends to 1 when the P_{tx} increases, which is due to the fact that for sufficiently high P_{tx} and finite R it is likely that all relays will be connected both to the source and to the destination. Specifically, for the given simulation setup, both intensity measures converge approximately to $125 = \lambda\pi R^2$, where λ was chosen to be 10 nodes per square km.

Finite value of R used in this experiment also allows the illustration of applicability of the result in Corollary 3.2.4 for infinite R. Specifically, the approximation tightly matches simulation results for transmit power values less than ≈ 6 dBm for the selected set of parameters, after which the deviation begins due to the discussed inability of the approximation in (3.16) to capture the convergence of intensity measure Λ_q for $R < \infty$. However, exact analytical results perfectly match simulations, as can be seen from Fig. 3.2.

Outage probability results for the case of exclusive access to the pool of relays will be presented in Section 3.4 together with the results for the case of shared pool of relays studied in the next section.

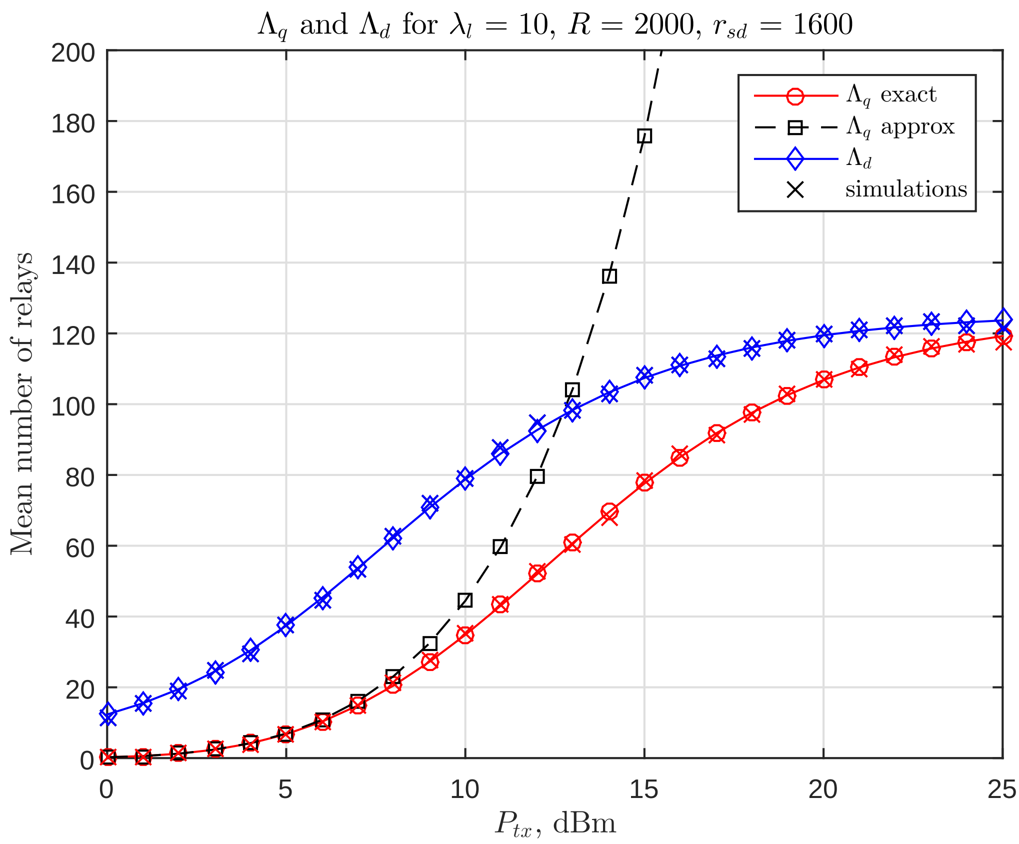

Figure 3.2: Intensity measures Λ_d and Λ_q of the Poisson point processes Φ_d and Φ_q respectively.

3.3 Shared relay pool

This section investigates communication outage probability in the case where multiple sources have access to the shared pool of relays to deliver a message to a single destination. Since there is a large number of possible user-relay scheduling combinations, the focus of this section will be on the performance a transmitter that is the last to access the pool of available relays. Combined with the results from Section 3.2, results from this section will provide performance bounds for the SC strategy.

3.3.1 System model and problem formulation

Consider a scenario as in Section 3.2.1, but with the difference that the source node s is now accompanied by a stationary Poisson point process Φ_s of rival source nodes with density λ_s as shown on Fig. 3.3. All sources contend for the shared resource of candidate relays Φ, however a *necessary* condition for a relay j to be qualified as a partner for any source in W is that $j \in \Phi$ must be connected to the destination. The set of relays connected to the destination is equivalent to the decoding set Φ_d discussed in Section 3.2.1 with the difference in the direction of communication. The

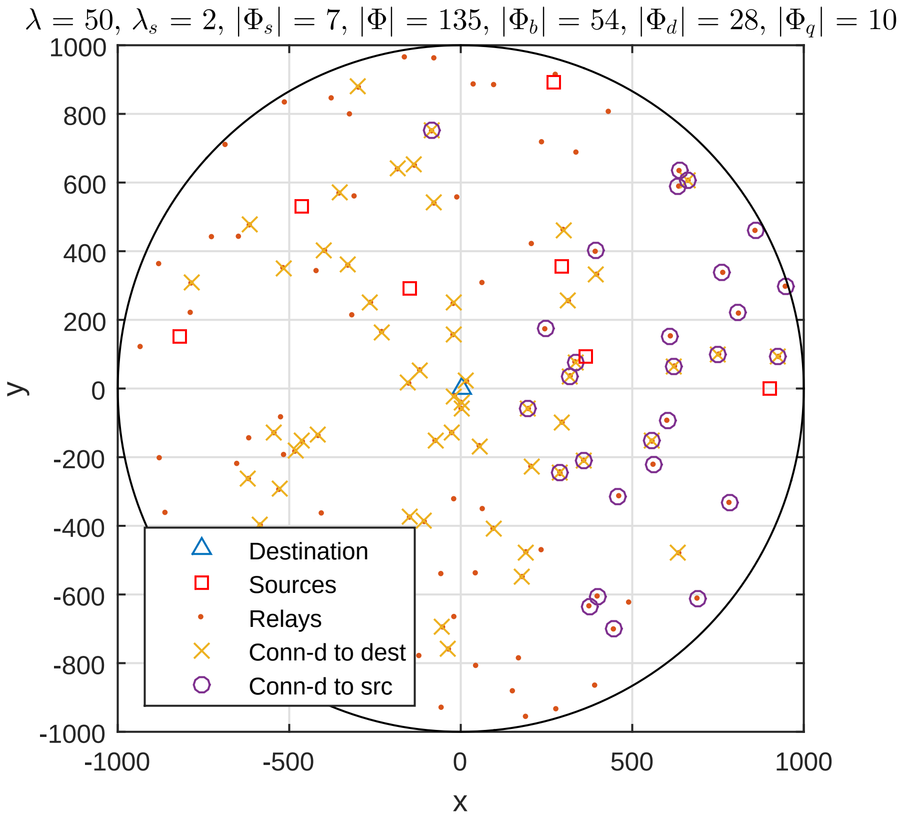

Figure 3.3: Network model: source aims to communicate with destination in presence of a realization of the process Φ of candidate relays and rival sources Φ_s. Notation is similar to the one used in Fig. 3.1

.

process of relays connected to the destination will be referred to as Φ_b:

$$\Phi_b = \left\{ j \in \Phi : \frac{|h_{jd}|^2}{1 + r_{jd}^{\alpha}} \geq \theta \right\}, \tag{3.17}$$

where h_{jd} is the small-scale fading coefficient between the relay and the destination, r_{jd} is the corresponding distance, and θ is the threshold for correct decoding.

Presented below are two key assumptions that will be used in derivation of the outage probability for the case when the source node s is the last to be assigned a relay:

A1 Firstly, all previous sources in the queue for relays are assumed to be assigned one relay. In other words it is assumed that none of the rival sources in Φ_s is in outage, and therefore each source Φ_s consumes one relay from Φ_b. In practice, some of the sources may be in outage, or may not need a relay to reach the destination. However, listing all possible combinations becomes restrictive in terms of computation and analysis.

A2 Secondly, it is further assumed that the effect from the presence of the process of sources Φ_s can be described by random deletion of $S = |\Phi_s|$ relays from the process of relay nodes connected to the destination Φ_b. This is motivated by the coupling between individual partner assignment decisions which heavily reduces tractability.

Effects these assumptions have on the accuracy of theoretical results will illustrated and discussed in Section 3.4.

3.3.2 Outage event and outage probability definition

Based on the discussion above, we are interested in the probability of outage for the cooperative transmission between the source and the destination separated by distance r_{sd} through a relay selected from the pool of candidate relays Φ. Communication outage occurs either when the set of qualified relays $\Phi_q = \emptyset$ for the considered source-destination combination, or when all qualified relays in Φ_q have been assigned to assist other sources. Associated outage event $\mathcal{A}$ can be expressed as

$$\mathcal{A} = \underbrace{(b \leq s)}_{\mathcal{A}_1} \cup \underbrace{(q = 0|b > s)}_{\mathcal{A}_2} \cup \underbrace{(\Phi_q \subseteq \Xi|b > s, q \neq 0)}_{\mathcal{A}_3}, \tag{3.18}$$

where b is the number of relays connected to the destination in a given network realisation, s is the number of rival sources and q is the number of points in the process Φ_q of qualified relays for the reference source, and $\Xi \subseteq \Phi_b$ is the set of relays that have been assigned to sources in Φ_s. Fig. 3.4 depicts the relationships between point processes involved in (3.18).

Fig. 3.5 illustrates the components outage the event defined in (3.18). Note that the events $\mathcal{A}_i, i \in \{1, 2, 3\}$ are mutually exclusive, therefore the outage probability $P_{\text{out,mult}}$ can be described as

$$P_{\text{out,mult}} = \Pr(\mathcal{A}) = \Pr(\mathcal{A}_1) + \Pr(\mathcal{A}_2) + \Pr(\mathcal{A}_3). \tag{3.19}$$

Probabilities for each component of $\mathcal{A}$ will be derived in the following.

Event $\mathcal{A}_1$

The event $\mathcal{A}_1$ can be interpreted as the situation when there are less relays connected to the destination than there are sources, i.e. that the cardinality of the set $|\Phi_b| = b$ is less than or equal to the cardinality of the set $|\Phi_s| = s$:

$$\Pr(\mathcal{A}_1) = \Pr(b \leq s) = \Pr(k \geq 0), \tag{3.20}$$

where $k = s - b$ is the difference between realizations of two Poisson random variables S and B. Such random variable $K = k$ is Skellam-distributed [57] with the PDF for

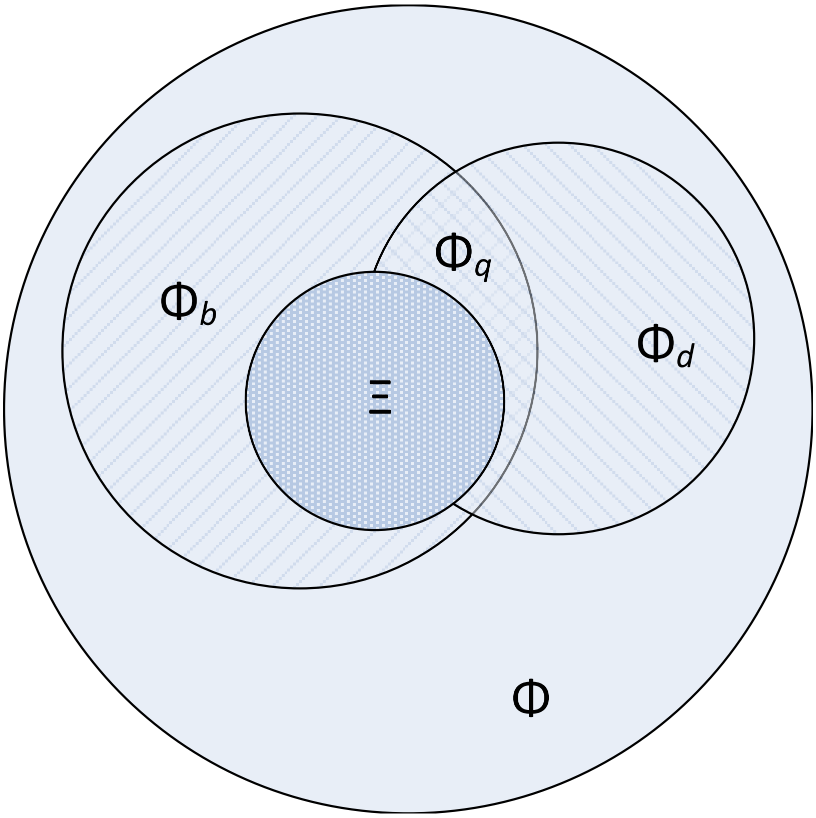

Figure 3.4: Possible realisation of relationships between point processes. Processes Φ_q and Φ_d are unique for each source in the system.

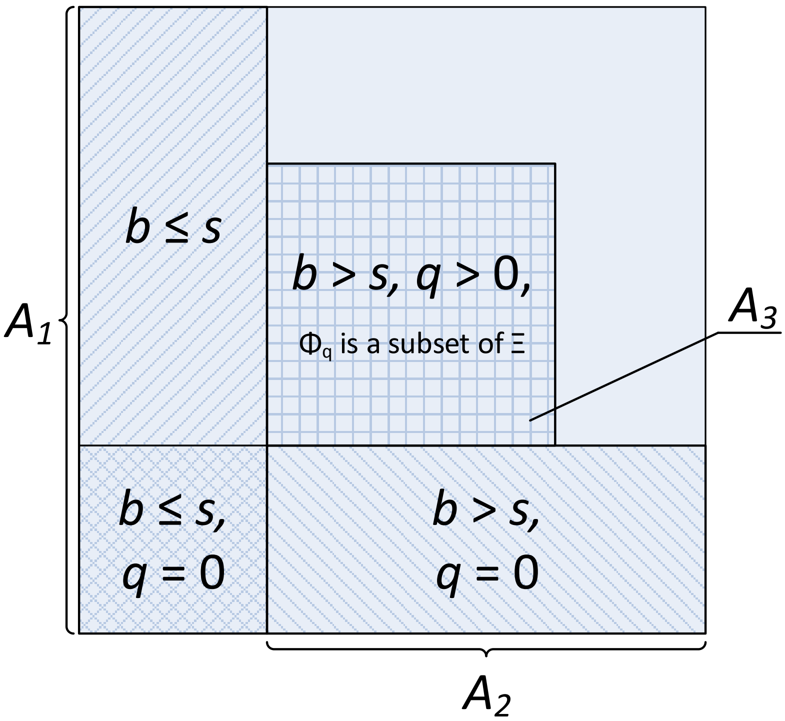

Figure 3.5: Illustration of outage event breakdown for the multi-source scenario. The whole square represents the space of possible outcomes for the transmission, with shaded areas denoting outage events associated with conditions for $\mathcal{A}_1$, $\mathcal{A}_2$ or $\mathcal{A}_3$, and the plain area standing for the successful transmission.

$k \in (-\infty, \infty)$ given as

$$\begin{aligned} f(k; \Lambda_s, \Lambda_b) &= \Pr(K = k) \\ &= e^{-(\Lambda_s+\Lambda_b)} \left(\frac{\Lambda_s}{\Lambda_b}\right)^{\frac{k}{2}} I_{|k|}\left(2\sqrt{\Lambda_s \Lambda_b}\right), \end{aligned} \tag{3.21}$$

where $I_n(\cdot)$ is modified Bessel function of n-th order. For the event $\mathcal{A}_1$ we are interested in the values of $k \in [0, \infty)$, so we can rewrite $\Pr(\mathcal{A}_1)$ as

$$\Pr(\mathcal{A}_1) = e^{-(\Lambda_s+\Lambda_b)} \sum_{k=0}^{\infty} \left(\frac{\Lambda_s}{\Lambda_b}\right)^{\frac{k}{2}} I_{|k|}\left(2\sqrt{\Lambda_s \Lambda_b}\right). \tag{3.22}$$

The mean measure Λ_s for the process of sources in the cell W with radius R is

$$\Lambda_s(W) = \int_W \lambda_s \mathrm{d}w = \lambda_s \pi R^2. \tag{3.23}$$

The mean measure Λ_b for the process of relay nodes connected to the destination can be obtained using $p_b(r_j)$ thinning as

$$\begin{aligned} \Lambda_b &= \lambda_l \int_W e^{-\theta(1+r_{jd}^2)} \mathrm{d}w = \lambda_l \int_0^R \int_0^{2\pi} r_{jd} e^{-\theta(1+r_{jd}^2)} \mathrm{d}r_{jd} \mathrm{d}\varphi \\ &= 2\pi\lambda_l \int_0^R r_{jd} e^{-\theta(1+r_{jd}^2)} \mathrm{d}r_{jd} \mathrm{d}\varphi = \frac{\lambda_l \pi}{\theta} e^{-\theta}\left(1 - e^{-\theta R^2}\right), \end{aligned} \tag{3.24}$$

With (3.23) and (3.24) one can easily find $\Pr(\mathcal{A}_1)$ from (3.22).

Event $\mathcal{A}_2$

Event $\mathcal{A}_2$ represents a situation when the reference source has no cooperative connections with the destination, given that there are more relays connected to the destination than there are sources. In other words the event $\mathcal{A}_2$ occurs when there are relays available to support the cooperative transmission by other sources in Φ_s, but none of them has a reliable link to the source s.

Note that the process of qualified relays Φ_q is not applicable to the analysis of this case since the number of relays connected to the destination is now capped by b, which is in contradiction with the properties of PPPs. Instead, let Ψ_q denote the binomial point process of relays chosen from the set of $|\Phi_b| = b$ nodes connected to the destination. Then the probability of the event $\mathcal{A}_2$ can be expressed as

$$\begin{aligned}\Pr\left(\mathcal{A}_2\right) &= \Pr\left(|\Psi_q| = 0 | b > s\right) \\ &= \sum_{b=1}^{\infty} \Pr\left(|\Psi_q| = 0 || \Phi_b| = b\right) \Pr\left(|\Phi_b| = b\right) \sum_{s=0}^{b-1} \Pr\left(|\Phi_s| = s\right).\end{aligned} \tag{3.25}$$

The probabilities $\Pr\left(|\Phi_b| = b\right)$ and $\Pr\left(|\Phi_s| = s\right)$ can be obtained using the standard PDF for Poisson-distributed random variables in [23]. The probability $\Pr\left(|\Psi_q| = 0 || \Phi_b| = b\right)$ can be interpreted as the frequency of the event that the BPP Ψ_q is empty conditioned on having exactly b relays connected to the destination. Note that this probability is independent of the number of sources in the network s, however s is included in the equation (3.25) due to the condition $b > s$.

Specifically, the probability $\Pr\left(|\Psi_q| = 0 || \Phi_b| = b\right)$ of getting exactly 0 relays connected both to the source and destination, given b relays in Φ_b, is:

$$\Pr\left(|\Psi_q| = 0 | \Phi_b = b\right) = \left(1 - p_x\right)^b, \tag{3.26}$$

where p_x is the probability of successful communication between a node in Φ_b and the reference source. Following proposition derives p_x applying thinning operation to a BPP, in a similar way it is applied to PPPs.

Proposition 3.3.1 (Thinning a BPP). *The conditional probability of success p_x in communication between a source and the destination through a randomly chosen cooperative relay $j \in \Phi_b$, connected to the destination is,*

$$p_x = \frac{\theta e^{-\theta(1+r_{sd}^2)}}{\pi\left(1 - e^{-\theta R^2}\right)} \int_0^{R^2} e^{-2\theta t} J_0\left(2\theta r \sqrt{t} i\right) \mathrm{d}t, \tag{3.27}$$

where θ is the threshold for correct decoding, $r \in [0, R]$ is the location of the source node and $J_0(\cdot)$ is zeroth order Bessel function. For $R \to \infty$ can be approximated as

$$p_x \approx \frac{1}{2} e^{-\theta\left(1 + r_{sd}^2/2\right)}. \tag{3.28}$$

Proof. The probability of successful communication between the source s and the relay j can be written as

$$p_x = \Pr\left(\frac{|h_{sj}|^2}{1 + r_{sj}^2} \geq \theta\right) = e^{-\theta} \mathbb{E}_{r_{sj}}\left(e^{-\theta r_{sj}^2}\right), \tag{3.29}$$

where the expectation is over the distance r_{sj} between the source s and a relay j.

Distance r_{sj} can be expressed as as

$$r_{sj}^2 = r_{jd}^2 + r_{sd}^2 - 2r_{jd}r_{sd}\cos(\varphi), \tag{3.30}$$

where the distance r_{jd} between the relay j in Φ_b and the destination treated as a random variable; r_{sd} is the distance between the source and the destination treated as constant, and φ is the angle between the source and the relay. We can therefore express p_x as

$$\begin{aligned} p_x &= e^{-\theta}\mathbb{E}_{r_{jd},\varphi}\left(e^{-\theta r_{sj}^2}\right) = e^{-\theta}\mathbb{E}_{r_{jd},\varphi}\left(e^{-\theta\left(r_{jd}^2+r_{sd}^2-2r_{jd}r_{sd}\cos(\varphi)\right)}\right) \\ &= e^{-\theta(1+r_{sd}^2)}\int\limits_{\varphi=0}^{\varphi=2\pi}\int\limits_{r_{jd}=0}^{r_{jd}=R} e^{-\theta\left(r_{jd}^2-2r_{jd}r_{sd}\cos(\varphi)\right)} f_{r_{jd}}(r_{jd})\cdot f_{\varphi}(\varphi)\,\mathrm{d}r_{jd}\mathrm{d}\varphi, \end{aligned} \tag{3.31}$$

where $f_{\varphi}(\varphi) = 1/2\pi$ is the density function of a uniformly distributed angle, and $f_{r_{jd}}(r_{jd})$ is the density function of distance from the BS to points in Φ_b. We now need to find $f_{r_{jd}}(r_{jd})$. As in [35]

$$f_{r_{jd}}(r_{jd}) = \frac{\lambda_b(r_{jd})}{\Lambda_b(W)}. \tag{3.32}$$

We know from (3.24) that $\Lambda_b(W) = \pi\lambda_b\frac{e^{-\theta}}{\theta}\left(1-e^{-\theta R^2}\right)$. Thus

$$\begin{aligned} \lambda_b(r_{jd}) &= \frac{\mathrm{d}\Lambda_b(r_{jd})}{\mathrm{d}r_{jd}} = \pi\lambda_b\frac{e^{-\theta}}{\theta}2\theta r_{jd}e^{-\theta r_{jd}^2} \\ &= 2\pi\lambda_l r_{jd}e^{-\theta(1+r_{jd}^2)}, \end{aligned} \tag{3.33}$$

therefore

$$\begin{aligned} f_{r_{jd}}(r_{jd}) &= \frac{2\pi\lambda_l r_{jd}e^{-\theta(1+r_{jd}^2)}}{\pi\lambda_l\frac{e^{-\theta}}{\theta}\left(1-e^{-\theta R^2}\right)} \\ &= \frac{2\theta r_{jd}e^{-\theta r_{jd}^2}}{1-e^{-\theta R^2}}. \end{aligned} \tag{3.34}$$

After substitution,

$$p_x = \frac{\theta e^{-\theta(1+r_{sd}^2)}}{\pi\left(1-e^{-\theta R^2}\right)}\int\limits_{r_{jd}=0}^{r_{jd}=R} r_{jd}e^{-2\theta r_{jd}^2}\int\limits_{\varphi=0}^{\varphi=2\pi} e^{-2\theta r_{jd}r_{sd}\cos(\varphi)}\mathrm{d}\varphi\mathrm{d}r_{jd}, \tag{3.35}$$

where the inner integral is equivalent to the one solved in Corollary 3.2.2, which after further substitution and change of variables leads to

$$p_x = \frac{2\theta e^{-\theta(1+r_{sd}^2)}}{\left(1-e^{-\theta R^2}\right)}\int\limits_0^{R^2} e^{-2\theta t}J_0\left(2\theta r_{sd}\sqrt{ti}\right)\mathrm{d}t. \tag{3.36}$$

For $R \to \infty$ the probability of success p_x can be approximated as

$$p_x \approx 2\theta e^{-\theta(1+r_{sd}^2)} \frac{1}{4\theta} e^{\frac{\theta r_{sd}^2}{2}} = \frac{1}{2} e^{-\theta\left(1+r_{sd}^2/2\right)}, \tag{3.37}$$

□

Substitution of the result for p_x into (3.26) gives us the desired probability of the event $\mathcal{A}_2$.

Event $\mathcal{A}_3$

Finally, outage also occurs when all connections for the reference source become busy because they are being assigned to other sources in the cell. Given the number of rival source nodes s, number relays b connected to the destination, number of connections q for the reference source, and following the assumption A2, one can find the probability that *all* q connections are busy as

$$\Pr\left(\Psi_q \subseteq \Xi | b, q, s\right) = \frac{\binom{b-q}{s-q}}{\binom{b}{s}}, \tag{3.38}$$

where Ξ is the point process of busy relay nodes as in (3.18). The probability of the event $\mathcal{A}_3$ can be then found as

$$\begin{aligned}\Pr\left(\mathcal{A}_3\right) = &\sum_{b=2}^{\infty}\sum_{s=1}^{b-1}\sum_{q=1}^{s} \Pr\left(\Psi_q \subseteq \Xi | b, q, s\right) \Pr\left(|\Psi_q| = q \,|\, |\Phi_b| = b\right) \\ &\times \Pr\left(|\Phi_b| = b\right) \Pr\left(|\Phi_s| = s\right),\end{aligned} \tag{3.39}$$

which can be evaluated using density functions for PPP [23] for $\Pr\left(|\Phi_b| = b\right)$ and $\Pr\left(|\Phi_s| = s\right)$; and $\Pr\left(|\Psi_q| = q \,|\, |\Phi_b| = b\right)$ can be found using the result of Proposition 3.3.1 and a generalised version of (3.26):

$$\Pr\left(|\Psi_q| = q \,|\, |\Phi_b| = b\right) = \binom{b}{q} p_x^q \left(1 - p_x\right)^{b-q}. \tag{3.40}$$

Recall, that our aim in this section was to estimate the probability $P_{\text{out,mult}}$ of the outage event $\mathcal{A}$ for the case when the reference source was the last to be assigned a relay. Having now estimated the probabilities of each of the component events $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$, one can find the desired conditional outage probability P_{shared} for the studied scenario using (3.19). Namely the results for $\Pr\left(\mathcal{A}_1\right)$, $\Pr\left(\mathcal{A}_2\right)$ and $\Pr\left(\mathcal{A}_3\right)$, are given in (3.22), (3.25) and (3.39) respectively.

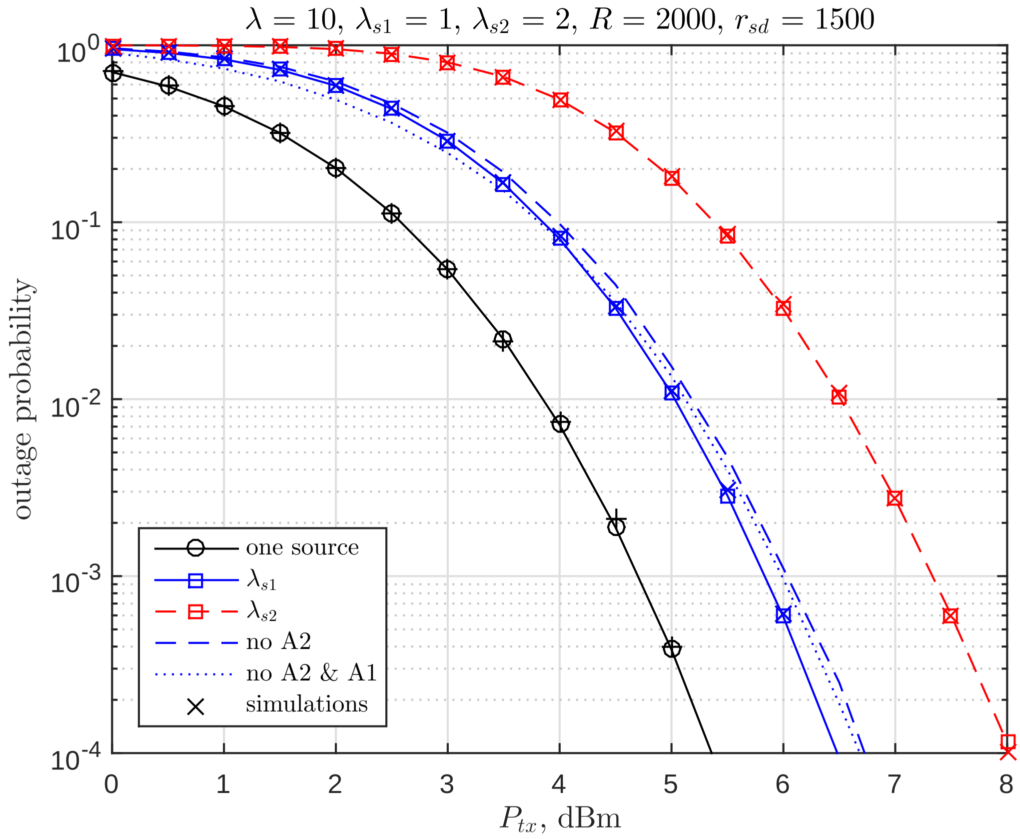

Figure 3.6: Outage probabilities for cooperative communication for the single- and multi-source scenarios. Depending on the density λ_s of source nodes in the network, outage probability increases as more sources compete for the finite number of relays with reliable connections to the destination in the cell centre.

3.4 Results

This section verifies the accuracy of system performance analysis methodology for the single- and multi-source scenarios developed in previous sections, and provides a characterisation of selection cooperation performance in terms of outage probability.

Fig. 3.6 depicts the communication outage probability as a function of transmission power for the cases of exclusive and shared access to the pool of relays connected to the destination. In the case of a single source node, the increase in power budget leads to a larger number of qualified relays in the cell, thereby reducing the probability of outage. The dynamics of the reduction of outage probability in this case can be estimated by taking first derivative of the general outage probability in (3.6). The result of this operation is shown on Fig. 3.7. First, the derivative of outage probability is non-positive, hence the chance of outage never increases as the number of qualified relays grows. In particular, the rate of outage probability decay increases rapidly as first qualified relays become available. However, as more relays become available the amount of additional benefit reduces dramatically.

Multiple sources contending for a shared set of relays negatively affect the per-

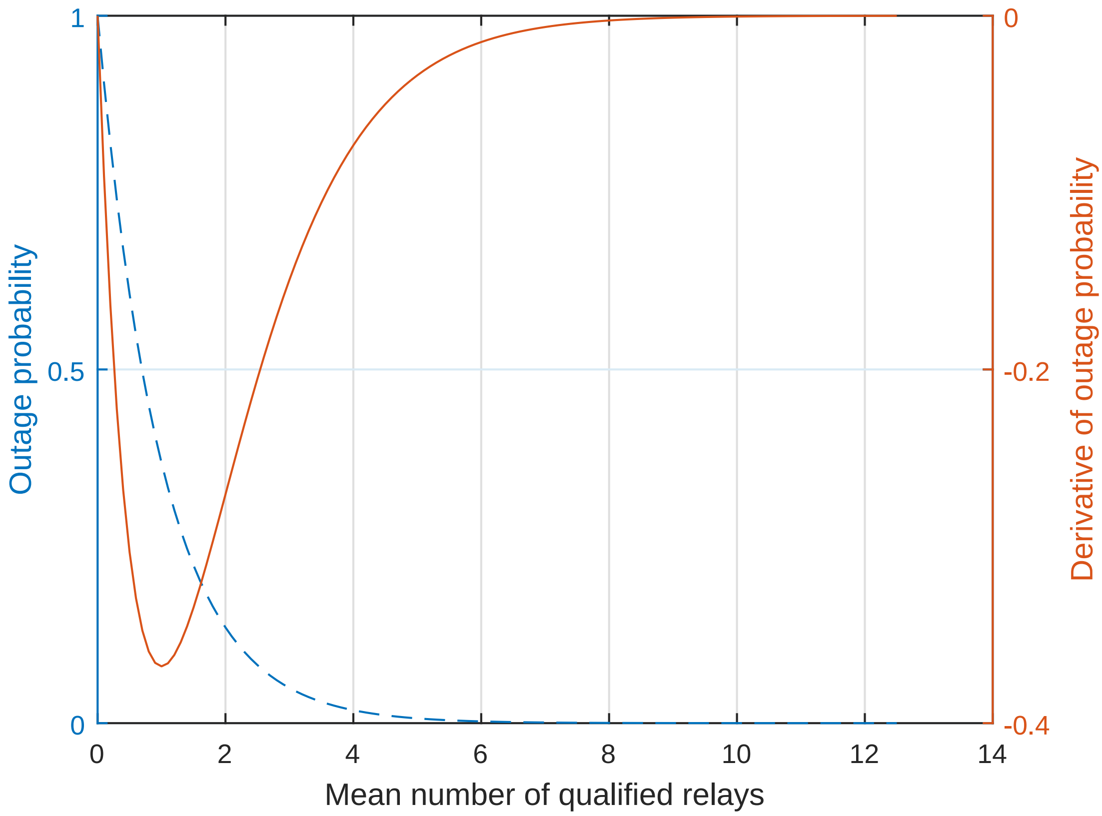

Figure 3.7: Dynamics of outage probability for the case of one source. larger mean number of qualified relays never makes harm in terms of outage probability, however additional benefits from every extra qualified relay diminish.

formance of the worst-case source. Blue and red curves on Fig. 3.6 illustrate the dependency of outage probability on transmission power budget when the process Φ_s of contending source nodes in the system has intensity $\lambda_{s1} = 1$ or $\lambda_{s2} = 2$ nodes per sq. km. As expected, the shared access to the cooperative resource results in a higher outage probability, with the degradation becoming more pronounced for the larger source intensity λ_{s2}. Specifically, the presence of rival sources with λ_{s1} results in ≈ 1 dB performance degradation compared to the exclusive access to relays, whereas the source intensity λ_{s2} causes the loss of ≈ 2.5 dB.

Theoretical results are in good agreement with conducted simulations, which verifies the accuracy of presented analytical approach. Fig. 3.6 includes additional simulation results for a system with relaxed assumptions A1 and A2. In particular, the assumption A2 was removed first, and the deletion of one relay per source was explicitly modelled. Second, assumption A1 was also removed, such that if a rival source is in outage no relay node is consumed. As follows from Fig. 3.6, the assumption A2 leads to an approximately 0.25dB more optimistic performance results compared to the explicit relay deletion model. The assumption A1 affects the results at lower transmission power levels, where the chance of a source outage is higher. Specifically, simulation results without the assumption A1 show a marginally better performance

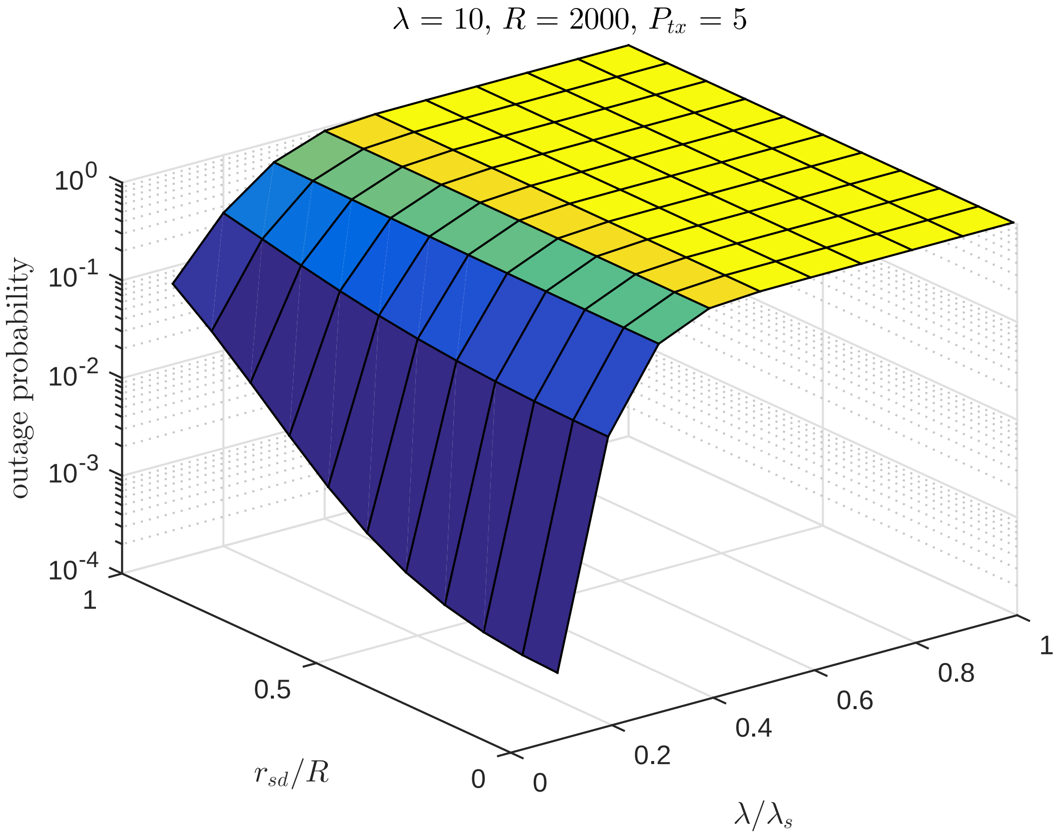

Figure 3.8: Outage probability as a function of the reference source location r_{sd} and the rival source intensity λ_s.

at low P_{tx} since some of the rival sources are in outage. It is also interesting to note that results for the cases with the two assumptions relaxed converge at high transmission power budget levels since there are fewer source nodes in outage.

Next set of results on Fig. 3.8 illustrates the dependence of the outage probability on the source location r_{sd} and on the ratio λ_s/λ. Fig. 3.8 was obtained using analytical results from Sections 3.2 and 3.3, specifically, surface points in Fig. 3.8 were calculated using (3.19) by varying the parameters λ_s and r_{sd} for a fixed transmission power P_{tx} of 5 dBm, $R = 2000$m and $\lambda = 10$ nodes per sq.km. The figure clearly shows that while for the case of exclusive access (i.e. $\lambda_s = 0$) the source location r_{sd} has significant impact, outage performance for the case of $\lambda_s > 0$ changes dramatically and for $\lambda_s/\lambda > 0.5$ almost no improvement in the worst-case can be achieved by shifting the source towards destination.

3.5 Conclusion

This chapter developed analytical methods for performance analysis of relay selection strategies with an explicit account for the spatial distribution of involved network elements. In particular, the focus was made on the derivation of outage probability expressions for the DF selection cooperation strategy, and covering two distinct cases where the source node (a) had an exclusive access to the pool of relays, or (b) was the last to access the pool of shared relays. Point process theory was used to develop analytical methodology in this chapter.

Results for both cases of access to candidate relays have shown good agreement between the developed methodology and simulations. Developed outage expressions allowed to uncover the non-trivial dynamics of outage probability for the single source case, with additional qualified relay bringing a diminishing return. Presence of additional sources has been observed to increase the chances of outage, especially for the case when the number of source nodes is comparable to the number of candidate relays in the system.

Next chapter extends the results obtained in this chapter to the case where relay selection decisions are made based on imperfect channel state information. Selection decisions based on such imperfect CSI can be suboptimal, which may or may not result in an outage in communication – source-destination communication may still be successful despite erroneous CSI if the chosen relay also happens to have a reliable link to the destination. Methodology to quantify the impact of CSI imperfection on cooperative network performance is developed in the next chapter.

Chapter 4

Relay selection with imperfect channel estimation

Channel estimation in practical communications is never perfect, therefore relay selection algorithms investigated in the previous chapter operate with imperfect CSI in practice. This chapter investigates the impact of different levels of CSI imperfections on communication outage probability in cooperative networks with relay selection.

4.1 Introduction

Relay selection (RS) reduces coordination overhead in cooperative systems with multiple relays while achieving full diversity [1, 10]. The objective of RS is to select one relay with the best channel state to the destination from a set of candidates that have previously decoded the source transmission successfully. With appropriate design of the RS mechanism, same diversity gain as in the case of coordinated all-relay transmission can be achieved [1]. In other words, a system utilising one optimally chosen relay may achieve performance comparable to the performance of a system utilising multiple relays in a coordinated fashion. Therefore, the challenge associated with achieving a better network performance through the use of relays has shifted from the process of coordinating multiple distributed relays to the process of selection of one optimal relay from a group.

RS mechanisms and analysis of their performance are especially important for emerging dense, multi-hop, multi-tier and decentralized network deployments to meet the expected exponential growth in mobile user traffic [58]. Some motivating application scenarios include uplink or downlink of a macro base station assisted by femto-access points, picocells or relays with possibly wireless backhaul. Other applications of relay-assisted source-destination communications include sensor networks or emergency communications

One of the key practical challenges associated with RS protocols is that relay-destination channel quality measurements must be available at the relays or some centralised decision point [12–14, 59, 60]. In practice such measurements are subject to errors, leading to suboptimal selection decisions and to performance loss in the employed RS strategy [52, 61].

The aim of this chapter is to assess outage performance of selection cooperation strategy in a scenario where the channel state information (CSI) used in the selection process is imperfect, while taking into account random spatial distribution of candidate relays.

4.1.1 Related works

The problem of RS based on imperfect CSI has been studied in literature, e.g. [52, 61–65]. Considered imperfection models include feedback delays, where the measured CSI becomes outdated at the selection instant [52,61–64], and noisy estimation where the estimated CSI contains unknown channel noise component [52, 65]. The effect of these imperfections and their combinations on outage probability and diversity order has been analysed extensively [52, 61–65]. However the impact of the spatial randomness of relay positions on RS system performance with imperfect CSI has not been considered to date.

In particular, references [20–22,48–50,66–68] have investigated RS with account for network topology, but assuming perfect CSI. Few available works on random networks with imperfect CSI used in the communication process do not consider RS [69, 70]. In all above works, Poisson point process is used to model node distributions in networks with different degrees of planning, e.g. macro-cellular systems and small-cell networks [58]. While a PPP is only an approximation of the real node deployments, tractable performance analysis is possible in contrast to uniform or regular node placement models, e.g. [71].

4.1.2 Chapter overview and contributions

The aims of this chapter are to (a) assess the impact of the accuracy of available CSI on outage performance of the selection cooperation strategy, and (b) develop appropriate methods for performance analysis. Specifically, two types of imperfect CSI availability are considered in this chapter:

- Only statistical CSI for the relay-destination links is available at the relays;
- Instantaneous local CSI with varying level of accuracy for the relay-destination links.

In both cases relays operate in a distributed fashion, with no information exchange between relays.

The main contribution of this chapter is in the proposed methodology for analysis of RS based on CSI with imperfections, and in the outage probability expressions developed for both cases of imperfect CSI. One key advantage of developed analytical approach is that explicit derivation of distance distributions as in [20, 21] is not required. Specific contributions of this chapter can be summarized as follows:

1. Exact outage probability expressions are derived for the considered cases of imperfect CSI used in the selection of the best relay from the pool of spatially distributed candidates;

2. A simplified analytical approach is demonstrated. Developed method (a) bypasses complicated calculation of distance distributions for the considered scenario, (b) is applicable both to the cases of perfect and imperfect CSI, and (c) allows extensions to different channel imperfection models.

3. Asymptotic analysis is conducted to highlight the impact of system parameters on outage performance at high SNR.

Material in this chapter was published in part in [27, 29].

4.2 System Model

This chapter partially re-uses and extends the network model described in the previous chapter. In summary, communication between a single source-destination pair is considered, with a set of candidate relays Φ available to support the communication process. Process Φ is a Poisson point process with uniform intensity function λ nodes per unit area. All transmitted signals in the network are subject to small-scale Rayleigh fading and propagation path loss.

In the first time-slot the source broadcasts a message, and all relays listen. Relays that are able to decode the message correctly form the decoding set Φ_d defined in (3.2), and run a distributed relay selection algorithm relying only on locally-available measurements. To be specific, this chapter will focus on selection cooperation strategy [12], where from the set Φ_d of relays that decode the source transmission successfully, one relay J that has the best *perceived* channel gain for the relay-destination channel is selected to forward the source's message in the second time slot. The selected relay J satisfies

$$J = \arg\max_{j \in \Phi_d} f\left(|\hat{g}_{jd}|^2\right), \tag{4.1}$$

where $f\left(|\hat{g}_{jd}|^2\right)$ is some function of the perceived channel gain. If the channel estimation is perfect, i.e. $\hat{g}_{jd} = g_{jd},\ \forall j \in \Phi_d$, then the selected relay J indeed has the best channel to the destination among all other relays. However, for the case of non-zero channel estimation error, the relay J with the best estimate does not necessarily have the best channel to the destination.

Following types of CSI will be assumed to be available at the relays in order to select the relay to retransmit the source message in the second time slot:

1. Statistics for CSI for relay-destination channel for each relay, i.e. $f\left(|\hat{g}_{jd}|^2\right) = \mathbb{E}\left\{|\hat{g}_{jd}|^2\right\}$ is known at each relay $j \in \Phi_d$;

2. Imperfect instantaneous CSI $f\left(|\hat{g}_{jd}|^2\right) = |\hat{g}_{jd}|^2$ for the relay-destination channel for each relay with varying degrees of accuracy, so that $\hat{g}_{jd} = g_{jd} + \epsilon$.

Outage probability for relay-assisted communication using the statistical CSI is investigated in Section 4.3. Performance of the case of imperfect instantaneous CSI is studied in Section 4.4.

4.3 Relay selection with statistical CSI

This scenario corresponds to the case when the relays have access to local statistics of the channels to the destination node, however no instantaneous CSI is available. Specifically, the case where each relay knows $\mathbb{E}\left\{|\hat{g}_{jd}|^2\right\}$ is considered. It is assumed that based on such channel statistics all relays are able to estimate the distance from the destination node with the *same level of precision*. Note that no exact distance estimation is required, any technique sufficiently effective to correctly order relays with respect to the channel quality to the destination is acceptable (eg. [22]).

The result of such distance-based ranking is an ordered sequence of relays $\{x_{(1)}, \ldots, x_{(j)}, \ldots, x_{(}$ where the relay $x_{(1)}$ has the shortest distance to the destination, and relay $x_{(J)}$ – the largest. Such distributed ordering formation can be realized via timer-based algorithm.

Generalising the original SC strategy, where one best relay is selected to retransmit the source message, in this section a set of k relays with lowest distance estimates is selected from Φ_d in a distributed fashion to forward the message to the destination. This is done to assess any gains from the retransmission by k nodes with the best perceived channel estimates instead of one.

A number of assumptions will be made in the derivation of the outage probability $P_{\text{out,stat}}$ for the considered scenario regarding the outcome of the source transmission and the properties of the coverage area W. These assumptions are discussed in the following.

Outcome of source transmission

It is assumed that there are always enough relays in the decoding set Φ_d to meet the demand of k relays, i.e. $|\Phi_d| \geq k$. In practice the cardinality of the decoding set $|\Phi_d|$

may be less that the requested number of relays k, or even be 0. However for high SNRs and small k, probability of such event can be shown to be small, while a precise account for $|\Phi_d| < k$ would involve conditioning on a specific outcome of the PPP Φ_d, which in turn would require using analytically more complicated Binomial point processes for the parts of derivation [23].

Extension of W

Edge effects is a long-standing problem in spatial statistics [23, p.132], associated with finite dimensions of the space where realizations of a point process take place. Consider a homogeneous PPP of candidate relays: strictly speaking, the points outside W do not belong to Φ, which is why observations from the origin of W will be different from those from the edge of W. While formally this is a contradiction to one of fundamental properties of a PPP, compensation for these effects increases analytical complexity. Fortunately for our scenario, impact of this formality is expected to be small because the number of points in Φ_d is expected to drop closer to cell edges. Therefore, it is assumed that the process Φ exists in the space beyond W.

Therefore, with the assumption of $|\Phi_d| \geq k$, outage event $\mathcal{A}$ for this scenario can be expressed as

$$\mathcal{A} = \bigcap_{j=1}^{k} (j\text{-th nearest to the destination relay } x_{(j)} \text{ fails}). \tag{4.2}$$

Note that the k components of the set intersection above are mutually independent events, since both fading and placement of one node give no information about fading and placement of another (recall that Φ_j is a Poisson point process). Therefore, overall outage probability can be found as

$$P_{\text{out,stat}} = \Pr(\mathcal{A}) = \prod_{j=1}^{k} P_j, \tag{4.3}$$

where P_j is the probability that j-th nearest to the destination node relay fails:

$$\begin{aligned} P_j = \Pr\left(\frac{|h_{jd}|^2}{1 + r_{jd}^{\alpha}} < \theta_k\right) &= 1 - \mathbb{E}_{r_{jd}}\left\{e^{-\theta_k\left(1+r_{jd}^{\alpha}\right)}\right\} \\ &= 1 - \int_{0}^{R+r_{sd}} e^{-\theta_k\left(1+r_{jd}^{\alpha}\right)} f_k\left(r_{jd}\right) \mathrm{d}r_{jd}, \end{aligned} \tag{4.4}$$

where h_{jd} is the complex baseband Rayleigh channel coefficient, r_{jd} is the distance from the destination to the nearest relay $x_j \in \Phi_d$, and $f_k(r_{jd})$ is the PDF for the distance to the k-th nearest to the destination relay. Using properties of a PPP, the

PDF $f_k(r_{jd})$ can be given as [23]

$$f_k(r_{jd}) = e^{-\Lambda_d^{'}(B)} \cdot \frac{2\left(\Lambda^{'}(B)\right)^k}{r_{jd}\Gamma(k)}. \tag{4.5}$$

Here $\Lambda_d^{'}(B)$ denotes the mean number of points of the PPP Φ_d of relays that can decode the source message inside a region $B \subseteq W$ with radius $r_{jd} \in [0, R + r_{sd}]$, centred at the destination location. Our second assumption is used here, as formally B cannot have circular shape, as Φ_d does not span beyond W.

Both $\Lambda_d^{'}(\cdot)$ and $\Lambda_d(\cdot)$ are mean measures of the same Poisson point process Φ_d with location-dependent intensity function $\lambda(w)$, with the key difference in the position of observation points. Specifically, for $\Lambda_d(\cdot)$, the observation point is located at the source, so that while the resulting PPP Φ_d is inhomogeneous, it is still isotropic with respect to the source. On the other hand, $\Lambda_d^{'}(\cdot)$ measures the number of points of the same process but from the destination's point of view, which makes PPP Φ_d anisotropic from such perspective. Indeed, when observed from the destination, it is more likely to find relays with reliable connections to the BS at angles φ_{jd} pointing towards the source, rather than in the opposite direction.

The mean number $\Lambda_d^{'}(B)$ of relays connected to the destination falling within a circular region B with radius r_{jd} can be expressed in terms of location-dependent, but universal intensity function $\lambda_d(w)$ as

$$\Lambda_d^{'}(r_{jd}) = \int_B \lambda(w)\mathrm{d}w = \int_0^{2\pi}\int_0^{r_{jd}} \lambda_d(r,\varphi) r \mathrm{d}r \mathrm{d}\varphi, \tag{4.6}$$

where r_{jd} is the distance from the destination to a relay j. The intensity function $\lambda_d(w)$ of the process of relays connected to the BS can be expressed as [23]

$$\lambda_d(w) = \lambda p_d(w) = \lambda e^{-\theta(1+r_{sj}^{\alpha})}. \tag{4.7}$$

Using standard trigonometry, we can rewrite $\lambda_d(r_{jd})$ in terms of integration variables $\lambda_d(r,\varphi)$ for $\alpha = 2$

$$\lambda_d(r,\varphi) = \lambda e^{-\theta\left(1+r_{sd}^2+r^2-2r_{sd}r\cos(\varphi)\right)}, \tag{4.8}$$

which after substitution into (4.6) gives

$$\Lambda_d^{'}(r_{jd}) = \lambda e^{-\theta\left(1+r_{sd}^2\right)} \underbrace{\int_0^{2\pi}\int_0^{r_{jd}} r e^{-\theta\left(r^2-2r_{sd}r\cos(\varphi)\right)} \mathrm{d}r\, \mathrm{d}\varphi}_{I}. \tag{4.9}$$

Unlike the estimation of outage probability in Section 3.2, approximation for large r_{jd} is inapplicable to the considered scenario, because we are interested in the behaviour of $\Lambda_d^{'}(B)$, including for small r_{jd} values. For this reason, we first take the inner integral to allow for subsequent numerical evaluation of (4.9) with respect to rotation angle φ around the destination node. After a straightforward but lengthy integration, I can be written as

$$\begin{aligned} I = & \frac{1}{2\theta_k}\left(1 - e^{-\theta_k\left(r_{jd}^2 - a r_{jd}\right)}\right) \\ & + \sqrt{\frac{\pi}{\theta_k}}\frac{a}{4}e^{\frac{a^2}{4}\theta_k}\left(\operatorname{erf}\left(\frac{a}{2}\sqrt{\theta_k}\right) - \operatorname{erf}\left(\frac{a}{2}\sqrt{\theta_k} - \sqrt{\theta_k}r_{jd}\right)\right), \end{aligned} \tag{4.10}$$

where $a = 2r_{sd}\cos(\varphi)$ and $\operatorname{erf}(\cdot)$ denotes error function. It is interesting to note that when the displacement of the observation point $r_{sd} = 0$, i.e. when the locations of the remote observation point and the source coincide, (4.10) reduces to $I = \left(1 - e^{-\theta r_{jd}^2}\right)/2\theta_k$. However for general $r_{jd} \in [0, R + r_{sd}]$, closed form solution of (4.9) can be overly complicated, and numerical solution will be used to obtain outage performance.

In order to illustrate the differences between the mean measures $\Lambda_d^{'}$ and Λ_d, Fig. 4.1 depicts the number of relays in the decoding set Φ_d observed within the distance r from two points: (a) from the source and (b) from the destination located at r_{sd} from the source. Cell radius R was chosen as 3000m, transmission power is 0 dBm, distance $r_{sd} = 2000$m and the free-space propagation path loss model was used. Fig. 4.1 shows that the destination can find a smaller number of relays from the decoding set within the same proximity compared to the source, which is due to exponential decay in the received power as the source-relay distance increases. For example, the destination can expect support from almost no relays within $r \approx 600$ m. As r increases, the circular region around the destination will eventually include all qualified relays, which can be seen from convergence of the curves for larger r.

Outage probability performance of the SC strategy based on statistical CSI is discussed in Section 4.6 utilising the relations (4.9) and (4.10) derived in this section. Next section develops communication outage probability expressions for RS based on more granular CSI yet with certain error component.

4.4 Relay selection with imperfect instantaneous CSI

This section investigates the performance of relay selection strategy in the case when relays have access to instantaneous, yet erroneous CSI.

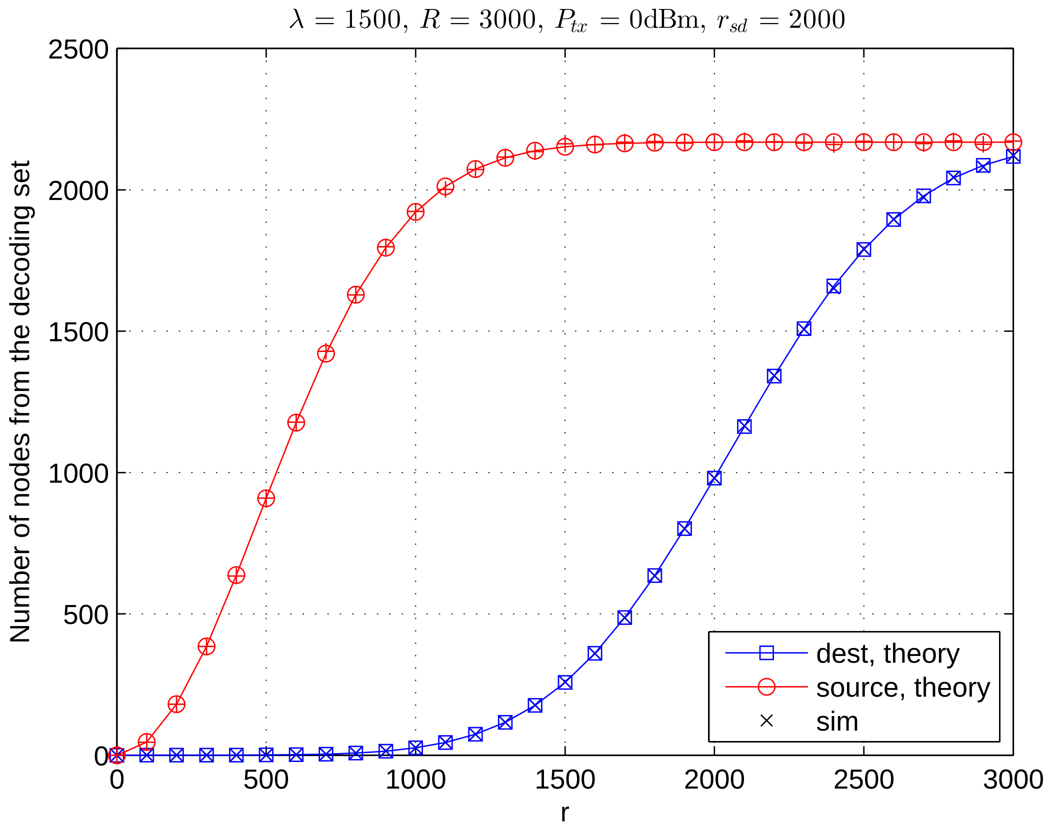

Figure 4.1: Mean number of relays connected to the source as a function of radius r, observed from the source (red circles) and the destination at r_{sd} (blue squares).

4.4.1 Channel estimation error model

Following the transmission of the source's message in the first time slot, all relays in the decoding set Φ_d run identical procedures based on the respective perceived channel gains $\hat{g}_{jd}$ at each relay j in order to select one relay to retransmit the message to the destination. Required CSI estimates can be obtained as the destination broadcasts a training sequence before the relay selection process takes place. An MMSE estimate of the channel can be expressed as [72–74]

$$\hat{g}_{jd} = g_{jd} + \epsilon, \tag{4.11}$$

where $\epsilon \sim \mathcal{CN}\left(0, \sigma_\epsilon^2\right)$ is the estimation error component. In order to focus on the system performance in the case of imperfect CSI, and to make the discussion specific, channel estimation error model from [72–74] is used, rather than the actual channel estimation process (e.g. MMSE).

As in [73], σ_ϵ^2 is assumed to be given a priori, through, for example, channel estimation using training sequence. Two simple models for estimation error variance σ_ϵ^2 are employed [73, 74]: (a) SNR-independent model where σ_ϵ^2 is selected as a constant, and (b) a relay transmission power-dependent model $\sigma_\epsilon^2 = \sigma_u^2 + \sigma_n^2 f(P_{\text{tx}})$. In the latter

σ_u^2 can be treated as a prediction error due to time variability of the channel, σ_n^2 is the measurement error caused by AWGN, and $f(P_{\text{tx}})$ is a function of the transmission power used in estimation process.

4.4.2 Outage probability formulation

For the case of selection cooperation, outage probability is typically defined as the conditional probability that for a given size of decoding set Φ_d, the channel gain $|g_{Jd}|^2$ for the chosen relay-destination channel falls below a threshold θ, averaged over all possible sizes of the decoding set [20, 50, 52]:

$$\begin{aligned} P_{\text{out,imp}} &= \Pr\left(\Phi_d = \emptyset\right) + \sum_{l=1}^{\infty} \Pr\left(|\Phi_d| = l\right) \Pr\left(|g_{Jd}|^2 < \frac{\theta}{1-\kappa} ||\Phi_d| = l\right) \\ &= \Pr\left(\Phi_d = \emptyset\right) + \Pr\left(\mathcal{O}|\Phi_d \neq \emptyset\right) \approx \Pr\left(\mathcal{O}\right), \end{aligned} \tag{4.12}$$

where the approximation in the last step is made for sufficiently high source transmission power, such that $\Pr\left(\Phi_d = \emptyset\right) \to 0$, and the event $\mathcal{O}$ can be defined as

$$\begin{aligned} \mathcal{O} &= \left(\nexists j : |g_{sj}|^2 > \frac{\theta}{\kappa}, |g_{jd}|^2 > \frac{\theta}{1-\kappa}, |\hat{g}_{jd}|^2 = \max_{i \in \Phi_d} |\hat{g}_{id}|^2\right) \\ &= \left(\exists j : |g_{sj}|^2 > \frac{\theta}{\kappa}, |g_{jd}|^2 < \frac{\theta}{1-\kappa}, |\hat{g}_{jd}|^2 > \max_{i \in \Phi_q} |\hat{g}_{id}|^2\right) \end{aligned} \tag{4.13}$$

where the first step corresponds to the event that there is no relay with a reliable connection both to the source and the destination, and with the channel estimate $|\hat{g}_{jd}|^2$ largest across the whole decoding set Φ_d. The second step corresponds to the event that there exists a relay with reliable connection to the source, but not to destination, yet with the channel estimate larger than any estimate at the relays in the set of qualified relays Φ_q.

Let $\Lambda_q\left(\lambda, \alpha, R\right)$ be the intensity measure of the process Φ_q, which we will denote as Λ_q for compactness. Further, let $\hat{\Lambda}_q(x)$ be the intensity measure of the process $\hat{\Phi}_q(x)$ of such relays in Φ_q that also have the estimation function $|\hat{g}_{jd}|^2 > x$, where $x \in [0, \infty)$. Note that $\hat{\Lambda}_q(x)$ is a decreasing function of x. Similarly, let Λ_u be the intensity measure of the process Φ_u of relays that have reliable connections to the source but *not* to the destination, so that $\hat{\Lambda}_u(x)$ is the intensity measure of relays in the process $\hat{\Phi}_u(x)$ that also have the function $|\hat{g}_{jd}|^2 > x$, where $x \in [0, \infty)$. Table 4.1 summarizes the properties of the point processes processes used above.

Then the probability of outage for the source-destination communication via the set of candidate relays can be expressed as in the following proposition.

Proposition 4.4.1 (Outage probability formulation)**.** *For sufficiently high transmission power, so that* $\Pr\left(\Phi_d = \emptyset\right) \to 0$*, the outage probability for SC strategy based on*

Table 4.1: Point processes and their properties

Notation	Explanation	Relay j qual. if
Φ	All candidate relays	Always
Φ_d	Relays with reliable connections to source (**decoding set**)	$\lvert g_{sj}\rvert^2 \geq \frac{\theta}{\kappa}$
Φ_q	Relays with reliable connections both to source and destination (**qualified set**)	$\lvert g_{sj}\rvert^2 \geq \frac{\theta}{\kappa}$, $\lvert g_{jd}\rvert^2 \geq \frac{\theta}{1-\kappa}$
$\hat{\Phi}_q(x)$	Relays with reliable connections both to source and destination, and with channel estimate $\lvert\hat{g}_{jd}\rvert^2$ to destination larger than x	$\lvert g_{sj}\rvert^2 \geq \frac{\theta}{\kappa}$, $\lvert g_{jd}\rvert^2 \geq \frac{\theta}{1-\kappa}$, $\lvert\hat{g}_{jd}\rvert^2 > x$
Φ_u	Relays with reliable connection to source, but not to destination	$\lvert g_{sj}\rvert^2 \geq \frac{\theta}{\kappa}$, $\lvert g_{jd}\rvert^2 < \frac{\theta}{1-\kappa}$
$\hat{\Phi}_u(x)$	Relays with reliable connection to source, but not to destination, and with channel estimate $\lvert\hat{g}_{jd}\rvert^2$ larger than x	$\lvert g_{sj}\rvert^2 \geq \frac{\theta}{\kappa}$, $\lvert g_{jd}\rvert^2 < \frac{\theta}{1-\kappa}$, $\lvert\hat{g}_{jd}\rvert^2 > x$

imperfect CSI can be expressed as

$$P_{out,imp} = 1 + \int_0^\infty \exp\left(-\hat{\Lambda}_u(x) - \hat{\Lambda}_q(x)\right)\hat{\Lambda}_q'(x)\mathrm{d}x, \tag{4.14}$$

where $(\cdot)'$ *denotes first derivative.*

Proof. Outage probability can be expressed as

$$\begin{aligned} P_{\text{out,imp}} \approx \Pr(\mathcal{O}) &= \int_0^\infty \Pr\Big(\mathcal{O}|M=x\Big) f_M(x)\mathrm{d}x \\ &= 1 - \int_0^\infty \Pr\left(\hat{\Phi}_u(x) = \emptyset | M = x\right) f_M(x)\mathrm{d}x, \end{aligned} \tag{4.15}$$

where $\mathcal{O}$ is defined in (4.13), and $M = \max_{j\in\Phi_q}(|\hat{g}_{id}|^2)$ is the maximal value of the function of the channel estimate $|\hat{g}_{id}|^2$ among all qualified relays in the process Φ_q. The estimate $|\hat{g}_{jd}|^2$ at some unqualified relay $j \in \Phi_u$ may still be larger than M because it is a *perceived* channel state at the relay. In other words, the outage event in Proposition 4.4.1 corresponds to the case when there exists at least one unqualified relay, whose perceived estimate of the channel to the destination is greater than any of the perceived estimates at qualified relays, provided the decoding set is non-empty.

The probability density function (PDF) $f_M(x)$ can be written as

$$\begin{aligned} f_M(x) &= \frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(\max_{j\in\Phi_q}\left(|\hat{g}_{id}|^2\right) < x\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(\hat{\Phi}_q(x) = \emptyset\right) \end{aligned} \tag{4.16}$$

The PDF of the outcome of a general Poisson point processes Φ_i [23] is

$$\Pr\left(|\Phi_i| = k\right) = e^{-\Lambda_i}\frac{(\Lambda_i)^k}{k!}. \tag{4.17}$$

Hence substituting (4.16) into (4.15) and invoking (4.17) on both processes $\Phi_q(x)$ and $\Phi_u(x)$, we obtain the result in (4.14) as

$$\begin{aligned} P_{\text{out,imp}} &= 1 - \int_0^\infty \Pr\left(\hat{\Phi}_u(x) = \emptyset|x\right)\frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(\hat{\Phi}_q(x)\right)\mathrm{d}x \\ &= 1 - \int_0^\infty \exp\left(-\hat{\Lambda}_u(x)\right)\frac{\mathrm{d}}{\mathrm{d}x}\exp\left(-\hat{\Lambda}_q(x)\right)\mathrm{d}x \\ &= 1 - \int_0^\infty \exp\left(-\hat{\Lambda}_u(x)\right)\exp\left(-\hat{\Lambda}_q(x)\right)\frac{\mathrm{d}}{\mathrm{d}x}\left(-\hat{\Lambda}_q(x)\right)\mathrm{d}x \\ &= 1 + \int_0^\infty \exp\left(-\hat{\Lambda}_u(x) - \hat{\Lambda}_q(x)\right)\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x)\mathrm{d}x. \end{aligned} \tag{4.18}$$

□

4.4.3 Discussion of the formulation

This section presents an initial asymptotic analysis of the outage probability formulation in (4.14).

First, note that in the case of no estimation error, there are no unqualified relays in Φ_u that have estimates for the channel to the destination larger than any estimate at the relays from the qualified set Φ_q, hence $\hat{\Lambda}_u(x) = 0$ for $\forall x$. Then (4.14) can be rewritten as

$$\begin{aligned} P_{\text{out,imp}} &= 1 + \int_0^\infty \exp\left(-\hat{\Lambda}_q(x)\right)\hat{\Lambda}_q'(x)\mathrm{d}x \\ &\overset{(a)}{=} 1 - \int_0^{\Lambda_q} \exp\left(-t\right)\mathrm{d}t = \exp(-\Lambda_q), \end{aligned} \tag{4.19}$$

where in step (a) integration by substitution was used:

$$\int_{a}^{b} f\left(g\left(x\right)\right) g'\left(x\right) \mathrm{d}x = \int_{g(a)}^{g(b)} f\left(y\right) \mathrm{d}y.$$

This result corresponds precisely to the outage probability for the case of perfect CSI-based RS in (3.6) in Section 3.2.1. The penalty for erroneous relay selection originates from the term $\hat{\Lambda}_u(x)$, which reduces the value of the integrand in (4.14), and consequently increases the outage probability $P_{\text{out,imp}}$.

4.5 Outage probability analysis

In this section outage probability of SC is analysed with account for imperfect channel estimates at randomly distributed relays. Exact expressions that can be evaluated numerically are derived first, followed by a development of asymptotic system performance afterwards.

Outage probability expression (4.14) offers an initial intuition on the impact of important factors on communication outage probability. In the following, exact outage probability expressions will be obtained utilizing (4.14) in the form of

$$P_{\text{out,imp}} = 1 + \int_{0}^{\infty} \exp\left(-\hat{\Lambda}_d(x)\right) \frac{\mathrm{d}}{\mathrm{d}x} \hat{\Lambda}_q(x) \mathrm{d}x, \tag{4.20}$$

where $\hat{\Lambda}_d(x)$ is the intensity measure of the process of relays in the decoding set with estimates of the channel to the destination larger than some value x. Similarly, $\hat{\Lambda}_q(x)$ is the intensity measure of the process of qualified relays with channel estimates to the destination larger than x. In the following we derive the quantities $\hat{\Lambda}_d(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x)$ individually.

4.5.1 Intensity measure $\hat{\Lambda}_d(x)$

This subsection is dedicated to the derivation of the intensity measure $\hat{\Lambda}_d(x)$ of relays that are in the decoding set and have estimates of the channel to the destination $|\hat{g}_{jd}|^2 > x$. The quantity $\hat{\Lambda}_d(x)$ can be expressed as [23]:

$$\hat{\Lambda}_d(x) = \lambda \int_{W} p_{dx}(w) \mathrm{d}w, \tag{4.21}$$

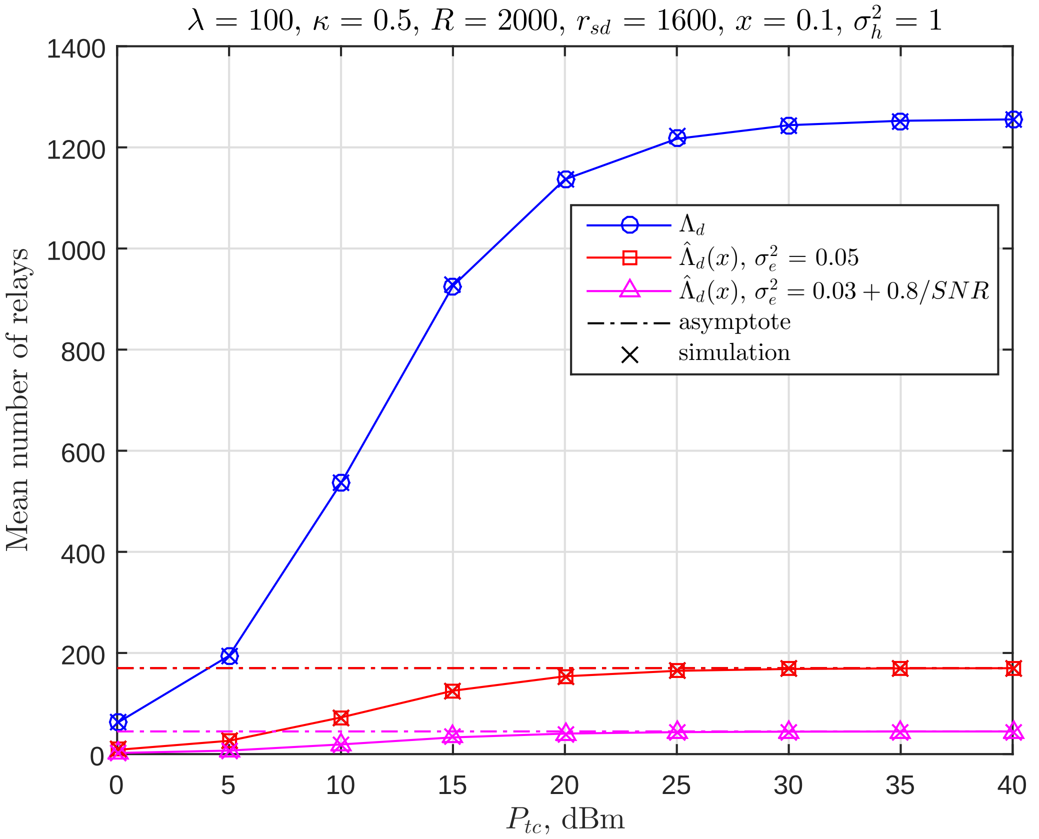

Figure 4.2: Exact and asymptotic plots of the intensity measure $\hat{\Lambda}_d(x)$. Intensity measure Λ_d is provided for reference.

where $p_{dx}(w)$ is the probability that a relay j at some location w will (a) be in the decoding set, i.e. $|g_{sj}|^2 > \frac{\theta}{\kappa}$, and (b) have the estimate $|\hat{g}_{jd}|^2 > x$:

$$\begin{aligned} p_{dx}(w) &= \Pr\left(|g_{sj}|^2 > \frac{\theta}{\kappa}, |\hat{g}_{jd}|^2 > x\right) \\ &= \exp\left(-\frac{1 + r_{sj}^{\alpha}}{\sigma_h^2}\frac{\theta}{\kappa}\right)\Pr\left(|\hat{g}_{jd}|^2 > x\right) \end{aligned} \tag{4.22}$$

where the last step follows from the fact that the two events in the probability are conditionally independent given the location w. The estimate of the channel $\hat{g}_{jd} = g_{jd} + \epsilon$ is distributed as $\hat{g}_{jd} \sim \mathcal{CN}\left(0, \hat{\eta}_{id}^2\right)$, where $\hat{\eta}_{id}^2 = \frac{\sigma_h^2}{1 + r_{jd}^{\alpha}} + \sigma_\epsilon^2$. Therefore, the second probability can be expressed as

$$\Pr\left(|\hat{g}_{jd}|^2 > x\right) = \exp\left(-\left(1 - \frac{\sigma_h^2}{\sigma_h^2 + \sigma_\epsilon^2\left(1 + r_{jd}^{\alpha}\right)}\right)\frac{x}{\sigma_\epsilon^2}\right). \tag{4.23}$$

Then the intensity $\hat{\Lambda}_d(x)$ can be rewritten as

$$\hat{\Lambda}_d(x) = \lambda e^{-\frac{x}{\sigma_\epsilon^2} - \frac{\theta}{\sigma_h^2(\kappa)}} \int_W e^{-\frac{r_{sj}^{\alpha}}{\sigma_h^2}\frac{\theta}{\kappa} + \frac{\sigma_h^2}{r_{jd}^{\alpha} + 1 + \sigma_h^2/\sigma_\epsilon^2}\frac{x}{\sigma_\epsilon^4}} \mathrm{d}w \tag{4.24}$$

Above integral can be easily evaluated using numerical methods, while closed form solutions for general α can be infeasible.

4.5.2 Derivative $\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x)$

This section calculates the derivative of the intensity measure $\hat{\Lambda}_q(x)$ of relays that are qualified to retransmit the source message to the destination, and have estimates of the channels to the destination $|\hat{g}_{jd}|^2 > x$:

$$\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x) = \lambda \int\limits_W \frac{\mathrm{d}}{\mathrm{d}x} p_{qx}(w)\mathrm{d}w, \tag{4.25}$$

where $p_{qx}(w)$ is the probability that a relay j at some location w will (a) be qualified for end-to-end transmission of the message, i.e. $|g_{sj}|^2 > \frac{\theta}{1-\kappa}$, $|g_{jd}|^2 > \frac{\theta}{\kappa}$, and (b) have the estimate $|\hat{g}_{jd}|^2 > x$:

$$\begin{aligned} p_{qx}(w) &= \Pr\left(|g_{sj}|^2 > \frac{\theta}{\kappa}, |g_{jd}|^2 > \frac{\theta}{1-\kappa}, |\hat{g}_{jd}|^2 > x\right) \\ &= \exp\left(-\frac{1+r_{sj}^{\alpha}}{\sigma_h^2}\frac{\theta}{1-\kappa}\right)\Pr\left(|g_{jd}|^2 > \frac{\theta}{\kappa}, |\hat{g}_{jd}|^2 > x\right). \end{aligned} \tag{4.26}$$

Hence the quantity of our interest can be expressed as

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x) = \lambda e^{-\frac{\theta}{\sigma_h^2(\kappa)}} \int\limits_W e^{-\frac{\theta}{\sigma_h^2(\kappa)} r_{sj}^{\alpha}} \\ \times \underbrace{\frac{\mathrm{d}}{\mathrm{d}x}\Pr\left(|g_{jd}|^2 > \frac{\theta}{1-\kappa}, |\hat{g}_{jd}|^2 > x\right)}_{f'}\mathrm{d}w, \end{aligned} \tag{4.27}$$

where the derivative f' of the probability can be found as

$$\begin{aligned} f' &= \frac{\mathrm{d}}{\mathrm{d}x}\int\limits_{\theta/(1-\kappa)}^{\infty}\int\limits_{x}^{\infty} f_{|\hat{g}_{jd}|^2||g_{jd}|^2}(y|t) f_{|g_{jd}|^2}(t)\mathrm{d}y\mathrm{d}t \\ &= -\int\limits_{\theta/(1-\kappa)}^{\infty} f_{|\hat{g}_{jd}|^2||g_{jd}|^2}(x|t) f_{|g_{jd}|^2}(t)\mathrm{d}t \end{aligned} \tag{4.28}$$

The conditional PDF $f_{|\hat{g}_{jd}|^2||g_{jd}|^2}(x|t)$ can be found following the methodology in [52, 62, 75] as

$$f_{|\hat{g}_{jd}|^2||g_{jd}|^2}(x|t) = \frac{1}{\sigma_\epsilon^2}\exp\left(-\frac{x+t}{\sigma_\epsilon^2}\right) I_0\left(\frac{2}{\sigma_\epsilon^2}\sqrt{xt}\right), \tag{4.29}$$

where $I_0(\cdot)$ is zero-order Bessel function of imaginary argument [56, 8.447]. The second PDF can be expressed as

$$f_{|g_{jd}|^2}(t) = \frac{1+r_{jd}^{\alpha}}{\sigma_h^2}\exp\left(-\frac{1+r_{jd}^{\alpha}}{\sigma_h^2}t\right). \tag{4.30}$$

Substituting the PDFs (4.29) and (4.30) into (4.28), the exact expression for the derivative of the intensity measure $\hat{\Lambda}_q(x)$ can be obtained as

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\hat{\Lambda}_q(x) &= -\lambda\exp\left(-\frac{\theta}{\sigma_h^2\kappa}-\frac{x}{\sigma_\epsilon^2}\right)\int\limits_W \frac{1+r_{jd}^{\alpha}}{\sigma_h^2}\beta \\ &\times \exp\left(-\frac{\theta}{\sigma_h^2\kappa}r_{sj}^{\alpha}+\beta\frac{x}{\sigma_e^2}\right)Q_1\left(\sqrt{\frac{2\beta x}{\sigma_e^2}},\sqrt{\frac{1}{\beta\sigma_e^2}\frac{2\theta}{1-\kappa}}\right)\mathrm{d}w,\end{aligned} \tag{4.31}$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q-function of first kind, and $\beta = \frac{\sigma_h^2}{\sigma_h^2+\sigma_e^2\left(1+r_{jd}^{\alpha}\right)}$.

Finally, the desired communication outage probability can be obtained numerically by substituting (4.24) and (4.31) into (4.20).

4.5.3 Asymptotic analysis

To understand the asymptotic outage performance in the case of relay selection with imperfect CSI, consider the high-SNR behaviour of the components $\hat{\Lambda}_d(x)$ and $\hat{\Lambda}_q(x)$ of the outage probability expression (4.20). When available transmission power P_{tx} is large, the mean number Λ_q of relays in the qualified set Φ_q approaches the mean number Λ_d of relays in the decoding set Φ_d. Consequently, $\hat{\Lambda}_q(x) \to \hat{\Lambda}_d(x)$. Then, at high SNR, (4.20) can be rewritten as

$$P_o \approx 1 + \int\limits_0^{\infty} \exp\left(-\hat{\Lambda}_q(x)\right)\mathrm{d}\hat{\Lambda}_q(x) = \exp\left(-\Lambda_q\right). \tag{4.32}$$

Therefore, outage probability of imperfect CSI-based RS at high SNR approaches outage probability of perfect CSI-based selection.

4.6 Results and discussion

This section presents and discusses the performance results for the selection cooperation strategy where relay selection decisions are based on CSI with imperfections discussed in Section 4.3 and 4.4. In addition to the discussion of system performance, this section highlights the accuracy and applicability of the expressions developed in this chapter.

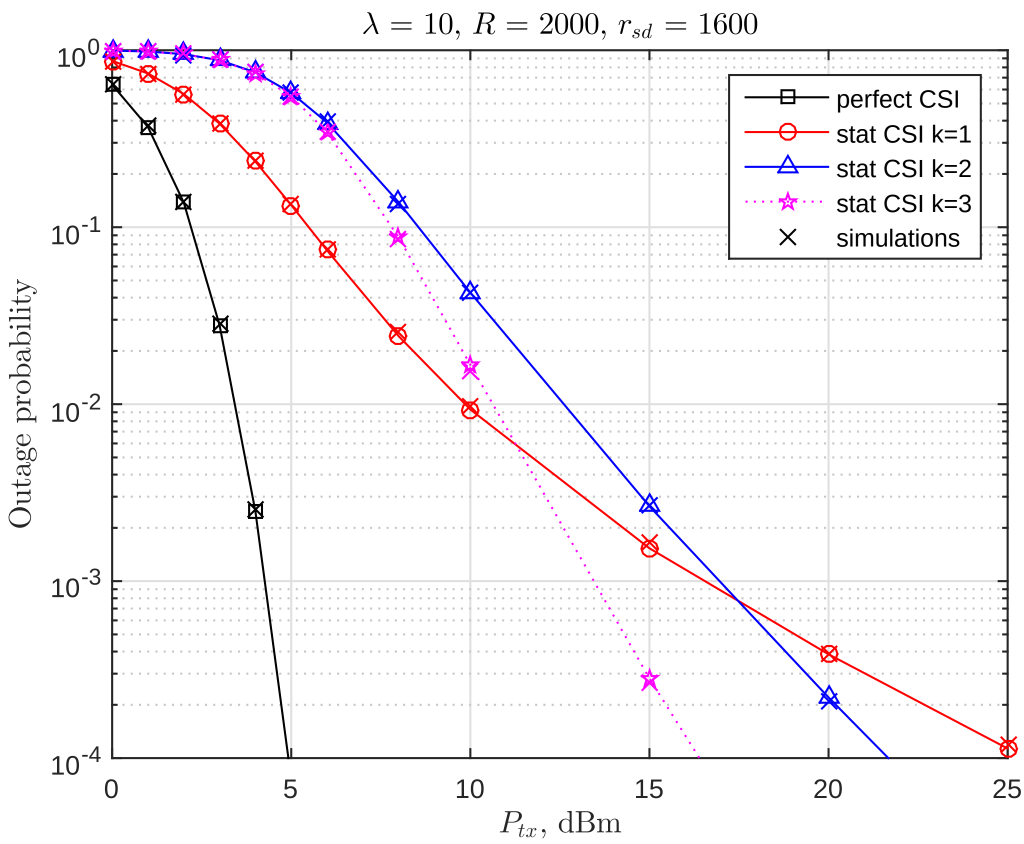

Figure 4.3: Outage probability for opportunistic relaying from the source to the destination for the cases of exact and statistical CSI at relays.

4.6.1 Statistical CSI-based selection

Section 4.3 investigated the performance of an RS strategy based on the longer-term statistics of channel gains at the relays, and presented expressions for communication outage probability calculation. Fig. 4.3 depicts the outage probability for communication between the source and the destination through either one or k relays from decoding set Φ_d selected based on such statistical CSI. A graph for the outage probability of RS based on perfect CSI is also included to help put the results in the context.

Fig. 4.3 shows a clear loss in performance between the relay selection based on perfect and statistical CSI. In particular, much larger transmission power is required for the latter to achieve the performance of the former. For example, outage probability of 10^{-2} is achieved at $P_{\text{tx}} \approx 3$ dBm in the case of perfect CSI, while for the same level of performance is achieved at $P_{\text{tx}} \approx 10$ dBm in the case of statistical CSI. Such increased power requirement to meet a certain QoS level can be viewed as a penalty for the lack of accurate channel state information.

In order to improve performance the source may ask $k > 1$ relays with the best channel estimates to retransmit the message. Performance of such scenario is illustrated in Fig. 4.3. For sufficiently high transmission power levels, multiple retransmitting relays outperform a single transmitting relay. On the other hand, for lower power budgets, splitting the power between relays leads to performance degradation. Indeed, larger k values mean that each relay can be allocated less power from the budget. In addition, since relay retransmissions take k additional time slots instead of 1, outage performance is further degraded by k-fold increase in the required data rate for each relay transmission. Nevertheless, the gain from increased diversity overweights the effect of resource splitting between k relays relatively quickly for $k = 3$.

Overall, when instantaneous CSI is unavailable, increasing the number of active relays can lead to a similar level of outage probability as in the case of perfect CSI at the cost of higher consumed power. Therefore, it may be beneficial to employ more relays when power budget is sufficiently high, rather than to invest all power into one nearest relay transmission when instantaneous CSI is unavailable. Simulation and analytical results were shown to be in good agreement in Figs. 4.3 and 4.1, corroborating the accuracy of developed relations.

4.6.2 Imperfect instantaneous CSI-based selection

This subsection presents the performance results for selection cooperation strategy utilising imperfect instantaneous channel knowledge. Analytical results in this subsection were obtained using relations derived in Section 4.4. The objective of this subsection is to investigate the outage behaviour of SC strategy, operating in described imperfect conditions, as a function of available transmission power P_{tx}, power allocation factor κ and channel estimation error variance σ_ϵ^2.

Results for two sets of experiments are presented in this subsection. The first set focusses on the outage probability as a function of the total available transmission power P_{tx} for the case of equal power allocation $\kappa = 0.5$ between the source and relay transmissions. These results are presented on Fig. 4.4 for different channel estimation error assumptions, together with additional graphs to help put the results in context. The second set of simulations, shown on Fig. 4.5, investigates the impact of power allocation coefficient κ on outage probability for a fixed value of total transmission power P_{tx}.

Two channel estimation error variance models are used as in [73,74] – the transmission power-dependent and independent models. For the power-independent model, the estimation error variance was chosen as $\sigma_\epsilon^2 = 10^{-6}$ or 10^{-7}. Power-dependent estimation error model was chosen as

$$\sigma_\epsilon^2 = 10^{-7} + 10^{-6}\left(1 - \frac{(1-\kappa)P_{\text{tx}}}{\max\left((1-\kappa)P_{\text{tx}}\right)}\right), \tag{4.33}$$

so that the value of the channel estimation error varies with the power available for the relay transmission. The source-destination distance is set as $r_{sd} = 1600$m, the

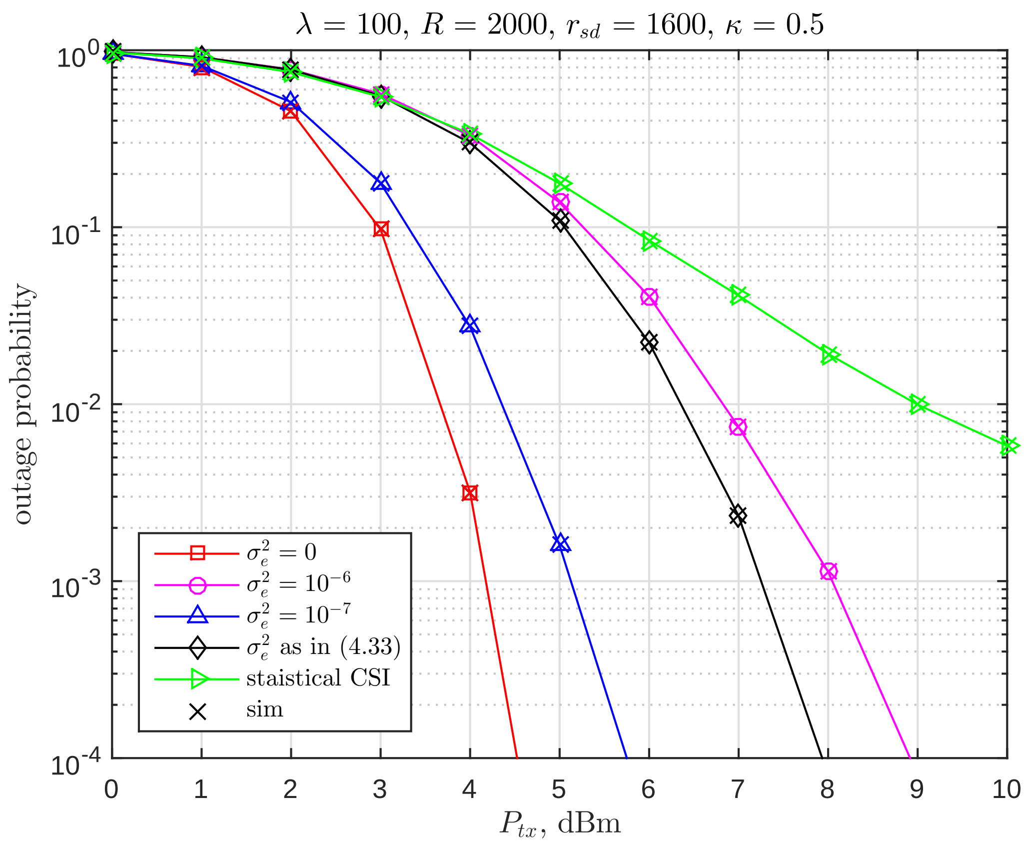

Figure 4.4: Outage probability as a function of transmission power budget P_{tx} for equal power distribution between transmission stages.

intensity function of the PPP of candidate relays is $\lambda = 100$ relays per sq. km, and the cell has radius $R = 2000$m. Free-space propagation path loss model is assumed.

Fig. 4.4 illustrates the outage probability behaviour of selection cooperation strategy as a function of total transmission power $P_{\text{tx}} \in [0, 10]$dBm allocated equally between the source and relay transmissions. The outage probability curves for different levels of the channel estimation error variance σ_ϵ^2 are shown together with the results for the perfect-CSI-based and statistical CSI-based selection methods. One can observe that for error variance of $\sigma_\epsilon^2 = 10^{-7}$ approximately 1 dB higher transmission power is required to achieve the performance of the perfect CSI-based selection since some relay selection decisions become suboptimal. For larger error variances, outage probability deteriorates significantly, so that for $\sigma_\epsilon^2 = 10^{-6}$ at lower transmission power budgets the system performance is identical to the statistical CSI-based selection, discussed in Section 4.6.1. Nevertheless at higher transmission power, relay selection decisions based on the imperfect instantaneous CSI outperform statistical CSI-based selection. In the case of power-dependent channel estimation error model, outage probability is high at lower P_{tx} values, however as the measurement error component diminishes with increasing P_{tx}, outage behaviour follows the case with power-independent error with $\sigma_\epsilon^2 = 10^{-7}$.

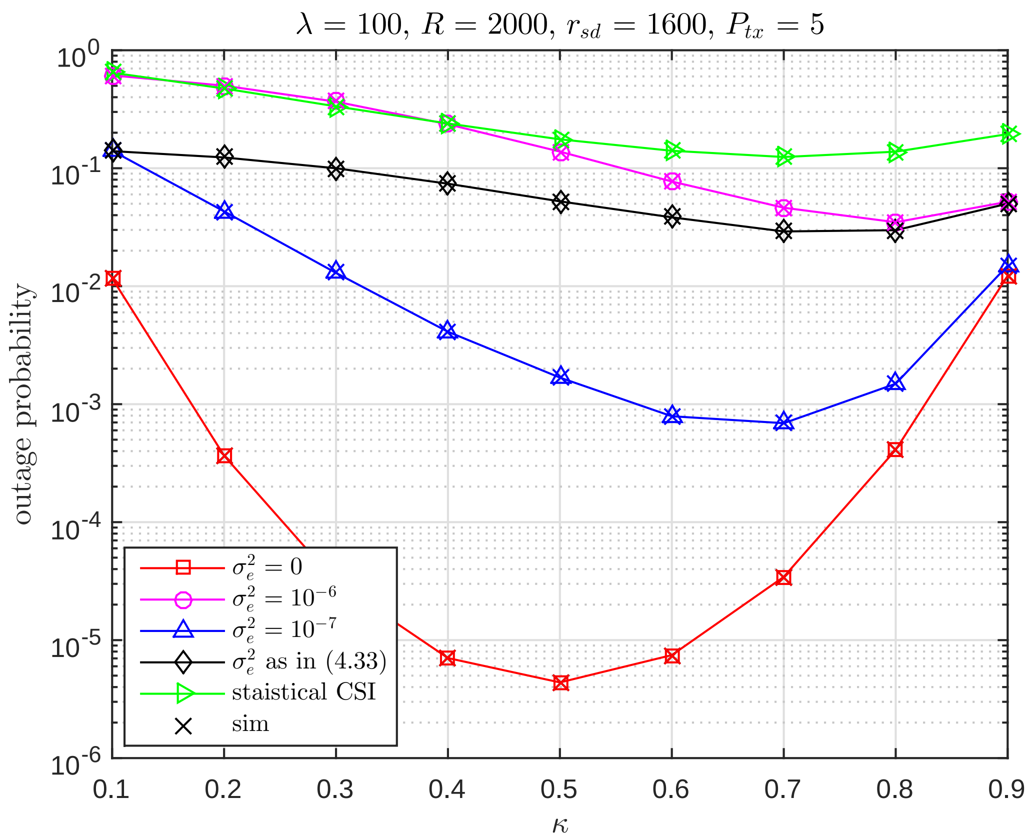

Figure 4.5: Outage probability as a function of power distribution coefficient κ for the SC strategy.

Fig. 4.5 depicts the dependence of outage probability on the power allocation coefficient κ. Results suggest that in presence of channel estimation errors, lower outage probability can be achieved if more power is allocated to the source transmission phase creating more relays in the decoding set Φ_d.

In particular, for the cases of $\sigma_\epsilon^2 = 10^{-7}$ and $\sigma_\epsilon^2 = 10^{-6}$, the minimum of outage probability is approximately at $\kappa = 0.7$ and $\kappa = 0.8$ respectively.

On the other hand, for the power-dependent channel estimation error model with σ_ϵ^2 defined in (4.33), additional power allocated to the source transmission only marginally improves performance, resulting in the performance equivalent to $\sigma_\epsilon^2 = 10^{-6}$ at higher values of κ. This can be explained by the reduction in the power used to estimate the channel in the model used in [73, 74]. Results for the performance of perfect CSI-based relay selection in similar conditions suggest that equal power allocation $\kappa = 0.5$ is favourable, while in the case of statistical CSI a larger source transmission power delivers only a marginal improvement in performance.

4.7 Conclusion

This chapter focussed on development of mathematical methods applicable to outage performance analysis of selection cooperation strategy where the retransmitting DF relay is chosen using CSI, which is not necessarily perfect. Different from existing works on imperfect CSI-based relay selection, network topology was explicitly considered, and unlike current literature on RS in random networks, the case of imperfect CSI was considered. Proposed methods allowed bypassing some analytical complexity involved in previous works, such as the derivation of distance distributions. In this way, exact outage probability expressions for the considered RS scenario were obtained.

Obtained system performance results indicated that when only statistical CSI is available in the relay selection process, increasing the number of retransmitting relays creates a trade-off between sacrificing performance at low transmission power values due to splitting resources among k relays, and the larger diversity order for high transmission power values. Availability of instantaneous imperfect CSI allows better performance compared to the statistical CSI-based decisions when the variance of the channel estimation error is small. When the error becomes large, statistical CSI-based selection may outperform a system based utilising errorneous instantaneous CSI.

The focus of this chapter was on systems with channel reciprocity, applicable to TDD systems. In frequency division duplex (FDD) mode correlation between uplink and downlink channels gains need to be established, which may result in a different channel estimation error model. In the case of Gaussian error, presented results are technically applicable to FDD case, however an explicit account for FDD channel estimation specifics is left as future work.

Chapter 5

Conclusions and further work

5.1 Conclusions

Cooperation is a form of interaction between network elements, which is expected to play a role in the design of communication technologies capable of meeting the explosive demand for mobile multi-media services and applications. The focus of this thesis was on the development of analytical methods applicable to performance assessment of cooperative networks. The motivation for this research has been in the observation that the impact of network geometry on cooperative system performance remains largely unexplored despite the breath and depth of existing literature on cooperative communications.

One of the methods used to account for the effect of network geometry and, consequently, path loss in previous literature was based on the treatment of inter-node distances as fixed parameters. In this way, performance of a system could be obtained as a function of a particular set of inter-node distance values. Another approach was to modulate channel impairments with the distance between communicating nodes. It was emphasized in this thesis that emerging cooperative network architectures cannot be analysed using above abstractions since cooperation is established between nodes on the basis of suitability, rather than on the sole basis of geographical location. New approaches capable of mathematically capturing trends in such suitability started to emerge in recent years, including the present work.

Analytical modelling is not the only approach to characterisation of the performance of cooperative networks. System level simulations have always been a valid method for assessment of the performance of communication systems, and cooperative networks are not an exception. However the main limiting factors of such a non-analytical approach are that a large computational power is required to conduct network simulations of sufficient scale, and that the insights into system performance are restricted to interpretation of a limited set of results. An alternative path taken in this thesis is in the development of analytical methods for statistical description of cooperative networks, explicitly incorporating the network topology.

This work utilised elements of stochastic geometry and point process theory to develop analytical methods capable of handling network geometry of cooperative communication systems explicitly. Research results based on SG have been published before and during the course of this work, however very few of them were made in the area of cooperative networks. This thesis has made a number of contributions that complement existing knowledge and open new approaches to further research on the subject of spatially-distributed cooperative networks.

Part of this thesis was dedicated to performance analysis of cooperative and non-cooperative broadcasting in networks with finite dimensions and finite number of users. Such selection of system parameters is often avoided in literature by focussing on geometrically infinite or infinitely dense networks, although practical networks are necessarily finite. In this thesis cooperative and non-cooperative broadcasting scenarios were characterised in terms of the latency in delivery of the source message to all nodes in the network. Particularly useful contributions of presented work include the PDF of αth powers of distances between a transmitter and the nearest receiver (or vice-versa). This further allowed obtaining the CDF of the path loss-inclusive channel gain from a network node to the nearest transmitter/receiver in a finite network. Presented framework for broadcast analysis in general, and specific contributions to statistical description of communication in random finite networks can be extended to other interactions, or other broadcasting protocols.

While broadcasting remains an important element in modern communication systems, many applications require personalised information delivery, for example on-demand video, file downloading or video-conferencing. Efficient realisation of cooperation in such scenarios requires coordination, volume of which can be reduced through the selection of cooperative relays. This thesis introduced an approach to obtain outage probability expressions for communication in cooperative networks utilising selection cooperation strategy. Proposed approach is based on thinning operation on point processes, and results in an intuitive flow of performance analysis and outage probability expressions that reduce to closed form expressions for special cases of system parameters, unlike the results in previous literature. In particular, outage probability for cooperative communication between a single source-destination pair was shown to be simply exponent to the negative power of the intensity measure of relays with reliable links both to the source and the destination. Methods to obtain such intensity measure have been developed in this thesis for different cases of system parameters. Performance of selection cooperation has also been considered for the case where multiple sources contend for relays with reliable links to the destination. Analysis of such a scenario for the case of a source that is the last to access the shared pool of relays has highlighted the fact that suitable relays can be treated as a scarce resource. In general, this part of the thesis presented a point process-based approach to outage probability calculation, that is alternative to the existing methods. Simplicity and flexibility of presented approach are expected to lead to further extension of presented methods to scenarios beyond those considered in this work.

Relay selection in practical systems is realised based on imperfect channel state information. This aspect has been captured in the final part of this thesis, which further extended presented analysis of selection cooperation using thinning procedure. To the best of author's knowledge, the combination of imperfect CSI and explicit account for network topology has not been considered before. In particular, relay selection based on imperfect CSI obtained either through long term averaging of channel measurements, or through instantaneous but noisy measurements was considered. Application of point process theory to such scenarios allowed obtaining exact outage probability expressions for different levels of CSI imperfections. It is expected that many important communications networking scenarios can benefit from the concepts developed in this work. In particular, it has been shown that thinning operation can be used as a spatial filter, looking for network nodes with certain properties. While the focus of this work has been on tuning such a spatial filter to the properties related to cooperation, there are numerous possible extensions of the concepts highlighted in this thesis. For example, statistical analysis of physical layer secrecy or energy-harvesting networking are the immediate candidates for such extensions.

Overall, this thesis has demonstrated that it is possible to analyse cooperative networks in an environment inclusive of network topology. Powerful instruments from spatial statistics exist and can help deepen the understanding of existing and emerging communication systems. This thesis has focussed on a number of specific protocols, yet presented instruments can be extended to a broad set of scenarios, for example to the analysis of opportunistic relaying, or utilising a different propagation path loss model. Following section highlights a number of interesting directions for further work.

5.2 Further work

5.2.1 Spatial capacity of cooperative networks

As communication networks are becoming more dense and multi-tiered, high area spectral efficiency becomes a particularly desirable characteristic of a communication system. Fundamental information-theoretic relations for different types of channel configurations are still valid in such new network architectures, however an interesting aspect is the aggregate performance of the system as a whole. Practical examples that could benefit from such an assessment include the Dual Connectivity within

LTE-Advanced, or various methods of beamforming.

It is expected that techniques developed in this thesis could help answer the questions on fundamental system performance for networks that are measured in terms of geometry-inclusive metrics.

5.2.2 Optimisation of physical and medium access layers based on spatial statistics

There could be a potential to optimise elements of the physical, medium access control, and other layers of different communication technologies based on the spatial statistics of operating environments and applications. For example, typical transmitter-receiver distances for a sensor network are likely to be different from the typical distances between mobile users and the elements of a cellular network. Likewise, communication in an urban environment could use a different set of handover parameters based on the statistics of proximity to network infrastructure elements.

Research in this direction could help optimise the operation of existing networks, such as LTE, and the methodology developed in this thesis could be used as a starting point in determining the initial sets of required system parameter values.

5.2.3 Spatial statistics and network dynamics

Self-organising functionality is intended to minimise operational expenditure and reduce the amount of human intervention in network control as a consequence of network densification. Distributed, centralised or hybrid algorithms running in the network elements essentially convert such a network in a dynamical system where nodes affect each other's behaviour through interactions. It is often important to understand the behaviour of such a dynamical system over a finite number of iterations, for example in order to estimate the speed of convergence of a power control algorithm. Spatial statistics in this case can provide an important addition to performance analysis of such network dynamics, typically conducted using graph theory and Markov chains.

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