EFFECTS OF H.V.D.C. TRANSMISSION LINE ON THE
TRANSIENT PERFORMANCE OF AN A.C. POWER SYSTEM
(A DIGITAL COMPUTER STUDY)

VOLUME 1

By

SYED MUHAMMAD AHMED

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SUMMARY

The aim of this work is a comparative study of the electro-
mechanical transient phenomena and of the stability limits of
synchronous power systems, when they are mainly interconnected by
a.c. transmission lines, but when one of the a.c. lines is replaced
by the d.c. one.

A thorough survey of the recent literature on h.v.d.c.
technology has been made in order to be conversant with latest
developments in the field. Digital computation and numerical
analysis are used to study mathematical models of the systems under
consideration.

For the h.v.d.c. system, three digital programmes have been
established to find the characteristics of the converter, to
simulate the h.v.d.c. link in the a.c. power system and to study
the transient behaviour of the system, respectively. For the
a.c. system the Park's equations describing the behaviour of the
synchronous machines are modified to suit the digital computer.
Then two digital programmes have been written, for the a.c. system,
one to draw the swing curves and the other to find the stability
boundaries which incorporate the A.V.R. and the speed governor.

To make a comparative study of the a.c. - d.c. system and of
the equivalent a.c. system, two new comprehensive programmes have
been established by incorporating the d.c. programme for the
transient studied in the modified a.c. programmes. One of the
digital programmes draws the swing curves while the other finds
the stability boundaries of both the systems, with or without
A.V.R. Thus, by performing different tests, it has been
established that from the stability point of view the a.c.-d.c.
system can be made superior to the equivalent a.c. one, provided
that provision is made for increasing the power through the d.c.
line when a fault is sensed.
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A.C. SYSTEM

Governor and Prime Mover Control Loops

\[ k_g \] - Speed governor control loop gain.
\[ r_g \] - Combined relay and main valve time constant, (sec.)
\[ r_p \] - Time constant of steam or water inlet-valve, (sec.)
\[ \Delta P_m \] - Change in input mechanical power, p.u.

Automatic Excitation Control Loops

\[ v_1 \] - Comparator output voltage, p.u.
\[ v_t \] - Voltage proportional to machine terminal voltage, p.u.
\[ v_1 \] - Output voltage of magnetic amplifier 1, p.u.
\[ v_2 \] - Output voltage of magnetic amplifier 2, p.u.
\[ v_f \] - Main exciter output voltage, p.u.
\[ v_{s1} \] - Output voltage of the stabiliser 1, p.u.
\[ v_{s2} \] - Output voltage of the stabiliser 2, p.u.
\[ v_s \] - Equivalent voltage of the stabiliser, p.u.
\[ v_R \] - Demanded or reference machine terminal voltage, p.u.
\[ k_4 \] - Gain of magnetic amplifier 1.
\[ k_5 \] - Gain of magnetic amplifier 2.
\[ k_7 \] - Gain of main exciter.
\[ k_6 \] - Gain of stabiliser 1.
\[ k_8 \] - Gain of stabiliser 2.
\[ r_4 \] - Time constant of magnetic amplifier 1, (sec.)
\[ r_5 \] - Time constant of magnetic amplifier 2, (sec.)
\[ r_7 \] - Time constant of main exciter, (sec.)
\[ r_6 \] - Time constant of stabilising loop 1, (sec.)
$T_8$ - Time constant of stabilising loop 2, (sec.)

$K$ - Overall gain of the main loop.

$K_s$ - Total gain of the stabilising loops.

$T_e$ - Overall time constant of amplifiers and exciter, (sec.)

$T_s$ - Overall time constant of stabilisers.

**Synchronous Generators**

$v_{d}, v_{q}$ - d-q axis components of terminal voltage, p.u.

$v_f$ - Field voltage, p.u.

$G_{e_f}$ - Terminal voltage on open-circuit at normal speed, p.u.

$i_{d}, i_{q}$ - d-q axis components of armature current, p.u.

$\psi_{d}, \psi_{q}$ - d-q axis components of total armature flux linkages, p.u.

$x_d$ - d-axis synchronous reactance, p.u.

$x_d'$ - d-axis transient reactance, p.u.

$r_1$ - Armature resistance (positive sequence), p.u.

$r_2$ - Armature resistance (negative sequence), p.u.

$i_2$ - Negative sequence armature current, p.u.

$\tau_{do}$ - Direct-axis open-circuit field time constant, (sec.)

$P_e$ - Positive phase-sequence armature power, p.u.

$P_{el}$ - Negative phase-sequence armature power, p.u.

$P_m$ - Mechanical power, p.u.

$T_m$ - Shaft torque, p.u.

$T_e$ - Electrical torque, p.u.

$T_a$ - Accelerating torque, p.u.

$H$ - Inertia constant.

$M$ - $H/\tau_f$

$K_d$ - Damping coefficient.

$\delta$ - Rotor-angle.

$\omega_0$ - Synchronous speed.
A.C. Network

\( x_t \) - Positive phase-sequence transmission line reactance, p.u.
\( r_t \) - Positive phase-sequence transmission line resistance, p.u.
\( x_c \) - Positive phase-sequence transmission line capacitive-reactance, p.u.
\( V_s \) - Sending end voltage of the transmission line, p.u.
\( V_r \) - Receiving end voltage of the transmission line, p.u.
\( Z_f \) - Fault impedance, p.u.
\( Z_o \) - Zero phase-sequence impedance of the network, p.u.
\( Z_1 \) - Positive phase-sequence impedance of the network, p.u.
\( Z_2 \) - Negative phase-sequence impedance of the network, p.u.
\( Z_{nn} \) - (\( n = 1, 2, 3 \ldots \)) are the self impedences of the "n" nodes, p.u.
\( Z_{ij} \) - are the mutual impedences, p.u. where \( i \neq j \) and \( i = 1, 2, 3 \ldots n \)
\( I_d, I_q \) - Injected currents, p.u.

D.C. System

Converters

\( x_c \) - Commutating reactance, p.u.
\( e_c \) - Voltage behind the commutating reactance, p.u.
\( \alpha \) - Delay angle of valve firing of a rectifier.
\( \beta \) - Advance angle of valve firing of an inverter.
\( r \) - Angle of commutation or angle of overlap.
\( \delta_o \) - Extinction angle.
\( \phi \) - The angle between the bus-bar voltage and the fundamental phase-current (displacement angle).
Converter Transformer

$x_v, x_l, x_t$ - Reactances in the valve, line and tertiary branches of the star equivalent circuit for the transformer, p.u.

$x_l$ - Leakage reactance of the converter transformer, p.u.

$n$ - Turn ratio of the ideal transformer.

$v_v, v_l, v_t$ - Nominal voltage of the valve, line and tertiary windings of the converter transformer, respectively, p.u.

$v_y$ - Fundamental component of the voltage at star point of the equivalent circuit, p.u.

$v_r$ - The voltage at which "$v_l$" is controlled, p.u.

$i_t, i_l, i_v$ - Fundamental components of tertiary, line and valve winding current, respectively, p.u.

Synchronous Compensators and Filters

$x_a$ - Reactance associated with the mean sub-transient reactance of the synchronous compensator, p.u.

$S$ - Capacitive reactance of the filter bank, p.u.

Supply System

$x_s$ - Effective supply system reactance, p.u.

$e_s$ - Effective voltage behind the supply system, p.u.

$i_s$ - The current drawn from the supply system, p.u.

$V_s$ & $V_r$ - Are sending end and receiving end a.c. bus-voltage, p.u., suffices d and q standing for direct and quadrature components, and the last suffices zero and 1 indicating common reference frame and Park's reference frame, respectively.
D.C. Network

\( R \)  - Resistance of the d.c. line, p.u.
\( R_t \)  - Equivalent resistance of the transformers, p.u.
\( V_{dl} \)  - Sending end average d.c. line voltage, p.u.
\( V_{d2} \)  - Receiving end average d.c. line voltage, p.u.
\( i_{dc} \)  - Current in the d.c. line, p.u.
\( P_{dc} \)  - Power transmitted by the d.c. line, p.u.
\( Q_{dc} \)  - Reactive power consumed by the d.c. system, p.u.

Controls

\( K \)  - Gain of the converter regulator.
\( K_{R} \)  - Gain of the rectifier regulator.
\( K_{I} \)  - Gain of the inverter regulator.
\( I_{r} \)  - Reference current of the regulator on rectifier side, p.u.
\( I_{ri} \)  - Reference current of the regulator on inverter side, p.u.
\( k \)  - Current margin, p.u.
\( e_{cr} \)  - Rectifier control signal, p.u.
\( e_{ci} \)  - Inverter control signal, p.u.

MATHEMATICAL NOTATIONS

\( \approx \)  - Approximately equal to.
\( \implies \)  - Implies.
\( \Re \)  - Real part of the quantity in the parenthesis.
\( \Im \)  - Imaginary part of the quantity in the parenthesis.
\( \bar{X} \)  - Vector of \( X \).
\( \bar{X} \)  - Conjugate of Vector \( X \).
\( \frac{d}{dt} \)  - Right hand side.
\( \frac{d}{dt} \)  - Left hand side.
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CHAPTER I

INTRODUCTION
Growth of Energy Consumption

The story of man's origin and continuing existence has been a story of capturing and storing energy, and subsequently converting it into certain desired forms. Out of all the basic forms of power which have been utilized so far by man in his day-to-day life, electrical energy heads the list, mainly because of its ease of control and transmission. Hence, the primary sources like wind, water power and fossil fuels (coal, oil and gas) are being converted into electrical energy which, in turn, being deployed to serve our needs. Recently, there has been added another basic source of energy on earth — the "fission-reaction" of uranium or thorium. It is hoped we may be able in the foreseeable future to have a controlled "fusion" on the earth, which may then be the major source of power in the world.

In the present technological age the importance of electrical energy has been enhanced so much that its per-capita consumption may safely be taken as an index for the socio-economic development of any country. In view of the growth in the consumption rate of electrical energy, many speculations regarding the projected sources and uses of energy in the foreseeable future have been made. One of the best of these forays into the future has been made by Brown, Bonner and Weir. The forecast is based on the hypothesis that there will be a population of seven billion on earth in the foreseeable future, and that the annual energy requirements for this civilisation, outside food production, will be $633 \times 10^5$ megawatt-years, for a "steady-state" civilisation. This would represent a 650 per cent increase in per capita consumption of energy when compared with the present figure.
and a total energy consumption increase of some 18-fold. At this rate the present fossil fuel reserves would last only for 35 years.

1-2 **Economics of Electrical Energy**

Speculations regarding the future consumption of electrical energy, though based on rough estimates, indicate the ever-increasing demand of electrical power. To face the challenge, all the available sources of energy are to be tapped, and the improved techniques of generation and transmission are being investigated to utilise these sources to their full capacity. At the same time world-wide research has been organized to harness light element fusion reaction for cheap power production.

Up to this time, however, water is considered to be the cheapest source of electrical energy; hence, hydroelectric generating stations are installed wherever a suitable head with a sufficient quantity of water is available. Another choice is thermogeneration, and in this case also, the power plants are installed where the fuel is economically available. These generating stations, individually or collectively, supply the power to the long distant load centres through the high-voltage (H.V) or extra-high-voltage (E.H.V.) transmission lines.

1-2-1 **Extra-High-Voltage**

In the transmission of electrical energy the power losses in the line are a function of impedance and the square of the current; thus, raising the voltage lowers the current proportionately, thereby reducing the cost of the conductor for power transmission at a certain percentage of power loss. Hence, extra-high-voltages are being
economically applied to transmit the bulk power from the generation sources to the distant load centres. The significance of E.H.V. transmission is better appreciated when it is considered that one 500-KV line could replace six 230-KV lines as far as loading capability is concerned. Recognising the economical feasibility of E.H.V. transmission, the International Electrotechnical Commission (I.E.C.) at its 1963 meeting in Venice, Italy, included 500/525-KV and 700-750/765-KV in its list of approved voltages.

## 1-2-2 Integrated System

To achieve an economical and reliable large-scale operation, power stations are interconnected so that their combined load can be treated as a unit system. The areas of mutual benefit that are usually exploited in interconnection and pooling are as given below:

1. Shared generation reserves.
2. Large economical generating units.
4. High transmission voltage.

and,
5. The benefit of economy energy exchange.

The "Pacific Northwest-Southwest Intertie" project is one of the best examples of an integrated system. It will connect directly and indirectly almost all the major power systems in eleven western states of America, thus providing a grid system at optimum electrical efficiency. After the completion of the dams in Canada, this integrated system will deliver 4600-MW of power and will comprise four E.H.V. lines in addition to several other supporting lines.
Two of the long lines will be 800-KV d.c. lines, the other two major lines will be 500-KV a.c. lines and the remaining lines of the system will be a short 800-KV d.c. line, two short 500-KV tie lines, two 345-KV a.c. lines and two relatively short 230-KV a.c. lines.

1-3 Stability Problems

In the integrated system, if one of the power stations fails the others share its load, thus reducing the power reserve required for reliable running of the system. But, the question whether or not the system can survive the first few seconds of loss of generation, while maintaining its stability, is to be answered by another set of calculations. For this study, which is the subject of this thesis, the dynamic performance of the system immediately following the disturbance is computed and the state of the system is found. The system is said to be stable, if it is operating or tending towards the state where the mechanical input power is equal to the electrical output power plus losses. If following a disturbance the system no longer satisfies the above conditions, the system is said to be unstable.

1-3-1 Disturbances in the System

The disturbances which could cause instability of the system are grouped as:

2. Changes in electrical loading.
3. Changes in generated power
The effects of all the four types of disturbances can be taken into account while designing the system, but the last one, in bringing about sudden and unexpected changes, is of great importance from the stability point of view.

1-3-2 Stability Studies

The first quantitative evaluations of stability problems were made by the long-hand method. \(^7^0\) In this method some assumptions were made to simplify the system equations, and the results thus obtained were only a guide to the actual performance of the system.

The use of a "Network-Analyser"\(^7^1\) for the solution of the system equations marked the first improvement in the study of stability problems. In this case results are obtained by representing the physical quantities of the network by the physical components in the network-analyser circuit. However, the representation of the synchronous machines is primitive, it being simulated as a voltage behind the synchronous reactance.

An alternative approach to solving the system equations is by simulating the entire physical system, where synchronous-machines are represented by micro-machines, and the network is simulated by a network analyser.\(^7^2\) To overcome the difficulties of scaling the electrical and magnetic quantities for micro-machines many ingenious methods have been put forward\(^7^3\), but the problems of control and metering of these machines have limited their use to research equipment only.

Aldred and others\(^7^4\) introduced the application of a "D.C Electronic Analogue Computer" to stability problems. In this approach, a full
representation of the system is possible, and simultaneous non-linear
differential equations can be solved in a more comprehensive way
than before; but the method is limited by operational difficulties,
such as component accuracy, drift of amplifiers, and its complex
setting and checking operation.

Sher and Lisser\textsuperscript{75} put forward a method which combines the
merits of the analogue computer with those of the network analyser;
and using this method, Aldred and Coreless\textsuperscript{76} at Liverpool and Humpage\textsuperscript{77}
at Newcastle upon Tyne, built the "Hybrid Analogue Network Analyser", though independently of each other. In this computer, the detailed
representation of the synchronous machines are made in the analogue
computer, whilst the network is simulated in the network analyser.
This hybrid computer requires specially designed coupling units to
convert both the d.c voltage levels into a.c quantities and alternating
currents into their d.c vector components.\textsuperscript{78}

\textbf{1-3-3 Digital Computation}

Since the early fifties, digital computers, because of their
speed, flexibility and accuracy, have been becoming more and more
popular for stability problem studies of the power system. In Ch. (2)
a brief description of the digital computer, (English Electric Leo
Marconi KDF9) used for the present work and the numerical methods of
solving differential equations and inversion of matrix, together with
their respective digital programmes, have been discussed.

In Ch. (5) Park's equations for the synchronous machines have
been developed, and the a.c system including speed governors and
excitation controls has been transformed into a mathematical model.
Two digital programmes have been written, one for drawing the swing curves of the synchronous machines when different types of faults have been applied on the network, and the other, known as "search programme", for drawing automatically the stability boundaries of the system.

1-4 High-Voltage-Direct-Current (h.v.d.c.) System

Recently the attention of power engineers has been diverted to the advantages of asynchronous power transmission over long distances by the h.v.d.c. system. The first commercial application of this system was brought into service in 1954 between the Swedish mainland and the island of Gotaland for delivering 20-MW at 100-KV over a single-conductor cable. Later on, other commercial h.v.d.c. systems were commissioned, and recently in U.S.A. two long h.v.d.c. lines of 1300-MW capacity at 800-KV each have been included in the ambitious project of "The Pacific Northwest-Southwest Intertie". These d.c lines will be the first in U.S.A. and will be the longest in the world, their individual lengths being 1375 km and 1325 km.

1-4-1 Operation of the h.v.d.c. System

In the h.v.d.c. system the power is rectified from the system frequency to direct current, and is transported to the load centres where it is inverted back to the system frequency for distribution to the consumers. In Ch. (3) major components, operational details, harmonic filters, reactive power demand and controls, etc., of the converters have been discussed in some detail.
In Ch. (4) a h.v.d.c. system has been simulated and two digital programmes have been written, one for studying the characteristics of the converter, and the other for observing the transient behaviour of converter variables because of a sudden change in one of its parameters, e.g., reference current or a.c bus-voltages on either end of the h.v.d.c. system. In the digital programme all the possible controls of the h.v.d.c. system have been included and provision for imposing limits on the controls has been made.

In this study it has been shown that, though the h.v.d.c. system has no significant inherent response characteristics of its own, it can be made to respond rapidly to controls, and thus it can be employed most effectively during a disturbance in the system.

1-4-2 Economics of h.v.d.c. Transmission

The efficiency of h.v.d.c. transmission over a long distance is evident in statistics on losses. When the comparisons are equal in terms of the amount of power, distance, size of conductor and peak voltage, a.c. losses are appreciably greater than those in a d.c line. Also, a d.c line with two conductors and its ground connections loses only about half its transmission capacity when one conductor fails, whilst in an a.c circuit, if one conductor breaks down, all the transmission ceases. If the terminal a.c equipment, such as a transformer fails, all the transmission is lost, but on the other hand, if half the terminal equipment in an h.v.d.c. system fails, the line can still transmit power at its one-half capacity. The fact that the earth can temporarily be used as a return conductor in case of the d.c line is an added advantage.
The terminal costs of the h.v.d.c. system are higher as compared with those of the a.c system, but, on the other hand, the costs of the d.c transmission line itself is two-thirds of the a.c one.

Hence, to justify the h.v.d.c. system, the length of the transmission distance is one of the decisive factors. B.P.A. (Bonneville Power Authority) engineers have calculated the break-even distance as 500 miles. It is reasonable to assume that as the technique continues to develop and h.v.d.c. becomes more generally used, the cost of its terminal stations under competitive mass production will be reduced to a greater extent than those of an a.c system.

1-5 Comparative Study of Transient-Behaviour of a.c-d.c and its Equivalent a.c System

In Ch. (6) the parallel a.c-d.c system and its equivalent a.c system were studied from the transient stability point of view. All the controls on the a.c and d.c sides have been simulated and two digital programmes written. One programme draws the swing curves of both the a.c-d.c and equivalent a.c. system, independently, and the other draws the stability boundaries of both the systems. A comparison of the swing curves and stability boundaries establishes that the transient stability in the case of the a.c-d.c system improves only if the d.c power is rapidly increased whenever there is a fault in the a.c system.

The verbatims of the digital programmes used in this work are given in the appendices in Vol. 2; whilst a description of the
identifiers used together with a brief explanation of the programmes are included in each individual appendix. There is great scope in this newly developed h.v.d.c. technology and the programmes written by the author may be another guide line for future work in power system analysis.
2-1 DIGITAL COMPUTER

2-1-1 Introduction

In recent years digital computers have gained great popularity in solving the problems of system engineering. This use is based on their ability to operate at great speed to produce accurate results, to store large quantities of information, and to carry out long and complex sequences of operation without human intervention. There are various makes of computers on the market but the author has worked on a KDF-9 digital computer\textsuperscript{10}, manufactured by English Electric Company, which has the storage capacity of 16-K, (16000 words) each word of 48 bits, arranged in four modules of 4000 words each. Each word is assessed in random fashion and the time taken to fetch or store a word is five microseconds whilst the binary addition of two such words takes one microsecond.

2-1-2 Steps for the Computer Programme

The following steps are to be carried out before a power system problem is solved by the computer:

(a) Numerical Analysis

For complicated problems, a considerable amount of simplification is necessary before information is fed to the computer. Generally, the computer in a single step can perform only one arithmetical operation or make only one logical decision, thus the majority of scientific problems involving integration, differentiating, vector manipulation, etc., are quite
(a) Cont'd ..

complicated for the computer to handle. Hence, a numerical method must be applied to translate these continuous functions using arithmetic difference, infinite series, continued fractions and iterative procedure etc.

(b) Programming

Programming implies all the planning which comes after numerical analysis, and is related specifically to the computer. A flow chart or block diagram is drawn and memory allocation is planned to keep accurate records of the coding procedure.

(c) Coding

Coding is the writing of detailed machine instructions which carry out the arithmetical operation called by the programme. The coding may be written in machine language or in any language that is in symbolic or abstract form.

There are many programming languages which are used for the computer. Though the fundamental ideas concerning alogarithm loops, the making of decisions, and the structure of arithmetic expressions, etc., are common to every computer language, some of them have been developed more than others and are widely used such as:

(a) MAD (Michigan Alogarithm Decoder).
(b) FORTRAN (Formula Translation).
and (c) ALGOL (Alogarithm Oriented Language), etc.
Of these languages Fortran is the oldest, and was introduced by IBM - another language "PL/i" is being introduced by IBM to meet the criticisms of Fortran. Algol is the preferred language for writing the programme for the KDF9 computer. Both "Algol" and "PL/i" are being used in the Computing Department of the University of Newcastle upon Tyne, though in the present work all the programmes, for the KDF9 computer, have been written in "Algol".

The Algol language was developed by the representatives of computer organizations and mathematicians from Europe and U.S.A. in an attempt to standardize the languages as much as practicable. As a result of a series of meetings from 1958 - 1960, a description of the Algol language was published. This language is relatively comprehensive and simple to understand, so it is quite suitable for the publication of computer programmes.

A programme written in Algol is composed of a number of instructions called statements, which specify how the problem is to be solved, and is termed as a source programme. A source programme is not readily comprehensible to a computer which can only obey the instructions if they are written in its own language, called machine language, and this is different for every type of computer. Thus, prior to obeying the source programme, a translation from Algol to machine language is carried out by a machine language programmer, called compiler, and the new programme is termed as object programme. For every source language there are different compilers, and for Algol a Whetstone Compiler or a Kidsgrove Compiler is used - the latter being much faster and more comprehensive than the former. When the object programme is run in the computer, it must be accompanied by
"data" on which it operates, the data of the problem being put into the computer either on punched cards or on punched tapes.

2-1-3  Check Out and Production

There are great chances of error in the written programme. The error can be in the translation which can be easily corrected; but sometimes the programme only solves a part of the problem and then fails to proceed further. In such a case the mathematical calculation requires checking. Even if the programme works up to the end, the results must be checked for the desired accuracy of the answers. When the programme is established, it remains only to obtain the required answers for different sets of input data and the operation becomes a routine which can be handled by an operator with less training than the programmer.

2-2  Solutions of Differential Equations

In Power System Analysis, some complicated differential equations are to be solved, which, if attempted by orthodox methods are quite cumbersome, and sometimes even impossible to solve. Due to this and also due to the advent of the digital computer, mathematicians and engineers have found some new numerical methods of solving various types of differential equations, and now more stress is being given to this new numerical approach. However, for engineering problems the methods dealing with the first order differential equations with initial conditions are of major interest, as higher order equations can be reduced to a set of simultaneous first order equations. For example, the second order equation:
\[ p^2 y = f(p, y, y, x) \]
can be written:
\[ p^2 z = f(z, y, x) \]
\[ p^2 y = z \]
where "z" is a new dependent variable defined by the second equation.
There are two simultaneous equations in "y" and "z" and their solution gives the function as well as the derivative.

**2-2-1 Taylor Series Solution**

This is one of the basic theoretical methods of solving differential equations. This method, though not used for computation, has its own importance as it provides the basis of evaluation and comparison of the other methods that are of considerable practical use.

To investigate the Taylor series solution the following simple differential equation is considered:
\[ p^2 y. = f(x, y) \] \hspace{1cm} (2-1)

and, \[ y(x_0) = y_0 \] \hspace{1cm} (2-2)

The Taylor series expansion of the solution \( y(x) \) about a point \( x = x_m \) is written as:
\[ y(x) = y_m + p^2 y_m (x - x_m) + p^2 y_m/2 (x - x_m)^2 \]
\[ + p^3 y_m/6 (x - x_m)^3 + \ldots \] \hspace{1cm} (2-3)

where \( p^j y_m \) is the jth derivative \( (j = 0, 1, 2, 3 \ldots) \)

The solution at the next point \( x = x_m + h \), which is at a distance "h" from "x"m, is approximated as:
\[ y_{m+1} = y_m + h \cdot p_y + h^2 \cdot p_{x} y_m + h^3 \cdot p_{x}^2 y_m + \ldots \ldots \ldots \ (2-4) \]

The derivatives from (2-1) are evaluated as:

\[ p_y y_m = f(x_m, y_m) \]

and

\[ p_x^2 y_m = f_x + f_fy \] \hspace{1cm} (2-5)

where

\[ f_x = \text{partial derivative with respect to } x. \]

and

\[ f_y = \text{partial derivative with respect to } y. \]

Substituting the values of derivatives in (2-4) we get:

\[ y_{m+1} = y_m + h \cdot \left[ f + h/2 (f_x + f_y) \right] + O(h^3) \] \hspace{1cm} (2-6)

where \( O(h^3) \) is read as "order \( h^3 \)" and means that all the succeeding terms contain "h" to the third or higher powers.

The truncation error is given as:

\[ e.T = K \cdot h^3 \] \hspace{1cm} (2-7)

where \( K \) is some constant.

The Taylor series solution has been classified as a one-step method, because the calculation of \( y_{m+1} \) requires information only at one preceding point \( (x_m, y_m) \). The practical difficulty of this method is that it may become difficult or sometimes even impossible to find "\( f_x \)" and "\( f_y \)" especially when the evaluation of higher derivatives is to be carried out. This method is therefore impractical from the computational point of view and is only used for judging other methods, the yardstick being the extent to which they agree with the Taylor series expansion.
2-2-2 Runge-Kutta Methods

The study of practical computing methods begins with a broad class of techniques known as Runge-Kutta methods. To establish the patterns of these methods, Euler's method, which by definition is classified as Runge-Kutta method of first order, is considered. However, this method is seldom used but like the Taylor-series solution it provides a necessary starting point for other more practical methods.

Let \( y_m \) be the solution of the differential equation (2-1) at \( x = x_m \).

The slope of the curve at the known point \( (x_m, y_m) \) is expressed as:

\[
p \cdot y_m = f(x_m, y_m) \tag{2-8}
\]

The equation of this slope is written as:

\[
y = y_m + p \cdot y_m (x - x_m) \tag{2-9}
\]

For the next solution of \( y_{m+1} \) consider \( x = x_{m+1} \) at a distance "h" from "x". The point \( y_{m+1} \) lies at the intersection of the slope of the curve(2-8) and the ordinate at \( x_{m+1} \) and its expression is found from (2-9) and (2-8) as:

\[
y_{m+1} = y_m + h \cdot f(x_m, y_m) \tag{2-10}
\]

This agrees with the Taylor-series expansion (2-3) through terms in "h" and so the truncation error is given as:

\[
e \cdot T = k \cdot h^2 \tag{2-11}
\]
Besides having a relatively large value of "e.T", Euler's method is also unstable because the error by this method magnifies as the value of "x" increases. To reduce the truncation error (e.T) this method has been improved in different ways. Two of the improved versions of Euler's methods, which belong to a family of second-order Runge-Kutta methods, are (a) improved Euler's method, and (b) modified Euler's method.

2-2-3 Runge-Kutta 4th Order Method

This method is so widely used for integrating differential equations that it is simply known as "Runge-Kutta" (R,K,) method without any qualification of order and type. R,K. method can be defined by the following five equations:

1. \[ y_{m+1} = y_m + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4) \]

2. \[ k_1 = f(x_m, y_m) \]

3. \[ k_2 = f(x_m + \frac{h}{2}, y_m + \frac{h}{2} \cdot k_1) \]

4. \[ k_3 = f(x_m + \frac{h}{2}, y_m + \frac{h}{2} \cdot k_2) \]

5. \[ k_4 = f(x_m + h, y_m + h \cdot k_3) \] (2-12)

Truncation error is given as:

\[ e.T = K.T^5 \] (2-13)
Computational Consideration

The computational considerations in using R.K. methods are given as:

(i) The application of R.K. methods is described in terms of an Algol procedure "Kutta" based on the set of five equations (2-12), together with formal parameters \( x, y, h \) and \( f \). The last parameter is a real procedure written by the user, which "Kutta" calls by \( f(x, y) \) and evaluates the R.H.S. of the differential equation \( p \cdot y = f(x, y) \) as necessary. The procedure gives an increment of "h" to the parameter "x" and replaces the initial "y" values by an approximation to \( y(x+h) \).

The procedure "Kutta" is written as:

```
Procedure Kutta (x,y,h,f); value h; real x, y, h;
real procedure f;

begin real a,b;
  a: = f(x,y); x: = x + h/2;
  b: = f(x,y + h x a/2);
  a: = a + 2 x b;
  b: = f(x,y + h x b/2); x = x + h/2;
  y: = y + h x (a + 2 x b + f (x,y + hxb))/6;
end;
```

(ii) In the R.K. method mentioned above the number of function evaluations is five, and as such it cannot work with a truncation error of \( O(h^5) \) or less. To increase the accuracy further, the only way is to decrease "h" within
(ii) Cont'd ..

a reasonable limit, and to follow the method with a truncation error of $O(h^5)$.

(iii) From the programming point of view the R.K. method is very simple and makes it extremely convenient for automatic computation.

(iv) The R.K. methods require roughly twice as many function evaluations per step for a given accuracy as some multi-step methods.

(v) To select the right interval "h" is the most difficult aspect of using any of the one-step methods. To achieve the required accuracy, a smaller value of "h" is necessary in the region where the solution is changing rapidly as compared with the region where the change is slow. Hence, a reasonable choice of "h" demands some knowledge of the behaviour of:

(a) The solution of the differential equation, and
(b) The method of solution.

Since the former is usually not available, it is necessary to check in some way that the interval in use or proposed is adequate, e.g., it can be achieved by repeating a step at half the interval and comparing the results, though it increases the amount of work.

2-2-4 Kutta Merson Method

Merson has shown that an estimate of truncation error can be obtained at the expense of one additional evaluation of $f(x,y)$, as below:
If \( k_1 = f(x_m, y_m) \)

\[
\begin{align*}
  k_2 &= f(x_m + h/3, y_m + h \cdot k_1/3) \\
  k_3 &= f(x_m + h/3, y_m + h(k_1 + k_2)/6) \\
  k_4 &= f(x_m + h/2, y_m + h(k_1 + 3k_3)/8) \\
  k_5 &= f(x_m + h, y_m + h(k_1 + 3k_3 + 4k_4)/2)
\end{align*}
\]

and assuming that for \( x \in (x_m, x_{m+1}) \)

\[ f(x, y) = Ax + By + C \]  \hspace{1cm} (2-14)

where \( A, B \) and \( C \) are some constants

the truncation error in

\[ y_{m+1} = y_m + h(k_1 + 4k_4 + k_5)/6 \]

is given as \( a_1 \)

which is dominantly \( \frac{1}{720} h^5(y)^5 \) for small "h"  \hspace{1cm} (2-15)

while the truncation error in

\[ y_{m+1} = y_m + h(k_1 - 3k_3 + 4k_4)/2 \]

which is dominantly \( \frac{1}{120} h^5(y)^5 \) for small "h"  \hspace{1cm} (2-16)

Consequently, the magnitude of error in (2-15) is

\[ |a_1 - a_2| = \frac{5}{720} |h^5(y)^5| \]  \hspace{1cm} (2-17)

In the Kutta-Merson method, equation (2-15) is used to calculate \( y_{m+1} \) and equation (2-16) is used only to estimate the error. The error estimate requires one extra function evaluation as compared with the R.K. method, Sect. (2-2-3).
If "o" is some prescribed accuracy requirement, an interval 
"h" used in (2-17) is deemed satisfactory if

\[
\frac{o}{32} < \frac{1}{5} \left| a_1 - a_2 \right| < o
\]

If the R.H.S. inequality is not satisfied the interval can be
halved and the step repeated. If the L.H.S. inequality is not
satisfied the interval can reasonably be doubled before taking the
next step.

The Kutta-Merson method has been used extensively, and the
assumption of linearity in \( f(x,y) \) does not cause troubles in practice -
although, for non-linear \( f(x,y) \) the truncation error in (2-15)
contains an \( h^4 \) term. The Kutta-Merson method seems to slightly
underestimate the optimum interval size "h", and hence, it errs on
the safe side which is an advantage.

Kutta-Merson Algol Procedure

The Algol procedure has the following heading, and uses the
Kutta-Merson method for solving "n" simultaneous first order
differential equations:

```
Procedure KM3 (n, t, y, range, fn, acc, h);
```

where

- \( n \) = number of differential equations
- \( t \) = time, an independent variable
- \( y \) = an array containing values of the dependent
  variables (solutions) at "t".
  (The initial values of the array and "t" are
  defined by boundary conditions).
- \( \text{range} \) = the procedure replaces the values in the
  array \( y \) by the values at "t + range" and "t"
  is replaced by "t + range".
fn = a procedure called by fn (t, y, k), is to be written by the user which leaves in k[i] the value of the R.H.S. of the ith equation for the values "t" and y[i] currently available.

acc = allowable truncation error

h = Interval of integration. "h" is called by name and its value can be altered by the procedure, also the corresponding actual parameter must be an identifier not an expression or number. Prior to calling KM3, the identifier should be given a value of less than or equal to "range". On exit, the value left in "h" is convenient for continuing the integration further, but it should not be altered between calls in this case.

2-3 Inverse of a Matrix

The matrix equivalent of division is given by the reciprocal or inverse of the divider. In the algebra of numbers b/a = a⁻¹ b, i.e., the division by "a", is exactly equivalent to the multiplication by a⁻¹, where a⁻¹ is the inverse of "a". Unless "a" is zero there is always a unique reciprocal of "a". The essential property of the reciprocal is that:

\[ a \cdot a^{-1} = a^{-1} \cdot a = 1 \]  \hspace{1cm} (2-18)
2-3-1 Methods of Inversion

The inverse of a matrix can be found by any of the standard elimination methods of solving simultaneous linear equations. This inversion method can be explained as below:

A set of "n" linear equations expressed as

\[ \sum_{j=1}^{n} a_{ij} \cdot x_j = b_i \quad (i = 1, 2, 3 \ldots n) \quad (2-19) \]

can be rewritten in the matrix notations as:

\[ A \cdot X = B \quad (2-20) \]

where

- \( A \) = square matrix (nxn) of the coefficient \( a_{ij} \)
- \( X \) = column matrix of "n" components of the variable \( x_j \)
- \( B \) = column matrix of "n" components of \( b_i \)

The set of matrix equations (2-20) can be solved by any of the elimination methods, e.g., Gauss, Jordan, Gauss and Jordan methods, etc.

If there are more than one terms in (R.H.S.) matrix B for the same (L.H.S.) matrix A, the above mentioned elimination methods are equally good to compute the answers. In this case, extra columns of B are treated together while separate back-substitution is carried out for each column to get the final answers.

Thus, by the basic definition (2-18) and the procedure mentioned above, the inverse of the matrix A can be found by replacing B in set (2-20) with the unit matrix. The rest of the procedure is the same as that of solving simultaneous equations as mentioned above.
2-3-2 Inversion Procedures

Many Algol inversion procedures are available in the library. However, for the present work the author has selected an Algol library procedure LUA 23, written by Mr. J. M. Watt (Liverpool University). This procedure has been proved very efficient for the inversion of the admittance matrix of the system and has been used in the stability studies of the a.c - d.c and the a.c. systems.

The above mentioned procedure is based on the Gauss and Jordan method of elimination, and its procedure heading is as follows:

```
procedure inversion (n,a,b,f,singular);
where n = dimension of the array
    b = required matrix inverse (A^{-1})
    a = given square matrix (nxn)
    f = minimum allowable magnitude of the pivot
      (generally taken as zero).
    singular = this is a label, where the programme exits
                in case there is a singularity in the matrix.
```

The special features of the inversion procedure are:

(i) The programme checks whether all the elements in each row are less than unity.

(ii) If any element is greater than unity, the programme applies the proper reduction.

(iii) After (i) and (ii) the programme selects the largest element in the row as "pivot".
(iv) The programme checks whether all the elements in any one row are zero or whether any pivot is smaller than "r"; if this is the case, the programme goes to the label "singular" where the singularity is printed out and the programme terminates.
CHAPTER III

HIGH VOLTAGE DIRECT CURRENT

(h.v.d.c.)

SYSTEM
Technically speaking the "h.v.d.c. mercury-arc-valve" is the key to all the progress made in the field of h.v.d.c. power system. The supremacy of the mercury-arc-valve was established because of its high ratio of hold-on voltage to conducting voltage and the ease of its grid control, so, in spite of its heavy cost, this is the only device which has been commercially used so far in all h.v.d.c. conversion-stations. Although the designers are seriously working to find some ways and means to use instead thyristors in series to reduce the conversion cost, they seem to have a long way to go and the mercury-arc-valve is expected to play a vital role in h.v.d.c. technology for some time to come.

At the same time to reduce the conversion cost, improvements are continuously envisaged to increase the voltage and current ratings of the valve. At present, commercial mercury-arc-valves are available only in selected voltage and current ratings, and they are connected to form a group of two or more than two valves as desired. The maximum rating of six-pulse converter groups which are at present commercially available are:

<table>
<thead>
<tr>
<th>D.C. Voltage per Group</th>
<th>D.C. Amperes (4 Anodes)</th>
<th>D.C. Amperes (6 Anodes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>133 Kv</td>
<td>1200</td>
<td>1800</td>
</tr>
<tr>
<td>150 Kv</td>
<td>1200</td>
<td>1800</td>
</tr>
<tr>
<td>200 Kv (under investigation)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These converter groups may be connected in series to attain higher voltages. So far, a maximum of 600 Kv to ground has been used by connecting four groups per pole. To achieve a higher
current rating, valve groups may also be connected in parallel
with a separate smoothing reactor in each line.

Mercury arc valves fulfill the main requirements necessary
for h.v.d.c. power transmission such as 12:

1. A mercury arc valve allows the current to pass
during the conducting interval with a very low
voltage drop across it.

2. The firing instant can be controlled, i.e., when
the anode is positive, the valve fires as soon
as the control grid is made also positive but will
not fire during the blocking period.

3. It can withstand a high negative voltage during
the period when the valve is inverting.

4. It has the provision to allow a desired commutation
margin for de-ionization during inversion and hence
avoids the possibility of re-establishing the
conductance in the outgoing valve.

3-1-1 Construction

A mercury arc valve rated for 133 Kv (d.c) and 1800 amp. (d.c)
is shown in Fig. 3-1, and brief description of the vital parts of
the valve is given below:

1. Anode Assembly

This is the most vital part of the valve. It consists of a
main anode, a fairly high number of grading electrodes and a control
grid, e.g., 20 electrodes are provided for the above mentioned
valve. 11 In this way, the voltage stress per gap is reduced and
the possibility of unwanted firing and arc-backs is minimised. The current rating of such an anode assembly is limited because of the thermal and arc quenching restrictions. It is therefore necessary to use a number of parallel anode assemblies to get the desired valve current rating.

2. **Anode Porcelain**

   The anode porcelain, sealed to the tank by a rubber gasket, forms a vacuum tight envelope which serves as a supporting insulator for the different electrodes in the anode assembly.

3. **Voltage Divider**

   Every second electrode is connected to an external capacitive-resistive divider which, together with the mutual electrode capacitances, determines the voltage distribution between the control grid and the anode. The capacitors of the voltage divider are stacked in three oil filled containers where each stack serves two anode assemblies.

4. **Tank**

   The tank is half a cylinder with a horizontal axis and a plane tope where six porcelains of the anode assembly are fixed. The major portion of this tank is equipped with a water jacket, made of stainless steel to avoid corrosion.

5. **Cathode**

   The insulated mercury pool is used as a cathode and is housed inside the tank. It contains two ignitors and is water cooled.

6. **Mercury Diffusion Pump**

   This is a high vacuum pump having no moving parts, and operates continuously to maintain the pressure of the residual gas at a fixed minimum level.
7. **Pre-Vacuum Tank**

The compressed gas from the high vacuum pump is transferred to the pre-vacuum tank which serves as a vacuum reservoir. The tank is pumped with a normal rotating pre-vacuum pump at least once a year.

8. **Vacuum Gauges**

A hot wire vacuum gauge monitors the residual pressure in the tank while a simple mercury U-tube gauge measures the pre-vacuum level.

9. **Temperature Control (Cathode)**

The temperature of the cathode tank is held in the range of 30°C - 35°C to get the right mercury vapour pressure. On no-load, the excitation current just heats the tank but on-load, the temperature increases so much that a temperature control equipment becomes essential. The cooling equipment circulates the soft water through the plastic pipes and thus keeps the temperature of the cathode at the required level.

10. **Temperature Control (Anode)**

To ensure that the temperature of the anode is always higher than that of the tank, for avoiding condensation of mercury in the anode assembly, some special control equipment is used. This equipment circulates hot air between the gap of the anode porcelain and the insulated glass fibre tube. The air is circulated by a fan, and it goes up around three of the anodes and down around the other three and thus keeps the temperature at the required level. On no-load, the temperature is kept at the required level by letting out hot air by a thermostatic control damper while cold air is sucked in through a vent.
11. **Ignition and Excitation**

In the multi-anode multi-phase valves, the arc once started is picked up by one of the anodes and is then transferred to the other anodes in sequence by commutation, the ignition being necessary while starting. In a single anode valve, however, the arc extinguishes after every cycle; and there are two ways of having continuous arc, namely:

(a) by exciting every cycle at the desired firing angle,
and, (b) by adopting some arrangement to maintain the arc once it has been started by the ignitor.

In the case of mercury-arc-valves used in the h.v.d.c. system, the ignition and excitation anodes are placed on the plain top of the tank in between the anode assemblies. A grid biasing device produces a negative direct voltage which is impressed on the control grid. The positive pulses are then super-imposed on the negatively biased grid. These pulses are transmitted from the ground potential either by an isolating transformer or by a pulse transmitting system. The exciting and re-ignition system is shown in Fig. 3-2.

12. **Current Divider**

It ensures that the six anodes in parallel fire at the same instant and that the current is equally shared among them. It consists of six current transformers with the turn ratio of 1:6. The primary windings of these transformers are connected in series and carry the valve current while the secondary windings are connected in series with the associated anodes as shown in Fig. 3-3.
The present technology of h.v.d.c. power system has been evolved from the characteristics inherent in the mercury-arc-valves. The supremacy of this valve was established because of its high ratio of hold-on-voltage to conducting voltage, and the ease of its grid control. But on the other hand, the cost of the valve which is about 25% of the whole converter station and the greater space required by it, diverted the attention of the designers to investigate and improve the working of other more economical alternatives.

The thyristor was found to be a possibility because it could solve the problem of space-charge-neutralization without having a vacuum. Research continued further trying to increase the hold-on-voltage and to devise simpler methods of its control.

The size of the heavy current mercury-arc-valve together with its auxiliaries, increases the shunt capacitance to ground, and hence, to increase the hold-on-voltage capacity of the valve dividing rings are provided. A similar technique has also been used in the construction of high voltage rectifiers by using strings of series connected diodes. These diodes are compensated for reverse leakage by resistors, and the voltage unbalance caused by the shunt capacitance is controlled by parallel capacitors. Impulse voltage distribution across such a network is a hyperbolic function of the distribution constant and is expressed as:

\[ \alpha = \sqrt{\frac{C_g}{C_s}} \]  

where \( \alpha \) = Distribution constant

\( C_g \) = Total shunt capacitance to ground

and, \( C_s \) = Total Series capacitance
The smaller the value of distribution-constant the more uniform will be the voltage distribution. A typical circuit arrangement for each thyristor is shown in Fig. 3-4. A shunting resistance $R_2$ is incorporated across each thyristor to avoid the unbalancing of the steady state voltage distribution caused by the leakage current. A shunting capacitor is included in the circuit with a small resistance $R_3$, which controls the build-up of reverse voltage across any one unit until such a time that all units are blocked. A saturating reactor controls the rate of rise of current at the time of discharge until the load current flows. Many practical design approaches have been made to control the series-connected thyristor having regard for its inherent characteristics of quick recovery time and freedom from arc-backs.

The advantages of a thyristor converter station are:

(a) A freedom from arc-backs in thyristor, reduces transformer size and minimises service interruptions.
(b) Compact size and out-door installations of the station reduce the size of the building.
(c) There is less radio interference.
(d) There is improved accuracy of commutation control.
(e) There is no warming up time.
(f) Rate of change of load is limited only by system characteristics.

and, (g) There is perfect flexibility in voltage and rating.

The comparison of projected costs of an h.v.d.c. terminal using mercury-arc-valves and thyristors-in-series is shown in a tabular form.
<table>
<thead>
<tr>
<th>Item</th>
<th>Mercury-Arc-Valve</th>
<th>Thyristor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projected cost Turnkey (5 years)</td>
<td>$ 20 per Kw</td>
<td>$ 16 per Kw</td>
</tr>
<tr>
<td>Projected Efficiency (5 years)</td>
<td>98.80%</td>
<td>98.50%</td>
</tr>
<tr>
<td>Terminal Space Required</td>
<td>100.00%</td>
<td>20.00%</td>
</tr>
<tr>
<td>Maintenance and operation cost</td>
<td>$ 1.2 per Kw</td>
<td>$ 0.4 per Kw</td>
</tr>
<tr>
<td>Evaluated cost (5 years projection)</td>
<td>$22.20 per Kw</td>
<td>$17.0 per Kw</td>
</tr>
<tr>
<td>Evaluated cost (10 years projection)</td>
<td>$22.20 per Kw</td>
<td>$15.40 per Kw</td>
</tr>
</tbody>
</table>

The feasibility of using thyristors in series for the h.v.d.c. system has been established, but it will take time to use it on a commercial basis; this will then revolutionise the h.v.d.c. technology.

### 3-3-1 Valve Connections

The following three different ways of arranging the valves in groups are shown in Figs. 3-5:

(a) 3-phase bridge connection.

(b) Double-star interphase transformers connection.

and, (c) 6-phase diametral connections.

The comparison of different arrangements is given in a tabular form:  

16
<table>
<thead>
<tr>
<th>Parameters</th>
<th>3-phase bridge</th>
<th>Double Star</th>
<th>6-phase diametral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer Secondary volts (r.m.s.)</td>
<td>0.427 $V_d$</td>
<td>0.855 $V_d$</td>
<td>0.741 $V_d$</td>
</tr>
<tr>
<td>Peak Inverse Voltage</td>
<td>1.045 $V_d$</td>
<td>2.09 $V_d$</td>
<td>2.09 $V_d$</td>
</tr>
<tr>
<td>Average Value Current</td>
<td>0.330 $I_{dc}$</td>
<td>0.167 $I_{dc}$</td>
<td>0.167 $I_{dc}$</td>
</tr>
<tr>
<td>Transformer Secondary rating</td>
<td>1.047 $P_{dc}$</td>
<td>1.481 $P_{dc}$</td>
<td>1.814 $P_{dc}$</td>
</tr>
<tr>
<td>Transformer Primary rating</td>
<td>1.047 $P_{dc}$</td>
<td>1.047 $P_{dc}$</td>
<td>1.283 $P_{dc}$</td>
</tr>
<tr>
<td>Ripple Factor</td>
<td>0.0404</td>
<td>0.0404</td>
<td>0.0404</td>
</tr>
<tr>
<td>Peak Valve Current</td>
<td>0.5 $I_{dc}$</td>
<td>$I_{dc}$</td>
<td>$I_{dc}$</td>
</tr>
</tbody>
</table>

where $V_d$, $I_{dc}$ and $P_{dc}$ are the direct voltage, current and power respectively.

Comparing bridge and diametral connections, the latter has comparatively large primary and secondary transformer ratings and twice the peak inverse voltage for the same peak value of the current. Therefore, bridge connections are preferred.

Comparing the bridge connections with the interphase double-star, the transformer-secondary-utilization for the latter is poor though the primary-rating is the same for both. The valve with a double star connected group has to withstand twice the peak-inverse-voltage, but it only carries half the current as compared to the bridge connected group. High stresses are not desirable, and thus, bridge connections are preferred.
3-3-2 **Bridge Connections**

In Fig. 3-5-1a, a 6-pulse double-way converter circuit is shown. Two valves are connected with each phase of the secondary winding of the transformer (the anode of the upper valve and the cathode of the lower). Thus, when the phase voltage is positive with respect to its cathode, the direct current flows out of the phase concerned through the upper valve. Similarly, when the phase voltage is negative the direct current flows into the phase winding.

In Fig. 3-5-1g, the voltages for the 3-phases are shown and the d.c. output is indicated with a thick line. From Fig. 3-5-1h, the presence of the sixth harmonic is quite apparent. Each valve carries the full value of direct current for 120°, and there are always two valves conducting in series.

The complex cycle of direct current can be explained by considering any starting point in Fig. 3-5-1g, say A:

At "A" current flows from phase "R" - valve 1 - load - valve 6 - ph.Y
At "C" " " " " " "R" - " 1 - " - " 2 - ph.B
At "D" " " " " " "Y" - " 3 - " - " 2 - ph.B
At "E" " " " " " "Y" - " 3 - " - " 4 - ph.R
At "F" " " " " " "B" - " 5 - " - " 4 - ph.R
At "G" " " " " " "B" - " 5 - " - " 6 - ph.Y

In this case the delay angle is taken to be zero, i.e., the firing of the valve starts without any delay. The arc drop and the angle of overlap are also neglected.
In Fig. 3-6, the effects of the different delay angles on the direct voltage with respect to neutral are shown in phase and magnitude. For values of the delay angle up to 90°, rectification of the converter takes place, i.e., the converter behaves as a d.c. power source and the current flows in the direction of the voltage rise. When the delay becomes equal to 90°, the average output voltage is zero. If the delay angle is further increased, beyond 90°, there is reversal of the voltage polarity indicating inversion of the converter, i.e., it is acting as a d.c. load with a current flowing in opposition to the voltage rise.

These bridge units can be connected in series to increase the capacity of the system. Connections are made by earthing the bottom unit or by earthing the centre point to obtain a positive and a negative voltage with respect to earth. Bridge units can also be connected in parallel through an interphase transformer, but it is not favoured due to the problem of ensuring the equal distribution of the current.

Every bridge unit has its own transformer and by-pass valve, though there is a possibility of connecting two units by using a transformer having two secondaries and one primary.

3-3-3 Commutation

In the bridge shown in Fig. 3-5-1, the top valves 1, 3, 5 and bottom ones 2, 4, 6 are connected in pairs with the 3-phases of the transformer. In Fig. 3-7, if the voltage $e_b$ of phase $Y$ connected with valve 3, becomes higher than the voltage $e_a$ of
the phase "R" connected with valve 1, the current starts flowing from valve 1 to 3. The current from the outgoing valve 1, changes from \( I_{dc} \) to zero and the current from valve 3, changes from zero to \( I_{dc} \), but these changes take some definite time owing to the commutation reactance. For an instant both the valves conduct simultaneously, and thus short-circuit their respective phases. This process is known as commutation and the period of delay is called the angle-of-overlap or the angle-of-commutation. Similarly, the commutation of lower valves takes place when the voltage of the incoming valve is more negative than that of the outgoing one.

As shown in Fig. 3-7, the commutation takes place only when the firing angle is less than 180°, and after this point the voltage "\( e_a \)" becomes negative stopping the commutation of the valves. But, if the range of the firing angle is increased by any external means, the efficiency of the system can be improved. Different methods are under investigation to increase the firing angle of the converter, e.g.,

(a) by reversing the valve-to-valve voltage during the commutation \(^{23}\);

and, (b) by connecting series capacitors in the secondary of the converter transformer \(^{24}\).

But, so far none of these methods has been commercially applied.
3-3-4 Commutation Oscillation

When the self and earth capacitances of the transformer, converter and the d.c. line combine with the inductances in the different parts of the system, an oscillatory circuit is formed. So, whenever the converter voltage rises suddenly during the commutation, oscillations are set up which are known as commutation oscillations.  

In case of an uncontrolled converter, i.e., when there is only self and earth capacitance in the system, inherited commutation oscillations start at the end of commutation overlap. These oscillations are principally voltage oscillations while the associated current oscillations are quite insignificant. In the case of controlled converters an additional jump, which is quite apparent both in the voltage and in the current oscillations, is produced at the start of the commutation due to external circuit elements.

3-4 CONVERTER PARAMETERS

To establish the relations between the a.c. and d.c. parameters of the converter, the following assumptions are made:

(a) The load on the d.c. side has infinite inductance to keep the d.c. output constant.

(b) The a.c. bus-bar impedance is zero, i.e., the a.c. system has infinite capacity.

and, (c) The arc drop of the valve, magnetising current, and the resistance of the transformer are negligible.
3-4-1 Rectification

The following expressions are used to establish the relationship between a.c. and d.c. parameters, when the converter is operating as a rectifier, i.e., when the delay angle is less than 90°.\(^1\)

\[
V_d = \frac{3}{\sqrt{2}} \cdot V_s \cdot \cos \alpha - I_{dc} \cdot \frac{3\omega L}{\pi} \tag{3.2}
\]

\[
I_{dc} = \frac{V_s}{\sqrt{2} \cdot \omega L} \{\cos \alpha - \cos (\alpha + \gamma)\} \tag{3.3}
\]

and, \(\cos \phi = \frac{1}{2} \{\cos \alpha + \cos (\alpha + \gamma)\} \tag{3.4}\)

The rectifier equivalent circuit is shown in Fig. 3-8.

3-4-2 Inversion\(^2\)

When the delay angle of the valve is increased beyond 60°, the negative voltage is quite pronounced. If the load on the rectifier is purely resistive, the conduction will be intermittent because the current cannot flow in the negative direction through the valve. But due to the presence of a large reactor in the line, the resultant of the positive and negative voltage output is continuously given out. When the delay angle is 90°, the resultant d.c. output is zero. Beyond this angle, theoretically the resultant output becomes negative, but in practice it remains zero as the current cannot pass through the valve from cathode to anode. If at this stage, the d.c. voltage which is larger than the negative resultant voltage is applied, it will force the current to flow from anode to cathode in opposition to the induced e.m.f. of the secondary of the transformer and the power will flow from the d.c. to the a.c. system. Hence,
the converter will now be acting as an inverter. Fig. 3-9, shows the inversion operation of the converter.

The commutation cannot be delayed beyond "A" because after this point the voltage of phase "R" becomes negative with respect to that of phase "Y". In fact, margins for commutation and de-ionization are to be kept, otherwise the firing of the previous valve will start again. The angles used in the inversion process are co-related by the following expressions:

\[ \beta = 180 - \alpha \]  \hspace{1cm} (3-5)

or \[ \beta = \gamma + \delta \]  \hspace{1cm} (3-6)

All the parameters of the inverter can be calculated as in the case of the rectifier. Alternatively, the equations may be obtained by putting \( \alpha = \pi - \beta \) and \( \gamma = \beta - \delta \) in the rectifier equations as:

\[ V_d = \frac{3\sqrt{2}}{\pi} \cdot V_R \cdot \cos\beta + I_{dc} \cdot 3\omega L/\pi \]  \hspace{1cm} (3-7)

\[ V_d = \frac{3\sqrt{2}}{\pi} \cdot V_R \cdot \cos\delta_o - I_{dc} \cdot 3\omega L/\pi \]  \hspace{1cm} (3-8)

and, \[ I_{dc} = \frac{V_R}{(\sqrt{2}\omega L)} \cdot (\cos\delta_o - \cos\beta) \]  \hspace{1cm} (3-9)

The equivalent circuit of the inverter is shown in Fig. 3-10.
3-5 CONVERTER TRANSFORMERS AND REACTANCES

3-5-1 Operations of Converters

When a large power is to be transmitted through the d.c. link, three separate units of single-phase transformers are used to obtain a 6-pulse operation of the converters. This operation produces a large number of harmonics which necessitates the use of harmonic filters. Therefore, to reduce the harmonics and consequently the cost of the filters, two bridges for each d.c. pole are used, where each bridge has its own transformer. The secondary windings of these transformers are displaced by connecting one in delta and the other in star as shown in Fig. 3-11. A 12-pulse operation is thus obtained with comparatively fewer harmonics than in the previous case of a 6-pulse operation. A 24-pulse operation is also possible but so far it has not been commercially used.

3-5-2 Commutation Reactance

The effect of commutation, as discussed in Sec. 3-3-4, appears as a resistance on the d.c. line and results in a voltage drop without loss of power. This equivalent resistance is mainly due to the reactance in the windings of the converter transformer, and is known as commutation reactance.

Commutation reactance, in addition to effecting the regulation of the d.c. line, controls the rate of change of current at the end of each valve conduction period of the converter. A low value of rate of change of current results in a poor displacement factor and
increases the reactive-power-demand. On the other hand, a too high value of rate of change of current may lead to an unacceptable rate of arc-backs.

Generally, the rate of change of current is specified by the manufacturer of the valve, and the inductance in the windings of the converter transformer is designed according to the requirements of the different systems. For example, if the regulation of the transmission line is the main criterion of design, then the inductance of the windings should be as low as possible but consistent with the given rate of change of current. If the converter is to be connected to an a.c. source of high fault level, then it may be necessary to have a high reactance in order to restrict the arc-backs to the specified limits.²⁰.

3-5-3 Winding Rating of Converter Transformer

The windings on the valve side of converter transformers are under strain due to the d.c. as well as to the normal a.c. voltages. Thus, the design of the converter transformer slightly differs from that of the ordinary power transformer. In this case, its secondary winding is always on a separate core and its rating is calculated as below:

(a) Approximate expression:

\[ MVA = 3. I \cdot K_y \]

also \[ I = I_{dc} \cdot 2/3 \]

hence \[ MVA = 2I_{dc} \cdot V_d \]

where \( I \) = alternating current of the valve winding of the transformer

\[ MVA \] = rating of the valve winding of the transformer
\( \text{KV} = \frac{\text{No-load valve winding voltage}}{\text{of the transformer}}. \)

(b) Accurate expression:

\[
\text{MVA} = \sqrt{P^2 + Q^2 + \Sigma (VA)^2} 
\]

where \( \text{MVA} = \) Rating of the valve winding of the transformer.

\( P = \) Rated power of the converter.

\( Q = \) Reactive demand of the converter at fundamental frequency, when delivering the rated power "P".

and, \( \Sigma (VA)^2 = \) The sum of the square of the "VA" demand at the various harmonic frequencies.

3-5-4 Tap Changer

When the converter is not directly connected to the generator, the terminal voltage of the converter station is controlled by the tap-changer to keep the direct voltage within the limits for a fixed delay angle.

3-5-5 Smoothing Reactance

In case of a fault on the a.c. side of the inverter, the converter voltage falls and causes a heavy current to flow in the line, which reduces the commutation angle to such an extent as to cause blocking of the inverter valves. Therefore, heavy reactors are to be provided one on each side of the d.c. lines, and these
are known as smoothing-reactances. Thus, the time constant of the line increases and delays any change in the flow of the current, whilst in the meantime the control of the invertor comes into operation so avoiding the blocking of the valves. Similarly, the blocking of the rectifier valves is checked when a heavy current tends to flow because of the fault in the d.c. line.

3-6 REACTIVE POWER REQUIREMENT - HARMONICS AND RADIO INTERFERENCE

3-6-1 Reactive Power Requirements

The reactive power is consumed in both the conversion operations; but, during inversion the active power is transmitted by the converter while during rectification the active power is taken from the a.c source. On normal load, the reactive power demand of the invertor goes up to 50-60% of the active power delivered which sometimes exceeds the rating of the generators, even if the reactive power generated by the harmonic filters is taken into consideration. Therefore, in such like cases an external source of reactive power becomes essential.

The choice of the reactive power source depends upon the functions of the converter stations, for example:

(a) If a d.c line is linking two large a.c systems, and the generators on both sides together with their respective filter banks are capable of supplying and absorbing the reactive power at heavy loads and light loads respectively, the external source of reactive power can be discarded. Otherwise, either synchronous
or static compensators are used to supplement the reactive power demand of the converters.

(b) If the d.c line is the only source of electric supply or is connected with a weak a.c system, then only synchronous compensators are used.

Synchronous Condensers

When a d.c line connects a weak a.c. system or is the only source of supply, synchronous compensators are used to supplement the reactive power demand of the converters. In the latter case, the synchronous compensators also supply the a.c. power which is essential for the valve commutation of the inverters, whilst in the former case, they improve the voltage regulation of the a.c. system.

For a practical example the synchronous compensators installed at Haywards sub-station can be considered. They are specified at 5 MVAR at zero p.f. (leading), so that together with 50 MVAR per pole provided by the filter banks, these compensators can feed the power from the invertor station to the a.c. system at unity p.f. The sub-transient reactance of the synchronous compensator is less than 16.2%.

The magnitudes of the various harmonic currents as a percentage of 50 c.p.s. rated current are given as:

<table>
<thead>
<tr>
<th>No. of Harmonic</th>
<th>5th</th>
<th>7th</th>
<th>11th</th>
<th>13th</th>
<th>17th</th>
<th>19th</th>
<th>23rd</th>
<th>25th</th>
<th>29th</th>
<th>31st</th>
<th>35th</th>
<th>37th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of the rated current at 50 c.p.s.</td>
<td>13.0</td>
<td>9.2</td>
<td>5.8</td>
<td>5.2</td>
<td>4.0</td>
<td>3.4</td>
<td>3.0</td>
<td>2.6</td>
<td>2.4</td>
<td>2.2</td>
<td>1.8</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Static Condensers

For a long time, the reactive power supply supporting the existing power system has been obtained from synchronous compensators, but now there are good prospects of their place being taken by static condensers.

Around 1937, experiments were made at the Laboratories of Siemens, Schuckertwerk A.G., Berlin and Selimstadt, and it was in established that a definite improvement/system stability is obtained by the use of saturated reactors, but it is only now that the real cause of this phenomenon has been explained. It has been found that the stability is due to there being different flux shapes in the saturated core from which the control over the reactive power was derived.

Static condensers, which are used to supply the reactive power to the converters, consist of fixed capacitors connected in parallel with a saturable reactor fed from a transformer. In Fig. 3-12, a schematic diagram of a reactor of this type (Quinn reactor) is shown. This reactor is used in conjunction with capacitors and provides the reactive power demand.

A comparison of synchronous and static compensators is given below:

<table>
<thead>
<tr>
<th>ITEMS</th>
<th>SYNCH. - COMP.</th>
<th>STATIC COMP.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Capital Investment</td>
<td>Comparatively high</td>
<td>Low</td>
</tr>
<tr>
<td>2. Maintenance cost</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>3. Starting-up conditions</td>
<td>&quot; difficult</td>
<td>Easy</td>
</tr>
<tr>
<td>4. Operating Gear</td>
<td>Complicated</td>
<td>No operating gear</td>
</tr>
<tr>
<td>5. Moving part (wear and tear)</td>
<td>Yes</td>
<td>No moving parts</td>
</tr>
<tr>
<td>6. Improvement in the stability of the system</td>
<td>Comparatively less</td>
<td>More</td>
</tr>
</tbody>
</table>
The drawback of static compensators is that they cannot be used to supplement the power of the inverter station connected with a weak a.c system.

3-6-2 Harmonic Filters

During the conversion operation, phase displaced bridges produce a substantial number of harmonics, for example, a 6-pulse bridge gives the harmonics of the order \((6n + 1)^{31}\), where "n" is an integer; and similarly when two bridges are phase displaced to give a 12-pulse operation the harmonics are of the order \((12n + 1)\).

To avoid the injurious effects of the harmonics entering into the a.c system, they are filtered at the converter terminal. The most adequate method of eliminating the harmonics is to connect a filter bank in shunt with the a.c system, thus providing a low impedance path for the harmonic currents.

There are two types of fixed-tune filters which are generally shunt connected to the h.v.d.c system, namely, high-Q series resonant filters, and damped low-path filters. Series blocking filters are generally avoided because:

(a) They increase the commutation reactance, and thus the regulation of the d.c line.

and, (b) The cost of the components, for full line insulation and line current rating, is very high.

In the harmonic filters, provision is to be made for harmonics of the order 5-25. Thus, eight series resonant arms are required where each arm, as shown in Fig. 3-13a is tuned to a separate
harmonic frequency. Alternatively, one damped low-pass arm shown in Fig. 3-13b which covers the entire frequency spectrum, is used. Both of these alternatives mentioned above are economically not feasible. Hence, in a practical scheme either a reduced number of series arms, up to a maximum of six, or a combination of series arms for lower harmonics and a damped arm for the higher frequencies are used.

The compound-series-resonant filters, as shown in Fig. 3-13c, can also be used to filter the harmonics. These filters are compact and use less space, but their tuning is comparatively difficult. Self-tuned filters are also used in the h.v.d.c. system, and they are comparatively less expensive when used for the converters of high ratings. It should however be noted, that where both frequency and temperature variations are small or where high reactive power generation at fundamental frequency is required, fixed-tuned filters are still preferred.

3-6-3 Radio Interference

Radio interference must be taken into consideration while designing the terminal station of the h.v.d.c. system. Though radio interference can easily be eliminated by using inexpensive means at the converter end, it is still advisable to provide the valve room with metallic screened walls.
The real strength of the h.v.d.c. power system lies in its quick control without needing the help of massive and expensive circuit breakers and other conventional control equipments used in the a.c system. This saving off-sets the high cost of the terminal station to some extent.

As a whole there are three means of control in a converter station, namely, through the converter delay angle, through the on-load tap-changer and through the A.V.R. (automatic voltage regulator). The delay angle can be varied to realize the control in a few cycles, and this has made the h.v.d.c. system more stable especially during the transient disturbance. The other two controls have a secondary importance for the h.v.d.c. system. An A.V.R. on the a.c generator acts within a time of the order of one second, and the on-load tap-changer takes several seconds to change each tap of the transformer, although each tap represents a voltage change of 2%. However, if the on-load tap-changer is used in conjunction with the converter transformer, it comes into action when the delay angle is at its extreme limits.

3-7-1 Converter Delay Angle Controls

In the converters, by varying the delay angle while keeping the a.c voltage fixed, the d.c voltage can be varied from zero to the value which corresponds to zero-phase delay in the grid. This unique characteristic of converters has been utilised in the applications of all types of delay angle controls.
Constant-Current (C-C) Control

In this case, the grid control is so arranged that the voltage of the converter is continuously matching with the counter e.m.f. of the converter on the other end, thus pushing a constant value of the current through the d.c line. But, if at any instant the maximum limit of the d.c voltage is exceeded and the converter voltage does not match with the counter e.m.f. the value of the current drops to zero unless C-C control is applied on the other converter. Hence, C-C controls on both rectifier and inverter are essential, though the inverter C-C control operates only when the rectifier voltage is not sufficient to push the required amount of current through the line. The setting of the inverter C-C control is always kept somewhat lower than that of the rectifier, the difference being called the current-margin.

Constant Extinction Angle (C.E.A.) Control

In this case, the grid control is so arranged that the converter has a sufficiently large margin of commutation. From the safety consideration the larger the angle the safer it is, but from the point of view of maximum utilization of the converter and the minimum demand of reactive power, it is desirable to keep the angle of advance to a minimum. Hence, the most desirable mode of operation is that which adjusts the angle of advance in such a way that the extinction angle always remains constant at a value slightly higher than the angle of deionization. This mode of control is known as Constant-Extinction-Angle Control.
In the normal operation, the rectifier and inverter are on C-C and C.E.A. controls respectively, but, when the rectifier voltage falls below a certain level and the C-C control is on the inverter side, the inverter in its efforts to reduce the current margin tends to raise the voltage unless it is controlled by the C.E.A. control on the rectifier side.

From the above discussion, it has been concluded that for the successful operation of the h.v.d.c. system both the rectifier and inverter must be equipped with C-C and C.E.A. controls each. In this particular case, if the voltage supplied by the rectifier drops due to some transient disturbance, the C-C control of the inverter intervenes and lowers its own voltage to such an extent that the current corresponding to its current-setting continues flowing. Hence, the power drops only by an amount that corresponds to the current-margin and initial voltage drop. But in the meantime, the on-load tap-changer on the rectifier transformer operates and eliminates any lasting deficiency in the d.c voltage of the rectifier, and the normal operation of the inverter resumes.

**Power Control**

Although C-C control as discussed above offers many features, it does not provide a direct means of regulating the power flow of d.c system. To realize a simple type of power control the C-C control is modified, the fixed current-setting being replaced by a current-setting which is continuously being calculated by dividing the reference power by the actual rectifier voltage.
Frequency Control

If the frequency of the h.v.d.c. system is to be maintained constant at either end, the current-setting is obtained from an automatic frequency discriminator.

3-7-2 Faults and Protection of the h.v.d.c. System

Faults due to Lightning

Though the h.v.d.c. transmission line is properly protected by lightning arrestors and ground wires, etc., as in the a.c line, the flashovers due to lightning still cannot be overruled.

The follow-current in the flashover, in the case of an a.c transmission line, is several times the normal value of the current (depending upon the short-circuit capacity of the system). But on the other hand, in the case of the h.v.d.c. line the follow-current in the flashover does not exceed the value of the current margin of the C-C controls. Hence, the outage due to the flashover in the h.v.d.c. system is considerably shorter than in the case of the a.c system in a similar situation. At the instant of flashover, the h.v.d.c. line is momentarily de-energized by automatically resetting the grid control of the rectifier into inverter operation.

Interior Faults

It is a peculiarity of the converters that their valves are exposed to occasional interior short circuits. These transient faults and the arc-backs in the valves, form a short circuit on the a.c side which has to be cleared by the grid blocking of all six valves of the affected converters. The blocking is so quick that
the short circuit lasts for only one-half cycle, and between 0.5 -
0.7 sec. normal operation is resumed. During the blocking period,
the line current is transferred to the by-pass valve in order that
the other converters of the pole can operate steadily.

The design of the valve is so co-ordinated with the parameters
of the circuit that they can stand the arc-back current without
any damage.

Commutation Failure

The commutation failure of the converter is caused either
by a disturbance in the voltage wave of the a.c system or by an
arc-through in the valve. In both the cases, the commutation
failure of the inverter causes a short circuit on the d.c side
and heavy current tends to flow, but due to the high value of
the time-constant of the d.c line and smoothing reactors, the
rise in current is delayed allowing the C-C control to come into
operation. Generally, within a fraction of a second the short
circuit due to the commutation failure is cleared.
FIG. 3-1 MERCURY-ARC-VALVE

1. Anode assembly
2. Anode porcelain
3. Voltage divider
4. Control pulse input
5. Grid bias device
6. Excitation and ignition set
7. Tank
8. Excitation anode
9. Ignitor
10. Cathode (mercury pool)
11. Chassis
12. Outlet
13. Inlet
14. Equipment for temperature control of anodes
15. Mercury diffusion pump
16. Pre-vacuum tank
17. Water
FIG. 3-2 EXCITATION AND RE-IGNITION CIRCUITS IN A HIGH VOLTAGE VALVE

1. Leakage flux transformer
2. Semi-conductor valve
3. Semi-conductor valve
4. Non-linear resistor
5. Ignitor
6. Cathode
7. Non-linear resistor

FIG. 3-3 CURRENT DIVIDER CIRCUITS FOR A SIX ANODE VALVE
FIG. 3-4 TYPICAL SCHEMATIC FOR A SERIES CONNECTOR THYRISTOR
FIG. 3-5-1 BRIDGE CONNECTION

(a) (b) (c) (d) (e) (f) (g) (h)
FIG. 3-5-2 DOUBLE STAR INTERPHASE TRANSFORMER CONNECTION

FIG. 3-5-3 6-PHASE DIAMETRAL CONNECTION
FIG. 3-6 EFFECT OF VARIOUS DELAY ANGLES

$\alpha = 0$  $\alpha = 30^\circ$  $\alpha = 60^\circ$

$\alpha = 90^\circ$  $\alpha = 120^\circ$  $\alpha = 150^\circ$
FIG. 3-7  COMMUTATION

FIG. 3-8  RECTIFIER EQUIVALENT CIRCUIT

\[ R_c = \frac{3 \omega L}{\pi} \]

\[ V_{do} \cos \alpha \]
FIG. 3-9 INVERTER OPERATION

FIG. 3-10 INVERTER EQUIVALENT CIRCUITS

\[ V_{d0} \cos \beta \]

\[ V_{d0} \cos \delta \]
Fig. 3.11  SIMPLIFIED SCHEMATIC DIAGRAM FOR DOUBLE-12-PULSE OPERATION
FIG. 3-12 QUIN-REACTOR SCHEME FOR HARMONIC STABILITY AT LOW IRON LOSS
FIG. 3-13  TYPES OF FILTER BRANCH

(a) Simple series resonant filter

(b) Damped low pass filter

(c) Double-tuned resonant filter
CHAPTER IV

MATHEMATICAL MODEL

OF

H.V.D.C. SYSTEM
4-1 REGULATION CHARACTERISTICS OF CONVERTERS

4-1-1 Equivalent Circuit for One Terminal

The basic unit of an h.v.d.c. system is the 6-pulse bridge, and in the terminal stations one or more than one pair of these bridges are connected in parallel on the a.c. side and in series on the d.c. side. On the a.c. side different arrangements of the circuits are used according to the requirements of the installations while the individual bridges of each pair are phase displaced to give a 12-pulse operation. The equivalent circuit for one terminal is shown in Fig. 4-1. The circuit is equivalent with respect to the fundamental components on the a.c. side, but on the d.c. side two assumptions are to be made:

(a) All 6-phase bridges are identical except for their phase displacement.

and, (b) Each bridge commutates independently.

4-1-2 Converter Functional Equations

In the process of commutation, reactances \(x_a\), \(x_v\), \(x_t\) and \(x_c\) as shown in Fig. 4-1 take part, while the total commutation reactance \(x_c\) can be calculated depending upon the circuitry arrangement on the a.c. side of the bridge. For example, if a synchronous compensator is connected with the tertiary winding of the converter transformer then

\[
x_c = x_v + (x_a + x_t) \times x_t/(x_a + x_t + x_L)
\]

(4-1)
and in the two-winding transformers:

\[ x_c = x_l \]  \hspace{1cm} (4-2)

In Fig. 4-2, an equivalent circuit is shown which forms the bases of converter calculations. The converter has been represented by the commutation voltage \( e_c \) behind the commutation reactance \( x_c \), and a node has been introduced to represent the star point of the converter transformer. The a.c. voltage and the current are calculated at this node, and these values give the starting point to find the details on the L.H.S. of the star point.

To express the performance equations in p.u., a.c. and d.c. quantities are referred to the rated MVA capacity of the transformers as a power base with the supply line-to-line voltage as unit voltage. Transformer connections and converters are arranged in such a way that with a fixed a.c. bus voltage the no-load rectifier output voltage is equal to the normal voltage of the d.c. line. The commutation voltage \( e_c \) is taken as a reference vector and other variables are calculated as:

The average d.c. output voltage in p.u. is expressed as:

\[ v_d = e_c \cdot \cos \alpha - \pi/6 \cdot x_c \cdot i_{dc} \]  \hspace{1cm} (4-3)

On the a.c. side of the converter, the cosine of the angle between the bus voltage and the fundamental phase current is approximated as:

\[ \cos \phi = \frac{1}{2} \{ \cos \alpha + \cos (\alpha + \gamma) \} \]  \hspace{1cm} (4-4)
or

\[ \cos \phi = \frac{v_d}{e_c} \quad (4-5) \]

Direct axis component of the current on the a.c. side of the converter is given as:

\[ i_p = i_{dc} \times \cos \phi \quad (4-6) \]

The current on the a.c. side of the converter is approximated as \( i_{ac} \):

\[ i_v = i_{dc} \quad (4-7) \]

Quad-axis component of the current \( i_v \) is calculated as:

\[ i_q = \sqrt{(i_v)^2 - (i_p)^2} \quad (4-8) \]

As the quad-axis component of \( e_c \) is zero, the reactive power component of the converter is expressed as:

\[ Q_{dc} = e_c \times i_q \quad (4-9) \]

From Equations 4-3 to 4-9, the voltage and current components at the star point \( 'Y' \) are calculated, when commutating voltage \( e_c \), delay angle \( \alpha \) and the direct current \( i_{dc} \) are known. The remainder of the calculations, on the L.H.S of \( 'Y' \) in Fig. 4-1, are in terms of fundamental components of voltage and current.
4-1-3  Reactive Power of Synchronous Compensator

If synchronous compensators are connected with the tertiary windings of the convertor transformers, as shown in Fig. 4-1, then the reactive power "\( q_y \)", supplied at the star point is also taken into consideration, and can be calculated as:

\[
q_t = q_y + (q_y/v_y)^2 
\]

and,

\[
q_y = v_y^2/2x_t \times (\sqrt{1 + 4x_t x_t/v_y^2} - 1) \quad (4-10)
\]

where voltage "\( v_y \)" at the star point is expressed as:

\[
\overline{v}_y = \overline{e}_c - j \cdot (x_c - x_y) \cdot \overline{I}_v \quad (4-11)
\]

If the reactive power component of the synchronous compensators "\( q_t \)" is not known it is calculated from the following set of equations, reference Fig. 4-1, as:

\[
j\cdot q_t = A \cdot B + \overline{V}_1^* \cdot B \cdot C + \overline{V}_1^* \cdot A \cdot D + \overline{V}_1^* \cdot V_1 \cdot C \cdot D \quad (4-12)
\]

where

\[
A = (i_v^* + j \cdot v_y/x_1^*)
\]

\[
B = (\overline{v}_y + j \cdot x_t \cdot \overline{I}_v + \overline{v}_y \cdot x_t/x_1)
\]

\[
C = 1/(j \cdot n \cdot x_1)
\]

and,

\[
D = x_t/n \cdot x_1
\]

or,

\[
j\cdot q_t = A \cdot R + v_r \cdot B \cdot C (\cos \theta - j \cdot \sin \theta) + v_r \cdot A \cdot D (\cos \theta + j \cdot \sin \theta)
\]

\[
+ v_r^2 \cdot C \cdot D \quad (4-13)
\]

where

\[
\overline{v}_1 = v_r \times (\cos \theta + j \cdot \sin \theta)
\]

and,

\[
v_r = \text{Magnitude of the bus-bar voltage.}
\]
or

\[ j \cdot q_t = A \cdot B + v_r \cdot B \cdot C (\sqrt{1-x^2} - j \cdot x) + v_r \cdot A \cdot D (\sqrt{1-x^2} + j \cdot x) \]
\[ + v_r^2 \cdot C \cdot D \]  \hspace{1cm} (4-14)

where

\[ x = \sin \theta \]
\[ \sqrt{1-x^2} = \cos \theta \]

To find \( \theta \), the real part of Equation 4-14 is equated to zero as:

\[ (q_t)_R = (A \cdot B)_R + v_r^2 (C \cdot D)_R + v_r \sqrt{1-x^2} \{(B \cdot C)_R + (A \cdot D)_R\} \]
\[ + v_r \cdot x \cdot \{(B \cdot C)I - (A \cdot D) I\} = 0 \]  \hspace{1cm} (4-15)

or

\[ a \cdot x + b \sqrt{1-x^2} + c = 0 \]  \hspace{1cm} (4-16)

where

\[ a = v_r \{(B \cdot C)I - (A \cdot D)I\} \]
\[ b = v_r \{(B \cdot C)_R + (A \cdot D)_R\} \]
\[ c = v_r^2 (C \cdot D)_R + (A \cdot B)_R \]

and

The quadratic Eqn. 4-16, is solved for \( x \):

\[ x = \left( -a \pm \sqrt{a^2 - c^2 + b^2} \right) / (b^2 + a^2) \]  \hspace{1cm} (4-17)

and, \( \theta = \arcsin x \)

where \( \theta \) = the argument of \( \sqrt{1} \) such that the real part of the Eqn. 4-14 is zero.
The smaller absolute value of "x" is substituted in Eqn. 4-14 to find the value of "q_c".

4-1-4 Harmonic Filter Banks

The main function of harmonic filters is to reduce the harmonic currents, consequently reducing the mutual interaction of bridges; but these filters also prevent the a.c. network on their L.H.S. from taking part in commutation, and thus the reactive power demand of the converters reduces. In addition to this, filters generate reactive power which is calculated on the assumptions that the voltage across the filters contains no harmonics, and they behave as pure capacitances at system frequency.

Referring to Fig. 4-1, the susceptance "S" of the filter bank is given as:

\[ S = \frac{\text{Rating of the Filter Bank in MVAR}}{\text{Base MVA}} \]  (4-18)

System current is given as:

\[ i_s = i_1 + jS \cdot v_1 \]  (4-19)

Voltage behind the system reactance "x_s" is expressed as:

\[ e_s = v_1 + jx_s \cdot i_s \]  (4-20)

4-1-5 Digital Computer Programme

For the analysis of the converter station, the required circuit data and the loading conditions are given in Table 4-1. Generally, in the load data the power transmitted by the rectifier station, the
direct voltage and the delay angle are given on the d.c. side, and either the supply voltage or the voltage at the primary bus-bar of the converter transformer is specified on the a.c. side. In the latter case, the supply voltage can be computed and is kept constant for further analysis at different loading conditions of the converter station. The flow diagram for the digital computer is given in Fig. 4-3, and the verbatim of the programme is given in Appendix (1).

The digital programme established by the author is quite versatile and has the following special features:

1. It can be used for both rectification and inversion operations of the converter.
2. Provision of synchronous compensator, if the converter station is equipped with them, is provided within the programme.
3. Tap changing routine can be incorporated in the programme if required.
4. The programme gives the results in both p.u and as actual values if required.

4-1-6 Study of Characteristics

For the study of converter characteristics, a hypothetical h.v.d.c. system is considered in which an a.c. system at 230-KV is aschronously connected to another a.c. system by a 500-KV d.c. link. The total rated capacity of the converter transformers, on each side, is 1000-MVA. Both converter stations consist of six 6-pulse bridges, each connected in parallel on the a.c. side
and in series on the d.c. side. Filter banks of 210-MVA capacity are connected with the a.c. bus-bars, one at each end. Other parameters of the h.v.d.c. system are given in Table 4-1.

Three sets of studies are made:

(a) to observe the converter d.c. voltage, power, displacement angle and the reactive power demand at various d.c. line currents,

(b) to study the variations in direct current, power delivered and the reactive power demand for various delay angles of the converter, and,

(c) to study the working current and voltage of the d.c. system.

Case (a)
The converter is simulated

(i) as a rectifier without any external source of reactive power,

(ii) as a rectifier with harmonic filter-banks,

and, (iii) as a rectifier having the synchronous compensator connected to the tertiary windings of the converter transformer in addition to the filter banks.

The direct current is varied from 0 to 1.2 p.u., although the normal current is 1.0 p.u., and the computer results are graphically shown in Figs. h-4, h-5 and h-6. From the results following observations are made:

1. When no external reactive power source is used, the required a.c. supply voltage for the normal operation is 1.41 p.u which gives a comparatively poor voltage
regulation on the a.c. side of the converter station. The regulation is improved when the filter-banks are used, and in this case the required supply voltage is 1.3056 p.u. The regulation is the best when synchronous compensators are used which reduce the required supply voltage to 1.024 p.u.

2. The filter banks and synchronous compensators reduce the reactive power demand on the generators. For normal operation, when no external source of reactive power is used, generators supply 1.0 p.u. reactive power, whilst demand on the generators reduces to 0.5 p.u. with the filters, and to 0.07 p.u. when both filters and synchronous compensators are used.

3. Synchronous compensators are capable of absorbing the reactive power at light-loads and supplying the reactive power to the converters at heavy loads. In this study, the synchronous compensators absorb 0.04 p.u. reactive-power when the d.c. line current is 0.1 p.u. and deliver 0.7 p.u. at the normal current of 1.0 p.u. Thus synchronous compensators keep the a.c. bus-bar voltage constant at a pre-set value of 1.1 p.u.

4. The reactive power demand of the converters varies directly with the load irrespective of the reactive power source. In this study, the reactive power demand goes up to 0.5 p.u. at normal current in the d.c. line in all the three cases.
5. The variation in displacement angle from light load operation to the normal one is very small, something of the order of 0.043 radian in all the three cases.

Case (b)

The converter is simulated as a rectifier having a harmonic filter-bank on the a.c. side. The delay angle is varied from 0° to 180° and various curves are drawn as shown in Fig. 4-7. From these results the following observations are made:

1. For the delay angle between 0°-90 degrees, rectification takes place, i.e., the converters take the power from the a.c. source and the direct current flows away from the converters. The magnitudes of d.c. power, voltage and current decrease sinusoidally with the increase of the delay angle. When the delay angle is between 90 - 180 degrees, inversion takes place and converters supply power to the external a.c. circuit.

2. Reactive power demand of the converters increases when the delay angle varies from 0 - 90 degrees, and then starts decreasing when the variation of the delay angle is between 90 - 180 degrees - although always remaining positive. This shows that the converters absorb reactive power for rectification as well as for inversion operations.
**Case (c)**

The converter is simulated first as an inverter to find the inverter inherent characteristics, and then as a rectifier to find the total voltage characteristics of the rectifier including that of the d.c. line. The results are graphically shown in Fig. 4-8 and the following observations are made:

1. The combined characteristics of the rectifier and d.c. line intersects the inherent characteristics of the inverter at "A" (current 0.5 p.u; voltage 1.17 p.u; power 0.585 p.u) which defines the working point of the d.c. line.

2. If the rectifier is already working on its minimum delay angle, for a certain tap position, then the current in the d.c. line can only be increased by reducing the inverter voltage.

3. The point "B" (current 0.85 p.u; voltage 0.93 p.u; power 0.79 p.u) shows the working point of the d.c. line. when synchronous compensators are used on the inverter side having the same normal conditions as in case (1).

**4-1-7 Conclusions**

1. If generators have a sufficient capacity of supplying the reactive power, only harmonic filter-banks are adequate to supplement the reactive power demand of the converters.
2. If the d.c. link is the only source of power or is connected to a weak a.c. system, the use of synchronous compensators becomes essential to supplement the reactive power demand of the inverters as well as to maintain the voltage regulation of the a.c. system at a required level.

3. Converters absorb reactive power both in rectification and inversion, although they take the active power from the a.c. mains during rectification and supply the power to the a.c. system during inversion.

4. The working point of the d.c. line is not well defined, and due to this we require some form of controls for the efficient transmission of power by h.v.d.c. system.

4-2 SIMULATION OF D.C. LINK IN A.C. LOAD-FLOW

4-2-1 Load Flow Study

In the load flow study when a d.c. link is a part of the a.c. system, generally we encounter two types of systems:

1. When two large a.c. systems are connected through a d.c. transmission line - such as the "Cross Channel Scheme" which electrically connects France with England. In such interconnected systems if the load flow study of one is required, it is quite adequate to assume the constant voltage at the other system throughout the study and to ignore its loading conditions.
2. When the d.c. transmission line is supplying power to an existing weak system or is the only power source - such as the "Gotaland Scheme" in which the power is being supplied to the island of "Gotaland" from the mainland. In such cases, the d.c. transmission line is the integral part of the system and the loading information on both sides is essential.

4-2-2 D.C. Transmission Line

The line diagram of a d.c. transmission line is shown in Fig. 4-9. The windings of the converter transformer take part in the commutation action, and hence they are considered with the d.c. system for the load flow study. Furthermore, to reduce the size of the immitance matrices the harmonic filters are also considered on the d.c. side. If the a.c. bus-voltage on both sides are known, the voltage, current and power on the d.c. side together with the active and reactive components of the current and the power factor on the a.c. sides, can all be computed as in Sec. 4-1. The current components, thus obtained, are represented either by current vectors for the nodal analyses or by the corresponding admittance or impedance, and are incorporated in matrices of the system. The author has selected the former method for the required calculations of the load flow.

4-2-3 Converter Characteristics (with C.C. and C.E.A. Controls)

It has been assumed, that both the converters of the h.v.d.c. system are equipped with constant current and constant extinction angle regulators each. Thus the four possible modes of control are given below:
The voltage drop due to d.c. line and transformer resistances, is either subtracted from the rectifier characteristics or one half added on the inverter side and the other half subtracted from the rectifier side. The following expressions are used to simulate the d.c. link characteristics:

When the converter is on C.E.A. Control

\[ v_d = -V \cdot \cos \delta_o + i_{dc} \left( \frac{\pi}{6} \cdot x_c + \frac{R}{2} + \frac{1}{2} \cdot R_t \right) \]  

(4-21)

When the converter is on C.C. Control

\[ v_d = -V \cdot \cos \delta_o + i_{dc} \left( \frac{\pi}{6} \cdot x_c - \frac{R}{2} - \frac{1}{2} \cdot R_t \right) + K \left( I_r - i_{dc} \right) \]  

(4-22)

When the converter is on N.V. Control

\[ v_d = V - i_{dc} \left( \frac{\pi}{6} \cdot x_c + \frac{R}{2} + \frac{1}{2} \cdot R_t \right) \]  

(4-23)
where \( R \) = resistance of the d.c. line.

\( R_t \) = equivalent resistance of the transformer.

\( K \) = Gain of the C.C. regulator.

and, \( I_r \) = Reference current for the C.C regulator.

Two of the above equations, based on the mode of control of each converter, are selected and are used to calculate the working voltage and current of the d.c. line.

4-3-4 Transformer On-Load Tap-Changer

The main purpose of the on-load tap-changer is to ensure that the rectifier operates on C.C. control and the inverter on C.P.A. one, so that the reactive power demand should be at its minimum and the system operates at a high p.f. Thus from the power factor, which can be obtained from the p.f. relay, the tap position of the transformer can be decided. The upper and lower limits of the p.f. are fixed quite close to each other to ensure the consistent high p.f. operation of the system. If the p.f. of the system falls below the lower limit, the tap-changer operates in such a way as to decrease the a.c. voltage \( V_s \), and if the p.f. is higher than the upper limit, the tap changer increases \( V_s \).

The percentage variation in voltage due to each tap change should be very small, but on the other hand it is not desirable to operate a tap-changer too frequently. So, a range of \( \pm 10\% \) in 2% steps has been taken. The tap-changer can also be provided on the inverter transformer but its action is opposite, i.e., it increases the voltage \( V_R \) at a low p.f. and decreases it at a high p.f. The d.c. system being operated on C.C control, any change in the voltage on the inverter
side alters the transmitted power; thus it is preferable to operate
the tap-changer on the rectifier side, whilst the tap-changer on the
inverter side should only come into operation when the tapping of the
rectifier transformer is at its extreme positions.

4-2-5 Current Setting

A change in current setting is often required to keep the frequency
or the power of the d.c. link constant. Hence a separate routine,
according to the need of the system, can be incorporated in the main
programme.

4-2-6 Digital Programme for the d.c. Link

From the current settings of the converters the mode of operation
can be found. Suppose the current setting of "A" is higher than
that of B, then the former will be operating as a rectifier and the
latter as an inverter. Normally, the inverter operates on C.E.A.
control and the rectifier on C.C. control. The current margin "k"
is kept constant, and this requires the current setting of only one
converter whilst that of the other can be calculated as:

\[ I_{rl} = I_r - k. \]

The inverter voltage is expressed as:

\[ v_{di} = -V_r \cos \delta_o + \pi/6 \cdot i_{dc} x c \]  

(4-24)
(The voltage drop due to the line resistance has been taken on the
rectifier side whilst the transformer resistance has been ignored).

The rectifier voltage is expressed as:

\[ v_{dr} = -V_s \cos \delta_o + i_{dc} (\pi/6 \cdot x_c - R) + KR \cdot (I_r - i_{dc}) \]  

(4-25)
The point of intersection of these two characteristics, expressed by Equations 4-24 and 4-25, defines the working point of the d.c. line, hence

\[ V_{dr} = -V_{di} \quad (4-26) \]

From Equations 4-24, 4-25 and 4-26, the current \( i_{dc} \) of the d.c. link is calculated. To verify the validity of the above mode of controls, the natural voltage characteristic of the rectifier, which is at zero -phase delay angle, is expressed as:

\[ V_{dn} = V_s - i_{dc} (\pi/6 \cdot x_c + R) \quad (4-27) \]

If the absolute value of \( V_{dn} \) is greater than the absolute value of \( V_{di} \), the above assumption for the controls is correct, otherwise the rectifier operates on N.V. control and the invertor on C.C. control. The expression for the rectifier on N.V. control is given in Equation 4-27 whilst that for the invertor, when operating on C.C. control, may be given as:

\[ V_{di} = -V_R \cos \delta + i_{dc} \pi/6 \cdot x_c + KI (I_{rl} - i_{dc}) \quad (4-28) \]

where the current setting of the invertor is expressed as:

\[ I_{rl} = I_r - k \quad (4-29) \]

When the d.c. voltage and current are known, the a.c. components are calculated as in Sec. 4-1. The load flow diagram for the digital programme is shown in Fig. 4-10, and the verbatim of this programme is given in Appendix (2).
Main Features of the Programme

The programme is quite versatile and can be used to study the various aspects of the steady-state operation of the d.c. system, such as:

1) It has the provision to include or exclude the synchronous compensators on either side of the d.c. system. If marker \( r_s \) is not equal to 1000, the programme will skip over the calculations of synchronous compensator; otherwise these calculations will be included.

2) Constant current controls of the system can be ignored by putting the gains of the current regulators equal to zero. Thus both the converters start working on fixed delay angles.

3) The system response, due to the variation in a.c. voltage on either side of the d.c. line, can be studied. If the marker Fl = 1, the programme will take the different values of \( V_R \) whilst \( V_s \) will remain constant at a predetermined value throughout the study. If Fl is given any other value, the action will be reversed, i.e., various values of \( V_s \) will be taken keeping \( V_R \) constant.

4) To reverse the direction of power flow, the sign of the "current-margin" is changed and this automatically swaps the necessary parameters of both the converters for the new study.

5) The routine for the transformer on-load tap-changer has been included in the programme and can be used by putting the value of marker F = 1. For any other value of "F", the converter transformers operate on fixed taps.
4-2-7 Studies and Results

The following studies are made on a h.v.d.c. system. The data for each study is given in Table 4-2.

(1) **Operation of the h.v.d.c. System with Fixed Delay Angles of both the Converters**

This study is required to appreciate the effect of C.C controls on the operation of the h.v.d.c. system. With reference to Fig. 4-9, converters "A" and "B" are simulated as a rectifier and as an inverter respectively. Both the converters are operated at fixed delay angles by putting KR and KI equal to zero. \( V_s \) is varied from 1.2 to 0.001 p.u. while \( V_R \) is kept constant at 1.0 p.u.

The digital programme discussed in Sec. 4-2-6 has a special subroutine which comes in the loop when C.C control is not used and compares the characteristics of the rectifier and the inverter at every interval. Thus, if the characteristic of the rectifier is greater than that of inverter, the programme continues though otherwise it terminates.

The data with the proper values of the markers is punched and the above mentioned digital programme is worked out. The relation between the a.c. voltage drop and the power delivered by the d.c system is graphically shown in Fig. 4-11.

(2) **Operation of the h.v.d.c. system with C.C and C.E.A. Controls - Power is being transmitted from A to B**

The operation is the same as in case (1) except KR and KI are taken as 20 each. The current setting of converter "A" is taken as 0.75 p.u with a current margin "k" of 0.1 p.u. The voltage \( V_s \) is
varied from 1.2 to 0.0001 p.u while \( V_R \) is kept at 1.00 p.u throughout the study.

The relationships between the voltage drop on the one hand, and current, power and voltage on the d.c. side on the other, are graphically shown in Fig. 4-12.

(3) **Operation of the h.v.d.c. system with C.C and C.E.A. Controls**

Power is being transmitted from B to A.

The system is operated as in case (2) except the values of "I_r" and "k" are taken as 0.65 and -0.1 p.u respectively; \( V_S \) is kept constant, while \( V_R \) is varied from 1.2 to 0.001 p.u. The results are shown graphically, as before, in Fig. 4-13. (These values of "I_r" and "k" are deliberately taken to compare the results with those of case (2).)

(4) **Operation of h.v.d.c. system with C.C and C.E.A. Controls**

Power is flowing from A to B and \( V_R \) is varying.

The operation is the same as in case (2) except that "V_R" on the inverter side is varied while \( V_S \) is kept constant at 1.2 p.u. The results are graphically shown in Fig. 4-14.

(5) **Operation of h.v.d.c. system with C.C and C.E.A. Controls with transformer on-load tap-changer**

The operation is the same as in case (2) except that the predetermined values for the maximum and minimum limits of the power factor and those of the transformer tap-changer have been included in the data. The sub-routine for the transformer tap-changer comes into operation only when the value of the marker \( F = 1 \), otherwise it remains out of the programme. The results are graphically shown in Fig. 4-15.
Conclusions

The main objects of these studies are:

1. To establish the versatility of fast acting grid controls. The definite influence of grid controls on the responses of the various variables of the h.v.d.c. system, on account of the changes in the a.c voltages on either side, has been observed as shown in Figs. 4-13 and 4-14. It has also been shown that the combination of C.C and C.E.A controls on each side of the h.v.d.c. system can achieve reversal of power just by changing the sign of the current-margin of both the regulators.

2. To establish the indispensability of the grid controls in the h.v.d.c. system. It has been shown that if C.C and C.E.A controls are not used, the working point of the d.c line is not well defined and the system fails to transmit power even when the a.c voltage falls by a small value, as shown in Fig. 4-11.

4-3 SIMULATION OF DC LINE FOR TRANSIENT STUDY

4-3-1 Simulation of Control Signals

As described in Sec. 4-2, the most efficient control on the h.v.d.c. system is achieved when the rectifier operates on C.C and the inverter on C.E.A controls respectively. In addition to these controls, current-override provision is also essential on the inverter side, so that when the delay angle of the rectifier is at its minimum for a certain tap position of the transformer, C.C control is transferred to the inverter side, overriding its C.E.A control.
The C.C control signal of the converters is the function of the actual current passing in the d.c line and the reference current \( I_r \)—the actual relationship depending upon the type of the regulator being used. In the case of a zero-time-lag regulator, the following expression is used to define the signal of C.C control of the rectifier:

\[
ecr = KR \times (I_r - i_{dc})
\]  

(4-32)

During normal operation, the required delay angle may be simulated as:

\[
\cos \alpha = \frac{ecr}{Vs}
\]

(4-33)

The following limits are imposed on the control signal of the rectifier:

1) If the current of the d.c line exceeds the reference current of the rectifier, the control signal becomes negative and inversion starts. Hence, to avoid this the delay angle is limited to a value which is a little less than 90°, and the signal is expressed as:

\[
ecr = Vs \times \cos \alpha_{\text{max}}
\]

(4-34)

where \( \alpha_{\text{max}} \) = Maximum limit of the delay angle.

2) The reference current of the invertor is kept smaller, by a fixed "current-margin", than that of the rectifier. If the current in the d.c line becomes less than or equal to the invertor reference current, C.C control is transferred to the invertor side and the control signal of the rectifier is simulated as:

\[
ecr = Vs \times \cos \alpha_{\text{min}}
\]

(4-35)
The following expression is used to calculate the control signals of the invertor:

\[ e_{ci} = -V_R \cdot \cos \delta + \pi/6 \cdot x_c \cdot i_{dc2} + e_{ci} \]  \hspace{1cm} (4-36)

and,

\[ e_{ci} = K_I \left[I_r - x_k - i_{dc}\right] \]  \hspace{1cm} (4-37)

where \( e_{ci} \) = Control signal when the invertor is on C.C control.

The following limits are imposed on the control signal of the invertor:

1. If the direct current of the d.c line is more than the reference current of the invertor, then its regulator works on C.E.A control and the control signal is expressed as:

\[ e_{ci} = -V_R \cdot \cos \delta \]  \hspace{1cm} (4-38)

2. The control signal of the invertor should always be negative otherwise rectification starts. Whenever the positive C.C control exceeds the negative C.E.A. control, the invertor signal is simulated as:

\[ e_{ci} = -V_R \cdot \cos \alpha_{imax} \]  \hspace{1cm} (4-39)

where \( \alpha_{imax} \) = Maximum limit of the invertor delay angle.

4-3-2 Operation of d.c System

In the conventional equation for the operation of converters, it has been assumed that any change in the delay angle instantaneously changes the d.c voltage, and that any change in the line current affects the commutation drop without any delay. In the first case, it is only possible if the number of converter phases are increased, but generally 6-phase units are being used in commercial h.v.d.c.
stations; and in the second case, the time constant due to
commutation reactance does not allow for the instantaneous change
in the current of the d.c line. Hence, to simulate the controllers
properly a small lag is to be taken into consideration. In a
6-phase unit, the change in d.c voltage due to a corresponding
change in the delay angle occurs only when the electrical system
has rotated 60 degrees, i.e., 1/300 seconds in a 50 c/s system. Hence, the time constant equal to 1/300 has been incorporated in
the conventional equation as:

\[ \dot{v}_d = v \cos \alpha - \frac{\pi}{6} \cdot \frac{x_c}{V} \cdot \dot{i}_{dc} \]  

(4-40)

where

\[ \cos \alpha = \frac{1}{T_p + 1} \cdot \frac{x_e}{V} \]  

(4-41)

\[ \dot{i}_{dc} = \frac{1}{T_p + 1} \cdot \dot{i}_{dc} \]  

(4-42)

and, \[ T = 1/300 \]

\section*{4-3-3 d.c. Transmission Line}

To simulate the d.c transmission line a "T" section representation, as shown in Fig. 4-16, has been assumed and the following differential
equations are used:

\[ v_{dr} = R \cdot \dot{i}_{dc1} + \frac{x_{dc}}{2\pi f} (p \cdot \dot{i}_{dc1}) + v_c \]  

(4-43)

\[ v_{di} = R \cdot \dot{i}_{dc2} + \frac{x_{dc}}{2\pi f} (p \cdot \dot{i}_{dc2}) - v_c \]  

(4-44)

and, \[ v_c = x_{cr} \cdot 2\pi f/p \cdot (i_{dc1} - i_{dc2}) \]  

(4-45)
4-3-4 Digital Programme

The parameters of the converter stations and those of the d.c line are given in the data which also include the normal delay angle of the rectifier, reference current or power, and the "current-margin". The comprehensive flow diagram for the computer is shown in Fig. 4-17, and the verbatim of the programme is given in Appendix (3).

The algebraic equations for the digital programme are rewritten as:

\[ \text{e}_{cr} = V_a \times \cos \alpha \quad \text{(During normal operation)} \tag{4-46} \]

\[ i_{dc} = (I_r - \frac{e_{cr}}{KR}) \tag{4-47} \]

\[ v_{dr} = V_s \cdot \cos \alpha - \frac{\pi}{6} \cdot x_c \cdot i_{dc} \tag{4-48} \]

\[ v_{di} = v_{dr} - i_{dc} \cdot R \tag{4-49} \]

\[ e_{ci} = -V_R \cdot \cos \delta_o + \frac{\pi}{6} \cdot x_c \cdot i_{dc} \tag{4-50} \]

and, \[ i_{dc3} = i_{dc2} = i_{dc1} = i_{dc} \quad \text{(During normal operation)} \tag{5-51} \]

where \( i_{dc1}, i_{dc2} \) and \( i_{dc3} \) are transient values of the direct currents in the d.c line, rectifier and inverter with respect to time.

For the transient calculations, both algebraic as well as differential equations are solved simultaneously. In this particular programme, the capacitive reactance of the d.c line, which is very small, has been ignored and the d.c line current is calculated directly. With reference to this current, the values of the rectifier and inverter control signals, within the permissible limits, are
computed, which further define the corresponding delay angles.

The functional equations are rewritten as shown below and Kutta Merson Algol procedure is used to solve them:

\[ p \cdot i_{\text{dc1}} = \frac{1}{T_1} \times (i_{\text{dc}} - i_{\text{dc1}}) \quad (4-52) \]

where \( \frac{1}{T_1} \) is the time constant of the d.c line.

\[ p \cdot i_{\text{dc2}} = \frac{1}{T} \times (i_{\text{dc1}} - i_{\text{dc2}}) \quad (4-53) \]

\[ p \cdot i_{\text{dc3}} = \frac{1}{T} \times (i_{\text{dc2}} - i_{\text{dc3}}) \quad (4-54) \]

\[ p \cdot (V_s \cdot \cos \alpha) = \frac{1}{T} \times (e_{cr} - V_s \cdot \cos \alpha) \quad (4-55) \]

and,

\[ p \cdot (V_R \cdot \cos \alpha_i) = \frac{1}{T} \times (e_{ci} - V_R \cdot \cos \alpha_i) \quad (4-56) \]

The algebraic equations are arranged in such a way as to get the values of the parameters in the right order for their use in further calculations. A switch routine is also provided within the programme, and this can change the reference current of the converters or the a.c bus voltages at any required interval of time.

4-3-5 Study of h.v.d.c. system

To analyse the h.v.d.c. system, a hypothetical 500 KV d.c. link is considered, which connects an a.c system at 230-KV, asynchronously to another large a.c system. The system parameters, referred to a 1000-MVA power base with 230-KV as the base line-to-line voltage, are given in Table 4-3. The transformer connections and converters are arranged so that, with 1 p.u. bus-voltage, the no-load rectifier voltage is 1.61 p.u. referred to the a.c system, while the voltage at the receiving end bus is kept constant at 1.0 p.u.
Studies and Results

The following two studies of the transient behaviour of the h.v.d.c. system have been made:

(a) Transient response of the h.v.d.c. system with a step change in the reference current or power, and, (b) Transient response of the h.v.d.c. system with a step change in the a.c. bus-voltage.

Case (a)

The h.v.d.c. system is operating at steady state with the normal a.c. voltages of 1.1 and 1.0 p.u. on the rectifier and invertor sides respectively. The normal value of the reference current $I_r$ of the rectifier has been considered as 1.0 p.u. with a current margin $k$ of 0.1 p.u. At time 0.05 sec., $I_r$ is given a step rise to 1.5 p.u., and then at time 0.15 sec. it is brought back to its normal value. Throughout the operation, the a.c voltages of the rectifier and invertor are restricted to positive and negative values respectively.

Transient responses with respect to time, of the following variables of the h.v.d.c. system are graphically shown as below:

(i) Direct current Fig. 4-19a
(ii) Rectifier voltage Fig. 4-19b
(iii) Invertor voltage Fig. 4-19c
(iv) Direct axis component of the alternating current on the rectifier side Fig. 4-19d
and, (v) Quadrature component of the alternating current on the rectifier side Fig. 4-19e
Case (b)

The h.v.d.c. system is operating as in case (a) with a constant current-reference of 1.1 p.u. At time 0.05 sec. the a.c voltage on the rectifier is stepped down to 0.11 p.u., and at time 0.15 sec. it is brought back to the normal value of 1.1 p.u. - this procedure simulates a short circuit on the a.c side of the rectifier.

Transient response with respect to time of the following variables are graphically shown as below:

(i) Direct current
(ii) Rectifier voltage
(iii) Invertor voltage
(iv) Direct axis component of the alternating current on the rectifier side
(v) Quadrature axis component of the alternating current on the rectifier side

Conclusions

The above studies have demonstrated how the modified functional relationships incorporating therein simple lags, can be used to investigate the transient behaviour of the h.v.d.c. system. At this stage it is not the intention to draw some broad or specific conclusions from these studies, but this technique is important inasmuch as it has been incorporated in the main h.v.d.c. digital programme in Chapter 6, where the comparative study of transient behaviour of the h.v.d.c. and its equivalent a.c system has been made.
Table 4-1  DATA
TO COMPUTE CONVERTOR CHARACTERISTICS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_j$</td>
<td>0.08 p.u.</td>
<td>$V_d$</td>
<td>1.00 p.u.</td>
</tr>
<tr>
<td>$x_a$</td>
<td>0.04 p.u.</td>
<td>$P_{dc}$</td>
<td>0.84 p.u.</td>
</tr>
<tr>
<td>$x_v$</td>
<td>0.07 p.u.</td>
<td>$Z$</td>
<td>0.940 p.u.</td>
</tr>
<tr>
<td>$x_t$</td>
<td>0.001 p.u.</td>
<td>$P_{dB}$</td>
<td>1000 MVA</td>
</tr>
<tr>
<td>$x_I$</td>
<td>0.10 p.u.</td>
<td>$mvarf$</td>
<td>210 MVAR</td>
</tr>
<tr>
<td>$x_s$</td>
<td>0.05 p.u.</td>
<td>$V_{AB}$</td>
<td>230 KV</td>
</tr>
<tr>
<td>$n$</td>
<td>1.00 p.u.</td>
<td>$V_{dB}$</td>
<td>500 KV</td>
</tr>
<tr>
<td>$R$</td>
<td>0.185 p.u.</td>
<td>$V_r$</td>
<td>1.00 p.u.</td>
</tr>
<tr>
<td>Study No.</td>
<td>$x_1$</td>
<td>$x_a$</td>
<td>$x_v$</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>I</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>II</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>III</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>IV</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>V</td>
<td>0.07</td>
<td>0.04</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Table 4-3  DATA
TO STUDY THE TRANSIENT BEHAVIOUR OF H.V.D.C. SYSTEM

<table>
<thead>
<tr>
<th>No</th>
<th>Parameter</th>
<th>Value</th>
<th>No</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$V_{dB}$</td>
<td>500 KV</td>
<td>13</td>
<td>$V_s$</td>
<td>1.10 p.u.</td>
</tr>
<tr>
<td>2</td>
<td>$V_{aB}$</td>
<td>230 KV</td>
<td>14</td>
<td>$V_r$</td>
<td>1.00 p.u.</td>
</tr>
<tr>
<td>3</td>
<td>R</td>
<td>0.185 p.u.</td>
<td>15</td>
<td>range</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>$X_{dc}$</td>
<td>15.48 p.u.</td>
<td>16</td>
<td>acc</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>KR</td>
<td>30</td>
<td>17</td>
<td>h</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>KI</td>
<td>30</td>
<td>18</td>
<td>computing</td>
<td>3 sec</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>time</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$I_r$</td>
<td>1.00 p.u.</td>
<td>19</td>
<td>Ang. 2 (rotor angle)</td>
<td>60 degrees</td>
</tr>
<tr>
<td>8</td>
<td>k</td>
<td>0.10 p.u.</td>
<td>20</td>
<td>T</td>
<td>0.003 sec</td>
</tr>
<tr>
<td>9</td>
<td>$Z_r$</td>
<td>0.960</td>
<td>21</td>
<td>$t_1$ (stepping up time interval)</td>
<td>0.05 sec</td>
</tr>
<tr>
<td>10</td>
<td>$Z_o$</td>
<td>0.940</td>
<td>22</td>
<td>$t_2$ (normalising time interval)</td>
<td>0.15 sec</td>
</tr>
<tr>
<td>11</td>
<td>$V_{d1}$</td>
<td>1.00 p.u.</td>
<td>23</td>
<td>K</td>
<td>Marker</td>
</tr>
<tr>
<td>12</td>
<td>$X_c$</td>
<td>0.10 p.u.</td>
<td>24</td>
<td>M</td>
<td>Marker</td>
</tr>
</tbody>
</table>
FIG. 4-1 EQUIVALENT CIRCUIT OF CONVERTOR CONNECTIONS

![Equivalent Circuit of Convertor Connections]

FIG. 4-2 EQUIVALENT CIRCUIT FOR CONVERTOR CALCULATIONS

![Equivalent Circuit for Convertor Calculations]
Fig. 4-3 FLOW DIAGRAM
TO COMPUTE CONVERTER CHARACTERISTICS

BEGIN
Read in data
Calculate commutation voltage
Enter procedure to calculate gen. voltage

Is synch. comp. used?

Yes
Is reactive power of synch. comp. known?

Yes
Calculate react. power
No
Calculate Gen. voltage

Is this the initial value?

Yes
Calculate the volt. at star point
Store and print

No
Take the first value of direct current

Is the difference between the new and initial values greater than tolerance?

Yes
Print
No
Take the next value of direct current

Is this the last value of direct current?

Yes
END

No
Take any suitable value of commutation voltage to start iteration

Calculate direct voltage
Change commutation voltage
Fig. 4-4 CHARACTERISTICS OF RECTIFIER
(without synchronous compensator and without filter bank)

PD (p.u)  VD (p.u)  Cos \( \phi \)

\( 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \quad 1.4 \quad 1.6 \)

\( 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \quad 1.2 \quad 1.4 \quad 1.6 \)

ES = Generator's Voltage (p.u)
PD = Output Power (p.u)
VD = Rectifier Voltage (p.u)
U = Displacement Factor
QS = Reactive Power of the Generators (p.u)
QR = Reactive Power taken by Converter
Fig. 4-5 CHARACTERISTICS OF RECTIFIER
(without synchronous compensators and with harmonic filters.)

VD (p.u)  
PD (p.u)  
Cos $\phi$

IDC (p.u)

FIGURE LEGEND:
VD = Rectifier Voltage (p.u)
PD = Rectifier Current (p.u)
QS = Reactive Power of Generators (p.u)
QR = Reactive Power of Rectifier (p.u)
QF = Reactive Power of Filters (p.u)
U = Displacement Factor
Fig. 4-6  CHARACTERISTICS OF RECTIFIER  
(with synchronous compensators and with harmonic filters)

VD (p.u.)
PD (p.u.)
Cos $\phi$

VD
PD
QR
QS
QF
ES = 1.02

0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6

$\text{IDC (p.u.)}$

VD = Rectifier Voltage (p.u)
IDC = Rectifier Current (p.u)
PD = Output Power (p.u)
QS = Reactive Power of Generators (p.u)
QR = Reactive Power of Rectifier (p.u)
QF = Reactive Power of Filters (p.u)
U = Displacement Factor
QY = Reactive power from synchronous compensator
FIG. 4-7 VARIATION OF DELAY ANGLE OF CONVERTOR
(WITH HARMONIC FILTERS)

PD = Output power (p.u.)
U = Displacement factor
VD = D.C. voltage (p.u.)
QR = Reactive power of convertor (p.u.)
QS = Reactive power of generator (p.u.)
**FIG. 4-8 WORKING POINT OF D.C. LINE**

VD1 = voltage regulation of Rectifier and d.c. line

VD2 = voltage regulation of Inverter without synchronous compensator

VD2' = voltage regulation of Inverter with synchronous compensator
FIG. 4.9 DIAGRAM OF D-C POWER SYSTEM WITH GRID CONTROLS ON BOTH SIDES
Fig. 4-10  FLOW DIAGRAM
TO SIMULATE H.V.D.C. SYSTEM IN LOAD FLOW STUDY OF A.C. SYSTEM

BEGIN
  Read in data
  Is current margin negative?  Yes
    Interchange the values of cosine of the delay angles, ref. currents, gains and transformers turn ratios of both the converters.
  No
    Take the first value of the desired A.C. voltage

  Find direct current of the line - invt. on C.E.A. and rect. on C.C. controls respectively
  Is tap-changer used?  Yes
    Tap changing calculations
  No
    Find rect. voltage on N.V. control

  Inv., on C.C. control
  rect. on C.E.A. control
  No
    Is this voltage inverter one?  Yes
      Interchange the values of A.C. components, power factors and A.C. bus-bar voltages on both ends.
      Is current margin negative?  Yes
        Interchange A.C. bus-voltages
        Print
      No
        Give an increment to the existing value of the A.C. voltage
        Is this the last value of the A.C. voltage?  Yes
        Print
        END
      No
      Print
      END
    Is this voltage inverter one?  No
      Print
      END
FIG. 4-12 STEADY STATE RESPONSE OF H.V.D.C. SYSTEM 'V_s' IS VARYING (power flows from convertor 'A' to convertor 'B')

- IDC = Current in D.C. line
- VD = Voltage of D.C. line
- PD = Power delivered by D.C. line

FIG. 4-13 STEADY STATE RESPONSE OF H.V.D.C. SYSTEM 'V_R' IS VARYING (power flows from convertor 'B' to convertor 'A')
FIG. 4-14 STEADY STATE RESPONSE OF H.V.D.C. SYSTEM 'VR' IS VARYING (power flows from convertor 'A' to convertor 'B')

FIG. 4-15 STEADY STATE RESPONSE OF H.V.D.C. SYSTEM 'Vs' IS VARYING (power flows from convertor 'A' to convertor 'B')
FIG. 4-16 ONE-LINE DIAGRAMME OF H.V.D.C. SYSTEM
BEGIN

Read in data

Set in initial conditions

Step integrating time 't' from 0 - computing time

Is \( t < t_1 \) (time to give step rise) or \( t > t_2 \) (time to normalise)?

Yes → Use initial values of A.C. voltage or D.C. ref. current

No → Give step rise to A.C. bus-voltage or D.C. reference current

Solve dif. eqns. for direct current, etc.

Apply limits and calculate reqd. values of rect. and invt. signals.

Calculate sending and receiving end D.C. voltages and powers

Print

Is t > computing time?

No → END

Yes → Go back to step integrating time
FIG. 4-19  TRANIENT RESPONSE OF H.V.D.C. SYSTEM  
(STEP CHANGE OF REFERENCE CURRENT)  

(a) Response of direct current with respect to time

(b) Response of DC voltage of rectifier with respect to time
(c) response of DC voltage of inverter with respect to time

(d) response of D-axis component of alternating current of rectifier

(e) response of Q-axis component of alternating current of rectifier
FIG. 4-20 TRANSIENT RESPONSE OF H.V.D.C. SYSTEM (STEP CHANGE OF AC VOLTAGE)

(a) Response of direct current with respect to time

1.2
1.0
0.8
0.6
0.4
0.2
0

Current (p.u.)

(b) Response of DC voltage of rectifier with respect to time

1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0

Voltage (p.u.)

Time (secs)

(c) Response of DC voltage of inverter with respect to time

-0.2
-0.4
-0.6
-0.8
-1.0
-1.2

Voltage (p.u.)
(d) response of D-axis component of alternating current of rectifier

![Graph of D-axis component](image)

(e) response of Q-axis component of alternating current of rectifier

![Graph of Q-axis component](image)
CHAPTER V

MATHEMATICAL MODEL OF A.C. SYSTEM
In the digital computation of an electric power system, the most important aspect is the formulation of equations defining the synchronous machines, controls and the inter-connecting network of the system. An elementary form of a mathematical model of the synchronous machine was presented by Blondeau. This mathematical representation involved the resolution of the machine variables in two axes, mutually displaced by 90°, and is termed as "Two-Reactance-Theory". With the help of this theory, the terms which alternate synchronously in the machine equations were eliminated and the resulting equations approached those of two-pole machines. Later on, Park in his work showed that the equations for the a.c machines become considerably simpler if they are expressed in terms of certain fictitious current and voltage components. These fictitious values of current and voltage can have a physical meaning in that they can be considered acting on the fictitious coils placed along the direct and quadrature axes of the machine reference frame. This representation is termed as "Park's Transformation". Adkins and others further extended Park's work and established the new set of equations, which are now generally used to simulate the synchronous machines for stability study.

5-1-1 Machine Equations with Assumptions

The voltage equations for the three-phases of the armature and that of the field of a synchronous machine are given as below:
\[ V_a = \frac{p}{\omega_0} \cdot \psi_a - r_a \cdot i_a \]
\[ V_b = \frac{p}{\omega_0} \cdot \psi_b - r_a \cdot i_b \]
\[ V_c = \frac{p}{\omega_0} \cdot \psi_c - r_a \cdot i_c \]
and, \[ V_f = \frac{p}{\omega_0} \cdot \psi_f + r_f \cdot i_f \]  

where \( V, i, \psi \) stand for voltage, current and flux-linkages respectively;

suffices \( a, b, c \) and \( f \) stand for three phases \((A, B, C)\) and the field respectively

and, \( r_a \) and \( \omega_0 \) represent the armature resistance and the normal speed in radians per second, respectively.

To convert the above mentioned equations into Park's equations in terms of direct and quadrature axes, the following assumptions have been made:

(a) The flux distribution in the air-gap is sinusoidal.
(b) The saturation effect is negligible.
(c) Hysteresis and harmonics are neglected.
(d) Only the positive sequence current is considered.
(e) Sub-transient phenomena is ignored.
(f) The small changes in speed, which do not effect the synchronism of the machine, are ignored.
and, (g) Armature resistance, which is very small, is ignored.
5-1-2 Park's Transformation

Park's transformation, which gives the direct and quadrature components in terms of the actual values of the armature variables, is expressed by the following matrix equations:

\[
\begin{bmatrix}
A_d \\
A_q \\
A_o
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos \theta & \cos (\theta - 2\pi/3) & \cos (\theta - 4\pi/3) \\
\sin \theta & \sin (\theta - 2\pi/3) & \sin (\theta - 4\pi/3) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
A_a \\
A_b \\
A_c
\end{bmatrix}
\]

where \(\theta\) is the phase displacement between phase A and the direct axis of the machine.

"A" stands for an identifier and suffices d, q and o indicate the direct component, quadrature component and zero sequence component respectively. By replacing "A" by "i", "v" and \(\phi\), the transformation of current, voltage and flux-linkages respectively can be obtained.

For the inverse transformation, giving the actual variables in terms of "d" and "q" axes components, the following matrix equations are used:

\[
\begin{bmatrix}
A_a \\
A_b \\
A_c
\end{bmatrix} = \begin{bmatrix}
\cos \theta & \sin \theta & 1 \\
\cos (\theta - 2\pi/3) & \sin (\theta - 2\pi/3) & 1 \\
\cos (\theta - 4\pi/3) & \sin (\theta - 4\pi/3) & 1
\end{bmatrix} \begin{bmatrix}
A_d \\
A_q \\
A_o
\end{bmatrix}
\]

Again by replacing "A" by the required variable, the inverse transformation can be obtained.
5-1-3 Induced Voltage in the Armature and Field

The impressed voltage $V_a$ of the phase A is the sum of the internal voltage induced by the main air-gap flux, the drops due to the local armature fluxes and the resistance drop.

The internal voltage due to the main air-gap flux is expressed as:

$$p \cdot (\psi_{md} \cos \theta + \psi_{mq} \sin \theta)$$

The voltage due to the local armature fluxes is expressed as:

$$l_1 \cdot p \cdot i_a - l_m \cdot p \cdot i_b - l_m \cdot p \cdot i_c$$

and the voltage drop due to resistance is expressed as:

$$r_a \times i_a$$

where $\psi_m$ stands for the air-gap flux linking with the coil on the corresponding axis,

and $l_1$ and $l_m$ are the leakage inductance, and the mutual inductance between the two armature coils due to the flux which does not cross the air gap, respectively.

Hence the impressed voltages in the 3-phases are given as:

$$V_a = p \left( \psi_{md} \cos \theta + \psi_{mq} \sin \theta \right) + (r_a + l_1 \cdot p) i_a - l_m \cdot p \cdot i_b - l_m \cdot p \cdot i_c$$

$$V_b = p \left( \psi_{md} \cos(\theta - 2\pi/3) + \psi_{mq} \sin(\theta - 2\pi/3) \right) + (r_b + l_1 \cdot p) i_b - l_m \cdot p \cdot i_a$$

and, $$V_c = p \left( \psi_{md} \cos(\theta - 4\pi/3) + \psi_{mq} \sin(\theta - 4\pi/3) \right) + (r_c + l_1 \cdot p) i_c - l_m \cdot p \cdot i_a - l_m \cdot p \cdot i_b$$

(5-3)
From the transformation equations (5-2a), and with some algebraic manipulation we can obtain the following set of equations for the armature voltage:

\[ V_d = \frac{p}{w_o} \cdot \psi_d - \psi_q \cdot \frac{p \cdot \theta}{w_o} - r_a \cdot i_d \]

\[ V_q = \frac{p}{w_o} \cdot \psi_q + \psi_d \cdot \frac{p \cdot \theta}{w_o} - r_a \cdot i_q \]

and, \[ V_o = \frac{p}{w_o} \cdot \psi_o - r_a \cdot i_o \]

Similarly, the field voltage can be expressed as:

\[ V_f = \frac{p}{w_o} \cdot \psi_f + r_f \cdot i_f \] (5-4)

For the balanced operation of the machine, \[ V_o \] and \[ i_o \] are zero, and hence the equation for \[ V_o \] can be ignored.

In the above equations, \[ \psi_d \] and \[ \psi_q \] represent the total flux linkages with the coil located on the appropriate axis, due to both the main air-gap flux and to the leakage flux, i.e:

\[ \psi_d = \psi_{md} + l_c \cdot i_d \]

and, \[ \psi_q = \psi_{mq} + l_c \cdot i_q \]

where \[ l_c \] is the effective leakage inductance of either of the axis coils, and is given by:

\[ l_c = l_1 + l_m \]
5-1-4 Component of Flux Linkages

If the operation of the machine is balanced, p.u scale is used throughout, time is measured in seconds, and the d-axis coil coincides with the pole of the machine which leads the q-axis by 90° (electrical) in the direction of rotation, then the components of the flux linkages are given as below:

\[
\psi_d = x_{af} \cdot i_r - x_d \cdot i_d
\]

\[
\psi_q = -x_q \cdot i_q
\]

and,

\[
\psi_r = x_r \cdot i_r - x_{af} \cdot i_d
\]

(5-5)

where \(x_{af}\) is the mutual inductance between each armature circuit and the field winding on the same axis.

5-1-5 Modified Equations of Flux Linkages

Eliminating the field current \(i_f\) from 5-5 and 5-4, the following new equations are obtained:

\[
\psi_d = \frac{x_{af}^2}{x_r + x_{af}^2 / \omega_o} \cdot v_f - \frac{x_d + (x_d - x_{af}^2 / x_r) \cdot x_r / r_f \cdot p \omega_o}{1 + x_f / r_f \cdot p \omega_o} \cdot i_d
\]

(5-6)

and,

\[
\psi_q = -x_q \cdot i_q
\]

where \(\frac{x_{af}^2}{r_f} v_f = G e_f\) (terminal voltage on open-circuit at normal speed).

\[
x_d = \frac{x_{af}^2}{x_r} = 'x_d\) (direct-axis transient reactance).\)
and, \( \frac{x_f}{\omega_0 \cdot r_f} = \tau_{do} \) (field time constant)

Hence \( \psi_d = \frac{G_f}{1 + \tau_{do} \cdot p} \cdot \frac{x_d + x_d \cdot \tau_{do} \cdot p}{1 + \tau_{do} \cdot p} \cdot i_d \),

and, \( \psi_q = -x_q \cdot i_q \) \( (5-7) \)

5-1-6 Modified Voltage Equations

Under the assumption that a small change in the machine speed does not affect the induced voltage, we can take:

\[ p \cdot \frac{\theta}{\omega_0} = 1 \]

and, \( p / \omega_0 \cdot \psi_d = p / \omega_0 \cdot \psi_q \to 0 \)

Hence, the set of equations (5-4) can be simplified as below:

\[ V_d = -\psi_q \]
\[ V_q = \psi_d \]

and, \( V_f = \frac{p}{\omega_0} \cdot \psi_x + r_f \cdot i_f \) \( (5-8) \)

From Equations 5-7 and 5-8 we get a new set as:

\[ V_d = x_q \cdot i_q \]
\[ V_q = \frac{G_f}{1 + \tau_{do} \cdot p} \cdot \frac{x_d + x_d \cdot \tau_{do} \cdot p}{1 + \tau_{do} \cdot p} \cdot i_d \]

and, \( V_f = \frac{p}{\omega_0} \cdot \psi_x + r_f \cdot i_f \) \( (5-9) \)
5-1-7 Accelerating Torque

When the machine is under constraints imposed by the abnormal current due to a fault in the line or overloading etc., accelerating torque starts varying with respect to time and effects the other variables of the machine. Hence in the stability study, the accelerating torque plays the most important role. The accelerating torque is defined as the difference between the shaft torque and the electrical torque developed across the air-gap, i.e.:

$$T_a = T_m - T_e$$

where $T$ stands for torque, and $a$, $m$ and $e$ indicate accelerating, shaft and electrical torques, respectively.

"$T_m$" is obtained from the prime mover and "$T_e$" is calculated from the terminal power "$P_e$" of the synchronous machine as shown below, provided the system is balanced:

$$T_e = P_e = V_d i_d + V_q i_q$$

By substituting the values of "$V_d$" and "$V_q$" from Equations 5-49, we get:

$$P_e = p \psi_d/\omega_o i_d + p \psi_q/\omega_o i_q + (\psi_d i_q - \psi_q i_d) P_e/\omega_o - r_a (i_d^2 + i_q^2)$$

(5-10)

i.e., $P_e = \text{(rate of decay of armature magnetic energy)} + \text{(torque developed across the air-gap x speed)} - \text{(armature copper losses)}$. 
Under the assumptions in Sec. 5-1-1, the first and the last terms will tend to zero. Hence, electrical torque can be expressed as:

\[ T_e = P_e = \psi_d i_q - \psi_q i_d. \]  

(5-11)

5-1-8 Damping Torque

Damping torque of the synchronous machines can either be calculated by Routh's Criterion or by Nyquiste's Criterion. However, for stability study the effect of the damping torque can adequately be incorporated in the electro-mechanical equation by adding a term proportional to rotor speed. This term is expressed as: "K_d p \delta", where "K_d" is the damping constant. From the field tests, the appropriate values of "K_d" have been found in the range of 3-10, which are quite adequate to simulate the damping torque in the stability study of the power systems.

5-1-9 Negative - Sequence Torque

Because of the negative-sequence current in the armature, a torque is developed which is known as negative-sequence torque. This torque is the same as that which would have been developed by an induction motor running backward with a slip of 2.

The negative-sequence torque is expressed as:

\[ T_{e2} = i_2^2 (r_2 - r_d) \]  

(5-12)

where \( i \) and \( r \) stand for current and resistance of the armature respectively, and the suffices 2 and 1 indicate the negative sequence and positive sequence respectively.
5-1-10 Axis Transformation

In the Park's transformation, all the equations have been defined by the rotor angle of the machine; however, the rotor angles of the different machines of the system may be phase displaced from each other. Hence, it is essential to refer all the machines to a common reference frame either by selecting one of the machines as a reference machine or by considering all the machines of the system with respect to any other synchronously rotating reference frame.

Suppose the machine "n" is the reference machine, the axes of machine "m" are transformed to the reference frame of machine "n", the angle of rotation between these machines is \( \delta_n - \delta_m \), as shown in Fig. 5-1, the notation \( K_{mn} \) is used to represent a quantity of the \( m \) th. machine expressed in the reference frame of the \( n \) th. machine, and \( K_m \) is used to represent the quantity of \( m \) th. machine in its own reference frame, then from the geometry of the figure the following equations are obtained:

\[
K_{dmn} = K_{dm} \cdot \cos(\delta_n - \delta_m) + K_{qm} \cdot \sin(\delta_n - \delta_m)
\]

\[
K_{qmn} = K_{qm} \cdot \cos(\delta_n - \delta_m) - K_{dm} \cdot \sin(\delta_n - \delta_m)
\]

(5-13)

In the matrice notation, the axis transformation may be expressed as:

\[
\begin{bmatrix}
K_{dmn} \\
K_{qmn}
\end{bmatrix} = \begin{bmatrix}
\cos(\delta_n - \delta_m) & \sin(\delta_n - \delta_m) \\
-sin(\delta_n - \delta_m) & \cos(\delta_n - \delta_m)
\end{bmatrix} \times \begin{bmatrix}
K_{dm} \\
K_{qm}
\end{bmatrix}
\]

(5-14)
5-2 SIMULATION OF CONTROLS

Amongst the controlling devices of the power system only speed governors and automatic-voltage-regulators (A.V.R.) are considered in this thesis, as both of them act continuously in the system. The governor maintains the constant speed of the prime-mover, and A.V.R. maintains the constant voltage of the generator; however, both of them definitely improve the stability of the system. In the case of a transient operation, the A.V.R., because of its high-speed action, is more effective than the governor.

5-2-1 Speed Governors

The speed governor controls the mechanical input to the generator in an effort to maintain its speed constant. In case of any variation in the absolute speed from the reference value, the governor adjusts the openings of the steam turbine or the gate of the hydraulic-turbine, thus varying the steam or water pressure to adjust the shaft torque. Full representation of the prime mover and governor is quite complicated and beyond the scope of this thesis, so only a simplified loop has been incorporated in the mathematical model of the power system, as shown in Fig. 5-2. In the simulation of the governor, the time constant of the steam or water inlet valves, and the time constant of the relay and valve controlling the response of the shaft torque, have only been included. Hence the governor is represented as:

\[
\Delta P_m = \frac{K_g}{(1 + \tau_{g,p})(1 + \tau_{p})} \chi \quad p, \delta
\]  
(5-15)
where \( \Delta P_n \) = change in power input.

\[ K_p = \text{gain of the governor.} \]

\[ t_p = \text{time constant of steam or water inlet-valve.} \]

and, \( \tau_g = \text{time constant of relay and valve.} \)

5-2-2 *Excitation Control System (A.V.R.)*

An automatic voltage regulator, feeding back error signals proportional to the difference between the reference and terminal voltages, keeps the terminal voltage constant and thus improves the transient response. The block diagram shown in Fig. 5-3 represents the action of an A.V.R. Subsidiary feed backs are added in the loop to improve the transient response further.

The error signal is amplified by the series magnetic amplifiers, and then it supplies the field voltage of the machine through the control exciter. Two derivative stabilisers are also used, one for the feed-back signal from the exciter and the other for the signal from the second amplifier, as shown in Fig. 5-3. The stability of the control loop formed by the A.V.R. and the machine is very essential, as otherwise unstable loops will inject oscillatory signals into the machine's field windings and may cause the instability of the whole system. To achieve this stability, the gain and time constants of the stabilisers are adjusted.

The A.V.R. mentioned above can be represented by the transfer functions which are summarised below:

**Magnetic Amplifier (1)**

\[ V_1 = \frac{K_h}{(1 + p \tau_h)} \times V_i \]
Magnetic Amplifier (2)

\[ V_2 = \frac{K_5}{1 + P \cdot \tau_5} \times V_1 \]

Exciter

\[ V_f = \frac{K_7}{1 + P \cdot \tau_7} \times V_2 \]

Stabilising feedback signal from exciter

\[ V_{s1} = \frac{p \cdot K_6}{1 + P \cdot \tau_6} \times V_f \]

Stabilising signal from amplifier (2)

\[ V_{s2} = \frac{p \cdot K_8}{1 + P \cdot \tau_8} \times V_2 \]

and,

\[ V_i = V_r - V_t - V_{s2} - V_{s1} \]

\[ V_t = \sqrt{V_d^2 + V_q^2} \quad (5-16) \]

where

- \( V_r \) = Reference voltage.
- \( V_i \) = Input voltage to the amplifier 1.
- \( V_1 \) = Output voltage of the amplifier 1 and input of amplifier 2.
- \( V_2 \) = Output voltage of the amplifier 2 and input of exciter and stabiliser 2.
- \( V_f \) = Output voltage of the exciter and input of stabiliser 1.
- \( V_{s2} \) = Output voltage of the stabiliser 2 and input of error detector.
\( V_{s1} \) = Output voltage of the stabiliser 1 and input of error-detector.

\( V_t \) = Terminal voltage of the machine.

\( K_4, K_5, K_7, K_6, \) and \( K_8 \) = gains of amplifier 1 and 2, exciter, and stabiliser 1 and 2, respectively.

and, \( \tau_4, \tau_5, \tau_7, \tau_6, \) and \( \tau_8 \) = the time constants of the amplifiers, etc. in the same order as above.

The above equations are further simplified for the computer's programme as below:

Exciter

\[
V_f = \frac{K}{(1 + T_e \cdot p)} \times (V_r - V_t - V_s)
\]

Stabiliser

\[
V_s = \frac{K_s \cdot p}{(1 + T_s \cdot p)} \times V_f
\]

and,

\[
V_t = \sqrt{V_d^2 + V_q^2}
\]

where \( K \) = Overall gain of the main loop.

\( K_s \) = Total gain of the stabilising loop.

\( T_e \) = Time constant of magnetic amplifiers and exciter.

and, \( T_s \) = Time constant of stabilisers.

Other quantities, which can also be used for feedback signals include:

(a) the rate of change of field voltage known as stabilising signal,

(b) rotor velocity,
(c) rate of change of rotor velocity,
(d) rate of change of field current,
(e) rate of change of terminal voltage,
and, (f) the magnitude of the fault current.

5-3 ELECTRO-MECHANICAL EQUATION

The swing of the rotor effects the other machine variables, which may exceed their design ratings, but they will normalise if the rotor swing decreases with time. Hence, for assessing the stability of the system it is the rotor angle which is to be examined. The rotor angle of the machine at any instant may be obtained from the basic electro-mechanical equation of the machine which is stated as:

\[ M \cdot p^2 \cdot \delta = (P_m - P_e). \]  

(5-18)

The comprehensive form of the electro-mechanical equation is obtained by incorporating the negative sequence, and damping torques together with the effect of the governor, as:

\[ M \cdot p^2 \cdot \delta = P_{m0} + AP_m - P_{e1} - P_{e2} - K_d \cdot P \cdot \delta. \]  

(5-19)

This equation is generally used in the digital programme to find the excursion of rotor angle with respect to time during the transient operation.

Where suffices 0, 1 and 2 indicate initial value, positive sequence and negative sequence, respectively.
5-4  EXTERNAL NETWORK

5-4-1 Transmission Lines

The simplest form of transmission line representation is a reactance connecting the machine to the infinite bus-bar - the resistance of the transmission line, which is comparatively very small, being ignored. This reactance is the sum of the reactances of the transmission line and transformers associated with the line. The equivalent "r" network representation is employed for the simulation of the transmission lines. In case of a large power system, the matrix representation is very convenient and takes less computing time for the solution of transmission lines.

The following assumptions are generally made to simulate the transmission lines:

1. Line shunt admittance is neglected.
2. Transformer excitation current is ignored.
3. Mutual coupling between the circuits is ignored.

Thus, the simplified equations for the transmission line may be written as:

\[ v_{sd} = v_{rd} + r_t \cdot i_{td} - x_t \cdot i_{tq} \]

\[ and, \quad v_{sq} = v_{rq} + r_t \cdot i_{tq} + x_t \cdot i_{td} \]  \hspace{1cm} (5-20)

where \( V, i, r \) and \( x \) stand for voltage, current, resistance and reactance respectively and first suffices \( s, r \) and \( t \) indicate sending end, receiving end and transmission line respectively.
There are generally two types of static loads which are to be simulated:

(a) resistance in series with the inductance, and,
(b) resistance in series with the capacitance.

In case (a), the load is simulated by the following equations:

\[ v_{ld} = -x_l \cdot i_{ld} + r_l \cdot i_{ld} \]

and, \[ v_{lq} = x_l \cdot i_{ld} + r_l \cdot i_{lq} \] (5-21)

In case (b), the load equations are:

\[ v_{ld} = x_{lc} \cdot i_{ld} - x_{lc} \cdot v_{lq}/r_l \]

and, \[ v_{lq} = -x_{lc} \cdot i_{ld} + x_{lc} \cdot v_{ld}/r_l \] (5-22)

where \( r_l \), \( x_l \), and \( x_{lc} \) are the resistance, inductive reactance and capacitive reactance of the load, respectively, and \( i_{ld} \), \( i_{lq} \) and \( v_{ld} \), \( v_{lq} \) are the direct and quadrature axes components of the load current and voltage respectively.

5-5 Faults on the Transmission Lines

5-5-1 Simulation of Unsymmetrical Faults

A three-phase network, which is symmetrical throughout except for one unsymmetrical short-circuit, can be represented by positive, negative and zero sequences connected at the point of fault, as shown in Fig. 5-1. Furthermore, as the synchronous power between the various synchronous machines of a power system is of positive sequence, a short circuit can be represented in this sequence by
connecting a shunt impedance $Z_f$ at the point of fault. The value of $Z_f$ depends upon the type of fault and upon the negative and zero sequence impedences, as viewed from the point of fault, and this can be calculated for various types of faults, as:

<table>
<thead>
<tr>
<th>Type of Fault</th>
<th>Impedence of the fault shunt $Z_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Line-to-ground</td>
<td>$Z_0 + Z_2$</td>
</tr>
<tr>
<td>2. Line-to-line</td>
<td>$Z_2$</td>
</tr>
<tr>
<td>3. Two lines-to-ground</td>
<td>$Z_0 \cdot Z_2 / (Z_0 + Z_2)$</td>
</tr>
<tr>
<td>4. Three phase</td>
<td>0</td>
</tr>
</tbody>
</table>

where $Z$ stands for impedance; and suffices $0$, $2$ and $f$ indicate zero-sequence, negative sequence and shunt-fault respectively.

As both $Z_0$ and $Z_2$ have been assumed to be reactive, $Z_f$ may be represented as $jx_f$ for further calculations.

5-5-2 Fault Clearing and Reclosure

Fault clearing involves the partial or total loss of synchronous contact between the sending and receiving ends of the line, so the large differential between the input and output powers of the machine tends to accelerate the rotor. Thus, the concept of transient stability is confined to the rapid reclosure of the line, which was disconnected at the instant of fault, to prevent the machine falling out of synchronism.

Modern high-speed circuit-breakers are capable of operating in a few cycles, but the minimum possible time taken by the circuit
breakers seems to be the lower limit of the deionising or dead time in which the arc can be extinguished. However, the longer the dead time, the greater is the likelihood of successful de-energisation which is essential for the continuity of supply under transient conditions - the minimum permissible deionisation time being calculated empirically.

Hence, the consideration of system stability and security of supply involves opposite requirements with respect to reclosing time, and demands the adoption of some measures to achieve a compromise between the two factors while designing the system.

For the above mentioned studies, the switching operation in case of fault is in the following sequence:

(a) Clearing, i.e., isolation of the faulted line or phase.
(b) Reclosure, i.e., reconnecting the line or phase previously isolated,
and, (c) Lockout, i.e., in case of unsuccessful reclosure the faulted line being kept out of circuit.

The timings for each operation are defined as:

1. **Clearing Time**
   The time between the fault occurrence and the isolation of the faulted line or phase is known as clearing-time.

2. **Reclosing Time**
   This is defined as the total time from the fault onset to reclosure, including the deionising or dead-time.

3. **Lockout Time**
   With unsuccessful reclosure the circuit-breakers are tripped off, and the time for the locking out operation approaches to the original clearing time.
The configuration of the system changes with the switching operation and requires that the representation of the system is to be modified accordingly. In the switching operation, the three-phase lock-out is identical to the three-phase clearing, unsuccessful reclosure is the same as the original fault clearance and the successful reclosure is equivalent to normal steady state operation. Hence, keeping in view these similarities, the equations for each configuration are written and the corresponding matrices are drawn up, while at the instant of switching the existing system matrix is replaced by the new one.

In the large systems, matrix modification technique may be employed to take care of switching performance, rather than to insert a complete new matrix, and thus save computing time. However, in the present work, where a small system is under study, this technique has not been used.

5-6 TRANSIENT STABILITY

5-6-1 Infinite Bus-Bar

An infinite bus-bar is defined as a source of voltage of constant magnitude and frequency, capable of absorbing or generating an infinite amount of power. In the study of an inter-connected power system there may exist a large remote generating group which will be slightly affected by the transient. It is, however, quite convenient to substitute this group by an infinite bus-bar to simplify the system representation without much loss of accuracy. This infinite bus-bar will also act as a frequency reference for the disturbed part of the system. Hence, for a transient study.
in practice, only those machines which are close to the point of disturbance need full representation, while the others are represented by infinite bus-bars.

5-6-2 Transient Stability Criteria

By definition, a system represented by a single machine connected to an infinite bus-bar is stable if the rotor angle of the machine "δ" with respect to bus-bar voltage remains within the range of "-π" and "+π" radians, maintaining at the same time its final value constant.

Prior to the disturbance, the system is assumed to be in steady state, and in consequence the initial value of the rotor velocity at t = 0 is zero. After the disturbance, the rotor of the machine starts swinging and may decelerate or accelerate depending on the decrease or increase in the electrical torque. At a certain value of "δ" in the first swing, the synchronising torque may be such as to cause the reversal of the rotor's direction, and if it does so while within the limits of stability, the machine is said to be first-swing stable. The conditions for first-swing stability are thus outlined as:

(a) \( \text{Sign} (\dot{\delta})_{t_1} \neq \text{Sign} (\dot{\delta})_{t_2} \)

(b) \( |\delta| < \pi \)

where \( t_1 = t_0 + \Delta t \)
and \( t_2 > t_1 \)

where \( t \) and \( \Delta t \) stand for time and time increment, and suffices 0, 1 and 2 indicate initial, after first increment and after second increment, respectively.
The above criteria are, however, not unique to decide stability, as according to these conditions, the machine which has the pole slip is not stable — and this is not correct. Thus, the concept of first swing stability is only meaningful when it is applied to the system having no pole-slip and no control devices. To study the transient stability of the modern power stations, which are equipped with speed governors and A.V.R., the first swing stability test may be of no significance. On the other hand, the test laid down in the basic definition of stable operation requires the computation for a large interval of time, and this is not economical. Hence, the conditions for transient stability of the machine are modified as below:

If $|\delta|$ first swing $\geq |\delta|$ second swing
and $|\delta| < \pi$ for all values of $t$,
then the machine is said to be stable.

Furthermore, there could be a possibility that the peak of the second swing may not be smaller than the peak of the first one, because of the slow action of the controls, but eventually it may start decreasing in the subsequent swings. Hence, to avoid this possible omission, any two consecutive peaks of the swing curve, rather than the first two peaks, are examined. With this modification the machine will be stable if:

(a) $|\delta|$ $n$th swing $\geq |\delta|$ $(n+1)$th swing for any "$n$",
and, (b) $|\delta| < \pi$ until (a) is satisfied.
Two programmes for "n" machines system have been written:
(a) to find the swing curve of the machine,
and, (b) to find the stability boundaries of the machine.

5-7-1 Swing Curve Programme

The digital programme has been written in such a way that it can easily be adopted for a system having one or more than one machine, and also for a system having a d-c link as one of its transmission lines. The analog representation has been used and the block diagram is shown in Fig. 5-5. The working of this programme is described below:

(a) Network Calculations

The network shown in Fig. 5-6, consists of a double circuit transmission line connecting a synchronous machine with the infinite bus-bar which has been considered as a reference frame. In a simple network, as that under consideration, the components of machine current are expressed as:

\[ i_q = \frac{v_b \cdot \sin \delta}{x_t + x_l} \]

and,

\[ i_d = \frac{e_z - v_b \cdot \cos \delta}{x_t + x_l} \]  \hspace{1cm} (5-23)

where \( v_b \) and \( e_z \) are the voltages of the infinite bus-bar and machine respectively, 
\( i_d \) and \( i_q \) are direct and quadrature components of the machine current, and \( x_t \) and \( x_l \) are reactances of the transformer and transmission line respectively.

But for the general programme of "n" machines "nodal-analysis" has been used, as below:

\[ I_n = Y_n \cdot V_n \]
and, \( V_n = Y_n^{-1} \cdot I_n \) \( (5-24) \)

where \( I_n \) (\( n = 1,2,3,... \)) are the injected currents at "n" nodes;

\( V_n \) (\( n = 1,2,3,... \)) are the nodal voltages at "n" nodes;

\( Y_{nn} \) (\( n = 1,2,3,... \)) are the self admittances of the "n" nodes;

\( Y_{ij} \) are the mutual admittances,

where \( i \neq j \) and \( i = 1,2,3 \ldots n, \)

and \( j = 1,2,3 \ldots \ldots n; \)

and, \( Y_n^{-1} \) is the inverse of matrix \( Y_n \).

All the matrices of Equations 5-24 are the complex matrices, where the admittance matrix is of the order \( n \times n \).

In order to find the nodal voltages and thus branch currents, the following steps are taken:

1. Admittance matrices are formed for the different configurations of the network, i.e.:
   (a) during normal conditions,
   (b) during the fault,
   (c) after the removal of the faulted line,
   and, (d) after reclosure of the faulted line.

2. All the admittance matrices are inverted by using the Algol library procedure of inversion.

3. Nodal currents are expressed as:

\[
I_n = \sum_{m=1}^{n} Y_{nm} x V_m \quad (5-25)
\]
where "n" is the node under consideration, and "m" is the branch connected with the node "n".

4. Nodal voltages are expressed as:

\[ V_n = Y_n^{-1} \cdot I_n \]  \hspace{1cm} (5-26)

5. From the nodal voltage, branch currents are calculated by the following expression:

\[ i_m = (V_m - V_n) \times Y_{mn} \]  \hspace{1cm} (5-27)

where \((V_m - V_n)\) is the voltage across the mth branch, and \(Y_{mn}\) is the admittance of the branch "m".

(b) Calculations of Synchronous Machine Variables

The calculations of the variables of the different synchronous machines are on the line of analog simulation of the machine, as shown in Fig. 5-5. Blocks in this figure represent the integrating amplifiers, summation amplifiers and multipliers. The differential and algebraic equations of the synchronous machine are modified to represent the input and the output of the amplifiers of the analog device, and as such, are used in the digital programme to find the transient response of the synchronous machine.

(c) Calculation of A, V, R Response

To include the A, V, R in the system the Eqns. 5-17 are modified, as described in case (b), and are included in the main programme.
(d) **Calculation of Speed Governor's Response**

The Eqn. 5-15 is modified as above and included in the torque Eqn. 5-19.

(e) **Initial Conditions**

To calculate the initial conditions of the multi-machine system a complete load-flow study is required. This involves the iterative-process to find the tap-settings on the transformers, generator loading, and rotor angles of the synchronous machine, etc. In the system under consideration, a single machine is connected to an infinite bus-bar, and thus the initial conditions can easily be calculated by solving the algebraic and transfer equations of the system simultaneously.

The flow diagram for the swing curve programme is shown in Fig. 5-7 and its verbatim is given in Appendix (4).

5-7-2 **Stability Boundary Programme**

Relative stability study of various machines and their interconnections within the system can be graphically exhibited as stability-boundaries. The graph is drawn between the fault-clearing time as x-axis with the y-axis representing the max power level or rotor angle at which a machine, when subjected to a given disturbance, operates without losing the stability. In earlier literature, a large number of calculations were carried out in the region of the boundary to find the critical points for a number of fault clearing times, but this was a cumbersome and time consuming procedure. Hence, to simplify the stability-boundary calculation a search programme has been developed, which automatically finds the required critical points for given values of the clearing-
time. The programme includes the swing-curve routine together with a sub-routine to test the stability state of the synchronous machine under study. A number of fault clearing times are selected, and the search programme finds the corresponding critical point by varying the rotor angle of the machine. Though this programme can cater with all forms of curves, yet the knowledge of the general form of the curve helps to reduce the computing time.

To use the search programme a guess for the critical rotor angle or power is made, and this value together with the constants of the system are supplied as data. The system is tested at the initial value of rotor angle or power. The result of this test tells whether the system is stable or unstable, and the programme alters the rotor angle accordingly to approach the stability limit. If the test is undecisive, i.e., the conditions for the stable or unstable position are not fulfilled, the rotor angle is given a prefixed increment, and the whole process is repeated until the stability-boundary is crossed. The increment of the rotor angle is reduced for every crossing, and its sign is reversed to continue the search for the required critical point for a given fault clearing-time within the fixed tolerance. At this stage, the pre-fault conditions for the last test are printed out as being the boundary conditions for this given clearing time. Similarly, the critical value for the other fault-clearing times are calculated and the pre-fault conditions are printed. With the knowledge of the form of curve, so the programme can roughly predict the critical points, narrowing down the search band for the subsequent points.
The computing time for this programme depends upon the stability criterion employed in the test sub-routine. For the present work the stability criteria, derived in Sec. 5-6-2, is used which defines the state of the system as:

(a) if the absolute value of the rotor angle goes beyond $\pm \pi$ radians during the computation time, the system is unstable,

and, (b) if the rotor angle remains within the range of $\pm \pi$ radians for one sec. and if its absolute value starts decreasing, the system is stable.

The flow diagram of the stability-boundary programme is shown in Fig. 5-3, and its verbatim is given in Appendix (5).

5.3 Power System Studies and Results

In this work the power system studies have been made under two main headings:

(a) Preliminary studies,

and, (b) Study of transient behaviour of the system during fault.

5.3.1 Preliminary Studies

To ascertain the accuracy of the mathematical model of the power system, it has been subjected to some step response tests.

Test I
To study the relative effects of the A.V.R. and speed governor

With the initial steady-state conditions corresponding to a rotor angle of $30^\circ$, a step increase of 0.5 p.u mechanical input power is applied to the machine, and the values of the rotor angle
with respect to time are printed in four different cases:

(a) when A.V.R. and speed governors are not included in the model,
(b) when only the speed governor is included,
(c) when only A.V.R. is included,
and, (d) when both A.V.R. and speed governor are included.

The results are shown graphically in Fig. 5-9.

Observations

The power level of the machine with and without A.V.R. being different, strict comparison cannot be made. But despite this, a comparison of the curves "a" and "b" shows that the speed governor tends to limit the first swing of the rotor angle to some extent and to increase the magnitude of the subsequent oscillations, thus implying that the speed governor introduces a small negative damping. In curve "c", the peak of the first swing increases due to the action of the A.V.R., while in curve "d" when both A.V.R. and speed governors are used, the peak is reduced comparatively. A comparison between the curves "a" and "c" shows that the A.V.R. has no effect on the damping of the system, and the relative increase in the first oscillation in curve "c" is the result of the unequal power angle characteristics with and without A.V.R.

Test II

Transient response of the main variables of the machine

The transient response of the main variables of the machine have been displayed by giving a step increment to the mechanical input-power of the machine, as in the previous case. This corresponds,
in practice, to the sudden isolation of one line of the double circuit supply system. The responses of the following machine variables with respect to time are shown in Fig. 5-10:

(a) Rotor angle.
(b) Rate of change of rotor angle,
(c) Electrical power output,
and, (d) Terminal voltage.

**Observations**

Electrical output "$P_e$" in curve "c" oscillates in time phase with the rotor angle and tends to approach the steady value of 0.5 p.u above the initial value of "$P_e$".

**Test III**

**Effects of various control parameters**

A step increase of 0.5 p.u mechanical input power was applied, as in the previous cases, and the following transient response studies are made:

(a) The rotor angle and the terminal voltage of the machine are computed for the A.V.R. open loop gains of 10, 40, 100, 500. The results are graphically shown in Fig. 5-11.

(b) As above, the rotor angle and terminal voltages are computed for the A.V.R. derivative stabilising loop gains of 0.1 and 0.01, and the results are shown in Fig. 5-12.

(c) Similarly, for the speed governor's gain of 20, 40 and 60, the results are shown in Fig. 5-13.
Observations

(a) When the open loop gain of the A.V.R. is 10, the first swing peak of the rotor angle is the highest, and it tends to approach the comparatively higher steady state position of about $65^\circ$. The terminal voltage tends to settle at a comparatively low value, Fig. 5-11-a.

When the open loop gain is 40, as in Fig. 5-11-b, the first-swing peak of the rotor angle decreases and tends to settle down at a new lower steady state value of $55^\circ$. In this way, the power angle characteristics and the terminal voltage regulation are both improved, while the damping is comparatively increased.

When the gain of the open loop is 100, Fig. 5-11-c, the oscillation of the rotor angle is more pronounced, and is not as symmetrical as in the previous cases; this is attributed to the A.V.R. action. The regulation of the terminal voltage is further improved, but its oscillations are larger than in the previous cases.

When the gain is 500, Fig. 5-11-d, substantial negative damping is introduced, and the main machine becomes unstable after two swings.

(b) Variation in the stabilising loop gain does not affect the rotor angle response much, except that with the higher loop gain of 0.1 the rotor angle approaches the new stable position a little faster than in the case of the lower loop gain of 0.01. The terminal voltage is sustained at a higher level for a considerable time with the higher value of the loop gain.
From the graphs in Fig. 5-13, it can be seen that by increasing the gain of the speed governors the peak of the first swing of the rotor angle is reduced and this is an advantageous factor in transient stability. But on the other hand, negative damping is quite substantial with the higher loop gains, and this is not desirable. Hence, some compromising value is to be found and consequently a loop gain of 20 has been used for the present study.

5-8-2 Transient Behaviour of the System During Fault

For this study the system as shown in Fig. 5-6, is subjected to different types of faults. From the sequence network diagram shown in Fig. 5-17, sequence and subsequently shunt fault admittances for different types of faults are calculated, and are represented in the network as discussed in Sec. 5-5-1.

The following tests are performed:

Test I - Effect of initial value of rotor angle

One of the double-circuit lines of the system is subjected to a three-phase fault in the vicinity of the sending end bus-bar. The fault is cleared in 0.15 sec. and the faulted line is reclosed in 0.3 sec. Various initial values for the rotor angle are taken and the results are graphically shown as below:

(a) For an initial value of rotor angle of 40° the graph is shown in Fig. 5-14-a.

(b) For an initial value of rotor angle of 45° the graph is shown in Fig. 5-14-b.
(c) For an initial value of rotor angle of 50° the graph is shown in Fig. 5-14-c.

and,

(d) For an initial value of rotor angle of 60° the graph is shown in Fig. 5-14-d.

Observations

(a) For the initial value of 40°, the rotor angle goes up to 100° in the first swing and then starts decreasing. But, in the consequent swings the peaks are simultaneously reduced, and the rotor angle approaches its normal position. Thus, the system is stable.

(b) For the initial value of 45°, though the system is still stable, the peak of the first swing is higher than in the previous case.

(c) When the initial value of the rotor angle is taken as 50°, the first swing exceeds the limit of 180°, and thus the system becomes unstable.

(d) For the initial value of 60°, the rotor angle overshoots the limits of stability faster than in the previous case. Thus, the system loses stability more rapidly.

Test 2 - Effects of various faults

One of the double circuit lines is subjected to various types of faults. In each operation the fault is cleared in 0.15 sec. and the switch is reclosed at 0.3 sec. Subsequently, for the initial value of the rotor angle as 50°, the swing curves are computed and are shown as:
(a) With line-to-ground fault, the swing curve is shown in Fig. 5-15-a.
(b) With line-to-line fault, the swing curve is shown in Fig. 5-15-b.
(c) With 2 lines-to-ground fault, the swing curve is shown in Fig. 5-15-c.
(d) With 3-phase fault, the swing curve is shown in Fig. 5-15-d.

Observations

(a) In the case of line-to-ground fault, the first swing of the rotor angle does not exceed 180°, and in the subsequent swings the rotor angle tends to settle down at the normal value; hence the system is stable.
(b) In the case of line-to-line and 2 lines-to-ground faults, the system still remains stable, though the peak of the first swing in both the cases increases, but the latter more so.
(c) When a 3-phase fault is applied, the rotor angle overshoots the limit of 180°, and the system becomes unstable.

Test 3 - Effects of reclosing time

One of the double circuit lines is subjected to a 3-phase fault with an initial value of the rotor angle of 50°. The fault clearing time is taken as 0.10 sec., and the swing curves are computed for different values of reclosing times. The results are graphically shown as below:

(a) For a reclosing time of 0.25 the graph is shown in Fig. 5-16-c.
(b) For a reclosing time of 0.20 the graph is shown in Fig. 5-16-b.

(c) For a reclosing time of 0.15 the graph is shown in Fig. 5-16-a.

**Observations**

- With the clearing time of 0.25, the rotor angle overshoots the limit of 180°, and hence the system is unstable. With the clearing times of 0.20 and 0.15, the rotor angle remains within the limit tending to approach the normal value, and hence the system is stable.

**Test 4 - Stability boundaries**

To compute the stability boundaries for the different faults, the computer program discussed in Sec. 5-7-2, is used. The reclosing time which is taken as 0.7, together with the other data given in Table 5-1, is fed in the computer and the results are graphically shown as:

(a) With line-to-ground fault the stability boundary is shown in Fig. 5-18-a.

(b) With line-to-line fault the stability boundary is shown in Fig. 5-18-b.

(c) With 2 lines-to-ground fault the stability boundary is shown in Fig. 5-18-c.

(d) With 3-phase fault the stability boundary is shown in Fig. 15-18-d.
Observations

From the graphs the following observations are made:

(a) The stability boundary curve of the synchronous machine, with a fault clearing time within the range of 0.1 to 0.7 sec., is highest in the case of line-to-ground fault and lowest in the case of 3-phase fault. For line-to-line and two lines-to-ground faults, stability boundary curves are in between the above-mentioned two cases.

(b) The diversity between the stability limits of the system with different types of faults is more at the higher clearing times than at the lower clearing times.

5-9 STATEMENT OF MODIFIED SYSTEM EQUATIONS

5-9-1 Machine Equations

\[ v_q = \frac{G_{er}}{1 + \tau_{do} \cdot p} - \frac{x_d + x_d \cdot \tau_{do} \cdot p}{1 + \tau_{do} \cdot p} \cdot i_d \]

\[ v_d = x_q \cdot i_q \]

\[ \psi_d = \frac{G_{er}}{1 + \tau_{do} \cdot p} - \frac{(x_d + x_d \cdot \tau_{do} \cdot p)}{1 + \tau_{do} \cdot p} \cdot i_d \]

\[ \psi_q = -x_q \cdot i_q \]

\[ P_{el} = v_d \cdot i_d + v_q \cdot i_q \]

\[ P_{e2} = i_d^2 \times (r_2 - r_a) \]

And, \( M_p^2 \delta = P_{mo} + \Delta P_m - P_{el} - P_{e2} - k_d \cdot p \cdot \delta \)
5-9-2 Speed Governor

\[ \Delta P_m = \frac{K}{(1 + r_g \cdot p)(1 + r_p \cdot p)} \times p \cdot \delta. \]

5-9-3 Excitation Control

\[ v_f = \frac{K}{1 + r_e \cdot p} \times (v_R - v_t - v_s). \]

\[ v_s = \frac{K_s \cdot p}{1 + r_g \cdot p} \times v_f \]

and, \[ v_t = \sqrt{v_d^2 + v_q^2} \]

5-9-4 Transmission Line

\[ v_{sd} = v_{rd} - x_t \cdot i_{tq} + r_t \cdot i_{td} \]

and, \[ v_{sq} = v_{rq} + x_t \cdot i_{td} + r_t \cdot i_{tq} \]

5-9-5 Static Load

(a) Resistance in series with inductance.

\[ v_{ld} = -x_l \cdot i_{lq} + r_l \cdot i_{ld} \]

and, \[ v_{lq} = x_l \cdot i_{ld} + r_l \cdot i_{lq} \]

(b) Resistance in series with capacitance.

\[ v_{ld} = x_{lc} \cdot i_{lq} - \frac{x_{lc} \cdot v_q}{r_l} \]

and, \[ v_{lq} = -x_{lc} \cdot i_{ld} + \frac{x_{lc} \cdot v_d}{r_l} \]
5-9-6 Axis Transformation

\[ k_{dm} = k_{dn} \cos(\delta_n - \delta_m) + k_{qm} \sin(\delta_n - \delta_m) \]

and,

\[ k_{qm} = k_{qm} \cos(\delta_n - \delta_m) - k_{dm} \sin(\delta_n - \delta_m) \]

5-9-7 Matrix Equations

\[ I_n = V_n \cdot Y_n \]

and,

\[ V_n = Y_n^{-1} \cdot I_n \]
Table 5-1 DATA
TO COMPUTE STABILITY BOUNDARIES OF AN A.C. SYSTEM

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Value</th>
<th>No</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Recycle</td>
<td>1</td>
<td>21</td>
<td>kmb (gain of the compensator)</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>Computing time</td>
<td>3 sec</td>
<td>22</td>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>$x_d$</td>
<td>0.8 p.u.</td>
<td>23</td>
<td>$k_{av}$ (overall gain of the main loop)</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>$x_q$</td>
<td>0.8 p.u.</td>
<td>24</td>
<td>$k_7$</td>
<td>0.066</td>
</tr>
<tr>
<td>5</td>
<td>$x_d$</td>
<td></td>
<td>25</td>
<td>$T_4$</td>
<td>0.04 sec</td>
</tr>
<tr>
<td>6</td>
<td>$K_d$</td>
<td></td>
<td>26</td>
<td>$T_5$</td>
<td>0.01 sec</td>
</tr>
<tr>
<td>7</td>
<td>$T_{do}$</td>
<td>4.67</td>
<td>27</td>
<td>$T_6$</td>
<td>0.15 sec</td>
</tr>
<tr>
<td>8</td>
<td>$H$</td>
<td></td>
<td>28</td>
<td>avr lim. (limit of the AVR)</td>
<td>6 p.u.</td>
</tr>
<tr>
<td>9</td>
<td>$n$ (no of diff. eqns. used)</td>
<td>0.01</td>
<td>29</td>
<td>$Ge_f$</td>
<td>1.5 p.u.</td>
</tr>
<tr>
<td>10</td>
<td>range</td>
<td>0.01</td>
<td>30</td>
<td>del min (min. value of rotor angle)</td>
<td>45 degrees</td>
</tr>
<tr>
<td>11</td>
<td>acc</td>
<td>0.01</td>
<td>31</td>
<td>del acc (reqd. accuracy of rotor angle)</td>
<td>1 degree</td>
</tr>
<tr>
<td>12</td>
<td>$h$</td>
<td>0.01</td>
<td>32</td>
<td>del inc (increment given to rotor angle)</td>
<td>10 degrees</td>
</tr>
<tr>
<td>13</td>
<td>$t_r$</td>
<td>0.7</td>
<td>33</td>
<td>$t_{fc}$ max. (max value of $t_{fc}$)</td>
<td>0.5 sec</td>
</tr>
<tr>
<td>14</td>
<td>Ang 3 (phase displacement of receiving end bus-bar)</td>
<td>0.00</td>
<td>34</td>
<td>$t_{fc}$ min (min value of $t_{fc}$)</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>15</td>
<td>Ang 1 (phase displacement of sending end bus-bar)</td>
<td>30 degrees</td>
<td>35</td>
<td>$Z_0$ (cos $\delta$)</td>
<td>0.996</td>
</tr>
<tr>
<td>16</td>
<td>$\pi$</td>
<td>3.1415</td>
<td>36</td>
<td>$Z_1$ (cos $\alpha$ min)</td>
<td>1.00</td>
</tr>
<tr>
<td>17</td>
<td>$x_{11}$</td>
<td>0.8 p.u.</td>
<td>37</td>
<td>$Z_2$ (cos $\alpha$ max)</td>
<td>0.087</td>
</tr>
<tr>
<td>18</td>
<td>$x_{12}$</td>
<td>2.22 p.u.</td>
<td>38</td>
<td>EF</td>
<td>Marker</td>
</tr>
<tr>
<td>19</td>
<td>$V_s$</td>
<td>1.1 p.u.</td>
<td>39</td>
<td>EA</td>
<td>Marker</td>
</tr>
<tr>
<td>20</td>
<td>kvt (gain of the machine)</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIG. 5-1 AXIS-TRANSFORMATION (VECTOR DIAGRAM)
FIG. 5-2 LOOP OF THE SPEED GOVERNOR

Steam or water inlet valve

Relay-valve

Centrifugal Governor

Input power

Turbine

p.s/w

manual speed setting
FIG. 5-4 CONNECTIONS BETWEEN THE SEQUENCE NETWORKS TO REPRESENT ASYMMETRICAL FAULTS

a) Phase-to-phase

b) Phase-to-ground

c) Two-phase-to-ground
FIG. 5-5 ANALOG REPRESENTATION OF THE MACHINE SYSTEM
FIG. 5-6 SCHEMATIC DIAGRAM OF DOUBLE CIRCUIT A-C TRANSMISSION SYSTEM
Fig. 5-7  FLOW DIAGRAM
TO DRAW SWING CURVES OF AN A.C. SYSTEM

BEGIN

Read in data

Set up matrices

Inverse admittance matrices

Calculate initial values

Print

Insert new matrices

Is integrating time = switching time?

Yes

No

Solve matrix eqns. for current components

Transform components to Parks reference frame

Perform step integration

Calculate dynamic elements of the system

Print out

Is integrating time = computing time?

Yes

No

END
Fig. 5-8 FLOW DIAGRAM
TO DRAW STABILITY BOUNDARIES OF AN A.C. SYSTEM

BEGIN
Read in Data

Set in initial conditions

Change initial value of Rotor Angle

Compute the Swing Curve using A.C. Swing Curve calculations

Is integrating time > 1 and is Rotor Angle decreasing

Yes
System is stable

No

Is Rotor Angle > 180

Yes
System is unstable

No

Is boundary crossed on the last computation

Yes

Reverse and half the Rotor Angle increment

No

Is the new increment required accuracy?

Yes
Print

No

Take next value of clearing time

Yes

Is another boundary point required?

No

Estimate new initial value of Rotor Angle and half its increment

Is another curve required?

Yes

No

END
FIG. 5-9 ROTOR ANGLE TRANSIENT FOLLOWING A STEP OF 0.5 p.u. MECHANICAL POWER INPUT

(a) without speed governor and A.V.R.

(b) with speed governor but without A.V.R.

(c) without speed governor but with A.V.R.

(d) with speed governor and A.V.R.
FIG. 5-10 MACHINE TRANSIENTS FOLLOWING A STEP OF 0.5 p.u. MECHANICAL POWER INPUT

(a) variation of rotor angle

(b) variation of rate of change of rotor angle

(c) variation of power output

(d) variation of terminal voltage
FIG. 5-11 THE EFFECT OF VOLTAGE REGULATOR GAIN
RESPONSE TO 0.5 p.u. STEP OF POWER INPUT

OPEN LOOP GAIN = 10
(a) variation of rotor angle

(b) variation of terminal voltage

OPEN LOOP GAIN = 40
(a) variation of rotor angle

(b) variation of terminal voltage

0 1 2 3 4 5 Time sec
OPEN LOOP GAIN = 100
(a) variation of rotor angle

OPEN LOOP GAIN = 500
(a) variation of rotor angle

(b) variation of terminal voltage

(b) variation of terminal voltage
FIG. 5-12 THE EFFECT OF STABILISING VOLTAGE RESPONSE TO 0.5 p.u. STEP OF MECHANICAL POWER INPUT
GAIN OF DERIVATIVE FEEDBACK LOOP = 0.01

(a) variation of rotor angle

(b) variation of terminal voltage

GAIN OF DERIVATIVE FEEDBACK LOOP = 0.1

(a) variation of rotor angle

(b) variation of terminal voltage

Time sec.
FIG. 5-13 EFFECT OF SPEED GOVERNOR GAIN RESPONSE TO 0.5 p.u. STEP OF MECHANICAL POWER INPUT

(a) variation of rotor angle - gain = 20

(b) variation of rotor angle - gain = 40

(c) variation of rotor angle - gain = 60
FIG. 5-14 ROTOR ANGLE TRANSIENT FOLLOWING 3-PHASE FAULT ON ONE OF THE DOUBLE CIRCUIT LINES

(a) Initial value of the rotor angle = 40°
(b) Initial value of the rotor angle = 45°
(c) Initial value of the rotor angle = 50°
(d) Initial value of the rotor angle = 60°

FIG. 5-15 ROTOR ANGLE TRANSIENT FOLLOWING DIFFERENT FAULTS ON ONE OF THE DOUBLE CIRCUIT LINES

(a) Line-to-ground fault
(b) Line-to-line fault
(c) Two-lines-to-ground fault
(d) Three-phase fault

FIG. 5-16 ROTOR ANGLE TRANSIENT FOLLOWING 3-PHASE FAULT ON ONE OF THE DOUBLE CIRCUIT LINES

(a) Reclosing time = .14 sec
(b) Reclosing time = .20 sec
(c) Reclosing time = .25 sec
FIG. 5-17 PHASE-SEQUENCE NETWORK FOR THE DOUBLE-CIRCUIT SYSTEM

$$\begin{align*}
\text{Infinite Bus} & \quad X_L & X_d & X_{ts} \\
\text{Gren} & 0.8 & 0.1 & 0.1 \\
\text{positive-phase sequence} & & 0.3 & & 0.3 \\
\text{negative-phase sequence} & & 0.5 & 0.1 & 0.1 \\
\text{zero-phase sequence} & & 0.15 & 0.1 & 0.1 \\
\end{align*}$$

FIG. 5-18 STABILITY BOUNDARIES FOR VARIOUS FAULTS

Stable region

- (a) L/G Line-to-Ground fault
- (b) L/L Line-to-Line fault
- (c) 2L/G2-lines-to-Ground fault
- (d) 3L 3-phase fault

Rotor angle (degrees)

Fault clearing time sec.
CHAPTER VI

MATHEMATICAL MODEL OF A.C.-D.C. SYSTEM
6-1 Comparative Study of A.C.-D.C. and A.C. Systems

Chapters 4 and 5 deal with stability problems of the d.c. system and the a.c. system independently. The prime interest in this chapter is the comparative study of electromechanical oscillations and stability limits of synchronous generators when they are connected by an a.c. and a d.c. transmission line, and when the equivalent system has two a.c. lines in parallel. In the former case, the damping of the rotor swing resulting from the d.c. power and its temporary increase during the disturbance is also investigated.

Two digital programmes have been written for this comparative study. One programme draws swing curves under various conditions and the other defines the stability boundary of the system.

6-2 Calculations of Initial Values

The calculations of the initial values are based on the knowledge of the terminal conditions of the synchronous generator, i.e., its terminal voltage and the complex power delivered. This information is obtained from the system or from the load flow study more readily than the theoretical variables like rotor angle, excitation voltage, power factors etc. which are the variables taken as the starting point in previous works on stability problems. 64-65

6-2-1 Initial Values of D.C.-System

The d.c. system is operated at the normal output voltage of the converter and is adjusted to carry a certain percentage of the power delivered by the generator. Under these conditions, as discussed in Sec. 4-2, the initial values of the d.c. system are calculated as below:
The current in the d.c. transmission line is given as:

\[ i_{dc} = \frac{P_{dc}}{V_{dl}} \]  

(6-1)

Control signals as well as the delay angles of the rectifier
and inverter, as discussed in Sec. 4-3-19, are expressed as:

- \[ e_{cr} = V_{dl} + \frac{\pi}{6} \cdot x_c \cdot i_{dc} \]
- \[ e_{ci} = -V_r \cdot \cos \delta_0 + \frac{\pi}{6} \cdot x_c \cdot i_{dc} \]
- \[ \cos \alpha_r = \frac{e_{cr}}{V_r} \]
- \[ \cos \alpha_i = \frac{e_{ci}}{V_r} \]  

(6-2)

The d.c. voltage on the inverter side is given as:

(See Ch. 4-1)

\[ V_{d2} = V_r \cdot \cos \alpha_i - \frac{\pi}{6} \cdot x_c \cdot i_{dc} \]  

(6-3)

Under Uhlmann's assumption\(^{37}\) which equates the magnitude of the
alternating current of the converter to its direct current, the
reactive power \(Q_{dc}\) of the rectifier can be expressed as:

\[ Q_{dc} = \sqrt{i_{dc}}^2 \left( \frac{i_{dc} \cdot V_{dl}}{V_b} \right)^2 \cdot V_s \]  

(6-4)

The equivalent a.c. components of the converter direct current
are calculated by Garilavic's equations\(^{36}\) in Sec. 4-1-29, and they
are transformed to the common reference frame as:

- \[ i_{bd} = i_{vq} \cdot \sin R - i_{vd} \cdot \cos R \]
- \[ i_{bd} = i_{vq} \cdot \sin R + i_{vd} \cdot \cos R \]

and,

\[ R = \tan^{-1} \frac{V_{sdo}}{V_{svo}} \]  

(6-5)
where $i_{vq}$ and $i_{vq}$ are the direct and quadrature components of the converter alternating current with the commutation voltage taken as reference vector.

$v_{vq}$ and $i_{vq}$, when transformed to the common reference frame, correspond to $i_{bq}$ and $i_{bd}$.

and $R$ is the phase difference between the quadrature axis of the common reference frame and that of the reference frame of the converter, and is computed for every step so that the $d$-axis component of the commutation voltage is always zero.

6-2-2- Initial Values of A.C.-System

The active and reactive powers delivered through the a.c. transmission line are given as below:

$$ P_{ac} = P_e - P_{dc} $$

and,  

$$ Q_{ac} = Q_e - Q_{dc} \quad (6-6) $$

where $P$ and $Q$ stand for active and reactive powers respectively,

and the suffices $e$, $ac$ and $dc$ indicate the power delivered by the generator, the a.c line and the d.c line respectively.

For the calculations of infinite-bus voltage, excitation voltage, rotor angle and power factor, the vector diagram shown in Fig. 6-1 is used where $V_s$ is taken as a reference vector. The calculations are based on the equations given below:

$$ V_{rq} = V_s + I_{td} \cdot x_l $$

$$ V_{rd} = -I_{tq} \cdot x_l \quad (6-7) $$
and, \( Ge_{fq} = V_s - I_{gd} \cdot X_g \)
\[
Ge_{fd} = I_{sq} \cdot X_g
\]  \( 6-8 \)

where \( V_r \) and \( V_s \) stand for receiving and sending end bus-bar voltages respectively,

\( Ge_{fq} \) and \( Ge_{fd} \) stand for direct and quadrature components of \( Ge_f \) respectively, with \( V_s \) taken as a reference vector,

\( I_l \) and \( I_g \) stand for the a.c line and generator currents respectively,

\( X_l \) and \( X_g \) stand for the reactances of the a.c line and generator respectively,

and the suffices \( d \) and \( q \) indicate the direct and quadrature components respectively, with \( V_s \) taken as reference vector.

Complex powers delivered by the generator and by the a.c line can also be expressed as:

\[
P_{ac} + jQ_{ac} = V_s^* I_l
\]

\[
P_e + jQ_e = V_s^* I_g
\]  \( 6-9 \)

From Equations 6-7 and 6-9, the following expressions for the infinite-bus voltage and its phase displacement with respect to the sending end voltage are obtained:

\[
V_{rq} = V_s - Q_{dc}/V_s \times X_l
\]

\[
V_{rd} = -P_{ac}/V_s \times X_l
\]
\[ V_r = \sqrt{V_{rq}^2 + V_{rd}^2} \]

and, \[ \delta_1 = \tan^{-1} \frac{V_{rd}}{V_{rq}} \] \hfill (6-10)

where \( \delta_1 \) is the phase displacement between \( V_r \) and \( V_s \).

From Equations 6-3 and 6-9, the following expressions for the open circuit terminal voltage and rotor angle are obtained:

- \[ G_{efq} = V_s + \frac{Q_e}{V_s} \times X_g \]
- \[ G_{efd} = \frac{P_e}{V_s} \times X_g \]
- \[ G_{e} = \sqrt{G_{efq}^2 + G_{efd}^2} \]
- \[ \delta_E = \tan^{-1} \frac{G_{efd}}{G_{efq}} \]

and, \[ \delta = \delta_E + \delta_1 \] \hfill (6-11)

where \( \delta_E \) is the phase displacement between \( V_s \) and \( G_{e} \).

The direct and quadrature components of the a.c bus-bar voltage and generator open-circuit terminal voltage with respect to a common reference frame are expressed as:

- \[ V_{sqo} = V_s \times \cos \delta_1 \]
- \[ V_{sdo} = V_s \times \sin \delta_1 \]
- \[ V_{rco} = V_r \]
- \[ V_{rdo} = 0 \]
- \[ G_{efco} = G_e \times \cos \delta \]

and, \[ G_{efdo} = G_e \times \sin \delta \] \hfill (6-12)
where suffix 0 indicates the common reference frame.

The generator current components with respect to the common reference frame are expressed as:

\[ I_{q0} = G_g \times (G_{e_{fqo}} - V_{sqo}) - B_g \times (V_{sdo} - G_{e_{fdo}}) \]

and,

\[ I_{do} = B_g \times (V_{sqo} - G_{e_{fqo}}) + G_g \times (G_{e_{fdo}} - V_{sdo}) \]  
(6-13)

where \( G \) and \( B \) stand for conductance and susceptance,

and suffices \( L \) and \( g \) indicate the a.c line and the generator.

\( I_{q0} \) and \( I_{do} \) components are transformed to Park's reference frame fixed in the rotor of the synchronous machine as below:

\[ I_{ql} = I_{q0} \times \cos\delta + I_{do} \times \sin\delta \]

and,

\[ I_{dl} = -I_{q0} \times \sin\delta + I_{do} \times \cos\delta \]  
(6-14)

where the suffix \( L \) indicates Park's reference frame.

Direct and quadrature components of the flux linkages with respect to Park's reference frame are given as:

\[ \psi_{ql} = I_{ql} \times x_q \]

and,

\[ \psi_{dl} = G_{e_r} \times x_d \times I_{dl} \]  
(6-15)

Generated voltage in the Park's frame is expressed as:

\[ e_{ql} = \psi_{dl} + \frac{1}{x_q} \times I_{dl} \]  
(6-16)
Direct and quadrature components of the generated voltage are transformed to the common reference frame as below:

\[ e_{q_0} = e_{q_1} \cos \delta \]

and, \[ e_{d_0} = e_{q_1} \sin \delta \] (6-17)

The initial values of the A.V.R. as discussed in Sec. 5-7-1 are also included.

When the system is in steady state, the acceleration of the rotor is taken as zero.

6-3 Transient Stability

When a short circuit or any other sudden change occurs in the power system, the rotor angle of the synchronous generator moves in the effort to cope with the transient state of the system. Consequently, other variables of the machine are also affected and may exceed their normal ratings if the rotor angle continues increasing with time. However, if the rotor swing starts decreasing, the machine variables assume their normal values. Hence, the excursion of the rotor angle is taken as the basis of assessment for the stability of the system.

For the present study, a three-phase fault close to the a.c bus-bar is simulated by a shunt fault impedance connected across the positive sequence network while the generator is represented by a voltage source behind the constant direct-axis component of the transient reactance of the generator. To compute the generator voltage and current during the transient state, matrix methods have been used to solve the nodal equations which can also take into consideration the d.c line. 39
In all stability studies, network equations are solved in the common reference frame, whilst machine equations are referred to Park's reference frame which is fixed in the rotor of the machine concerned. However, during the computation the direct and quadrature components can be transformed from one reference frame to another as required.

6-3-1 Transient Calculations of the D.C. System

During the transient disturbance, when the voltage of the a.c bus-bar drops, variables of the d.c system connected with the same bus are also affected. Hence, for every step integration to evaluate the rotor angle, the current components on the a.c side of the converter are to be calculated simultaneously. For the calculation of d.c variables, the procedure given in Sec. 4-3 has been modified to give a step increment to the d.c power whenever the fault is sensed and bringing it back gradually when the faulted line is reclosed.

6-3-2 Transient Calculations of the A.C. Network

During the transient disturbance and the consequent switching operations, the parameters of the a.c network change, which necessitates the writing of equations for the following configurations:

(a) Before the fault — the normal parameters are used.
(b) During the fault — the shunt-fault susceptance is included.
(c) After removing the fault — the admittance of faulted a.c line is excluded.
and, (d) After reclosure - as in case (a).

The procedure given in Sec. 5-7-1a is used with some modifications to include the alternating current components of the rectifier.

The modified equations are as below:

\[ I_q = e_q + e_{do} x B_q + V_{rq} x G_l + V_{rd} x B_l - i_{pq} \]
\[ I_d = -e_q x B_q + e_{do} x G_q - V_{rq} x B_l + V_{rd} x G_l - i_{bd} \]
\[ V_{sqo} = I_q x Z_{11} + I_d x Z_{12} \]
\[ V_{sdo} = I_q x Z_{21} + I_d x Z_{22} \]
\[ I_{qo} = G_q x (e_q - V_{sqo}) - B_q x (V_{sdo} - e_{do}) \]
\[ I_{do} = B_q x (V_{sqo} - e_q) + G_q x (e_{do} - V_{sdo}) \]
\[ I_{q1} = I_{q0} x \cos\delta + I_{do} x \sin\delta \]
\[ I_{d1} = -I_{q0} x \sin\delta + I_{do} x \cos\delta \]

and, \( I_{d1} \) = \( -I_{q0} x \sin\delta + I_{do} x \cos\delta \) (6-18)

6-3-3 Machine Equations

For calculating the machine variables during the transient condition, the equations of Sec. 5-9-1 are used.

6-3-4 Excitation and Speed Control

To include the A.V.R. and the speed governor, the procedures given in Sec. 5-2 are used.

6-3-5 Swing Curve Programme

The swing curve programme, as discussed in Sec. 5-7-1, has been modified to include the d.c. system and the A.V.R. subroutines.
The function procedure, required by the procedure KM3, has been so arranged that only the required equations take part in the computation, computing time thus being reduced. This programme can be used both for the a.c.-d.c. and the equivalent a.c. systems independently by feeding the appropriate values of the markers.

The following markers have been used for the different instructions to the computer:

(i) FE - if FE = 1, then the programme skips over the d.c. subroutine.
(ii) EF - if EF = 1, then the alternating current components of the converters, as well as their control signals are punched on the tape.
(iii) EA - if EA = 1, then the programme skips over the A.V.R. subroutine.

The flow diagram of the swing curve programme is shown in Fig. 6-29 and its verbatim is given in Appendix (6).

6-4 Stability Boundaries

The stability limits of the various systems can be well exhibited by drawing the curves between the fault clearing time as the abscissae, and the maximum power delivered by the generator without losing stability as the ordinate. For the present comparative study of the stability boundaries of the a.c.-d.c. system and the equivalent a.c. system, the programme in Sec. 5-7-2 is modified as:

(a) The swing curve programme discussed in Sec. 6-3-5, has been included as a subroutine in the main programme.
(b) The criterion for the stability is redefined as:

If the rotor angle of the synchronous machine begins to decrease, and the advanced value of the d.c. reference current normalises, then the system is stable.

The programme has been written in such a way that it can be used for both the a.c.-d.c. system and its equivalent a.c. system, and also the systems with or without A.V.R. can be computed without any alteration in the programme. Thus, the stability boundary programme retains all the versatilities of the a.c.-d.c. swing curve programme. The flow diagram of the stability boundaries programme is shown in Fig. 6-3, and the verbatim is given in Appendix (7).

6-5 Studies and Results

For the stability study, a hypothetical a.c.-d.c. system shown in Fig. 6-4, is considered. In this system, a 1000-MVA equivalent synchronous machine is connected to an infinite bus by 500 miles of a 500-KV a.c. transmission line, and asynchronously by 500 miles of a 500-KV d.c. transmission line. The system parameters are referred to 1000 MVA power base with 230-KV as the base line-to-line voltage. The transformer connections and converters are so arranged that with 1.0 p.u. bus-voltage, the no-load rectifier output voltage is 1.65 referred to the a.c. system. The parameters of the system are given in Table 6-1.

6-5-1 Study No. 1 (Damping Effect of D.C. Line)

In this study, the complex power delivered by the generator during steady state is taken as \(0.7 + j0.3\) at the normal terminal
voltage of 1.1 p.u. A 3-phase short-circuit is applied to the a.c. transmission line close to the generator bus and the fault is represented by a fault susceptance of $10^6$ p.u. The admittance matrices are formulated to represent the following configurations of the system:

(i) Before the occurrence of the fault.
(ii) During the fault.
(iii) After disconnecting the faulted line.
and (iv) After reclosing the faulted line.

The fault clearing time and the reclosing time of the circuit-breaker are taken as 0.1 and 0.2 sec. respectively. Four differential equations are required for this computation, and the instruction is carried out by putting $n = 4$ in the data. The d.c. power increment is not considered, and this can be achieved by taking $A = 1$ in the data where $A$ is the multiplying factor of the d.c. reference current. The integrating time is taken as 2 sec. with step intervals of 0.01 sec. The range and accuracy are taken as 0.01 each.

The computer results are obtained by considering the initial d.c. power as 0.3, 0.4 and 0.5 p.u. separately. The Kalgol version of the swing curve programme takes about 1 min. 20 sec. of computing time for each run. The results are shown graphically in Fig. 6-5.

Observations:

From the summary of the results tabulated below, a comparative study is made of the damping effects for different initial values of d.c. power.
<table>
<thead>
<tr>
<th>No.</th>
<th>Initial Value of $P_{dc}$</th>
<th>Initial Value of Rotor Angle</th>
<th>Whether Rotor Angle exceeds $180^\circ$</th>
<th>Time taken to exceed $180^\circ$ sec.</th>
<th>Maximum Value of Rotor Angle</th>
<th>State of System</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.3</td>
<td>$67^\circ$</td>
<td>Yes</td>
<td>0.75</td>
<td>X</td>
<td>Unstable</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>$56^\circ$</td>
<td>Yes</td>
<td>1.1</td>
<td>X</td>
<td>Unstable</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>$44^\circ$</td>
<td>No</td>
<td>X</td>
<td>$115^\circ$</td>
<td>Stable</td>
</tr>
</tbody>
</table>

**Conclusion:**

From the above results, it is obvious that the more the initial value of the d.c. power the more is the damping effect on the rotor swing and so the stability of the system improves.

6-5-2 Study No. 2 (Effects of D.C. Power and its Increment)

For a comprehensive study of the effects of various initial values of d.c. power and its increment during the disturbance, a number of sets of swing curves are obtained:

- **1st Set**  
  Initial value of $P_{dc} = 0.1$ p.u.  
  $A = 2, 3, 6, 8$
- **2nd Set**  
  Initial value of $P_{dc} = 0.2$ p.u.  
  $A = 2, 3, 4, 8$
- **3rd Set**  
  Initial value of $P_{dc} = 0.3$ p.u.  
  $A = 1.5, 2, 3, 4$
- **4th Set**  
  Initial value of $P_{dc} = 0.4$ p.u.  
  $A = 1.0, 1.5, 2, 3$
- **5th Set**  
  Initial value of $P_{dc} = 0.5$ p.u.  
  $A = 1, 2, 2.5, 4$
In this study, the total power delivered by the generator is assumed to be $0.7 + j0.3$, and the fault clearing and reclosing times are taken as 0.15 and 0.3 sec. respectively. Time taken by the computer is approximately 1 min. 20 sec. for each swing curve, and the results are graphically shown in Fig. 6-6.

Observations:

(a) From the graphs it is obvious that the initial value of the rotor angle reduces with the increase in initial d.c. power.

(b) The peak of the swing curve for the same d.c. power increment increases with the decrease in initial d.c. power, so much so that at lower values the rotor angle increases more than 180° and the system becomes unstable. For example, when $A = 2$, the peaks of the swing curves for different initial values of $P_{dc}$ and the states of the system are summarised as below:

<table>
<thead>
<tr>
<th>Initial Value of $P_{dc}$</th>
<th>Peak of the Swing</th>
<th>State of the System</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 0.5</td>
<td>87°</td>
<td>Stable</td>
</tr>
<tr>
<td>2 0.4</td>
<td>103°</td>
<td>Stable</td>
</tr>
<tr>
<td>2 0.3</td>
<td>123°</td>
<td>Stable</td>
</tr>
<tr>
<td>2 0.2</td>
<td>-</td>
<td>Unstable</td>
</tr>
<tr>
<td>2 0.1</td>
<td>-</td>
<td>Unstable</td>
</tr>
</tbody>
</table>

(c) For the same initial value of $P_{dc}$, the peak of the swing curve decreases with the increase in $A$. For example, in Fig. 6-6-d, for the initial value of $P_{dc} = 4$ peaks of the swing curves and the states of the system for different values of $A$ are tabulated as:
<table>
<thead>
<tr>
<th>Initial Value of $P_{dc}$</th>
<th>A</th>
<th>Peak of the swing</th>
<th>State of the System</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>1.0</td>
<td>-</td>
<td>Unstable</td>
</tr>
<tr>
<td>0.4</td>
<td>1.5</td>
<td>123°</td>
<td>Stable</td>
</tr>
<tr>
<td>0.4</td>
<td>2.0</td>
<td>103°</td>
<td>Stable</td>
</tr>
<tr>
<td>0.4</td>
<td>3.0</td>
<td>98°</td>
<td>Stable</td>
</tr>
</tbody>
</table>

(d) When the initial value of $P_{dc}$ is high, the higher value of A produces overdamping and delays the rotor angle from normalising, even though the system is stable. For example, in Fig. 6-6-c, for $A = 4$ the system is stable but the rotor has been overdamped.

6-5-3 Study No. 3 (Comparison of Different Systems)

For the comparative study of the transient behaviour of the a.c.-d.c. system and its equivalent a.c. system for the same terminal conditions, the total power delivered by the generator is taken as $0.7 + j 0.3$ at its normal terminal voltage of 1.1 p.u. The programme is run for both the systems under different conditions as discussed in Sec. 6-5-1. The results are graphically shown in Fig. 6-7 and for comparison they are summarised as below:

<table>
<thead>
<tr>
<th>Type of System A.V.R. D.C. Increment No. of diff. equations used Whether time max. rotor taken value computing to exceed Rotor Angle 180° 180° Angle State of the System Ref. Fig.</th>
<th>(sec) (min) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.c. No X 3 Yes</td>
<td>1.25</td>
</tr>
<tr>
<td>a.c. Yes X 7 No</td>
<td>X</td>
</tr>
<tr>
<td>a.c.-d.c. No No 4</td>
<td>Yes</td>
</tr>
<tr>
<td>a.c.-d.c. Yes No 4</td>
<td>No</td>
</tr>
<tr>
<td>a.c.-d.c. Yes No 8</td>
<td>Yes</td>
</tr>
<tr>
<td>a.c.-d.c. Yes Yes 8</td>
<td>No</td>
</tr>
</tbody>
</table>
Observations

1. From the results (a) and (b), as tabulated above, it can be observed that the use of the A.V.R. stabilises the equivalent a.c. system which is unstable otherwise. Also the results (d) and (f) show that the A.V.R. is also effective in the case of the a.c.-d.c. system, where the swing curve has been damped with the action of the A.V.R.

2. From the results (d) and (f), it has been observed that the a.c.-d.c. system remains stable if the initial value of the d.c. power is raised during the disturbance. But, the equivalent a.c. system is stable only if the A.V.R. is used, as observed from the results (a) and (b), even then the peak of the rotor swing is high as compared with that recorded in the case of the a.c.-d.c. system.

3. The results (c) and (e) show that when the initial d.c. power is not increased during the disturbance, the a.c.-d.c. system becomes unstable even if the generator is equipped with the A.V.R.

6-5-4 Study No. 4 (Stability Boundaries)

For the comparison of the stability boundaries of the a.c.-d.c. system and the equivalent a.c. system, the Kalgol version of the stability boundary programme as discussed in Sec. 6-4 is used. A 3-phase fault on the a.c. transmission line is simulated by a fault susceptance of $10^6$ p.u., and admittance matrices are formulated for the different configurations as in Sec. 6-5-1. Reclosing time for the faulted circuit is taken as 0.7 sec. and the fault clearing time is varied from 0.1 - 0.5 sec. The minimum value of the rotor angle is arbitrarily taken as $45^0$ and is given an initial increment.
of $10^\circ$ within the search routine. This search routine, computes
the desired value of the rotor angle as well as the corresponding
initial power on the stability boundary within an accuracy of
$0.2^\circ$.

The stability boundaries of the following systems are computed
and the results are graphically shown in Fig. 6-8.

(a) The a.c.-d.c. system with A.V.R. and power control
on the d.c. side.
(The reference current is increased to 1.5 of its
normal value).

(b) The equivalent a.c. system with A.V.R.

(c) The a.c.-d.c. system with A.V.R. but without power
control. (The reference current remains constant
throughout).

Observations and Conclusions

The following observations are made:

(i) The stability boundary for case (a) is higher than
that for case (b) as shown in Fig. 6-8-a. Moreover,
the difference between these boundaries begins to
increase rapidly when the fault clearing time is less
than 0.2 sec.

(ii) The stability boundary for case (c) is lower than
that for case (b).

The diversity in the stability limits of the different systems
can be explained as follows: The 3-phase short circuit at the
a.c. line results in a serious decrease in the d.c. power delivered,
thus causing the generator to accelerate. At this stage, if the d.c. power is not increased, the a.c.-d.c. system loses the stability at comparatively lower initial values of the power delivered by the generator. Hence, the stability boundary for case (c) is lower than that for case (b). But on the other hand, if the d.c. power is increased as soon as the fault is detected, the rotor acceleration is damped and the a.c.-d.c. system can work for higher initial values of power without losing stability. Hence, the stability boundary in case (a) is higher than that for case (b).
Fig. 6-1 DATA
TO COMPUTE STABILITY BOUNDARIES OF AN A-C D.C. SYSTEM AND A.C. SYSTEM

<table>
<thead>
<tr>
<th>No</th>
<th>Parameters</th>
<th>Values</th>
<th>No</th>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Recycle</td>
<td>1</td>
<td>28</td>
<td>T7</td>
<td>6 p.u.</td>
</tr>
<tr>
<td>2</td>
<td>Computing time</td>
<td>2 sec</td>
<td>29</td>
<td>Avrlim (limit of the A.V.R)</td>
<td>1.5 p.u.</td>
</tr>
<tr>
<td>3</td>
<td>xd</td>
<td>0.8 p.u.</td>
<td>30</td>
<td>GeF</td>
<td>0.185</td>
</tr>
<tr>
<td>4</td>
<td>xq</td>
<td>0.8 p.u.</td>
<td>31</td>
<td>R</td>
<td>0.1 p.u.</td>
</tr>
<tr>
<td>5</td>
<td>xq</td>
<td>0.8 p.u.</td>
<td>32</td>
<td>xc</td>
<td>15.48 p.u.</td>
</tr>
<tr>
<td>6</td>
<td>Kq</td>
<td>0.05</td>
<td>33</td>
<td>xdc</td>
<td>1.00 p.u.</td>
</tr>
<tr>
<td>7</td>
<td>Td</td>
<td>4.67</td>
<td>34</td>
<td>vd1</td>
<td>1.00 p.u.</td>
</tr>
<tr>
<td>8</td>
<td>H</td>
<td>6</td>
<td>35</td>
<td>KR</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>n (no of diff. eqns. used)</td>
<td>0.01</td>
<td>36</td>
<td>KI</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>range</td>
<td>0.01</td>
<td>37</td>
<td>k</td>
<td>0.1 p.u.</td>
</tr>
<tr>
<td>11</td>
<td>acc</td>
<td>0.01</td>
<td>38</td>
<td>VdB</td>
<td>500 KV</td>
</tr>
<tr>
<td>12</td>
<td>h</td>
<td>0.01</td>
<td>39</td>
<td>VaB</td>
<td>230 KV</td>
</tr>
<tr>
<td>13</td>
<td>t_r</td>
<td>0.7 sec</td>
<td>40</td>
<td>M (no. of steps to normalise Ir)</td>
<td>45 degrees</td>
</tr>
<tr>
<td>14</td>
<td>Ang 3 (Phase displacement of receiving end bus-bar with respect to reference frame)</td>
<td>0.00</td>
<td>41</td>
<td>Al (multiplying factor to raise Ir)</td>
<td>45 degrees</td>
</tr>
<tr>
<td>15</td>
<td>Ang 1 (Phase displacement of sending end bus-bar)</td>
<td>25 degrees</td>
<td>42</td>
<td>del min (min value of rotor angle)</td>
<td>0.2 degrees</td>
</tr>
<tr>
<td>16</td>
<td>Pi</td>
<td>3,14159</td>
<td>43</td>
<td>del acc (reqd accuracy for rotor angle)</td>
<td>10 degrees</td>
</tr>
<tr>
<td>17</td>
<td>xi1</td>
<td>0.8 p.u.</td>
<td>44</td>
<td>del inc (increment of rotor angle)</td>
<td>0.5 sec</td>
</tr>
<tr>
<td>18</td>
<td>xi2</td>
<td>2.22 p.u.</td>
<td>45</td>
<td>tfc max (max value of tfc)</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>19</td>
<td>Vs</td>
<td>1.1 p.u.</td>
<td>46</td>
<td>tfc min (min value of tfc)</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>20</td>
<td>G</td>
<td>1.00</td>
<td>47</td>
<td>tfc dec (decrement of tfc)</td>
<td>0.1 sec</td>
</tr>
<tr>
<td>21</td>
<td>kvt (gain of the machine)</td>
<td>1.00</td>
<td>48</td>
<td>Z_0 (cos 50)</td>
<td>0.966</td>
</tr>
<tr>
<td>22</td>
<td>kmb (gain of the compensator)</td>
<td>1.00</td>
<td>49</td>
<td>Z_1 (cos α min)</td>
<td>1.00</td>
</tr>
<tr>
<td>23</td>
<td>kav (overall gain of the main loop)</td>
<td>200</td>
<td>50</td>
<td>Z_2 (cos α max)</td>
<td>0.087</td>
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<tr>
<td>24</td>
<td>k7</td>
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<td>51</td>
<td>Pdc</td>
<td>0.4 p.u.</td>
</tr>
<tr>
<td>25</td>
<td>T4</td>
<td>0.04 sec</td>
<td>52</td>
<td>FE</td>
<td>Marker</td>
</tr>
<tr>
<td>26</td>
<td>T5</td>
<td>0.01 sec</td>
<td>53</td>
<td>EF</td>
<td>Marker</td>
</tr>
<tr>
<td>27</td>
<td>T6</td>
<td>0.15 sec</td>
<td>54</td>
<td>EA</td>
<td>Marker</td>
</tr>
</tbody>
</table>
Fig. 6-1 VECTOR DIAGRAM TO CALCULATE INITIAL VALUES OF VOLTAGES AND ROTOR ANGLE
BEGIN

Read in data

Set up matrices of A.C. system

Inverse admittance matrices

Calculate initial values of A.C.-D.C. systems

Print

Insert new matrices

Is integrating time switching time? (Yes/No)

D.C. routine calculates the components of convertor alternating current

Solve matrix eqns. for current components of generator

Transformation to Park's reference frame

Perform step integration

Calculate dynamic elements of the system

Print

Is integrating time computing time? (Yes/No)

END
BEGIN

Read in Data

Set initial conditions

Change initial value of Rotor Angle

Is integrating time > 1 and is Rotor Angle decreasing?

Yes

System is stable

No

Compute the Swing Curve Using A.C.-D.C. Swing Curve calculations

Is Rotor Angle > 180?

Yes

System is unstable

No

Is boundary crossed on the last computation?

Yes

Reverse and half the Rotor Angle increment

No

Is the new increment required accuracy?

Yes

Print

No

Take next value of clearing time

Is another boundary point required?

Yes

No

Estimate new initial value of Rotor Angle and half its increment

Is another curve required?

Yes

No

END
FIG. 6-4 ONE-LINE DIAGRAM OF AC-DC SYSTEM

1000 MVA

1.1 p.u.

500kv-AC-line

500kv-DC-line

Infinite Bus
FIG. 6-5 SWING-CURVES WITH VARIOUS INITIAL VALUES OF D.C. POWER (WITHOUT D.C. POWER INCREMENT)

(a) when initial d.c. power = 0.5 p.u.
(b) when initial d.c. power = 0.4 p.u.
(c) when initial d.c. power = 0.3 p.u.
FIG. 6-6 SWING-CURVES WITH INCREMENTS IN D.C. POWER

(a) $P_{dc} = 0.1$ p.u., $A=2$

$A = $ Multiplying factor for the reference current
$P_{dc} = $ Initial value of the d.c. power

(b) $P_{dc} = 0.2$ p.u.

$A = $ Multiplying factor for the reference current
$P_{dc} = $ Initial value of the d.c. power

(c) $P_{dc} = 0.3$ p.u.

$A = $ Multiplying factor for the reference current
$P_{dc} = $ Initial value of the d.c. power
FIG. 6-6 SWING-CURVES WITH INCREMENTS IN D.C. POWER

(d) $P_{dc} = 0.4 \text{ p.u.}$

(e) $P_{dc} = 0.5 \text{ p.u.}$
FIG. 6-7 SWING-CURVES OF A.C.-D.C. SYSTEM AND A.C. SYSTEM (SAME TERMINAL CONDITIONS)

(a) A.C. System without A.V.R.
(b) A.C. System with A.V.R.
(c) A.C.-D.C. System without A.V.R.
   (without D.C. Power Increment)
(d) A.C.-D.C. System without A.V.R.
   (with D.C. Power Increment)
(e) A.C.-D.C. System with A.V.R.
   (without D.C. Power Increment)
(f) A.C.-D.C. System with A.V.R.
   (with D.C. Power Increment)
FIG. 6-8 STABILITY-BOUNDARIES (AC-DC AND AC SYSTEMS)

(a) AC-DC system with power inc. (A.V.R. included)
(b) Equivalent A.C. system (A.V.R. included)
(c) AC-DC system without power inc. (A.V.R. included)
<table>
<thead>
<tr>
<th>List of References</th>
</tr>
</thead>
</table>


21 I.E.C. Publication 84: "Recommendation for Mercury Arc Converters".


DISCUSSION

The results of the operational study of a parallel a.c.-d.c. transmission system and an equivalent a.c. system, as derived in Ch. 6, form the basic part of this work and may be discussed under two headings:

(i) Damping characteristics of a D.C. line
and, (ii) Comparison of an A.C.-D.C. system and equivalent A.C. system.

Damping Characteristics of a D.C. Line

The effects of various initial d.c. inputs have been studied in Sec. 6-5-1, in which the power delivered by the generator has been taken as 0.7 + j0.03 at the normal terminal voltage of 1.1 p.u., a 3-phase a.c. fault near the sending end bus-bar has been represented by a fault susceptance of 10^6 p.u. which has been cleared in 0.1 sec., and the line has been reclosed at 0.2 sec. From the swing curves, as shown in Fig. 6-5, the following observations are made:

(a) that the peak of the rotor swing is maximum with an initial input of 0.3 p.u.; that with an initial input of 0.4 p.u., though the system is still unstable, the rotor swing is damped; and that with an initial d.c. input of 0.5 p.u., the rotor swing is damped further and the system becomes stable.

and, (b) that the initial value of the rotor angle decreases with the increase of initial d.c. input in the same order.

In Sec. 6-5-2, the effects of increments to initial d.c. input have been investigated and a set of results with an initial d.c. input of 0.4 p.u. has been tabulated in observation (c) of Sec. 6-5-2. It is observed that the system is unstable when the
initial d.c. input is not increased during the disturbance, i.e. the multiplying factor of the d.c. reference current "A" = 1.0. But with the higher values of "A" the peak of the rotor swing decreases proportionately.

The above results can be justified by the fact that, after clearing the fault, the generator deceleration power depends either upon the initial input of the d.c. line, which is still in service, or upon its increment during the disturbance. Hence in a parallel a.c.-d.c. transmission system, if a major portion of a generator output can be transmitted through a d.c. line or if a sufficient increment is given to the initial d.c. input, the generator regains its stability, even after a most critical fault on the a.c. line, as exhibited in Figs. 6-5 and Fig. 6-7 respectively. A similar study has been made by Machida\(^79\) on an a.c.-d.c. transmission model. His test results are in full agreement with the digital investigations of Sec. 6-5-1 and Sec. 6-5-2.

In the above a.c.-d.c. parallel operation a balance between the two methods of improving generator transient limits is essential. Thus, for each specific system, the critical values of the initial d.c. input and its required increment during the disturbance must be optimised by drawing a series of swing curves, like those shown in Fig. 6-6.

**Comparison of an A.C.-D.C. System and an Equivalent A.C. System**

For this study, three equivalent transmission systems have been considered in Sec. 6-5-4.
(a) when a generator is connected with an infinite bus-bar through two identical a.c. lines in parallel.

(b) when one of the a.c. lines has been replaced by an equivalent d.c. system.

and, (c) when a rapid power control has been introduced on the rectifier side of the d.c. system.

To compare stability limits of the above systems, a 3-phase a.c. fault close to the sending end bus bar, has been considered. The fault has been represented by a fault susceptance of $10^6$ p.u. as before, the reclosing time has been taken as 0.7 sec. and in system (c) an increment of 50% has been given to the initial d.c. input at the time of disturbance. The stability boundaries of the above systems have been shown in Fig. 6-8, from which the following observations are made:

(1) Under similar operational conditions and for a fault clearing time of e.g. 0.2 sec., the transient limit of the generator in system (a) is 9.5% higher than in system (b). This can be attributed to the inherited resynchronising characteristics of the a.c. line which is still in service after clearing the fault and provides more deceleration power to the generator than the d.c. line of system (b) in a similar situation.

(2) In system (c), the transient limit of the generator for the same fault clearing time of 0.2 sec. is 10.9% higher than in system (b) and 1.6% higher than in system (a). This can be attributed to the extra deceleration power provided to the generator by the incremental operation of the d.c. transmission system.
From the above observations the following conclusions are drawn:

(1) In a parallel operation of an a.c.–d.c. transmission system, if the bulk of the generator output is delivered through a d.c. line, the transient limit of the generator increases as shown in Fig. 6-5.

(2) When a generator is connected through an a.c.–d.c. transmission system or through an equivalent a.c. double-circuit system, the transient limit of the generator is higher in the latter. But, if the initial d.c. input is given a sufficient increment during the disturbance, the transient limit of the former is higher. Though, in the initial and incremental operations of an a.c.–d.c. system the improvement of transient stability has been established, still economic feasibility for each specific project is the overriding factor.

Power transmission by an a.c.–d.c. parallel operation, keeping in view stability and economy of the system, may be suggested for the following cases:

(1) For a long overhead power transmission and for the interconnection of two distant power systems, the a.c.–d.c. parallel operation could be considered where:

(a) the d.c. line is designed to carry the bulk of the power and is not tapped,

and, (b) an a.c. line is run in parallel which may have intermediate stations, if required.
This system, with its capability of improving the stability limits, may have its economic justification even for smaller distances than the "break-even distance" of 300-500 miles generally mentioned.

(2) The load centres of some densely populated areas are connected by a.c. underground cables, but with ever-increasing load growth, they require reinforcements from time to time. A d.c. underground cable system in parallel with the existing one could be a worthwhile economic proposition which could be further investigated. By improving the transient limits of the generator the above mentioned parallel operation may establish its economic justification even for shorter distances than the accepted "break-even distance" for the underground transmission system of 40-60 miles. Furthermore, the capital expenditure can be deferred by phasing out the installation of the converter bridges in line with the load growth while starting from a mono-polar arrangement.

**REFERENCES**