Asset Returns and the Real Economy

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ABSTRACT

This thesis presents an empirical investigation of the behaviour of financial markets and also the relationship on the real economy. The thesis will focus on Ireland, a small open economy with increased dependence on international developments. Two important aspects of the Irish economy, the term structure of interest rates and impact of exchange rate volatility, will be analysed. The motivation for the analysis of the term structure of interest rates in part I is two fold. Central banks can control very short-term interest rates, but of course the real economy will only really be affected by the long-term interest rate. Therefore the transmission mechanism from monetary policy to the real economy will depend on the relationship between short-term interest rates and long-term interest rates, i.e. the term structure of interest rates. The second important issue is that of market efficiency, and whether asset prices and returns are correctly valued by the market. A number of different interest rate maturities will be used to test the Expectations Hypothesis (EH) of term structure. The EH will also be tested assuming constant and time varying term premia. The results give support for the EH, and find no evidence of a time varying term premium. Given the recent extraordinary growth in the share of Irish exports in GDP, the impact of exchange rate volatility on Irish exports is analysed in part 2. The motivation behind part 2 is to test whether the resulting monetary union will lead to a rise in exports, as a result of the end of exchange rate risk. Using the cointegration-ECM methodology I find that in the long-run there is no significant effect on Irish exports to the UK, while there is actually a positive impact on exports to European countries (UK included). I tentatively conclude that in the long-run the involvement in a single European currency will have no impact on trade.
# Contents

Abstract

Contents

List of Tables

List of Figures

Acknowledgements

## Chapter 1

**Introduction and Overview** 1

## Chapter 2

**Econometric Methodology** 4

2.1 Introduction 4

2.2 Unit Root Processes 5

2.3 Testing the Order of Integration 6

2.4 Vector Autoregression 10

2.4.1 Lag Length 13

2.4.2 Generalised Method of Moments 15

2.4.3 Granger Causality 18

2.5 Error Correction Models and Cointegration 18

2.5.1 Engle-Granger 20

2.5.2 Phillips Hansen 22

2.5.3 Johansen 23

2.5.4 Identification of Cointegrating Relationships 27

2.6 Conclusions 29
PART I:
THE EXPECTATIONS HYPOTHESIS OF THE TERM STRUCTURE

Chapter 3

Introduction to the Expectations Hypothesis

3.1 Introduction 30
3.2 Theoretical Overview 31
3.3 Outline of Part I 35

Chapter 4

Modelling the EH using Single Equation Estimation 38

4.1 Introduction 38
4.2 The Spread predict Future Changes in the Short Rates 40
4.2.1 Theoretical Review 40
4.2.2 Empirical Evidence 42
4.3 Alternative Single Equation Analysis 44
4.3.1 Theoretical Review 44
4.3.2 Empirical Evidence 46
4.4 Empirical Results 49
4.4.1 Unit Root 49
4.4.2 The Spread and the predictability of Future Changes in Short Rates 49
4.4.3 Alternative Single Equation Test Results 50
4.5 Monte Carlo Experiments 53
4.6 Conclusions 55

Chapter 5

Modelling the EH using the Cointegration Approach 57

5.1 Introduction 57
5.2 Theoretical Models: Cointegration and ECM's 58
5.3 Empirical Evidence 61
5.4 Empirical Results 64
5.4.1 Unit Root Test Results 64
Chapter 6

The VAR Approach to Testing the Expectations with a Constant Risk Premium

6.1 Introduction 68
6.2 Theoretical Model 69
   6.2.1 Derivation of the VAR Approach 69
   6.2.2 Testable Restrictions' 74
6.3 VAR Empirical Evidence 76
6.4 Empirical Results 79
   6.4.1 The Data 79
   6.4.2 The Theoretical Spread and VAR Results 79
6.5 Interpretation of Results 81
6.6 Conclusions 83

Chapter 7

The VAR Approach to Testing the Expectations with a Time Varying Risk Premium

7.1 Introduction 84
7.2 Theoretical Model 87
7.3 Testing the Model 89
7.4 Empirical Evidence using Long-Rates 94
7.5 VAR Results 97
   7.5.1 The Data 97
   7.5.2 Unit Root Results 97
   7.5.3 VAR Analysis 98
   7.5.4 Interpretation 99
7.6 Conclusions 100
PART II:  
THE IMPACT OF EXCHANGE RATE VOLATILITY AND TRADE

Chapter 8
Modelling Exchange Rate Volatility and Trade  101
  8.1 Introduction  101
  8.2 Theoretical Models  102
  8.3 Empirical Models  105
    8.3.1 Stationarity Issues  106
    8.3.2 Exchange Rate Volatility: Measurement and Estimation  108
    8.3.3 Data Sample Period  113
  8.4 Irish Evidence
  8.5 Conclusions and Outline of Part II  117

Chapter 9
Exchange Rate Volatility and Exports to the UK  119
  9.1 Introduction  119
  9.2 Modelling Exports to the UK  121
  9.3 Empirical Results  123
    9.3.1 The Data  123
    9.3.2 Unit Root Results  124
    9.3.3 Cointegration Results  125
    9.3.4 ECM Results  126
  9.4 Conclusions  128

Chapter 10
Exchange Rate Volatility and Exports to the EU  130
  10.1 Introduction  130
  10.2 Modelling Exports to the EU  132
  10.3 Empirical Results  134
    10.3.1 The Data  134
    10.3.2 Unit Root Results  135
    10.3.3 Cointegration Results  136
    10.3.4 ECM Results  137
    10.3.5 Interpretations and Further Results  139
  10.4 Conclusions  142
Chapter 11

Conclusions

11.1 Conclusions of Part I 144
11.2 Conclusions of Part II 146

Appendix 1: Data Description 148
Appendix 2: Monte Carlo Programme 151
Appendix 3: Vector Autoregression Programme 158

Bibliography 171
LIST OF TABLES

PART I

4.1 Unit Root Test Results
4.2 Regression Results: Perfect foresight spread regressed on the actual spread
4.3 Monte Carlo Bias
4.4 Test Rejection Frequencies

5.1 Shea’s (1992) Cointegration Results: Testing the EH in the USA
5.2 Hall, Anderson and Grangers (1992) Cointegration Results: Testing the EH in the USA
5.3 Cuthbertson’s (1996a) Cointegration Results: Testing the EH in the UK
5.4 Unit Root Test Results
5.5 Engle-Granger Cointegration Test Results
5.6 Phillips-Hansen Cointegration Test Results
5.7 Johansen Test Results
5.8 Testing the EH Restrictions in the Cointegrating System
5.9 Error Correction Model Results

6.1 Bivariate VAR Model Results: $S_t^{(n,m)}$, $\Delta r_t^{(m)}$
6.2 Regression Results: Actual Spread on the Theoretical Spread

6.3 Bivariate VAR Results: Testing the EH Using Weakly Rational Expectations

7.1 Unit Root Test Results

7.2 Multivariate VAR Model Results: $S_t^{(n,m)}$, $\Delta r_t^{(m)}$, $h(n,t+1)-r_t$

7.3 Test Results: Are Excess One Period Returns Time Varying?

7.4 Regression Results: Actual Spread on the Theoretical Spread

7.5 Multivariate VAR Results: Testing the EH

7.6 Test Results Variance Decomposition

PART II

8.1 Summary of Estimation Techniques and Results from Exchange Rate Volatility and Trade Literature

8.2 Summary of Exchange Rate Volatility Measures

9.1 Unit Root Test Results

9.2 Cointegration Test Results
10.1 Unit Root Test Results
10.2 Cointegration Test Results
10.3 Cointegrating Vector and Likelihood Ratio Test Results
10.4 Error Correction Model Test Results
10.5 Cointegration Test Results (Revised Sample)
10.6 Cointegrating Vector and Likelihood Ratio Test Results (Revised Sample)
10.7 Error Correction Model Test Results (Revised Sample)
LIST OF FIGURES

PART I

4.1 Irish 1 Month Money Market Rate
4.2 Irish 3 Month Money Market Rate
4.3 Irish 6 Month Money Market Rate
4.4 3 and 1 Month Interest Rate Spread
4.5 6 and 1 Month Interest Rate Spread
4.6 6 and 3 Month Interest Rate Spread

5.1 Impulse Response Function – Equation R1
5.2 Impulse Response Function – Equation R3
5.3 Impulse Response Function – Equation R6

6.1 3 and 1 Month Interest Rate Spread
6.2 6 and 1 Month Interest Rate Spread
6.3 6 and 3 Month Interest Rate Spread
6.4 Actual and Theoretical Interest Rate Spread (3 Month, 1 Month)
6.5 Actual and Theoretical Interest Rate Spread (6 Month, 1 Month)
6.6 Actual and Theoretical Interest Rate Spread (6 Month, 1 Month)

7.1 5 Year and 1 Month Interest Rate
7.2 10 Year and 1 Month Interest Rate
7.3 15 Year and 1 Month Interest Rate
7.4 5 Year and 1 Month Interest Rate Spread
7.5 10 Year and 1 Month Interest Rate Spread
7.6 15 Year and 1 Month Interest Rate Spread
7.7 Actualse and Theoretical Interest Rate Spread (5 Year, 1 Month)
7.8 Actual and Theoretical Interest Rate Spread (10 Year, 1 Month)
7.9 Actual and Theoretical Interest Rate Spread (15 Year, 1 Month)

PART II

9.1 Total Irish Exports
9.2 Irish Trade Weights
9.3 Irish Exports (Real) to the UK
9.4 UK Real Income
9.5 Competitiveness Measure
9.6 Volatility Measure

10.1 Total Irish Exports
10.2 Irish Trade Weights
10.3 Irish Trade Weights (Revised)
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CHAPTER 1

INTRODUCTION

This thesis presents an empirical investigation of the behaviour of financial markets and also the relationship on the real economy. The focus of the thesis will be on Ireland, a small open economy with increased dependence on international developments. Modern econometric methods will be used to model the time series properties of the asset returns and a detailed commentary on the implications of the reported results is presented. Multivariate vector autoregression (VAR) models have proved to be valuable in the financial economics literature, and in Part I of the thesis I present results using both a two variable and a three variable VAR. An important additional element is that a generalised method of moments (GMM) estimator is adopted in order to correct for possible serial correlation and heteroscedasticity. Monte Carlo experiments are also carried out in part one of the thesis.

The motivation for the analysis of the term structure of interest rates is twofold. Similar to the US Federal Reserve, the German Bundesbank or the European Central Bank, most central banks use the day-to-day interest rate on the inter-bank money market as its operational target. Using modern monetary instruments central banks can control very short-term interest rates. However, the real economy, e.g. investment and consumption, will only really be affected by the long-term interest rate. Therefore the transmission mechanism from monetary policy to the real economy will depend on the relationship between short-term interest rates and long-term interest rates, i.e. the term structure of interest rates. The expectations hypothesis (EH) of the term structure of interest rates is the main theory behind the analysis of the link between interest rates of different maturities. The theory states that
the interest rate on a long-term bond will equal an average of short-term interest rates that people expect to occur over the life of the bond. Therefore, the reason why interest rates on bonds of different maturities differ is that short-term interest rates are expected to have different values in the future.

A further incentive to analyse the EH is the issue of market efficiency, which has important consequences in the allocation of resources throughout the economy. One interpretation of market efficiency is that it is not possible for agents to earn abnormal profits by trading in a specific market, based on a particular information set. In other words, asset prices and returns are correctly valued by the market participant at all time. If on the other hand, markets are not efficient, then expenditure on real investments will be misallocated between firms and there may also be a non-optimal aggregate level of investment. As a result, it not surprising that this area has become an extensive research area.

The first part of the thesis will use a number of different maturity interest rates to test the Expectations Hypothesis of term structure. Although research in this area has attracted a large amount of academic research, this is the first known study on the EH using Irish data. This part on the thesis will focus on the latest econometric techniques to analyse both short-term and long-term Irish interest rate data. The EH will also be tested assuming constant and time varying term premia. The extension of the two variable VAR to the three variable VAR represents an important contribution to the literature, as it accounts for the possibility of a time varying term premium. Monte Carlo experiments are reported in Part I of the thesis. Given the importance of time varying term premia, I use the Monte Carlo experiments to test whether the rejections of the EH found in the literature may be due to inaccurate statistical tests, rather than the incoherency of the EH model with the data.

Of course of vital importance to a small open economy will be factors that have a detrimental effect on its exports. Given that good policy decisions are assisted by having relevant information on the factors that determine the level of exports, I will carry out a detailed analysis of the impact of exchange rate volatility on Irish
exports. This will have important implications for an economy whose exports have grown dramatically in recent times. A further issue of concern is the fact that the main policy organisations, e.g. the Group of Twenty-Four and the Group of Ten in reports tabled by the IMF, continue to view exchange rate volatility as having the traditional effect on trade, i.e. a negative impact. A number of studies in the literature have shown that this is in fact not the case, exchange rate volatility may lead to greater trade. The motivation behind Part 2 is to test whether the resulting monetary union will lead to a rise in exports, as a result of the end of exchange rate risk.

The cointegration-ECM modeling approach is again adopted in this part of the thesis, and the main focus will be on developing long-run relationships and short-run interactions. Initially the estimation is carried out on exports to the UK alone. I then test the impact of exchange rate volatility on exports to 5 European countries (UK, Germany, France, Netherlands and Italy). These countries represent Ireland's key trading partners. The export decisions of the multinational corporation (MNC) sector and the indigenous sector are also compared. Given that a large portion of Irish exports are from the MNC sectors, this separate analysis may have important implications.

A detailed outline is given at the start of each part of the thesis.
CHAPTER 2
ECONOMETRIC METHODOLOGY

2.1 INTRODUCTION

In this chapter I will concentrate on a number of econometric techniques used to
analysis macroeconomic and financial data. There are a number of important
econometric issues associated with analysis of financial data. Traditional tests of
economic models assume that the error term is stationary, and that the mean is zero and
has a constant variance. However this is generally not the case, as most economic time
series tends to move up or down over time. In order to ensure that the error in the
regression is stationary, the modeller must either include stationary variables or
variables that are cointegrated in the model. Unit root tests will be used to test the order
of integration of the variables. Cointegration is then used to test for a long-run
relationship among non-stationary variables. If there is a linear combination of
integrated variables that is stationary, such variables are said to cointegrated. Therefore
a set of nonstationary variables may drift together in the long-run and so could lead to a
stationary error term.

I also discuss the issues of multivariate time series models where the variables
are stationary and depend on their own past values and the past values of other
variables, vector autoregressions (VAR's). The VAR models will be used primarily as a
forecasting tool. The main advantage of the VAR approach, is that for tests of the
rational expectations restrictions all that is required is to estimate the unrestricted
model. The test of the restrictions can then be set up. Given the possibility of serial
correlation and heteroscedastic errors, a generalised method of moments (GMM)
estimator will be adopted.
The chapter will take the following form. Section 2.2 will analyse issues of stationary and non-stationary data, while the appropriate tests of the order of integration will be discussed in section 2.3. The VAR approach and the corrections for moving average errors and possible heteroscedasticity, will be considered in section 2.4. Section 2.5 will analyse the modelling of non-stationary variables, issues of cointegration and error correction models. Finally section 2.6 will give a summary.

2.2 Unit Root Processes

Issues of stationarity are of importance to the modeller as they deal with the behaviour of a particular time series of data. Shocks to a stationary time series will have a temporary effect, if over time the series will return to its long-run (mean) level. On the other hand, shocks to a non-stationary series will have permanent effect and the mean (and/or the variance) will be dependent on time. Much of the economic and financial data, which I analyse has a stochastic trend and is therefore non-stationary.

Considering the regression model below;

\[ x_t = \beta_0 + \beta_1 z_t + e_t \]  

(2.1)

where the classical regression model would assume stationarity of the variables \([x \text{ and } z]\) and so the errors would have zero mean and finite variance. However given the possible non-stationary nature of the variables this model would be a spurious regression\(^1\). An intuitive example is the random walk process,

\[ x_t = \alpha_0 + \alpha_t x_{t-1} + \varepsilon_t \]  

(2.2)
where, $\alpha_1$ - parameter coefficient

$\varepsilon_t$ - stationary white noise process

The above random walk process is clearly an AR(1) process. If $\alpha_1 < 1$ ($\alpha_1$ is positive) then $x_t$ is a stationary series. A special case of a non-stationary variable is an integrated variable. An integrated variable can be transformed into a stationary series by differencing the variable a number of times, depending on its order of integration.

If $\alpha_1 = 1$, then $x_t$ is I(1) and needs to be differenced once to yield a stationary series:

$$\Delta x_t = \alpha_0 + \varepsilon_t \quad (2.3)$$

From Equation 2.3, $\Delta x_t$ is stationary given that $\varepsilon_t$ is stationary. A series is said to be integrated of order $d$, I($d$), if it needs to be differenced $d$ times in order to become a stationary series. Most of the economic and financial data which we deal with is integrated of order 1, I(1).

### 2.3 Testing for the Order of Integration

An integrated series is defined as having at least one unit root in its autoregressive (AR) process. Tests for the order of integration are set up with a null hypothesis that the series contains a unit root, i.e. is non stationary.

---

1 A spurious regression tends to have a high $R^2$, but the results will have no economic meaning. For a detailed discussion of this issue, see Granger and Newbold (1974).
Taking Equation 2.2 and subtracting $x_{t-1}$ from both sides will yield the following:

$$\Delta x_t = \alpha_0 + \gamma x_{t-1} + \epsilon_t$$

(2.4)

where $\gamma = \alpha_1 - 1$. As can be seen testing that $\alpha_1 = 1$ is equivalent to testing $\gamma = 0$. Dickey and Fuller (1979) have developed a test of the null hypothesis that $\gamma = 0$ (or $\alpha_1 = 1$), and the series $x_t$ contains a unit root, against a general alternative. An extension to the Dickey-Fuller (DF) test is the augmented Dickey-Fuller (ADF) where additional lagged dependent variables are added to Equation 2.4 in order to remove any serial correlation. Therefore if the error term in Equation 2.4 is serially correlated then lagged dependent variables should be added until the serial correlation is eliminated.

$$\Delta x_t = \alpha_0 + \gamma x_{t-1} + \sum_{i=2}^{p} \beta_i \Delta x_{t-i+1} + \epsilon_t$$

(2.5)

where $\gamma = -\left(1 - \sum_{i=1}^{p} \alpha_i \right)$ and $\beta_i = \sum_{j=1}^{p} \alpha_j$

Again the coefficient of interest is $\gamma$, and if $\gamma = 0$, then the series has a unit root. The t statistic on the coefficient $\gamma$ in Equations 2.4 and 2.5 is the Wald test of the null hypothesis that the series contains a unit root. This t statistic however is not the standard t-distribution and the critical values cannot be obtained from the conventional student t-tables.

The Phillips-Perron (PP) test is an alternative to the ADF test when serial correlation may be present in Equation 2.4. As can be seen from the above discussion the Dickey-Fuller approach aims to retain the validity of tests which assume white noise

---

2 In their Monte Carlo study Dickey and Fuller (1979) found that the critical values for $\gamma = 0$ depend upon the form of the regression and also the sample size.
errors, by adding terms to the regression and so ensuring that the errors are in fact white noise. Phillips and Perron (1988) show a simple transformation of the DF statistic, which accounts for serially correlated and heteroscedastic residuals. Given that the PP is just a transformation of the DF, the tests are very similar and the test statistics will be equal if \( \varepsilon_t \) is NIID (Banerjee, Dolado, Galbraith and Hendry, 1993).

The PP has become a popular approach, as it only requires one to estimate a first order autoregressive process of the following form:

\[
x_t = \mu + \alpha x_{t-1} + \varepsilon_t
\]

(2.6)

The PP test calculates the unit root test from the following regression and tests the null hypothesis, \( H_0: \beta = 0 \)

\[
\Delta x_t = \mu + \beta x_{t-1} + u_t
\]

(2.7)

The coefficient estimates and their t-statistics are calculated and the statistics are then adjusted to reflect autocorrelation in the error.
The t statistic is calculated as:

$$Z(t_{\beta}) = \left( \frac{\hat{\beta}}{\hat{\sigma}} \right) t_{\beta} - \frac{1}{2} (\hat{\sigma}^2 - \hat{s}^2) T \left[ \hat{\sigma}^2 \sum_{i=1}^{T} (x_i - \mu)^2 \right]^{\frac{1}{2}}$$  \hspace{1cm} (2.8)

where

$$\hat{s}^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{u}_t^2 \quad \text{and} \quad \hat{\sigma}^2 = \hat{s}^2 + \frac{2}{T} \sum_{j=1}^{r+1} \left( \frac{1}{r+1} \right) \sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}$$

$r$ is a truncation parameter, $r \sim T^{1/2}$

$T$ is the sample size

$\hat{s}^2$ is the estimated error variance of the regression

$\hat{\sigma}^2_t$ is $\hat{s}^2$ plus a weighted average of the covariance between $\hat{u}_t$ and $\hat{u}_{t-j}$

In the case where the error term is white noise, in Equation 2.7, i.e. $u_t = e_t$, this then implies that covariance of the error term is zero and $\hat{\sigma}^2_t = \hat{s}^2$.

As a result, Equation 2.8 becomes $Z(t_{\beta}) = t_{\beta}$. The second term in the above equation will only matter if the error term contains serial correlation. The magnitude of serial correlation determines the size of the difference between, $\hat{\sigma}^2_t = \hat{s}^2$ and so the correction to the DF test statistic.

The problems associated with the spurious regression and the availability of procedures for testing unit roots, has led a major growth in the testing of the order of integration. Initially, most studies could not reject that most economic series contained a unit root, Nelson and Plosser (1982). However there are a number of problems associated with these testing procedures, e.g. the low power of the tests and their sensitivity to possible breaks in the series, Pesaran and Smith (1998). If for example the series has an approximate unit root, i.e. the series is stationary, but has an autoregressive parameter, $\alpha$, that is close to unity, then the DF and the PP have relatively low power. The tests are also unable to detect a structural break in the series.
Recently Diebold and Senhadjji (1996) provided evidence that suggests US GNP, the series most investigated, is trend stationary, I(0), rather than I(1). A further complication with the unit root tests, is that they assume linear adjustment processes. Therefore series that are assumed to be I(1), may be in fact be I(0), once non-linearities in the adjustment process are considered, Michael, Nobay and Peel (1997). Based on these difficulties associated with unit root tests, I will also perform the Johansen procedure (discussed in full below) which tests the whole VAR for stationarity.

2.4 Vector Autoregressions (VAR's)

Vector autoregression (VAR) models are multivariate time series models and can be seen as extensions of the univariate autoregressive moving average (ARMA) models of Box and Jenkins (1970). The key to time series modelling is the Wold representation, which states that if the variables are weakly stationary, non-stochastic processes, then they can be written as a linear combination of a sequence of uncorrelated random variables.

If we consider the simple bivariate example, VAR(1),

\[ X_{t+1} = a_{11}x_t + a_{12}y_t + \varepsilon_{1t+1} \]
\[ Y_{t+1} = a_{21}x_t + a_{22}y_t + \varepsilon_{2t+1} \]

Where it is assumed that both \( x_{t+1} \) and \( y_{t+1} \) are stationary, \( \varepsilon_{1t+1} \) and \( \varepsilon_{2t+1} \) are white noise disturbances with standard deviations of \( \sigma_x \) and \( \sigma_y \), respectively and \( \varepsilon_{1t+1} \) and \( \varepsilon_{2t+1} \) are uncorrelated white noise disturbances. Given that the right hand side of both equations are identical, the model may be estimated efficiently and consistently by OLS.
Since there are no exogenous variables in VAR models, once a vector of variables has been chosen there are no restrictions placed on the feedback effects between them.

Using matrix algebra I can write the system of Equations 2.9 and 2.10 in a more compact form:

$$\mathbf{z}_{t+1} = A \mathbf{z}_t + \varepsilon_{t+1}$$  \hspace{1cm} (2.11)

where $\mathbf{z}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$ and $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

The VAR system is perfectly suited for forecasting. If one wishes to forecast $x_{t+j}$ based only on information at time $t$ and earlier,

$$E_x x_{t+j} = e_1 A^j \mathbf{z}_t$$  \hspace{1cm} (2.12)

Where $e_1$ is a 2x1 vector with 1 as its first element and zero as its second.

---

4 It is quite easily shown that Equation 2.12 is true. For example, if one wishes to forecast $x_{t+j}$; where $j=2$, if we lead Equation 2.9 by one period and take expectations at time $t$,

$$E_x x_{t+2} = a_{11} E_x x_{t+1} + a_{12} E_y_{t+1}$$

If we replace the expectations with the forecast from Equation 2.9 and 2.10 then,

$$E_x x_{t+2} = (a_{11}^2 + a_{12} a_{21}) x_t + (a_{11} a_{12} + a_{12} a_{22}) y_t$$

The coefficients on $x_t$ and $y_t$ are the exact same as is picked out for the first row of the $A^j$ matrix.
The VAR of higher order can be written as, VAR(p);

\[
x_{t+1} = \sum_{i=0}^{p-1} a_i x_{t-i} + \sum_{i=0}^{p-1} b_i y_{t-i} + \varepsilon_{t+1}
\] (2.13)

\[
y_{t+1} = \sum_{i=0}^{p-1} c_i x_{t-i} + \sum_{i=0}^{p-1} d_i y_{t-i} + \varepsilon_{2t+1}
\] (2.14)

This can be written where \( z_t \) includes the past values of \( x \) and \( y \) and may be written in companion form as;

\[
\begin{bmatrix}
  x_{t+1} \\
  \vdots \\
  x_{t-p+2} \\
  y_{t+1} \\
  \vdots \\
  y_{t-p+2}
\end{bmatrix} =
\begin{bmatrix}
  a_1 & a_2 & \ldots & a_{p-1} & a_p & b_1 & b_2 & \ldots & b_{p-1} & b_p \\
  1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
  0 & 1 & \ldots & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & \ldots & 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
  0 & 0 & \ldots & 0 & 1 & 0 & \ldots & 0 & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & \ldots & 0 & 0 & 0 & \ldots & 1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
  x_t \\
  \vdots \\
  x_{t-p+1} \\
  y_t \\
  \vdots \\
  y_{t-p+1}
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_{t+1} \\
  \vdots \\
  \varepsilon_{t-p+1} \\
  \varepsilon_{2t+1} \\
  \vdots \\
  \varepsilon_{2t-p+1}
\end{bmatrix}
\] (2.15)

And j ahead forecasts can be obtained from;

\[
E_tZ_{t+1+j} = A^{j+1} Z_t
\] (2.16)

Forecasts from Equation 2.16 will be very important especially in part 1 of the thesis.
Since the publication of Sims's early work (1972, 1980a, 1980b, 1982) on the methodology, the VAR approach has caused much debate. Critics of the approach claim that it bears little relationship with economic theory and relies on unsustainable assumptions, Canova (1995). However, the VAR methodology has proved to be a popular tool in the applied economics literature. A major advantage of the VAR approach is that it imposes no explicit theoretical restrictions on the variables in the system, Cuthbertson (1996b). For example in part 1 of the thesis, when focusing on tests of the rational expectations restrictions all that is required is to estimate the unrestricted model. The test of the restrictions can then be set up.

2.4.1 Lag Length

An important issue when estimating the VAR is the appropriate lag length, $p$. If the lag length is too large, the VAR is more likely to 'pick-up' within sample random variation as well as any systematic relationship, due to the greater number of parameters that need to be estimated. Abadir, Hardi and Tzavalis (1998) noted that even moderate values of $p$ will lead to substantial biases in the VAR. On the other hand if the lag length is too small, then important lag dependencies may be omitted from the VAR and if serial correlation is present the estimated coefficient will be inconsistent. The applied econometrician is left with 2 options; choose a particular lag length and verify that the results are independent of this auxiliary assumption or let the data choose a particular lag length using some optimal statistical criteria, Canova (1995). Examples are the Akaike Information Criteria (AIC), Akaike (1974), or the Swartz Bayesian Criterion (SBC), Schwartz (1978). It has been noted by a number of studies that little is known about the small sample properties of these selection procedures, and that in many cases they may give conflicting conclusions, Pesaran and Smith (1998).

A likelihood ratio test of the VAR system with $p_0$ lags versus $p_1$ lags will be used, where $p_0 < p_1$, to test the significance of the extra lagged variables. The likelihood ratio test is;
\[(T-c)(\log|\Sigma_{\rho}| - \log|\Sigma_{\rho}|)\]  \hspace{1cm} (2.17)

Where

- \(T = \) number of observations
- \(c = \) number of estimated parameters in each equation
- \(\log|\Sigma_{\rho}| = \) natural log of the determinant of the variance covariance matrix of the residuals

The above likelihood ratio statistic has an asymptotic \( \chi^2 \) distribution, where \( q \) is the degrees of freedom, such that \( q = k'(p_1 - p_2) \) (Hamilton, 1996).

Given that the likelihood ratio tests are based on asymptotic theory, they may not be very useful in small sample situations. Alternative tests are the AIC and the SBC;

\[
\text{AIC} = T \log |\Sigma| + 2N \hspace{1cm} (2.18)
\]
\[
\text{SBC} = T \log |\Sigma| + N \log(T) \hspace{1cm} (2.19)
\]

Where, \( N = \) total number of parameters estimated in all equations
- \(\log|\Sigma_{\rho}| = \) natural log of the determinant of the variance covariance matrix of the residuals

If each equation in the system has \( n \)-variables with \( p \) lags and a constant, then \( N = n^2p + n \). Addition regressors will reduce \( \log|\Sigma| \) at the expense of raising \( N \). The lag will be based on the lowest value for the AIC or the SBC.
2.4.2 Generalised Method of Moments

Given that multi-period expectations are taken, this leads to the possibility of serially correlated errors.

Given that \( u_t \) is a white noise error

\[
y_t = \beta_1 x_{t+1} + \beta_2 x_{t+2} + u_t \tag{2.20}
\]

and

\[
x_{t+j}^e = E(x_{t+j} | \Omega_t) \quad (j=1,2) \tag{2.21}
\]

Given rational expectations (RE);

\[
x_{t+j} = x_{t+j}^e + \omega_{t+j} \quad (j=1,2) \tag{2.22}
\]

Substituting Equation 2.22 into Equation 2.20 leads to;

\[
y_t = \beta_1 x_{t+1} + \beta_2 x_{t+2} + q_t \tag{2.23}
\]

\[
q_t = (u_t - \beta_1 \omega_{t+1} + \beta_2 \omega_{t+2}) \tag{2.24}
\]

There are two possible problems with the estimation of Equation 2.23. The first is associated with the fact that the error term and the ex-post variables \( x_{t+j} \) are correlated. Therefore instrumental variables (IV) estimation is required to obtain consistent estimates of the parameters. However, given that one is dealing with
multi-period expectations, IV estimation is not appropriate. One possible solution is to estimate using the generalised method of moments (GMM) approach of Hansen (1982), which 'corrects' the covariance matrix to take account of serially correlated errors.

Putting Equation 2.23 into matrix notation;

$$y = X\beta + q$$ \hspace{1cm} (2.25)

Given that in the above equation the error term and the ex-post variables $X$ are correlated;

$$E(X'q) \neq 0$$ \hspace{1cm} (2.26)

In this case one must choose a set of instruments, $Z$, that are correlated with $X$, but uncorrelated with $q$, such that;

$$E(Z'q) = 0$$ \hspace{1cm} (2.27)

Let $Z$ contain $J$ variables that can be used as valid instruments for $X$. It is unlikely that there will be a unique choice of instruments, and the more instruments used, the more efficient the estimator. GMM estimation will involve minimising a quadratic form based on the $J$ orthogonality/moment conditions, Equation 2.27. This will give an estimate that is efficient amongst all estimators defined by the moment conditions. A good approximate of the population moment is the sample counterpart;

$$\frac{1}{n}Z'(y - X\beta) = 0$$ \hspace{1cm} (2.28)

A least squares equivalent is found by minimising the following objective function with respect to $\beta$;
where \( I \) is an identity matrix. This gives a consistent, but inefficient estimate of \( \beta \).

The GMM objective function will include a weighted matrix, \( W \);

\[
(1/n) [Z'(y - X\beta)]' \cdot W^{-1} \cdot (1/n)[Z'(y - X\beta)]
\]  
(2.30)

Hansen (1982) showed that the optimal choice of weighting matrix, \( W \), is the asymptotic covariance matrix of the moment conditions, which for the above case is equal to \((1/n)(Z'q)\). The weighting matrix will give less weight to the estimates that have larger variances compared to those that have been estimated more precisely. Given the possibility of both serial correlation and heteroscedastic errors, the Newey and West (1987) estimator will be used to calculate a consistent estimate of the covariance matrix. The objective is to minimise Equation 2.30 with respect to \( \beta \).

\[
(X'Z)W^{-1}(Z'y - Z'X\beta) = 0
\]

\[
(X'Z)W^{-1}Z'y - (X'Z)W^{-1}Z'X\beta = 0
\]

\[
(X'ZW^{-1}Z'X)^{-1}(X'Z)W^{-1}Z'y = \beta
\]  
(2.31)

In the homoscedastic and serially independent case the above estimator is just the least squares estimator. What is clear is that the least squares estimator is a special case of the GMM case.
2.4.3 Granger-Causality

Given that in the VAR system the lagged values of all the variables appear in every equation, of particular interest would be whether specific variables, or groups of variables, determine other variables.

Taking Equation 2.11 and suppose the VAR is specified as;

\[
\begin{bmatrix}
  x_{t+1} \\
  y_{t+1}
\end{bmatrix}
= \begin{bmatrix}
  a_{11} & 0 \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  x_t \\
  y_t
\end{bmatrix}
+ \begin{bmatrix}
  e_{t+1}^1 \\
  e_{t+1}^2
\end{bmatrix}
\]  

(2.32)

In the above system \( y_t \) does not have any explanatory power on \( x_{t+1} \), i.e. \( y_t \) does not Granger-cause \( x_{t+1} \). This can be tested by regressing \( x_{t+1} \) on \( x_t \) and \( y_t \) and examining whether the coefficients on \( y_t \) is significantly different from zero. An F-test will be used to test Granger-causality in the VAR, Enders (1995). In the \( n \) variable case \( A_y(L) \), represents the coefficients of the lagged values of variable \( j \) on variable \( i \), variable \( j \) does not Granger-cause variable \( i \) if all coefficients in the polynomial \( A_y(L) \) can be set equal to zero.

2.5 Error Correction Models & Cointegration

In section 2.2, it was noted that if a time series variable has a single unit root, then the first difference would be taken in order to obtain a stationary series. Given that one is interested in the relationship between variables, it is useful to consider differencing in term of a regression model (Holden and Perman, 1994).

\[ \text{A detailed discussion is provided in Davidson and McKinnon (1993) and Green (1990).} \]
Given that,

$$\Delta y_t = \beta \Delta x_t + u_t$$  \hspace{1cm} (2.33)

Where both $y$ and $x$ are I(1) and so the first differences are I(0). The problem associated with Equation 2.33 is that it does not contain a long-run solution. If $y$ in the long-run is given by $y^*$, such that $y = f(x)$, then Equation 2.33, can be written as;

$$\Delta y_t = \beta \Delta x_t + \theta (y_{t-1} - f(x_{t-1})) + u_t$$  \hspace{1cm} (2.34)

Equation 2.34 shows both the short-run and long-run relationships. Re-writing Equation 2.34;

$$\Delta y_t = \beta \Delta x_t + \theta (y_{t-1} - \lambda x_{t-1}) + u_t$$  \hspace{1cm} (2.35)

Where it is assumed that $f$ is a linear function, $y^* = f(x) = \lambda x$. The above is the error correction model (ECM), which has proved to be a popular approach in applied econometric research\(^6\) \(^7\).

Although it has been argued that Equation 2.33 is not appropriate, as it has no static equation and so may be inconsistent with economic theory even though, on statistical grounds it is appropriate. However on statistical grounds there is a question over the use of Equation 2.35. Given that $y$ and $x$ are both I(1), and that in general a linear combination of I(1) variables is itself I(1), this then raises questions over the validity of Equation 2.35. The cointegration approach shows how a linear combination of integrated variables is stationary, i.e. are the variables cointegrated\(^8\). In effect I look

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\(^6\) ECM's have become a popular tool since studies by, Sargan (1964), Hendry and Anderson (1977), and Davidson, Hendry, Srba, and Yeo (1978).

\(^7\) The ECM approach will be used in detail in part two and three of the thesis. The general to specific modeling strategy will be adopted. I start with a general model and test down, and at each step in the testing down procedure, I check the diagnostics (e.g. tests for serial correlation, autoregressive heteroscedasticity, normality, etc.) of the model. The general to specific methodology is in sharp contrast to the classical statistical methodology, which starts with a very definite statistical model (Cuthbertson and Hayes, 1994).

\(^8\) I assume throughout the thesis that variables are at most I(1). There are however a growing number of studies within an I(2) framework, Johansen (1995a, chapter 9, 1995b) and Kitamura (1995).
at the estimation of a VAR using non-stationary variables. Before looking at the multivariate approach, I will focus on the bivariate approach. Three procedures will be used to test for cointegration;

i) Engle-Granger Approach

ii) Phillips-Hansen

iii) Johansen Procedure

Which test to adopt will depend on the particular model and the underlying economic theory. There are however, a number of statistical difficulties associated with each approach, (e.g. distribution problems for hypothesis tests of the cointegrating vector, heteroscedasticity problems when used with the Johansen procedure). It is understandable therefore, that there is scope for conflicting results based on the different approaches, particularly in small samples.

2.5.1 Engle-Granger

Given that there are 2 variables, $y_t$ and $x_t$, which are integrated of order one, I(1), the Engle-Granger approach can be used to test whether they are cointegrated. The Engle-Granger approach proposes a test of whether 2 variables, that are I(1), are cointegrated. The variables are first tested for the order of integration. The Dickey-Fuller and the Phillips-Perron tests, discussed earlier in the chapter, can be used to infer the number of unit roots for each variable. Given that the variables are integrated of order one, I(1), the first step is to estimate the following static OLS regressions;

$$x_t = \alpha_t + \beta y_t + \epsilon_{1t}$$ (2.36)
or

\[ y_t = \alpha_2 + \beta x_t + \epsilon_{2t} \]  \hspace{1cm} (2.37)

The second step is that the residuals of the above Equations 2.36 and 2.37 are tested for a unit root. If the residuals are I(0), then \( x_t \) and \( y_t \) are cointegrated, with a cointegrating parameter of \( \beta \). When testing the residual for a unit root, it is not possible to use the Dickey-Fuller tables. This is due to the fact that the \( [\epsilon_t] \) sequence is generated from a regression equation, i.e. it is the forecast of \( [\epsilon_t] \) and not the actual \( [\epsilon_t] \) that is known. Engle and Granger do provide test statistics that can be used.

The Engle-Granger approach does however suffer from a number of drawbacks. The estimation procedure requires that one variable is placed on the left hand side, while the others are used as regressors. It is possible that one regression indicates cointegration, while reversing the order indicates no cointegration. This is certainly a concern given that the test for cointegration should be invariant to the choice of variables for normalisation. A second limitation is that the procedure does not take account of tests of cointegration of multiple variables, three or more variables. Finally, the fact that the Engle-Granger procedure is a two-step procedure is also a limitation. Given that in the first step errors are generated, and then used to estimate a regression in the second step, this may lead to any error introduced in the first step being carried over into the second step, Enders (1995).

The Engle-Granger approach however, has become a very popular approach in the literature. The main advantage of the approach is that it provides an intuitive method of investigating cointegration without imposing a particular cointegrating vector. An important consideration is that the regressions in Equations (2.28) and (2.29), lead to super consistent estimators, as demonstrated by Engle and Granger (1987). The cointegrating parameter converges to its true parameter ‘more quickly’ than the OLS estimator does in the standard regression, involving stationary variables.
2.5.2 Phillips-Hansen

As a comparison with the Engle-Granger approach, I will also adopt the Phillips-Hansen (1990) procedure. The Phillips-Hansen approach estimates the parameters of a single cointegrating relationship using a fully-modified OLS (FM-OLS) procedure. The long-run covariance matrix can be corrected for serial correlation and for endogeneity and so fully modified Wald tests can be derived. The advantage of the Phillips-Hansen approach is that the inference due to the nuisance parameter dependencies, which is present in cointegrating systems, has been removed. It is also of note that the bivariate case of Phillips-Hansen approach is closely related with the OLS single equation approach of Engle and Granger (1987).

The Phillips-Hansen estimator is only appropriate when there exists a single cointegrating relationship between a set of I(1) variables. Given that the model is as follows,

\[ y_t = \beta_0 + \beta_1 X_t + u_t \quad (2.38) \]

Where \( y_t \) is an I(1) variable and \( X_t \) is a \( k \times 1 \) vector of I(1) regressors. It is assumed that the regressors in \( X_t \) are not cointegrated among themselves. It is also assumed that \( X_t \) has the following first order difference stationary process;

\[ \Delta X_t = \mu + v_t \quad (2.39) \]

Where \( \mu \) is a \( k \times 1 \) vector of drift parameters and \( v_t \) is a \( k \times 1 \) vector of I(0) variables. It is also assumed that \( (ut, v_t)' \) is strictly stationary. As has been shown in Engle and Granger (1987) and Stock (1987), the OLS estimates of \( \beta \) are consistent estimates, even if \( v_t \) and \( u_t \) are correlated. However, the asymptotic distribution of the OLS estimator involves the unit root distribution and is non standard. Therefore the usual t-test will be invalid. The Phillips-Hansen approach takes account of the these
correlation's in a semi parametric manner. Phillips and Hansen (1990) show that these corrections work effectively in sample sizes as small as 50\(^9\).

### 2.5.3 Johansen

In the previous two section's, the cointegration procedures implicitly restrict the number of cointegrating relationships to one. The extension of the analysis to three are more variables, and so more than one cointegrating relationship, is possible. Johansen (1988) developed a procedure for estimating and testing multiple cointegrating vectors. An intuitive understanding of the Johansen procedure can be drawn from the fact it can be represented as a general formulation of the Dickey-Fuller unit root test, which is not restricted to the univariate case.

By extending the univariate case to a / multivariate case;

\[
X_t = A_1X_{t-1} + A_2X_{t-2} + \ldots + A_pX_{t-p} + \varepsilon_t \quad (2.40)
\]

Where,

- \(X_{t-i} = l\) vector of I(d) variables (where \(i = 0, \ldots, p\))
- \(A_i = l \times l\) matrix (where \(i = 0, \ldots, p\))
- \(\varepsilon_t = l \times l\) vector of white noise errors

\(^9\) Pesaran and Shin (1995) provide evidence on small sample properties of these estimates using Monte Carlo.
This may be reparameterised as an error-correction model:

\[ \Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_p \Delta X_{t-p+1} - \Pi X_{t-p} + \varepsilon_t \]  
(2.41)

and

\[ \Delta X_t = \Theta_1 \Delta X_{t-1} + \Theta_2 \Delta X_{t-2} + \ldots + \Theta_p \Delta X_{t-p+1} - \Pi X_{t-1} + \varepsilon_t \]  
(2.42)

where, \( \Gamma_i = -I + A_1 + A_2 + \ldots + A_i \)

\[
\Theta_{p-1} = -A_p \\
\Theta_{p-2} = -A_p - A_{p-1} \\
\vdots \\
\Theta_1 = -A_p - A_{p-1} - \ldots - A_2 \\
\Pi = I - A_1 - A_2 - \ldots - A_p
\]

The matrix \( \Pi \), determines whether or not, Equation 2.42, contains a set of cointegrating vectors. There are three possibilities:

(i) The \( \Pi \) matrix has zero rank and so indicates that there are no cointegrating vectors.

(ii) The \( \Pi \) matrix has full rank and so indicates that all the variables in \( X_t \) are I(0).

(iii) The \( \Pi \) matrix has rank \( k \), where \( k \) is less than full rank, but greater than rank zero, i.e. there are \( k \) cointegrating vectors.
If the rank of the $\Pi$ matrix is equal to $k (< l)$, case (iii), this implies that $\Pi$ can be represented by:

$$\Pi = \alpha\beta'$$  \hspace{1cm} (2.43)

Where $\alpha$ is an $l \times k$ matrix of error correction weights and $\beta$ is an $l \times k$ matrix of cointegrating vectors. In the case of (iii) a set of $l$ variables, will have at the very most $k (= l-1)$ cointegrating vectors. The variables in $X_t$ will form a cointegrating relationship, which is referred to as the cointegrating matrix, $\beta$. The $\beta$ matrix has the property that $\beta'X_t$ is I(0), whereas $X_t$ is I(1). The $\alpha$ matrix can seen as a speed of adjustment parameter.

The number of cointegrating vectors is determined by the rank of $\Pi$. It is not possible to estimate $\alpha$ and $\beta$ by OLS, due to the cross equation restrictions. It is possible to estimate by maximum likelihood. The Johansen procedure tests the number of cointegrating vectors by determining the rank of $\Pi$. The rank of a matrix is equal to the number of non-zero eigenvalues of that matrix. The Johansen procedure consists of estimating the $k$ eigenvalues of $\Pi$, and testing their significance from zero. If none are significant, then we have case (i). If all $k$ eigenvalues are significant, then we have case (ii). If $k < l$, then the system has $k$ unique cointegrating vectors, case (iii).
Johansen describes two different tests. They are the Maximum Eigenvalue Test and the Trace Test:

\[ \text{Max. Eigenvalue Test} = -T \ln(1 - \hat{\lambda}_{k+1}) \] (2.44)

\[ k = l-1, l-2, \ldots, 0 \]

\[ \text{Trace Test} = -T \sum_{i=k+1}^{n} \ln(1 - \hat{\lambda}_{i}) \] (2.45)

\[ k = l-1, l-2, \ldots, 0 \]

Where \( \hat{\lambda}_i \) are the eigenvalues obtained from the estimated \( \Pi \) matrix, while \( T \) is the number of usable observations. The maximum eigenvalue test is set up such that the null tests that the number of cointegrating vectors is \( k \) against an alternative of \( k+1 \) cointegrating vectors. The second statistic, trace test, tests the null hypothesis that the number of distinct cointegrating vectors is less than or equal to \( k \) against a general alternative. The critical values for both statistics is given in Johansen and Juselius (1990).

Although, Johansen procedure does overcome some of the problems associated with the Engle-Granger approach, it too has some drawbacks. The main advantage of the Johansen procedure is that it estimates the cointegrating vectors freely and allows hypothesis tests to be carried out. A limitation of the procedure is that it requires the residuals from the VAR to be white noise. If the residuals are serially correlated or are heteroscedastic, then the cointegration approach is invalid. As has been mentioned earlier, the lag length of the VAR will be determined by the SBC and the AIC. By increasing the lag length, serial correlation often can be eliminated.
2.5.4 Identification of Cointegrating Relationships

As has been discussed in detail in the last section the estimation of the system, subject to the rank restrictions on the long-run multiplier matrix, $\Pi$, does not lead to a unique choice for the cointegrating relations. An important aspect of the Johansen procedure is that it enables the testing of restrictions on the cointegrating vectors. It is however, important to remember when carrying out such hypothesis tests that if there are $k$ cointegrating vectors, only these $k$ linear combinations are stationary. All other linear combinations are non-stationary. Having determined $\alpha$ and $\beta'$, testing various restrictions on $\alpha$ and $\beta'$ can easily be carried out. The test statistics involve comparing the number of cointegrating vectors under the null and alternative hypothesis. The ordered characteristic roots (eigenvalues) of the unrestricted $\Pi$ matrix are denoted by $\lambda_1, \lambda_2, \ldots, \lambda_n$ and $\lambda^*_1, \lambda^*_2, \ldots, \lambda^*_n$ are those from the restricted model. The test statistic used in order to test the restrictions on $\beta$ is the following;

$$ T \sum_{i=1}^{k} \left[ \ln(1-\lambda^*_i) - \ln(1-\hat{\lambda}_i) \right] $$

(2.46)

The above statistic has a $\chi^2$ distribution, with degrees of freedom equal to the number of restrictions placed on $\beta$. The restriction embedded in the null hypothesis is binding if the calculated value above exceeds the $\chi^2$ critical value.$^{10}$

The identification of $\beta$ requires at least $k$ restrictions per each of the $k$ cointegrating relations.$^{11}$ When $k = 1$, the only restriction required is the 'normalizing' restriction in terms of the unit coefficient on any one of the integrated variables which enter the cointegrating relation. When the number of cointegrating vectors is greater than one, the number of 'normalizing' restrictions is equal to $k$, while there should also be a further $k^2 - k$ a priori restrictions. Pesaran (1997) emphasises the need to use

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$^{10}$ Restrictions on $\alpha$ can be tested in the same way.
$^{11}$ A detailed discussion is contained in Pesaran and Shin (1999).
economic theory to guide the choice of identification restrictions on the cointegrating vectors\textsuperscript{12}.

Although the Johansen procedure does implicitly impose $k^2$ just-identifying restrictions on $\beta$, these are adopted from a purely statistical perspective and have little bearing on economic theory (Pesaran and Shin, 1999). As a result Pesaran (1997) favours the approach of estimating exact or over-identifying a priori restrictions obtained from the long-run equilibrium based on a suitable economic model. In the case where the restrictions are based on individual vectors, the restriction can be set as:

$$R_i\beta_i = b_i, \quad i = 1, 2, \ldots k$$ (2.47)

Where $\beta_i$ is the $i$th cointegrating vector, $R_i$ is the $i$th block in matrix $R$, where $R$ is a $r \times k_l$ matrix ($\text{Rank}(R) = r$) and $b_i$ is defined by $b' = (b_{1i}, b_{2i}, \ldots, b_{ki})$. In the above case the necessary conditions for identification of the cointegrating vectors are:

$$\text{Rank}(R_i\beta_i) = k, \quad i = 1, 2, \ldots, k$$ (2.48)

This verifies the point made earlier in this section, that there must be at least $k$ independent restrictions on each of the $k$ cointegrating vectors. In the case where the restrictions are applied to parameters of two or more cointegrating vectors, the matrix $R$ is not block diagonal and is written as:

$$\text{Rank}\{R(I, \otimes \beta)\} = k^2$$ (2.49)

where $\otimes$ is the Kronecker product. A necessary condition for the above to hold is given by the order condition $r \geq k^2$. There are three possible cases;

a) $r < k^2$, the under-identified case
b) $r = k^2$, the exactly identified case

\textsuperscript{12}Pesaran (1997) argues in favour of using intertemporal optimisation techniques from economic theory when identifying long-run relationships in applied economics, as opposed to purely the statistical approach of the cointegration analysis.
c) \( r > k^2 \), the over-identified case

Log-likelihood ratio statistic for testing the over identifying restrictions can easily be set up.

2.6 Conclusions

In this chapter I have discussed a number of econometric techniques used to analyse macroeconomic and financial data. There are two main features associated with the empirical analysis in this thesis. The first is using VAR analysis, containing stationary variables, to forecast financial variables. The second is to use VAR analysis, containing nonstationary variables, to establish long-run relationships that are stationary. There are a number of important econometric issues associated with both models.

For example in part one of the thesis a number of tests of the Expectations Hypothesis of the term structure of interest rates will be analysed. Although in many cases estimation can be conducted using OLS, there are certain formulations of the tests which induce serial correlation. If for example the change in the short interest rate is over a longer horizon than the data frequency, this will result in overlapping observations and hence moving average errors. As a result a GMM correction is required in the presence of overlapping observations.

Establishing a long-run relationship that is stationary, among integrated variables is also an important tool. When possible the Engle-Granger, the Phillips-Hansen and the Johansen procedure will be adopted. However, for example, in part two of the thesis tests of cointegration will primarily use the Johansen procedure. This is due to the fact that I am interested in the long-run relationship among a number of variables and there may be more than one cointegrating vector.
PART I
CHAPTER 3

INTRODUCTION TO THE EXPECTATIONS HYPOTHESIS

3.1 INTRODUCTION:

In the main body of Part I, an empirical investigation is carried out on the term structure of interest rates, at the short end of the maturity spectrum. The final chapter focuses on various testable extensions using long maturity bonds. The term structure of interest rates (also referred to as the yield curve) describes the relationship between debt instruments with different times to maturity at a particular point in time. Modern theory has developed a number of hypothesis's to explain the relationship between different interest rates and yields to maturity, pure expectations hypothesis (PEH), expectations hypothesis (EH), liquidity preference hypothesis (LPH). The PEH of the term structure assumes that agents are risk neutral, and instantaneous risky arbitrage ensures that expected returns on bonds of different maturities are equalised. Taking spot yields and assuming the PEH as the correct description of market participants behaviour, the terminal value of a (long) investment in an n-period bond should be equal to n consecutive 1-period (short) investment in a safe asset:

\[(1 + R_t^{(n)})^n = (1 + r_t^{(1)}) (1 + E_t r_{t+1}^{(1)}) \ldots (1 + E_t r_{t+n}^{(1)}) \quad (3.1)\]

If the terminal value of the investment in the long bond exceeds the expected terminal value, investors would buy long bonds and sell short ones. The result would be a rise in the current market price of the long bond, and given a fixed and known
redemption value, this implies a fall in the long interest rate \((R_t(n))\). Simultaneously, as a result of the sales of the short-term bonds, there would be a drop in their current price and a rise in the short-term interest rate \((r_t^{(l)})\). A linear approximation of Equation 3.1;

\[
R_t^{(n)} = 1/n \left( r_t^{(l)} + E_t r_{t+1}^{(l)} + ... + E_t r_{t+n-1}^{(l)} \right) \tag{3.2}
\]

which is known as the 'fundamental equation' of the PEH.

The PEH, as stated in Equation 3.1 and 3.2, does not take into consideration any risk or liquidity premia in the term structure. However, investors holding long positions would presumably require compensation for this extra risk, i.e. a risk premium. This point can be seen by the lower liquidity of a long-term bond, as it has to be held for \(n\) periods to receive a certain return. Assumptions regarding risk premia take a number of forms. If we assume the risk premia is constant over time, then a constant is added to Equations 3.1 to 3.2 and this constitutes the expectation hypothesis (EH). If the term premium increases as the time to maturity increases, then we refer to this as the liquidity preference hypothesis (LPH). Apart from the constant term, the main testable implications of the PEH, EH and the LPH, using regression analysis, are identical.

### 3.2 Theoretical Overview:

The REHTS states that, after adjusting for risk, the expected return from holding for one period a bond that has \(n\) periods to maturity is the same as a certain return from a one period bond, i.e.;

\[
E_t[h(n,t+1)] = E_t[\ln P(n-1,t+1) - \ln P(n,t)] = r_t + T_t^{(n)} \tag{3.3}
\]
CHAPTER 3: INTRODUCTION TO THE EXPECTATIONS HYPOTHESIS

Where, \( h(n,t+1) \), equals the capital gain from holding an \( n \)-period bond for one period\(^1\). The \( h(n,t+1) \) is approximated by \( \ln P(n-1, t+1) - \ln P(n,t) \), where \( P(n,t) \) is the price at time \( t \) of a pure discount bond with a face value of £1 and \( n \) periods to maturity. \( E_t \) is the rational expectations operator conditional on information available in period \( t \), \( r_t \) is the (one period) risk free rate \( r_t \) and \( T^{(n)}_t \) is a risk premium, perceived at time \( t \), which compensates investors for the risk of investing in long bonds. Under risk neutrality, we assume \( T^{(n)}_t = 0 \)\(^2\).

For continuously compounded rates, substituting \( \ln P_t^{(o)} = \ln M - nR^{(o)}_t \) into Equation 3.3 yields:

\[
nR_t^{(n)} = (n-1)E_t R_{t+1}^{n-1} + r_t + T_t^{(n)} \tag{3.4}
\]

Leading Equation 3.4 by one period yields:

\[
(n-1)R_{t+1}^{(n-1)} = (n-2)E_{t+1} R_{t+1}^{(n-2)} + r_{t+1} + T_{t+1}^{(n-2)} \tag{3.5}
\]

\(^1\) Although Equation 3.3 is based on the holding period yield (HPY), it leads to a term structure relationship in terms of spot yields.

\(^2\) Campbell (1986) has shown that this assumption is also a good approximation in a general equilibrium model.
CHAPTER 3: INTRODUCTION TO THE EXPECTATIONS HYPOTHESIS

Expectations of Equation 3.5 are taken, using $E_t E_{t+1} = E_t$ and substituting into Equation 3.4 yields;

$$nR_t^{(n)} = (n-2)E_t R_t^{(n-2)} + E_t (r_{t+1} + r_t) + E_t (T_t^{(n-1)} + T_t^{(n)})$$  \hspace{1cm} (3.6)

After continuous substitution for the first term on the right hand side of Equation 3.6, and noting that $(n-j)E_t R_t^{(n-j)} = 0$ for $j=n$ yields;

$$R_t^{(n)} = E_t R_t^{*(n)} + \Phi_t^{(n)}$$  \hspace{1cm} (3.7)

where;

$$R_t^{*(n)} = \left( \frac{1}{n} \right) \sum_{i=0}^{n-1} r_{t+i}$$  \hspace{1cm} (3.8a)

$$\Phi_t^{(n)} = \left( \frac{1}{n} \right) \sum_{i=1}^{n-1} T_t^{(n-i)}$$  \hspace{1cm} (3.8b)

Equation (3.7) implies that the yield from holding a long bond to maturity equals the expected return from rolling over a series of one period bonds plus the average risk premium on an n-period bond until it matures. The variable $R_t^{*(n)}$ is
referred to as the perfect foresight rate since it is the average of the outturn values for the short rates, \( r_{t+i} \).

Subtracting \( r_t \) from both sides and re-arranging:

\[
S_t^{(n,1)} = E_t S_t^{*(n,1)} + E_t \Phi_t^{(n)}
\]  

(3.9)

where:

\[
S_t^{(n,1)} = R_t^{(n)} - r_t
\]  

(3.10a)

\[
E_t S_t^{*(n,1)} = \sum_{i=1}^{n-1} (1-i/n) E_t \Delta r_{t+i}
\]  

(3.10b)

Equation 3.9 states that the actual spread between an \( n \)-period and one-period rate, equals a weighted average of expected future changes in short rates plus future changes in the average term premium.

The PEH applied spot yields assumes that investors are risk neutral, i.e. they are indifferent to risk and base their investment primarily on expected returns. Given that the PEH assumes risk neutrality, uncertainty regarding returns will not affect investment decisions. The PEH implies that \( \Phi^{(n)} = 0 \), for Equation 3.7 and 3.9, for all \( n \).
However it is certainly plausible that the return on holding period return of a long bond, held over one period, will be uncertain. Given that the price at the end of the period on the long bond will be uncertain, therefore the capital gain will be uncertain. Therefore in the case of long bonds the excess holding period return should take into consideration the compensation for this extra risk, i.e. the risk premium.

3.3 Outline of Part I:

In the first 3 chapters of part I, I focus on tests of the EH using Irish interest rates at the short end of the maturity spectrum. I carry a number of tests using a high quality data set from a major commercial bank. The final chapter focuses on various testable extensions using long maturity bonds. The data set for the long maturity bonds is from a major stockbroking firm. Appendix 1 gives a complete account of the data used in Part I of the thesis.

In chapter 4, I focus on single equation methods of testing the EH. The first part of this chapter will concentrate on the perfect foresight spread equations. As shall be seen, variables in Equation 3.7, namely $R_{t}^{(o)}$ and $r_{t+i}^{m}$ are usually found to be non-stationary whereas $\Delta r_{t+i}^{m}$ and $S_{t}$ in Equation 3.9 are found to be stationary, I(0), variables. Hence econometric tests on Equation 3.9 can be based on standard distributions, whereas those in Equation 3.7 cannot. I will focus on tests of whether the spread predicts future changes in short rates. This chapter will also look at some alternative single equation tests of the EH.

Although the theory suggests that the long-short spread should have some predictive power for the future change in the short rate, ordinary least squares (OLS) estimation of the actual change in the short rate on the lagged yield spread plus a constant results in coefficients which are often the wrong size and sign (Mankiw and Miron, 1986). However, empirical evidence has found that, when the same relationship
is estimated using instrumental variables (IV) to allow for errors in variables or a random error in the term structure relationship, the rejection of the model is less conclusive. We will also estimate this model using generalised methods of moments (GMM) to account for any heteroscedasticity in the error term or serial correlation due to the use of overlapping errors. Finally the model is estimated the other way around, i.e. the yield spread is regressed on the expected future changes in the short rate, where the actual future change in the short rate proxy the expected change. In the final part of chapter 4, I run a number of Monte Carlo experiments in order to verify the fact that the tests based on the regression of the change in the short-term rate on the lagged spread is prone to severe over-rejection of the EH. However I also show that tests of the spread on the first difference of the short-rate reject at the correct rate.

In chapter 5, I will summarise methods of testing the EH under the relatively weak assumptions of a stationary term premium and stationary forecast errors. Given that long rates $R_t$ and short rates $r_t$ are taken to be integrated of order one, I(1), then a weak test of the PEH + RE is that $R_t$ and $r_t$ are cointegrated with a cointegration parameter of unity. We test this hypothesis and compare with previous empirical results. Drawing on some recent empirical evidence, Hall, Anderson and Granger (1992), Shea (1992), and Cuthbertson, Hayes and Nitzsche (1998), I will also use the cointegration approach to test the EH on the entire yield curve. If one has $\gamma$ interest rates which are I(1) then the EH implies that $(\gamma-1)$ linearly independent spreads $S_{t(n,m)}$ are cointegrated. The Johansen procedure (Johansen, 1988, 1989) will be used which can contain two or more interest rates in the variable vector. I can then test whether the number of cointegrating vectors in the system equals $(\gamma-1)$ and then test the joint null that the cointegrating vector complies with the theory. Based on the assumption that the variables will cointegrate, then an error correction model can be used to test the term structure of interest rates.

In chapter 6, I use the Campbell and Shiller (1991) VAR methodology to test the EH, assuming a constant risk premium. The VAR methodology, which is a popular for forecasting economic variables, is purely a forecasting technique, which performs well over short horizons. Campbell and Shiller (1991) have developed a number of
metrics, which can be tested using the VAR analysis, and so is an important tool for our purposes. Using the Campbell-Shiller VAR methodology on data at the short end of the maturity spectrum (i.e. up to one year) Cuthbertson (1996) finds reasonable support for the EH on UK data. To my knowledge the expectations hypothesis (EH) using the VAR approach has not been examined using Irish data. I test parameter restrictions on the VAR models and assess the results in comparison to the previous evidence.

Given the findings in chapter 6 and the recent empirical evidence, chapter 7 will test the EH of the term structure at the long end of the maturity spectrum. Taylor (1992) focusing on longer maturities, 5, 10 and 15 years, finds strongly against the EH (see also MacDonald and Speight 1991). Taylor (1992) noted that the failure of the EH at the long end of the maturity spectrum may be due to the presence of a time varying (yet stationary) risk premium. Based on the results of Taylor (1992), where the excess holding period yield is found to be time varying when using a single equation format, and Cuthbertson and Nitzsche (1998) we modify the standard 2-variable VAR to allow for a time varying risk premium. The main insight in this chapter is the use of the excess holding period return to provide a proxy for a possible time varying term premium. Drawing on Tzavalis and Wickens (1998) and Cuthbertson and Nitzsche (1998) I estimate a 3-variable VAR which incorporates a time varying risk premium.
CHAPTER 4

MODELLING THE EXPECTATIONS HYPOTHESIS USING SINGLE EQUATION METHODS

4.1 INTRODUCTION

In this chapter a number of single equation tests are applied to the Irish short-term interest rate market. From the previous chapter the long rate, under the PEH, can be written as an average of expected future short rates,

\[ R_t^{(n)} = \frac{1}{k} \sum_{i=0}^{k-1} E_t r_{t+i} \quad (4.1) \]

while the long-short spread can be written in terms of a weighted average of expected changes in future short rates;

\[ S_t^{(n,m)} = E_t \sum_{i=1}^{k-1} (1-i/k) \Delta_t^{m} r_{t+i} \quad (4.2) \]

where \( k = n/m \), is an integer.
CHAPTER 4: MODELLING THE EH USING SINGLE EQUATION METHODS

The first part of this chapter will concentrate on the perfect foresight spread equations. As shall be seen, variables in Equation 4.1, namely $R_t(n)$ and $r_{t+i}^m$ are usually found to be non-stationary whereas $\Delta r_{t+i}^m$ and $S_t^{(n,m)}$ in Equation 4.2 are found to be stationary, I(0), variables. Hence econometric tests on Equation 4.2 can be based on standard distributions, whereas those in Equation 4.1 cannot. I will focus on tests of whether the spread predicts future changes in short rates. Section 2 will present the theory and the empirical evidence for the predictive power of the spread, in terms of future changes in the short rates. I will then report the results, in section 4, using a number of maturity combinations for the Irish data set.

This chapter will also look at some alternative single equation tests of the EH. Although the theory suggests that the long-short spread should have some predictive power for the future change in the short rate, ordinary least squares (OLS) estimation of the actual change in the short rate on the lagged yield spread plus a constant, results in coefficients which are often both the wrong size and sign (Mankiw and Miron, 1986). However, empirical evidence has found that, when the same relationship is estimated using instrumental variables (IV) to allow for errors in variables or a random error in the term structure relationship, the rejection of the model is less conclusive. I will also estimate this model using generalised methods of moments (GMM) to account for any heteroscedasticity in the error term or serial correlation due to the use of overlapping errors. The model is also estimated the other way around, i.e. the yield spread is regressed on the expected future changes in the short rate, where the actual future change in the short rate proxy the expected change. Section 3 will give a detailed description of the theory and the empirical evidence on the alternative methods of single equation analysis. The reported results concentrate on the 6 and the 3 month combination and are reported in section 4.

In the final section of the chapter, I present the results from a number of Monte Carlo (MC) experiments. The experiments focus on the fact that the alternative single equation tests based on the regression of the change in the short-term rate on the lagged spread are prone to severe over-rejection of the EH, even when it is true. However tests of the spread on the first difference of the short-rate reject at the correct rate. The results from the MC experiments are reported in section 4.
4.2 The Spread Predicts Future Changes in Short Rates

4.2.1 Theoretical Review

From Equation 4.2, one can see that the spread is an optimal predictor of (weighted average) future changes in short rates. A single equation model of this hypothesis is the perfect foresight spread regression;

\[ S_{t}^{(n,m)} = E_{t} \sum_{i=1}^{k-1} (1 - i/k) \Delta r_{t+i}^{(m)} = E_{t}[PF S_{t}^{(n,m)}] \] (4.3)

Where \( S_{t}^{(n,m)} = (R_{t}^{(n)} - r_{t}^{(m)}) \) is the yield and \( PF S_{t}^{(n,m)} \) is the perfect foresight spread (Campbell and Shiller, 1991). Perfect foresight spread is the spread that would be predicted by agents if they had perfect foresight about future movements in interest rates.

For simplicity lets consider the above equation for \( n = 6, m = 3 \),

\[ S_{t}^{(6,3)} = E_{t}S_{t}^{*(6,3)} \]

where;

\[ S_{t}^{*(6,3)} = (1/2) \Delta^{3} r_{t+3}^{(3)} \]
and $\Delta^m Z_t = Z_t - Z_{t-m}$. The term $S_t^{(6,3)}$ is the perfect foresight spread, the optimal predictor of future changes in short rates. By optimal predictor, I mean that under the expectation hypothesis no variable other than $S_t$ can improve the forecasts of future changes in short rates. If the $S_t^{(6,3)} > 0$, then agents expect that on average short rates should rise.

Turning now to the testable implications of Equation 4.3. A weak test of the EH, would be whether or not the spread Granger causes future changes in short rates\textsuperscript{1}, since from Equation 4.3 the spread is an optimal forecast of future changes in short-term interest rates, conditional on the full information set, $\Omega_t$. If agents have additional information in the prediction of short rates, then this will be reflected in the actual spread. The EH does not make such predictions and so the spread should be an exact linear function of current and future expected changes in short rates.

If I now add the assumption of rational expectations (RE):

\begin{equation}
E_t r^{(m)}_{t+im} = r^{(m)}_{t+im} + \varepsilon_{t+im}
\end{equation}

this gives the 'pure expectations hypothesis plus rational expectations', PEH plus RE;

\begin{equation}
PFS_t^{(n,m)} = S_t^{(n,m)} + \varepsilon_t^*\end{equation}

where $\varepsilon_t^*$ is a moving average error of order $(n-m-1)$ consisting of a weighted sum of future values of $\varepsilon_{t+im}$. Under RE, $\varepsilon_t^*$ is independent of information at time $t$, $\Omega_t$, and

\textsuperscript{1} Strictly, failure of Granger causality does not constitute a rejection of the EH, but a failure to confirm it.
in particular is independent of the yield spread. The fact that $\varepsilon^*_t$ may be heteroscedastic is not ruled out.

A testable extension from Equation (4.6) is the following,

$$PFS_t^{(n,m)} = \alpha + \beta S_t^{(n,m)} + \gamma \Omega + \varepsilon^*_t \quad (4.6)$$

$$H_0: \alpha = \gamma = 0, \beta = 1$$

Testing the null hypothesis of $H_0: \alpha = \gamma = 0$ and $\beta = 1$ implies that the spread is an optimal predictor of future changes in short-term rates. I set up the null such that information at time $t$ or earlier doesn’t incrementally add to the predictions of future interest rates. If there is a constant term premia or if there are differential yet constant transactions costs (between investing ‘long’ and in a series of rolled-over short-term investments) then $\alpha \neq 0$. Under RE the right hand side variables in Equation 4.6 are independent of $\varepsilon^*_t$, and hence does not require IV estimation. However a GMM estimator is employed to correct the covariance matrix for the moving-average error of order $(n-m-1)$ and possible heteroscedasticity (Hansen, 1982; Newey and West, 1987).

### 4.2.2 Empirical Evidence

The study which will be drawn on frequently in part I of the thesis is, Campbell and Shiller (1991). The authors use a wide variety of maturities from 1 to 12 months and for 2, 3, 4 ... 10-years test the EH on monthly data from January 1952 to February 1987. For the perfect foresight spread equations Campbell and Shiller find slope coefficients ranging between 0 and 0.5 for maturities up to 2 years and for maturities greater than 2 years the slope coefficients increase to around 1. Overall they find little
support for the EH at the short end, but do find some support at the long end of the maturity spectrum.

However for my purposes the studies carried out using UK data will prove important for comparison purposes. Hurn, Moody and Muscatelli (1993) also test the term structure of interest rates at the short end of the maturity spectrum using UK LIBOR for 1, 3, 6, and 12 month maturities. In contrast to Cuthbertson (1996a), the authors use monthly data over a longer sample period covering January 1975 to December 1991. The findings from the perfect foresight spread regression are supportive of the EH, with slope coefficients ranging between 0.816 for the S(12,3) months spread and 1.168 for the S(3,1) months spread combination.

Two recent studies give evidence in favour of the EH using UK data, Cuthbertson (1996) and Cuthbertson, Hayes and Nitzsche (1996). Cuthbertson (1996a) uses London Interbank (offer) rates (LIBOR) with maturities of 7 days, 1, 3, 6, and 12 month to test the EH at the short end of the maturity spectrum. The data set is sample weekly beginning on the 2nd Thursday in January 1981 and ending on the 2nd Thursday of February 1992. The perfect foresight spread equations yield evidence in favour of the economic theory. The null $H_0: \beta=1$ (given $\gamma = 0$) is not rejected in all cases except that for the 4-week/1-week spread.

Cuthbertson, Hayes and Nitzsche (1996) using UK Certificates of Deposit (CD) rates for maturities of 1, 3, 6, 9 and 12 months also find evidence in favour of the EH + RE. The data set used in the study is sampled weekly and covers the period from the 1st of October 1975 to 14th of October 1992. In all cases the authors do not reject the null of $H_0: \beta=1$, or that information, available at $t$ or earlier does not incrementally add to the predictions of future interest rates. Therefore the results are consistent with the EH + RE and previous empirical evidence on UK rates, Cuthbertson (1996a).

---

2 Although the Sterling/Irish pound link has been broken, with Ireland joining the ERM in March 1979, the two countries financial systems are still very closely linked, Walsh (1993).

3 The only exception here is the 9 month CD rate which ends on the 27th of January 1988.
4.3 ALTERNATIVE SINGLE EQUATION ANALYSIS

4.3.1 THEORETICAL REVIEW

In this section a number of alternative single equation tests will be described. If I focus on the (6,3) month combination from the last section, under the EH the 6 month rate is equal to an average of current plus expected future 3 month rates plus a term premium.

\[
R_{t}^{(6)} = (1/2)[r_{t}^{(3)} + E_{t}^{r_{t+3}^{(3)}}] + \theta
\]

While the 6 month - 3 month spread is equal to a weighted average of the expected change in future 3 month rates.

\[
S_{t}^{(6,3)} = (1/2)E_{t}^{\Delta^{2}r_{t+3}^{(3)}} + \theta
\]

One can see that an appropriate test would be, the regression of the change in the short rate on the lagged yield spread and a constant (Mankiw and Miron, 1986). Taking the inverse of Equation (4.8) and from theory the short rate should differ from its predicted value only by a forecast error, which would be orthogonal to all information at t or earlier, yields;

\[
\Delta^{2}r_{t+3}^{(3)} = -2\theta + 2S_{t}^{(6,3)} + \eta_{t+3}
\]
where $\eta_{t+3}$ is the forecast error.

The EH implies that the coefficient on $\Delta R_{t+3}$ should be insignificantly different from $1/2$, in Equation 4.8, while the coefficient on $S_t$ in Equation 4.9, should be insignificantly different from 2. Empirical evidence would suggest that the EH does not hold; OLS estimation yields coefficients that are of the wrong magnitude and sometimes the wrong sign (Shiller, 1990).

There are two possible explanations for the empirical rejection of the theory. The first assumes that market expectations are rational but that the information contained in the term structure is affected by non-stationary risk premia. The second explanation assumes that risk premia are stationary, but that market expectations are not strictly rational and so long rates tend to overreact to future short rates. I will focus on the weaker form of the EH where we have a term premium that contains elements which vary randomly over time, independent of the short-term rates. Equation (4.7) is replaced by:

$$R_t^{(6)} = (1/2)[r_t^{(3)} + E_t \Delta r_{t+3}^{(3)}] + \theta + \varepsilon_t$$  \hfill (4.10)

and Equation 4.8 by

$$S_t^{(6,3)} = (1/2)E_t \Delta^3 r_{t+3}^{(3)} + \theta + \varepsilon_t$$  \hfill (4.11)

The $\varepsilon_t$ is a zero-mean random term and is uncorrelated with $r_{t+3}$. The $\varepsilon_t$ may be thought of as representing a time varying term premium or some other error.
Equation 4.9 may be re-written as;

\[ \Delta^3 r_{t+3}^{(3)} = -2\theta + 2S_t^{(6,3)} + \eta_{t+3} + 2\varepsilon_t \]  (4.12)

4.3.2 EMPIRICAL EVIDENCE

Mankiw and Miron (1986) run an OLS regression of the change in the short-run rate \( \Delta R_t \) on the lagged yield spread \( S_{t-1} \) and a constant over a number of different sample sizes. The total sample period runs from 1890 – 1958, however the authors take into account the different monetary regimes over the total sample period. In all the predictive power of the spread is tested over 4 different regimes for the (6m, 3m) maturity combination using quarterly data\(^4\).

\[ (r_{t+1} - r_t) = \alpha + \beta(R_t - r_t) + \nu_{t+1} \]  (4.13)

Initially Mankiw and Miron (1986) test the predictive power of the spread over a relatively recent sample period, 1959 – 1979. The authors find a coefficient on the spread that is insignificantly different from zero and significantly different from the theoretical value of 2. The authors also find an adjusted R-squared of 0.01, which implies that the spread has negligible predictive power. Overall the results are not

\(^4\) The 4 sub-samples for each of the different regimes are the following. 1890Q4 – 1914Q4 which ends with the founding of the Federal Reserve System. 1915Q1 – 1993Q4 ends with both the introduction of the New Deal banking reforms and the approximate end of the gold standard and approximate beginning of interest rate begging. 1934Q1 – 1951Q1 which ends with the Fed no longer pegging interest rates and finally 1951Q2 – 1958Q4 ending at the time when the active market in 3 and 6 month Treasury bills begins.
supportive of the expectations hypothesis and suggest that the slope of the yield curve
does not contain information regarding the future path of the short rate.

The results for the various sub-samples offer some interesting comparisons. For
the sub-samples over the period 1915 – 1958 the results are remarkably similar to the
recent sample and the spread is not significantly different from zero. However for the
sample period 1890 – 1914, the results are in marked contrast and, although the
coefficient on the spread is still statistically different from its theoretical value, it is
three times larger that that for the recent sample. The predictive power is also much
greater, the adjusted R2 is 0.40 compared to 0.01 for the recent sample. Therefore the
sample period 1890-1914 confirm that expectations are an important factor of
fluctuations in the yield curve.

Two recent studies include Sola and Driffill (1994) and Driffill, Psaradakis and
Sola (1997). Sola and Driffill (1994) test the expectations hypothesis for the 3 and 6
month combination using quarterly data for US treasury bills for the sample 1962Q1 –
1987Q3. Given the change in the Federal Reserves operating procedure towards
monetary base control in the late 70’s the authors initially test the theory from 1962Q2
– 1979Q3 and so avoid any possible rejection based on the shift in regime\(^5\). The
authors initially estimate Equation 4.9 using OLS and find the coefficient on the
lagged spread is significantly different from the theoretical value, 2. However as has
been discussed above, the failure of Equation 4.9 may be due to time varying term
premia, fads, measurement errors, or other random deviations from the pure
expectations hypothesis.

The authors re-estimate the model using instrumental variables (IV), using \(S_{t,i},\)
i= 2,\ldots,4, and \(\Delta r_{t+i}, i = 1,\ldots,4\) as instruments. The results do not find evidence against
the weak version of the EH. Sola and Driffill (1994) also estimate Equation 4.12 using

\(^{5}\) There are 2 possible reasons why the theory may be rejected given the switch in regime. There may be
a break in the time series properties of the data given the regime switching. Secondly there may be a
perception of future shifts which may affect the behaviour of market participants, i.e. ‘peso problem’.
S_{ti}, i=1,\ldots,4, \text{ and } \Delta r_{ti}, i = 0,\ldots,4 \text{ as instruments. The coefficient on the change on the short rate is not statistically different from the theoretically correct value of 0.5. The authors conclude that Equation 4.12 is the correct model and there is a measurement or other error in Equation 4.9. In a Monte Carlo study Driffill, Psaradakis and Sola (1998) confirm that the results found in real data are likely to emerge when Equation 4.12 is the correct model. The authors also estimate the models over the full sample period 1962Q2 – 1987Q3 and find unsurprisingly that the theory is rejected when there is no allowance made for the regime switch.

Driffill, Zacharias and Sola (1997) test the expectations hypothesis for UK and US 1 and 3 month combinations using monthly data. The sample period for UK covers the period 1975M2 - 1994M12 and 1982M12 - 1991M2 for the US. The authors also analyse the theory for the UK using a shorter sample period, starting 1982M11, given the possible structural breaks in the preceding period\(^6\). The authors recognise the possible measurement or other error in the relationship between the spread and the expected future change in the short rate and so focus on;

\[
S_t = (2/3)E_t \Delta r_{t+2} + (1/3)E_t \Delta r_{t+4} + \theta + \epsilon_t
\]

Again the authors run the single equation tests using IV, and find that the results are well within the statistically significance level. Also of note is the fact that non-rejection of the theory was found in both samples for the UK.

\(^6\) There were a number of causes for these structural changes; the change in monetary policy regime that accompanied the Medium Term Financial Strategy, and the elimination of all foreign exchange controls.
CHAPTER 4: MODELLING THE EH USING SINGLE EQUATION METHODS

4.4 EMPIRICAL RESULTS

4.4.1 UNIT ROOTS

In order to run the above tests I need to establish the order of integration of the interest rate series. The two main testing procedures used, are the Dickey-Fuller Test and the Phillips-Perron Test\(^7\). Table 4.1 gives the results of tests for a unit root on the individual series, \( R_t \) and the various spreads of the interest rate combinations, \( S_t^{(n,m)} \). The results indicate that I cannot reject the null hypothesis that changes in short rates \( \Delta r_t^{(m)} \) and the yield spread \( S_t^{(n,m)} \) are I(0), i.e. stationary series. As a further indication, I present the plots of each of the individual series and the spread. Figures 4.1 - 4.3 show the path of short rates over the sample period, while 4.4 - 4.6 show the long-short spread.

4.4.2 THE SPREAD AND THE PREDICTABILITY OF CHANGES IN SHORT RATES

The regression of the perfect foresight spread, \( PFS_t^{(n,m)} \) on the actual spread \( S_t^{(n,m)} \) and the limited information set \( H_t \) (consisting of lags of \( S_t^{(n,m)} \) and \( \Delta r_t^{(m)} \)) are shown in table 4.2. Under the null hypothesis of PEH + RE, we expect \( H_0 : \alpha = \gamma = 0, \beta=1 \). The method estimation is GMM with a correction for heteroscedasticity and moving average errors using the Newey-West (1987) declining weights\(^8\).

I first run the model including the information set and test, \( H_2 : \gamma = 0 \). As can be seen from table 4.2, I cannot reject the null and so is consistent with the theory. In all cases I do not reject the null of \( H_0 : \beta=1 \) or that information, available at time \( t \) or earlier does not incrementally add to the predictions of future interest rates. This is the

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\(^7\) Chapter 2 has a detailed account of both stationarity and the various testing procedures.
case for the each of the chosen interest rate maturity combinations, at the short end of the spectrum. The results therefore do not reject the EH + RE.

4.4.3 ALTERNATIVE SINGLE EQUATION TEST RESULTS

I now focus on alternative single equation tests, described earlier in this chapter. I focus on the 6 month and the 3 month interest rate combination, as a comparison with Sola and Driffill (1994). The method of estimation includes IV and GMM. Based on previous empirical evidence, Shiller (1990), OLS estimation yields coefficients that are often both the wrong sign and magnitude. Therefore I focus on the errors in variables, IV estimation and GMM. I also run a Wald test of the restriction implied by the EH.

I first consider Equation 4.8, where the 6 month – 3 month spread (S_t) is regressed on the change (3 month) in the future 3 month short rate (Δr_{t+3}) and a constant. The numbers in parenthesis are the standard errors.

\* A detailed account of the estimation procedure is given in chapter 2.
Regressing $S_t$ on a constant and $\Delta r_{t+3}$, using $S_{t-i}$, $i = 1, \ldots, 4$ and $\Delta r_{t-i}$, $i = 0, \ldots, 4$, as instruments yields:

**Instrumental Variables (IV)**

$$S_t = 0.0005 + 0.37\Delta R_{t+3}$$

$[0.0005] + [0.05]$ 

**Instruments:** $S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}, \Delta r_t, \Delta r_{t-1}, \Delta r_{t-2}, \Delta r_{t-3}, \Delta r_{t-4}, \Delta r_{t-9}, \Delta r_{t-12}$.

Sample Size $= 147$, Standard Error $= 0.005$, $R^2 = 0.31$

$AR[\chi^2(12)] = 45.77$, $HET[\chi^2(1)] = 0.31$

Wald Test of the Expectations Hypothesis Restriction: $\chi^2(1) = 5.41$

I also give a summary of the diagnostic tests for each regression. The AR test is a test for serial correlation up to order 12, while the HET test is a test of unconditional heteroscedasticity. As can be seen from the above equation the estimated coefficient on the change in the short rate appears to be different from the theoretically correct value. The Wald test result rejects the restriction implied by the EH. As has already been discussed these results have been taken as evidence against the EH.

As has already been discussed earlier in the chapter, $\varepsilon^*_t$ may have a moving average error and be possibly heteroscedastic. As a result I also estimate the model using a GMM estimator to correct the covariance matrix for the moving-average error of order $(n-m-1)$ and possible heteroscedasticity (Hansen, 1982; Newey and West, 1987).
CHAPTER 4: MODELLING THE EH USING SINGLE EQUATION METHODS

Generalised Method of Moments (GMM)

\[ S_t = 0.001 + 0.25\Delta r_{t-3} \]

\[ [0.0004] + [0.07] \]

Instruments: \( S_{t-1}, S_{t-2}, S_{t-3}, S_{t-4}, \Delta r_t, \Delta r_{t-3}, \Delta r_{t-6}, \Delta r_{t-9}, \Delta r_{t-12} \).

Sample Size = 147, Standard Error = 0.006, \( R^2 = 0.31 \)
\( \text{AR}[\chi^2(12)] = 92.64, \text{HET}[\chi^2(1)] = 41.11 \)
Wald Test of the Expectations Hypothesis Restriction: \( \chi^2(1) = 4.08 \)

Overall the results are quite similar to those using the IV estimation. As can be seen from both the regression results and the Wald test, the estimated coefficient on the change in the short rate appears to be different from the theoretically correct value.

I also estimate Equation 4.9, by regressing the change (3 month) in the 3 month short rate (\( \Delta r_t \)) on the 3 month lagged spread (\( S_{t-3} \)). Again, I estimate using both IV and GMM estimation.

Instrumental Variables (IV)

\[ \Delta r_t = -0.001 + 3.02S_{t-3} \]

\[ [0.001] + [0.71] \]

Instruments: \( \Delta r_{t-1}, \Delta r_{t-2}, \Delta r_{t-3}, S_{t-6}, S_{t-9}, S_{t-12} \).

Sample Size = 150, Standard Error = 0.01, \( R^2 = 0.31 \)
\( \text{AR}[\chi^2(12)] = 87.23, \text{HET}[\chi^2(1)] = 2.91 \)
Wald Test of the Expectations Hypothesis Restriction: \( \chi^2(1) = 293.18 \)
As can be seen from the above regression results, the Wald test overwhelmingly rejects the restriction implied by the EH. I also report the results with a correction for possible heteroscedasticity and moving average error.

**Generalised Method of Moments (GMM)**

\[
\Delta r_t = -0.002 + 2.25S_{t-3} \\
[0.001] + [0.47]
\]

**Instruments:** \(\Delta r_{t-1}, \Delta r_{t-2}, \Delta r_{t-3}, S_{t-6}, S_{t-9}, S_{t-12}\).

Sample Size = 150, Standard Error = 0.01, \(R^2 = 0.31\)

\(AR[\chi^2(12)] = 92.64, HET[\chi^2(1)] = 41.11\)

Wald Test of the Expectations Hypothesis Restriction: \(\chi^2(1) = 298.30\)

Overall the results are quite similar to those using the IV estimation. As can be seen from both the regression results and the Wald test, the estimated coefficient on lagged spread is significantly different from the theoretically correct value.

**4.5 Monte Carlo Experiments**

In the previous section of this chapter, I used two alternative methods to test the weak version of the EH, considering time varying term premium, using single equation estimation. I now focus on the findings that the tests based on the regression of the change in the short-term rate on the lagged spread is prone to severe over-rejection of the EH. However tests of the spread on the first difference of the short-rate reject at the correct rate. I will use Monte Carlo (MC) experiments to show this point\(^9\).

\(^9\) The GAUSS programme used in this section of the thesis is shown in Appendix 2.
This fact has been briefly discussed in the section dealing with the empirical results on these alternative single equation tests and can be seen at first hand from the reported results in the previous section. The procedure used will be to estimate both methods of the single equation tests for 3 alternative samples: \( T = 100, 200 \) and \( 500 \). A 1000 series of these regressions will be produced. I set up the experiments based on the Irish data set already analysed here. In the experiments I take the long rate as the 2 period rate (6 month), and the short rate as the 1 period rate (3 month). The generating process is determined by the following equations;

\[
R_t = (1/2)[r_t + E_t r_{t+1}] + \theta + \sigma_t \varepsilon_t
\]

\[
r_{t+1} = \mu + \sum_{i=0}^{2} b_i r_{t-i} + \sigma_u u_{t+1}
\]

As has been mentioned the simulations are carried out based on the previously used Irish data set. Based on the data set, \( \theta = 0.0077, \mu = 0.0088, b1 = 1.11, b2 = 0.47, b3 = 0.24, \sigma_e = 0.011, \sigma_u = 0.012 \). As has been mentioned 1000 series of regressions will be generated, and the pseudo-random deviates \( u_t \) and \( \varepsilon_t \) will be obtained using the RNDN function in GAUSS. As a direct comparison with the previous section, estimation will be by IV and GMM.

**Model 1: The DS Test**

\[ \Delta r_t = \alpha_1 + \beta_1 S_{t-1} + \epsilon_{1t} \]

**Instruments:** \( S_{t-2}, S_{t-3}, S_{t-4} \)

\( \beta_1 = 2 \)

**Model 2: The SD Test**

\[ S_t = \alpha_2 + \beta_2 \Delta r_{t-1} + \epsilon_{2t} \]

**Instruments:** \( \Delta r_t, \Delta r_{t-1}, \Delta r_{t-2} \).

\( \beta_2 = 0.5 \)

---

10 The sample size will be \( T + 50 \), in each replication. Then the first 50 data points will be dropped in
In table 4.3, I present the results for the mean bias for models. As can be clearly seen from the table of results, model 1 is prone to a large amount of over-rejection of the EH, even when it is true. This is the case for both the IV and the GMM estimator. As can be seen the bias for model 1 continues to be sizeable, even with a larger sample size. On the other hand, model 2's bias is much smaller and is not significant in any of the cases. In table 4.4, I report the findings for the test rejection frequencies for the 2 models. Again the results are consistent with that in table 4.3. The results are based on both the t test and the Wald test at the 5% significance level, that the beta value is equal to its theoretical value. As can be seen the fraction that reject for model 1, is much greater than that for model 2. Therefore, model 1 rejects the EH even when it is true. The results reported in this section are consistent with those reported in a similar study by Driffill, Psaradakis and Sola (1998), which uses MC experiments.

4.6 Conclusions

The first part of this chapter concentrated on the perfect foresight spread equations. The unit root tests on the Irish interest rate series at the short end of the maturity spectrum are non-stationary, but integrated of order one. We have focused on tests of whether the spread predicts future changes in short rates at a number of interest rate maturities. In all cases we do not reject the null of $H_0: \beta=1$ or that information, available at time t or earlier does not incrementally add to the predictions of future interest rates. This is the case for the each of the chosen interest rate maturity combinations, at the short end of the spectrum. The results therefore do not reject the EH + RE.
This chapter has also looked at some alternative single equation tests of the EH. Focusing in particular on the 6 month and 3 month maturities, I initially test the model using OLS. However given the previous evidence, that OLS estimation of the actual change in the short rate on the lagged yield spread plus a constant, results in coefficients which are often both the wrong size and sign (Mankiw and Miron, 1986), I also use IV and GMM estimation.

Finally, in section 5 I report the results for the MC experiments which show that the single equation tests, reported earlier in section 4, based on the regression of the change in the short-term rate on the lagged spread are prone to severe over-rejection of the EH. However the tests of the spread on the first difference of the short-rate reject at the correct rate. These findings are consistent with those from Driffill, Psaradkis and Sola (1998) using US data.
Table 4.1:
Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Maturity</th>
<th>ADF</th>
<th>PP-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Interest Rate: ( R_t^{(n)} )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td></td>
<td>-2.64</td>
<td>-2.62</td>
</tr>
<tr>
<td>3 month</td>
<td></td>
<td>-2.35</td>
<td>-2.20</td>
</tr>
<tr>
<td>6 month</td>
<td></td>
<td>-2.20</td>
<td>-1.82</td>
</tr>
<tr>
<td><strong>Change in interest rate: ( \Delta R_t^{(n)} )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 month</td>
<td></td>
<td>-6.56</td>
<td>-11.49</td>
</tr>
<tr>
<td>3 month</td>
<td></td>
<td>-6.42</td>
<td>-11.49</td>
</tr>
<tr>
<td>6 month</td>
<td></td>
<td>-5.74</td>
<td>-12.07</td>
</tr>
<tr>
<td><strong>Spread: ( S_t^{(n,m)} )</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,1) month</td>
<td></td>
<td>-4.49</td>
<td>-6.33</td>
</tr>
<tr>
<td>(6,1) month</td>
<td></td>
<td>-4.40</td>
<td>-5.31</td>
</tr>
<tr>
<td>(6,3) month</td>
<td></td>
<td>-4.32</td>
<td>-4.89</td>
</tr>
</tbody>
</table>

Notes:
The sample period is from January 1984 to October 1997. ADF(5) is the augmented Dickey-Fuller statistic with 5 lags, which ensures the regressions are free of serial correlation. 'PP' is the Phillips-Perron (1988) statistic with correction for up to 5th order serial correlation. The critical value for both test statistics is -2.86 at the 5% significance level.
Table 4.2:
Does the Spread Predict Future Changes in Short-Rates?

Regression: \( PFS_t^{(n,m)} = \alpha + \beta S_t^{(n,m)} + \gamma \Omega_t \)

<table>
<thead>
<tr>
<th>Lead (n,m)</th>
<th>Coefficients</th>
<th>Wald</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha ) s.e.( ( \alpha ) )</td>
<td>( \beta ) s.e.( ( \beta ) )</td>
<td>H(_0) : ( \beta = 1 ) [p-value]</td>
</tr>
<tr>
<td>(6,3) Month</td>
<td>-0.0006 (0.0008)</td>
<td>1.02 (0.23)</td>
<td>0.007 [0.93]</td>
</tr>
<tr>
<td>(3,1) Month</td>
<td>-0.001 (0.001)</td>
<td>0.87 (0.21)</td>
<td>0.38 [0.54]</td>
</tr>
<tr>
<td>(6,1) Month</td>
<td>-0.001 (0.001)</td>
<td>1.04 (0.15)</td>
<td>0.09 [0.77]</td>
</tr>
</tbody>
</table>

Notes:
The regression coefficients reported in columns 2 and 3 are from the regression with \( \gamma = 0 \) imposed. The method of estimation is GMM with a correction for heteroscedasticity and moving average errors using the Newey-West (1987) declining weights. The last 3 columns report Wald statistics and marginal significance levels for the null hypothesis stated. For \( H_0 : \gamma = 0 \) the reported results are for an information set which includes 4 lags of the change in the interest rates and the interest rate spread. The null \( H_0 : \beta = 1 \), is conditional on \( \gamma = 0 \) while the null \( H_1 : \alpha = 0, \beta = 1 \) is also conditional on \( \gamma = 0 \).
Table 4.3:
Monte Carlo Bias

<table>
<thead>
<tr>
<th>T</th>
<th>Mean Bias of the IV Estimators</th>
<th>Mean Bias of the GMM Estimators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1.896 (0.0396)</td>
<td>-0.0370 (0.0709)</td>
</tr>
<tr>
<td>200</td>
<td>-1.844 (0.0398)</td>
<td>-0.0172 (0.0702)</td>
</tr>
<tr>
<td>500</td>
<td>-1.830 (0.0336)</td>
<td>-0.0095 (0.0667)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>-1.888 (0.0363)</td>
<td>-0.0356 (0.069)</td>
</tr>
<tr>
<td>200</td>
<td>-1.868 (0.0338)</td>
<td>-0.0197 (0.0675)</td>
</tr>
<tr>
<td>500</td>
<td>-1.785 (0.0365)</td>
<td>-0.0119 (0.0669)</td>
</tr>
</tbody>
</table>

Notes:
The reported results give the mean bias of the slope estimators and its Monte Carlo standard error in parenthesis.
Table 4.4:
Test Rejection Frequencies:

<table>
<thead>
<tr>
<th>T</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.540</td>
<td>0.063</td>
</tr>
<tr>
<td>200</td>
<td>0.548</td>
<td>0.054</td>
</tr>
<tr>
<td>500</td>
<td>0.524</td>
<td>0.051</td>
</tr>
</tbody>
</table>

Test Rejection Frequencies - GMM Estimators (Wald-Test)

<table>
<thead>
<tr>
<th>T</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.580</td>
<td>0.069</td>
</tr>
<tr>
<td>200</td>
<td>0.575</td>
<td>0.065</td>
</tr>
<tr>
<td>500</td>
<td>0.540</td>
<td>0.058</td>
</tr>
</tbody>
</table>

Notes:
The reported results give the fraction of MC replications that fail the t-test and the Wald test at the 5% significance level. Both the t-test and the Wald test, test whether the value for the beta for both model 1 and model 2 are equal to the theoretical value.
Figure 4.1: Irish 1 Month Money Market Rate
Figure 4.2: Irish 3 Month Money Market Rate
Figure 4.3: Irish 6 Month Money Market Rates
Figure 4.4: 3 Month - 1 Month Interest Rate Spread
Figure 4.5: 6 Month - 1 Month Interest Rate Spread
CHAPTER 5

MODELLING THE EXPECTATIONS HYPOTHESIS USING THE COINTEGRATION APPROACH

5.1 INTRODUCTION

In this chapter I summarise methods of testing the EH under the relatively weak assumptions of a stationary term premium and stationary forecast errors. Traditional methods only allow the estimation of the EH for the bivariate cases, i.e. a long-term rate with a short-term 'safe' asset. Based on the standard approach it is not possible to focus on the relationship of all the interest rates on the whole yield curve. Hall, Anderson and Granger (1992), Shea (1992), and Cuthbertson, Hayes and Nitzsche (1998) use the cointegration approach to test the EH on the entire yield curve. The Johansen procedure (Johansen, 1988) will be used which can contain two or more interest rates in the variable vector. Based on the assumption that the variables will cointegrate, then an error correction model can be used to test the term structure of interest rates.

Assuming the yields are integrated of order one, I(1), under the cointegration approach, the EH implies that yield variables should be spanned by yield spreads. A high quality data set using Irish short-term interest rates will be used to test the above hypothesis. Given that I use pure discount bonds, I avoid the approximations used in extracting spot rates from the yield curve, (see Barr and Pesaran (1994) and McCulloch (1990) or to use an approximation to the EH based on yields to maturity (see Shiller (1979), Shiller, Campbell and Schoenholtz (1983)).

---

1 The term premium in principle needs to be constant in order to confirm with the EH.
2 I can therefore focus on the 1 month and 3 month as well as the 3 month and the 6 month.
Using a number of cointegration test methods, e.g. Phillips-Hansen and Johansen, and error correction models, I find that in general the results are supportive of the EH. The chapter is organised as follows. In section 5.2 I outline the theoretical models which form the basis of the cointegration and ECM approach. The results from previous empirical studies are analysed in section 5.3. The results using Irish short-term interest rates are provided in section 5.4. Section 5.5 concludes the chapter.

5.2 Theoretical Models: Cointegration & ECM's

For any two yields $R_t^{(n)}$ and $r_t^{(m)}$ where $n > m$, the EH implies that the spread $S_t^{(n,m)}$ is the optimal predictor of future changes in yields. Using the usual logarithmic approximation and by assuming that expectations are formed rationally the 'fundamental term structure relationship' can be represented by:

$$ S_t^{(n,m)} = E_t \sum_{i=1}^{k-1} (1-i/k) \Delta^{m/r^{(m)}} r_{t+i+m} $$

Which holds for all $(n,m)$ where $k (\leq n/m)$ is an integer$^3$. Given that I expect $(R_t^{(n)}, r_t^{(m)})$ to be I(1), then $\Delta r_t^{(m)}$ is I(0), which would imply that the spread $S_t^{(n,m)}$ should also be I(0) from Equation 5.1, and therefore $(R_t^{(n)}, r_t^{(m)})$ should be co-integrated with a co-integrating parameter of unity$^4$. If I have $\gamma$ - interest rates which are integrated of order one, I(1), then Equation 5.1 implies that a set of $(\gamma-1)$ should span the cointegrating space and so will have rank $\gamma-1$. The Johansen approach will be used to test for the rank of the system$^5$. Given Equation 5.1 the EH suggests that any $m$-period yield that is cointegrated, will have cointegrating vectors of the form $(-1,1,0,...,0), (-1,0,1,0,...,0)$ etc. and the spreads will form a basis for the cointegrating space.

---

$^3$ This representation of the EH is only valid for pure discount bonds.

$^4$ Strictly, for this to hold, forecast errors and any term premia must also be I(0).
Under the EH the null hypothesis states

$$\beta = H\phi$$  \hspace{1cm} (5.2)

where $\phi$ is the estimated matrix of the cointegrating vectors.

Under the EH the null hypothesis states that the cointegrating vectors spans the cointegrating space;

where $H$, the $(y \times y-1)$ restriction matrix contains the restrictions of the EH.

$$H = \begin{bmatrix} -1 & -1 & -1 & \cdots & -1 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \hspace{1cm} (5.3)$$

The null of the EH posits that the cointegrating vectors should span the cointegrating space. The $H$ matrix above is the $(y \times y-1)$ restriction matrix which contains the EH restrictions. The approach which will be used is the maximum likelihood of Johansen (1988) and Johansen and Juselius (1990). Given that $y-1$

---

5 A detailed account of the Johansen approach has been given in chapter 2.
cointegrating vectors exist, a likelihood ratio test (LR) will be used in order to test the cointegrating vectors. Both the Phillips-Hansen (1990) fully efficient estimator (which gives asymptotically valid standard errors) and the Johansen (1988) approach will be used to carry out the appropriate tests.

An extension to the cointegration analysis, when testing the EH across different maturities, is to estimate a system of ECM equations. Engle and Granger (1987) posit that if rates are integrated of order one, I(1), and there is a stochastic link, they are cointegrated, then they can be expressed in an ECM. Therefore some of the γ-1 yield spreads should enter the ECM which explains the change in the set of interest rates. The set-up of the vector error correction model (VECM) should be of the following form:

\[
\begin{bmatrix}
\Delta r_t^{(1)} \\
\Delta r_t^{(2)} \\
\end{bmatrix} = \begin{bmatrix}
c_1 \\
c_2 \\
\end{bmatrix} + \begin{bmatrix}
a_1(L) & b_1(L) \\
a_2(L) & b_2(L) \\
\end{bmatrix} \begin{bmatrix}
\Delta r_{t-1}^{(1)} \\
\Delta r_{t-1}^{(2)} \\
\end{bmatrix} - \begin{bmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22} \\
\end{bmatrix} \begin{bmatrix}
S_{t-1}^{(2,1)} \\
S_{t-1}^{(3,1)} \\
\end{bmatrix} + \begin{bmatrix}
u_{1t} \\
u_{2t} \\
\end{bmatrix} (5.4)
\]

This shows the ECM equation. One can impose the long-run relationship, as is the case in Equation 5.4, where the cointegrating vector is imposed on the yield spread, \((S^{(m)})_{t-1}\). Alternatively I can estimate the ECM, without the restrictions imposed, \((R^{(o)}) - \beta_r^{(m)})_{t-1}\).

---

5 The ECM approach is not inconsistent with agents being forward looking since the spread vectors, under the EH, are optimal predictors and hence should Granger-cause changes in interest rates.
5.3 Empirical Evidence

The expectations hypothesis (EH) of the term structure implies that the yield spread between the long rate and short rate is an optimal predictor of future changes in short rates over the life of the 'long bond'.

There are a growing number of papers which use the cointegration-ECM framework to test the EH plus RE for the entire yield curve. The empirical evidence based on tests of the above restrictions is mixed. In table 5.1 – 5.3, I summarise the findings of Shea (1992), Hall, Anderson and Granger (1992) and Cuthbertson (1996). Both Shea (1992) and Hall, Anderson and Granger (1992) use monthly US rates. The data used by Shea is from McCulloch (1990) and covers the period January 1952 to February 1987 on zero coupon bonds with a maturity of 1 month to 25 years. Hall, Anderson and Granger (1992) focus on the short-end of the maturity spectrum using monthly data from March 1970 to December 1988 with a maturity of 1 to 12 months. The EH implies that a set of $\gamma - 1$ yield spreads should span the cointegration space which comprises of the level of the $\gamma$ yield variables.

A summary of Shea (1992) is the following:

- the number of cointegrating vectors increases as the number of interest rates, included in the vector, rises
- as the number of interest rates in the vector increases, the number of common stochastic factors tends to rise as well (this result holds for different sub-periods)

The latter result violates the EH and is in contrast to the results of Hall, Anderson and Granger, who analyse the long-run relationship between 11 interest rates and find that the number of cointegrating vectors in the system is equal to $l-1 (=10)$.

---

Shea (1992) also constructs quarterly data series, in order to test the relationship between 3 month holding period yields on different long-term bonds and the 3 month short-term interest rate.
The tests of the restrictions based on the EH, the cointegrating vector is represented by Equation 5.2, are also reported in table 5.1-5.3. In order to test Equation 5.2 the spreads must form a basis for the cointegration space. Both Shea (1992) and Hall, Anderson and Granger (1992) find mixed results when testing the null hypothesis that the cointegrating vector is defined by Equation 5.2. As can be seen from the tables both studies reject the null. As can be seen from the Hall, Anderson and Granger (1992) study, when the tests are repeated on sample sub-sets (e.g. less than 10 restrictions holding jointly) the results improve. The authors also estimate the models over smaller sample periods and a smaller system. Using 1 month, 2 month, 3 month and 4 month interest rates over the sample period October 1979 to September 1982 the authors find only 1 or 2 cointegrating vectors, i.e. 3 or 4 common trends. Although rates still cointegrated, the cointegrating relationship is no longer defined by the spread. A possible explanation, is that as a result of volatility in money growth, interest rates, and economic activity over this period the term premium has become non-stationary. A non-stationary term premium would lead to a break down in the cointegrating relationship stated in Equation 5.2.

The authors also estimate an ECM and find that the error correction terms are statistically significant in each of the equations. Therefore supporting the hypothesis that cointegration exists among the various interest rates. Hall, Anderson and Granger (1992) also find that more interest rate spreads are required to explain the change in short rates at shorter maturities. The author's comment that this may be due to more information being contained in the current interest rate spreads at the long end of the maturity spectrum and current short rates adjust instantaneously.

The McCulloch (1990) data on pure discount bonds is also used by Engsted and Tanggaard (1994a). Using maturities of up to 10 years, the author's find more evidence in favour of the EH, compared to the Shea (1992) results. Engsted and Tanggaard (1994a) find that the restriction imposed, by the EH, cannot be rejected in any 2 variable system. When a 3 and a 4 variable system are considered, they find similar results, once the interest rate vector does not contain the 1 month rate. The

---

8 Hall, Anderson and Granger (1992) find that when the Federal Reserve Bank targets interest rates as it's instrument of monetary policy, the tests generally support the theory.
author's find consistent evidence, with the results of Hall, Anderson and Granger (1992), of a breakdown in the cointegrating relationship between long and short term interest rates during the period of monetary base control.

Using monthly data for Denmark Engsted and Tanggaard (1994b) carry out tests of the EH for the period January 1976 to December 1991 using data from Denmark. The authors also focus on two sub-periods, January 1976 to July 1985 and from August 1985 to December 1991. The author's use zero coupon bonds for maturities of 1, 3 and 6 months and 1, 2, 5, 10 and 15 years. The first period was determined by monetary base control, which means that interest rates were highly volatile. The central bank switched to a policy of interest rate targeting for the second period. The authors find for both the full sample and the two sub-periods that the number of cointegrating vectors is in fact γ-1. However, tests of the restriction imposed by the Equation 5.2 are consistently rejected at the 5% significance for the whole sample period, except for the 1 and 3 month combination. The authors conclude that their results find more support for the two sub-periods. This is especially so from the 2 and 3 variable systems and in particular from the maturities at the short end, which do not exceed 1 year. Engsted and Tanggaard (1994b) argue that the improved results from the sub-periods are due to the fundamentally different policy regimes.

For the UK, Cuthbertson (1996) uses London Interbank (offer) rates (LIBOR) with maturities of 7 days, 1, 3, 6, and 12 month to test the EH at the short end of the maturity spectrum. The data set is sample weekly beginning on the 2nd Thursday in January 1981 and ending on the 2nd Thursday of February 1992. Overall the results are consistent with the EH. The author tests the restrictions using both the likelihood ratio (LR) statistic and the wald test. The wald test is used given the possible presence of heteroscedastic errors, where the covariance matrix is corrected for possible heteroscedasticity of unknown form (White, 1980)\textsuperscript{10}. For each of the maturity

\textsuperscript{9} It should be noted that Shea (1992) included long maturity bonds (i.e. 25 years) as well as short and medium term maturity bonds.

\textsuperscript{10} Tests based on the Johansen procedure assume serially uncorrelated and homoscedastic errors. Given that I use the Johansen procedure to test the number of cointegrating vectors, implicitly assuming white noise errors, and then run a wald test based on the presence of heteroscedastic errors is an inconsistency. However, these test statistics are to be viewed as indicative. Also of note is the fact that to date the Johansen tests are not available under the assumption of heteroscedastic disturbances.
combinations the LR test results are consistent with the theory, the exception being when the 6 and the 12 month yields are included as a pair in the multivariate tests. The Wald test results are supportive of the LR results, except for the bivariate 6 and 12 month combination.

5.4 EMPIRICAL RESULTS

5.4.1 UNIT ROOT TEST RESULTS

Table 5.4 gives the results of tests for a unit root on the individual series $R_t$ and $S_t^{(n,m)}$. Both the augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) test results are reported. The results indicate that I cannot reject the null hypothesis that changes in short rates $\Delta r_t^{(m)}$ and the yield spread $S_t^{(n,m)}$ are I(0), i.e. stationary series. These results indicate that the spread is a stationary process which represents weak evidence of a stable long term equilibrium relationship between the short and the long interest rate.

5.4.2 CO-INTEGRATION ANALYSIS

Table 5.5 and 5.6 show the bivariate cointegration results. Table 5.5 reports the results using the standard OLS estimation, while table 5.6 adopts the Philips-Hansen approach. Both tables show the cointegration results of the regression $R_t^{(n)}$ on $r_t^{(m)}$ and vice versa. Hall (1986) argues that the point estimates from the of regression $R_t^{(n)}$ on $r_t^{(m)}$ and vice versa should ‘band’ the true cointegrating vector. Given this rather

---

11 The bivariate case of the 6 and 12 month is supportive of the EH.
12 See chapter 2 for a detailed discussion on the approaches to testing for cointegration.
crude rule of thumb, I can accept that all cointegrating vectors are cointegrated with a cointegrating vector of (-1,1). This result provides weak evidence in favour of the EH under the assumption of a constant or stationary term premium and any expectation scheme that yields I(0) forecast errors.

However as can be seen from table 5.5 and 5.6, the Wald test of the restriction that a cointegrating vector of (-1,1) exists, is rejected for each case. Therefore although the estimated β’s would appear to be close to the theoretical value of unity, statistical tests clearly reject this.

I now turn to tests based on the Johansen (1988) procedure. The EH of the term structure implies that the number of cointegrating vectors in a system containing γ interest rates should be γ-1. I focus on both the bivariate cases, but also the multivariate case. As can be seen from table 5.7, the Johansen procedure indicates that the rank of the cointegrating space is indeed γ-1, i.e. 1 cointegrating vector for the case of the two interest rate combinations and 2 for the case of the three interest rate combination13. I initially focus on the bivariate results. The Johansen results, shown in table 5.7, provide strong evidence that $R_t^{(n)}$ and $r_t^{(m)}$ are co-integrated and from table 5.8, I cannot reject the hypothesis that the co-integrating vector is given by the theoretical value (-1,1). The (normalised) point estimates for the co-integrating vectors from the Johansen procedure are (-1, 0.99) for each case. This is backed up by the test restrictions results, where I cannot reject that the co-integrating spread vector is of the form (-1, 1).

As can be seen from table 5.7, the Johansen procedure indicates that the rank of the cointegrating space is γ-1 for the multivariate case, 2 cointegrating vectors. Both the maximum eigenvalue test and trace test statistics for γ≤2 versus γ=3 is 3.34 with a critical value of 9.16 (at the 5% significance level). As with the bivariate case I test the cointegrating spread vector complies with the theory. The joint test of the restrictions is the following form, (-1, 1, 0) and (-1, 0, 1). As can be seen from table 5.8, I cannot reject the restriction for the multivariate case.

13 Both the Akaike information criterion (1973) and the Schwartz Bayesian criterion (1978) are used to choose the appropriate lag length of the VAR model.
I also run a number of impulse response functions for the multivariate system based on a shock to each of the variables. The horizon for the impulse response is 50 months. As can be seen from figures 5.1 to 5.3 the effect of a unit shock has only a small effect on the interest rates and they converge within 10 months.

Table 5.9 reports the results for the ECM equations. The lag for the general ECM is set to two, which correspond to the lag length of 4 for the Johansen VAR model. The diagnostic tests on the ECM are also reported and there appears to be no sign of mis-specification. The error correction term's (ECT) are statistically significant, which is consistent with the finding of cointegration. Finally, the sign on the ECT is negative, indicating that it is in fact an equilibrium correcting term.

5.5 CONCLUSIONS

The EH of the term structure of interest rates under the weak assumptions of an I(0) term premium and I(0) forecast errors, implies that given \( y \) yields then \((y-1)\) yield spreads should form the basis of the cointegrating space. This hypothesis is tested using short-term interest rate data from Ireland. Overall the results are favourable. Using a number of different estimation techniques, the data would appear to be consistent with the theory. The bivariate testing approaches adopted were OLS and Phillips-Hansen. The cointegration results of the regression \( R_t(n) \) on \( r_t(m) \) and vice versa are banded, which provides weak evidence in favour of the EH.

Based on the Johansen VAR methodology and given that if I have \( y \) interest rates, I find that the rank of the cointegrating space is \((y-1)\). When testing the hypothesis that the cointegrating parameters estimates are of the form \((-1, 1, 0, ...\)), I

---

14 The exception is ECM, for the 2nd and 3rd equation.
15 In chapter 7 the VAR methodology will be augmented to incorporate the possibility of a stationary time varying term premia for long maturity data. However, cointegration analysis of the returns will still yield superconsistent estimates of the long-run relationships, even if the term premia is omitted (Tzavalis and Wickens, 1998).
find in favour of the theory. This is true for both the bivariate and multivariate equations. The results for the ECM are also supportive of the EH. The ECT’s are statistically significant, which is consistent with the finding of cointegration earlier in the chapter. The reported results would therefore appear to be consistent with the EH.
Table 5.1
Summary of Shea's (1992) Cointegration Results: US Data

<table>
<thead>
<tr>
<th>Interest Rate Vector</th>
<th>Data Frequency</th>
<th>Sample Period</th>
<th>Spread Restriction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Rejected</td>
</tr>
<tr>
<td>1M, 6M</td>
<td>MONTHLY</td>
<td>Jan. 52 – Dec. 78</td>
<td>X</td>
</tr>
<tr>
<td>1M, 10Y</td>
<td></td>
<td>Jan. 52 – Dec. 78</td>
<td>X</td>
</tr>
<tr>
<td>1M, 10Y</td>
<td></td>
<td>Jan. 52 – Feb. 87</td>
<td>X</td>
</tr>
<tr>
<td>1M, 5Y, 10Y</td>
<td></td>
<td>Jan. 52 – Dec. 78</td>
<td>X</td>
</tr>
<tr>
<td>1M, 5Y, 10Y</td>
<td></td>
<td>Jan. 52 – Feb. 87</td>
<td>X</td>
</tr>
<tr>
<td>1M, 3Y, 5Y, 10Y</td>
<td></td>
<td>Jan. 52 – Dec. 78</td>
<td>X</td>
</tr>
<tr>
<td>1M, 3Y, 5Y, 10Y</td>
<td></td>
<td>Jan. 52 – Feb. 87</td>
<td>X</td>
</tr>
<tr>
<td>1M, 5Y, 10Y, 15Y</td>
<td></td>
<td>Jan. 52 – Apr. 70</td>
<td>X</td>
</tr>
<tr>
<td>1M, 5Y, 10Y, 15Y</td>
<td></td>
<td>Jan. 73 – Feb. 87</td>
<td>X</td>
</tr>
<tr>
<td>1M, 5Y, 10Y, 15Y, 20Y</td>
<td></td>
<td>Jan. 53 – Feb. 87</td>
<td>X</td>
</tr>
<tr>
<td>1M, 5Y, 10Y, 15Y, 20Y</td>
<td>QUARTERLY</td>
<td>May 73 – Feb. 87</td>
<td>X</td>
</tr>
<tr>
<td>3M, 5Y, 10Y, 15Y</td>
<td>QUARTERLY</td>
<td>52Q(1) – 70Q(1)</td>
<td>X</td>
</tr>
<tr>
<td>3M, 5Y, 10Y, 15Y</td>
<td>QUARTERLY</td>
<td>53Q(1) – 70Q(1)</td>
<td>X</td>
</tr>
<tr>
<td>3M, 5Y, 10Y, 15Y, 25Y</td>
<td>QUARTERLY</td>
<td>55Q(1) – 67Q(3)</td>
<td>X</td>
</tr>
</tbody>
</table>

Note: M in column 1 refers to the maturity of interest rates in months, while Y denotes years. The tests of the spread restrictions, in column 4 and 5 are conducted for the 5% significance level. Data used in Shea's study is the McCulloch (1990) data set.

Source: Shea (1992)
Table 5.2
Hall, Anderson, and Granger’s (1992) Results
Tests of the EH using cointegration analysis

i) Single spread restrictions

<table>
<thead>
<tr>
<th>Interest Rate Vector</th>
<th>LR Statistic</th>
<th>Interest Rate Vector</th>
<th>LR Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1M, 2M</td>
<td>4.39</td>
<td>2M, 3M</td>
<td>6.95</td>
</tr>
<tr>
<td>1M, 3M</td>
<td>6.56</td>
<td>3M, 4M</td>
<td>0.0</td>
</tr>
<tr>
<td>1M, 4M</td>
<td>4.36</td>
<td>4M, 5M</td>
<td>0.26</td>
</tr>
<tr>
<td>1M, 5M</td>
<td>2.48</td>
<td>5M, 6M</td>
<td>0.73</td>
</tr>
<tr>
<td>1M, 6M, 7M</td>
<td>1.54</td>
<td>6M, 7M</td>
<td>5.49</td>
</tr>
<tr>
<td>1M, 7M</td>
<td>0.65</td>
<td>7M, 8M</td>
<td>2.51</td>
</tr>
<tr>
<td>1M, 8M</td>
<td>0.34</td>
<td>8M, 9M</td>
<td>0.72</td>
</tr>
<tr>
<td>1M, 9M</td>
<td>0.21</td>
<td>9M, 10M</td>
<td>1.15</td>
</tr>
<tr>
<td>1M, 10M</td>
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<td>10M, 11M</td>
<td>4.57</td>
</tr>
<tr>
<td>1M, 11M</td>
<td>0.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii) Interest Rate Vector LR Statistic Critical Value

<table>
<thead>
<tr>
<th>Interest Rate Vector</th>
<th>LR Statistic</th>
<th>Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1M, 2M, 3M</td>
<td>7.27</td>
<td>5.99</td>
</tr>
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<td>1M, 2M, 3M, 4M</td>
<td>13.31</td>
<td>7.81</td>
</tr>
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<td>1M, 2M, 3M, 4M, 5M</td>
<td>14.34</td>
<td>9.49</td>
</tr>
<tr>
<td>1M, 2M, 3M, 4M, 5M, 6M</td>
<td>14.34</td>
<td>11.07</td>
</tr>
<tr>
<td>1M, 2M, 3M, 4M, 5M, 6M, 7M</td>
<td>21.96</td>
<td>12.59</td>
</tr>
<tr>
<td>1M, 2M, 3M, 4M, 5M, 6M, 7M, 8M</td>
<td>21.96</td>
<td>14.07</td>
</tr>
<tr>
<td>1M, 2M, 3M, 4M, 5M, 6M, 7M, 8M, 9M</td>
<td>21.97</td>
<td>15.51</td>
</tr>
<tr>
<td>1M, 2M, 3M, 4M, 5M, 6M, 7M, 8M, 9M, 10M</td>
<td>22.58</td>
<td>16.92</td>
</tr>
</tbody>
</table>

Notes:
M refers to monthly maturity in the interest rate vector. In part i) the LR test statistics are $\chi^2$ distributed with one degree of freedom. The critical value at 5% significance level is 3.84. In part ii) the critical values reported in column 3 are for the 5% significance level.

Data used in the Hall, Anderson, and Granger (1992) is taken from the ‘Fama Twelve Month Treasury Bill Term Structure File’

Source: Hall, Anderson and Granger (1992)
### Table 5.3
Cuthbertson’s (1996a) Cointegration Results
UK Inter-bank Rates

<table>
<thead>
<tr>
<th>Interest Rates</th>
<th>1W</th>
<th>1M</th>
<th>3M</th>
<th>6M</th>
<th>12M</th>
<th>LR Test</th>
<th>Wald Test</th>
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<tr>
<td><strong>Single Spread Restrictions</strong></td>
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<td></td>
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</tr>
<tr>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
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<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>0.08</td>
<td>0.15</td>
</tr>
<tr>
<td>X</td>
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<td></td>
<td></td>
<td>0.08</td>
<td>0.14</td>
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<td>5.99*</td>
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<td>X</td>
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<td>4.59</td>
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<td>X</td>
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<td>20.30*</td>
<td>43.48*</td>
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<td>29.17*</td>
<td>72.96*</td>
</tr>
<tr>
<td><strong>Four Spread Restrictions</strong></td>
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<td>X</td>
<td>X</td>
<td>29.17*</td>
<td>72.86*</td>
</tr>
</tbody>
</table>

Notes:
W refers to interest rates of weekly maturity and M refers to monthly maturity. LR is the likelihood ratio test, while the Wald test uses a variance-covariance matrix corrected for heteroscedasticity (White, 1980).

Source: Cuthbertson (1996a)
<table>
<thead>
<tr>
<th>Variable</th>
<th>Maturity</th>
<th>ADF</th>
<th>PP-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate: $R_t^{(n)}$</td>
<td>1 month</td>
<td>-2.64</td>
<td>-2.62</td>
</tr>
<tr>
<td></td>
<td>3 month</td>
<td>-2.35</td>
<td>-2.20</td>
</tr>
<tr>
<td></td>
<td>6 month</td>
<td>-2.20</td>
<td>-1.82</td>
</tr>
<tr>
<td>Change in interest rate: $\Delta R_t^{(n)}$</td>
<td>1 month</td>
<td>-6.56</td>
<td>-11.49</td>
</tr>
<tr>
<td></td>
<td>3 month</td>
<td>-6.42</td>
<td>-11.49</td>
</tr>
<tr>
<td></td>
<td>6 month</td>
<td>-5.74</td>
<td>-12.07</td>
</tr>
<tr>
<td>Spread: $S_t^{(n,m)}$</td>
<td>(3,1) month</td>
<td>-4.49</td>
<td>-6.33</td>
</tr>
<tr>
<td></td>
<td>(6,1) month</td>
<td>-4.40</td>
<td>-5.31</td>
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<tr>
<td></td>
<td>(6,3) month</td>
<td>-4.32</td>
<td>-4.89</td>
</tr>
</tbody>
</table>

Notes:
The sample period is from January 1984 to October 1997. ADF(5) is the augmented Dickey-Fuller statistic with 5 lags, which ensures the regressions are free of serial correlation. 'PP' is the Phillips-Perron (1988) statistic with correction for up to 5th order serial correlation. The critical value for both test statistics is -2.86 at the 5% significance level.
### Table 5.5
Engle-Granger Cointegration Analysis

\[ R_t(n) = \alpha + \beta R_t(m) + \varepsilon_t \]

<table>
<thead>
<tr>
<th>Term to Maturity</th>
<th>Coefficients</th>
<th>Hypothesis Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha )</td>
<td>Wald Test ( H_0: \beta = 1 )</td>
</tr>
<tr>
<td>Dependent Variable</td>
<td>(Standard Error)</td>
<td>(Standard Error)</td>
</tr>
<tr>
<td>1 month</td>
<td>3 month</td>
<td>-0.00005</td>
</tr>
<tr>
<td>3 month</td>
<td>1 month</td>
<td>0.00007</td>
</tr>
<tr>
<td>1 month</td>
<td>6 month</td>
<td>-0.00008</td>
</tr>
<tr>
<td>6 month</td>
<td>1 month</td>
<td>0.00016</td>
</tr>
<tr>
<td>3 month</td>
<td>6 month</td>
<td>-0.00005</td>
</tr>
<tr>
<td>6 month</td>
<td>3 month</td>
<td>-0.00007</td>
</tr>
</tbody>
</table>

Notes:
The Wald statistic in column 5 tests the hypothesis which is imposed by the EH. It is the \( \chi^2 \) distribution one degrees of freedom (number of restrictions). The critical value for a \( \chi^2 \) distributed test with one degree of freedom is 3.84 (at the 5% level). In column 6, the cointegrating test, tests the residuals from the Phillips-Hansen estimation for a Unit root. The critical value for the ADF statistic (at the 5% level) is -2.88. The critical values can be found in MacKinnon (1991).
Table 5.6
Phillips-Hansen Cointegration Analysis

\[ R_t^{(n)} = \alpha + \beta R_t^{(m)} + \epsilon_t \]

<table>
<thead>
<tr>
<th>Term to Maturity</th>
<th>Coefficients</th>
<th>Hypothesis</th>
<th>Wald Test</th>
<th>Cointegration Test (ADF-stats)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(\alpha) (Standard Error)</td>
<td>(\beta) (Standard Error)</td>
<td>(H_0: \beta = 1)</td>
</tr>
<tr>
<td>1 month</td>
<td>3 month</td>
<td>-0.000052 (0.000014)</td>
<td>1.05 (0.015)</td>
<td>9.77</td>
</tr>
<tr>
<td>3 month</td>
<td>1 month</td>
<td>0.000072 (0.000013)</td>
<td>0.93 (0.012)</td>
<td>28.60</td>
</tr>
<tr>
<td>1 month</td>
<td>6 month</td>
<td>-0.000085 (0.000003)</td>
<td>1.08 (0.03)</td>
<td>8.62</td>
</tr>
<tr>
<td>6 month</td>
<td>1 month</td>
<td>0.00013 (0.00003)</td>
<td>0.85 (0.03)</td>
<td>52.52</td>
</tr>
<tr>
<td>3 month</td>
<td>6 month</td>
<td>-0.000052 (0.000012)</td>
<td>1.05 (0.012)</td>
<td>19.07</td>
</tr>
<tr>
<td>6 month</td>
<td>3 month</td>
<td>-0.000007 (0.000011)</td>
<td>0.93 (0.011)</td>
<td>41.91</td>
</tr>
</tbody>
</table>

Notes:
The Wald statistic in column 5 tests the hypothesis which is imposed by the EH. It is the \(\chi^2\) distribution one degrees of freedom (number of restrictions). The critical value for a \(\chi^2\) distributed test with one degree of freedom is 3.84 (at the 5% level). In column 6, the cointegrating test, tests the residuals from the Phillips-Hansen estimation for a Unit root. The critical value for the ADF statistic (at the 5% level) is -2.88. The critical values can be found in MacKinnon (1991).
### Table 5.7
Johansen Procedure: Testing the Number of Cointegrating Vectors

<table>
<thead>
<tr>
<th>Interest Rate Combinations</th>
<th>Maximum $r = 0$ (critical value)</th>
<th>Eigenvalue $r \leq 1$ (critical value)</th>
<th>Test $r \leq 2$ (critical value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,3) Month</td>
<td>33.63* (15.87)</td>
<td>5.61</td>
<td></td>
</tr>
<tr>
<td>(3,6) Month</td>
<td>32.99* (15.87)</td>
<td>3.81</td>
<td></td>
</tr>
<tr>
<td>(1,6) Month</td>
<td>36.05* (15.87)</td>
<td>3.83</td>
<td></td>
</tr>
<tr>
<td>(1,3,6) Month</td>
<td>41.55* (22.04)</td>
<td>30.60* (15.87)</td>
<td>3.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Interest Rate Combinations</th>
<th>Maximum $r = 0$</th>
<th>Eigenvalue $r \leq 1$</th>
<th>Test $r \leq 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,3) Month</td>
<td>39.24* (20.18)</td>
<td>5.61</td>
<td>(9.16)</td>
</tr>
<tr>
<td>(3,6) Month</td>
<td>36.80* (20.18)</td>
<td>3.81</td>
<td>(9.16)</td>
</tr>
<tr>
<td>(1,6) Month</td>
<td>39.88* (20.18)</td>
<td>3.83</td>
<td>(9.16)</td>
</tr>
<tr>
<td>(1,3,6) Month</td>
<td>75.49* (34.87)</td>
<td>33.94* (20.18)</td>
<td>3.34</td>
</tr>
</tbody>
</table>

**Notes:**
The Johansen procedure is estimated for the *non-trended case*, no trend in the cointegrating relationship and no trend in the DGP. The lag length is set to 3 for all cases except the (1,6) month combination, where the lag length is equal to 2. The Schwartz criterion is used to choose the appropriate lag length.
Table 5.8: Bivariate and Multivariate Johansen Procedure

<table>
<thead>
<tr>
<th>Interest rates (n,m)</th>
<th>Lag length</th>
<th>Cointegrating Vector</th>
<th>LR Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,3) Months</td>
<td>3</td>
<td>1</td>
<td>(-1, 0.99)</td>
</tr>
<tr>
<td>(3,6) Months</td>
<td>3</td>
<td>1</td>
<td>(-1, 0.99)</td>
</tr>
<tr>
<td>(1,6) Months</td>
<td>2</td>
<td>1</td>
<td>(-1, 0.99)</td>
</tr>
<tr>
<td>(1,3,6) Months</td>
<td>3</td>
<td>2</td>
<td>(-1, 1.84, -0.86)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-1, 3.44, -2.47)</td>
</tr>
</tbody>
</table>

Notes: In the Johansen procedure both the maximum eigenvalue test and the trace test do not reject the null of γ-1 co-integrating vectors. The likelihood ratio (LR) statistic in column 4 tests the null that the co-integrating vector(s) is (-1,1) for the bivariate case and is (-1, 1, 0) and (-1, 0, 1) for the multivariate case. Under the null, the reported test statistics have a critical values (at 5% significance level) of 3.84 for $\chi^2(1)$ and 5.99 for $\chi^2(2)$. 
## Table 5.9
Error Correction Models

### ECM Model (dependent variables)

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>$\Delta r_{1t}$</th>
<th>$\Delta r_{3t}$</th>
<th>$\Delta r_{6t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta r_{1t-1}$</td>
<td>0.16</td>
<td>-0.03</td>
<td>-0.21</td>
</tr>
<tr>
<td>$\Delta r_{1t-2}$</td>
<td>-0.36</td>
<td>-0.15</td>
<td>-0.10</td>
</tr>
<tr>
<td>$\Delta r_{3t-1}$</td>
<td>0.41</td>
<td>0.60</td>
<td>0.92</td>
</tr>
<tr>
<td>$\Delta r_{3t-2}$</td>
<td>1.35</td>
<td>0.78</td>
<td>0.61</td>
</tr>
<tr>
<td>$\Delta r_{6t-1}$</td>
<td>-0.57</td>
<td>-0.57</td>
<td>-0.75</td>
</tr>
<tr>
<td>$\Delta r_{6t-2}$</td>
<td>-0.85</td>
<td>-0.47</td>
<td>-0.41</td>
</tr>
<tr>
<td>ECM1&lt;sub&gt;t-1&lt;/sub&gt;</td>
<td>-0.04*</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>ECM2&lt;sub&gt;t-2&lt;/sub&gt;</td>
<td>-0.04*</td>
<td>-0.04*</td>
<td>-0.03*</td>
</tr>
</tbody>
</table>

### Diagnostic Statistics

<table>
<thead>
<tr>
<th>Type</th>
<th>$\Delta r_{1t}$</th>
<th>$\Delta r_{3t}$</th>
<th>$\Delta r_{6t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.30</td>
<td>0.20</td>
<td>0.13</td>
</tr>
<tr>
<td>Serial Correlation</td>
<td>$\chi^2(12)$</td>
<td>$\chi^2(12)$</td>
<td>$\chi^2(12)$</td>
</tr>
<tr>
<td>Heteroscedasticity</td>
<td>2.74</td>
<td>1.78</td>
<td>1.70</td>
</tr>
</tbody>
</table>

Notes:

* denotes significance at the 5% level.
Figure 5.1: Generalised Impulse Response to a one Standard Error Shock in the Equation for R1
Figure 5.2: Generalised Impulse Response to a one Standard Error Shock in the Equation for R3
Figure 5.3: Generalised Impulse Response to a one Standard Error Shock in the Equation for R6
CHAPTER 6

THE VAR APPROACH TO TESTING THE EXPECTATIONS HYPOTHESIS WITH A CONSTANT RISK PREMIUM

6.1 INTRODUCTION

In this chapter I use the Campbell and Shiller (1991) VAR methodology to test the EH, assuming a constant risk premium. The VAR methodology, which is a popular for forecasting economic variables, is purely a forecasting technique, which performs well over short horizons. Campbell and Shiller (1991) have developed a number of metrics, which can be tested using the VAR analysis, and so is an important tool for my purposes. All variables entering the VAR system must be stationary\(^1\).

There is a great deal of evidence on the EH based on US data. The Campbell and Shiller (1991) VAR methodology has become an important tool to analyse the EH. In general, for a wide variety of maturities from 1 to 12 months and for 2,3,4 \ldots\ 10-year maturities, Campbell and Shiller (1991) reject the EH. The (long-short) interest rate spread does not predict the direction of changes in the long-term interest rate that is consistent with the EH, and future changes in short-rates are not often closely correlated with the long-short spread (Campbell and Shiller, 1991).

As can be seen from the previous chapter there is broad support for the EH using long-run tests, e.g. cointegration analysis. This is true for both the US (Hall, et al., 1992) and the UK (Cuthbertson, 1996a). However there is much less support using

\(^1\) See chapter 2 for a detailed account of this issue.
short-run tests (Tzavalis and Wickens, 1998). In this chapter I will test the EH using the VAR methodology under the assumption of a constant term premium. In the next section I will focus on the theoretical model and the various testable implications. In section 6.3 I will analyse the empirical evidence using the VAR methodology for a number of different data sets. The empirical results for Irish spot rates at the short end of the maturity spectrum will be reported in section 6.4. This section will also interpret the results in terms of the previously discussed empirical evidence. Section 6.5 will summarise the chapter's findings.

6.2 THEORETICAL MODEL: VAR METHODOLOGY

6.2.1 DERIVATION OF THE VAR APPROACH

If $Z_t = (S_t^{(n,m)}, \Delta r_t^{(m)})$ is stationary, then there exists a bivariate Wold representation (Hannan, 1970), which implies that the bivariate process has a unique infinite order moving average representation.
CHAPTER 6: THE VAR APPROACH WITH A CONSTANT TERM PREMIUM

This can be approximated by a vector autoregression (VAR) model of the appropriate lag length\(^2\):

\[
\begin{bmatrix}
S_t^{(n,m)} \\
\Delta r_t^{(m)}
\end{bmatrix}
= \begin{bmatrix}
a(L) & b(L) \\
c(L) & d(L)
\end{bmatrix}
\begin{bmatrix}
S_{t-1}^{(n,m)} \\
\Delta r_{t-1}^{(m)}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\tag{6.1}
\]

Where \(a(L), b(L), c(L),\) and \(d(L)\) are \(p^{th}\) order scale polynomials in the lag operator. While the error term \(\varepsilon_t\) are white noise error such that;

\[
E\left(\varepsilon_i \varepsilon_i^{'}
\right) = \begin{cases}
\sigma_i^2 & \text{for } i = 1, 2 \\
0 & \text{for } j \neq 0
\end{cases}
\tag{6.2}
\]

Where \(\sigma_i\) denotes the constant error standard deviation. The VAR can now be used to forecast future \(\Delta r_t^m\). The spread variable will also be included in the VAR, which itself is the optimal forecast of future changes in short rates.

\(^2\) The appropriate lag length should be chosen so that there is no serial correlation in the residuals of the VAR. The AIC and the SBC will be used to test for the appropriate lag. Extra lags will be added in case of serial correlation.
The VAR can be written as a first order system,

\[
\begin{bmatrix}
S_t^{(n,m)} \\
S_{t-1}^{(n,m)} \\
\vdots \\
S_{t-p+1}^{(n,m)} \\
\Delta r_t^{(m)} \\
\Delta r_{t-1}^{(m)} \\
\vdots \\
\Delta r_{t-p+1}^{(m)}
\end{bmatrix}
= 
\begin{bmatrix}
a_1 & a_2 & \cdots & a_{p-1} & a_p & b_1 & b_2 & \cdots & b_{p-1} & b_p \\
1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 & 0
\end{bmatrix}
\begin{bmatrix}
S_{t-1}^{(n,m)} \\
S_{t-2}^{(n,m)} \\
\vdots \\
S_{t-p}^{(n,m)} \\
\Delta r_{t-p}^{(m)} \\
\Delta r_{t-2}^{(m)} \\
\vdots \\
\Delta r_{t-p+1}^{(m)}
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\tag{6.3}
\]

which in companion form is:

\[
Z_t = AZ_{t-1} + \nu_t
\tag{6.4}
\]

and where the A matrix represents the companion matrix of the VAR. The vector \(Z_t\) contains the variables of the VAR, which are the long-short interest rate spread and the change in the short rate.
Forecasting future values of $Z_t$, conditional on the limited information set $\Omega_t$ can be achieved by:

$$E_t(z_{t+j}|H_t) = A'z_t \tag{6.5}$$

For bivariate case, where $Z_t = (S_t^{(n,m)}, \Delta r_t^{(m)})$, this can be written as:

$$E_t(S_{t+j}^{(n,m)}|H_t) = e1'Az_t \tag{6.6}$$

and

$$E_t(\Delta r_{t+j}^{(m)}|H_t) = e2'Az_t \tag{6.7}$$

where $e1$ and $e2$ are $2p \times 1$ selection vectors with unity in the first and second rows, respectively and zeros everywhere (Campbell and Shiller, 1991). Projecting the yield spread at time $t$ ($S_t^{(n,m)}$) and the change in short-term interest rates at time $t$ ($\Delta r_t^{(m)}$) on the restricted information set $H_t$ ($H_t \subset \Omega_t$), gives for $j = 0$:

$$e1'z_t = S_t^{(n,m)} \tag{6.8}$$

and

$^3$ The limited information set consists of current and lagged values of long and short-term interest rates.
\[ e^{2'}z_t = \Delta r_t^{(m)} \]  

(6.9)

The above equations assume that agents form their expectations rationally. The key to this forecasting process is the fact that Equations 6.6 and 6.7 represent the weakly RE predictions of \( S_t^{(n,m)} \) and \( \Delta r_t^{(m)} \) respectively, since only the limited information set \( H_t \), which is based on publicly available information at time \( t-i \) (where \( i = 0, 1, ... \)) is used.

In order to derive the cross equation restrictions for a finite horizon, \( n \) and \( m \) are expressed as spot rates, which requires the following identity:

\[ \Delta m_t^{(m)} = \sum_{j=q}^{im} \Delta r_t^{(m)} \]  

(6.10)

where \( q = m(i-1) \). Using Equation 6.7 and Equation 6.10, one obtains:

\[ E_t \Delta m_t^{(m)} = e^{2'} \sum_{i=1}^{k-1} \left( 1 - \frac{i}{k} \right) \sum_{j=q}^{im} A^j z_t \]  

(6.11)

Substituting Equation 6.11 into the EH formula and using Equation 6.9 leads to the following:

\[ e^{l'}z_t = S_t^{(n,m)} = e^{2'} \sum_{i=1}^{k-1} \left( 1 - \frac{i}{k} \right) \sum_{j=q}^{im} A^j z_t \]  

(6.12)
Following some tedious algebra the right hand side (RHS) of Equation 6.12 can be written more succinctly and the VAR parameter restrictions for the finite horizon follow directly;

\[ e_1' = e_2' A \left[ I - (m/n)(I - A^n)(I - A^m)^{-1} \right] (I - A)^{-1} \]  

(6.13)

The above cross equation restriction will be used to test the EH using the VAR approach. The restriction is stated in matrix notation where \( n \) and \( m \) are scalars, denoting the time to maturity of the long and short term asset respectively, and \( I \) represent the identity matrix.

### 6.2.2 Testable Restrictions

There are a number of metrics to test the EH from Campbell and Shiller (1991). Campbell and Shiller use the VAR restrictions to construct the theoretical spread; the optimal forecast of future changes in short-term interest rates, given the limited information set, \( H_t \). The theoretical spread can be defined as;

\[ S_t^{(n,m)} = e_2' A \left[ I - (m/n)(I - A^n)(I - A^m)^{-1} \right] (I - A)^{-1} Z_t \]  

(6.14)
Using the VAR methodology there are a number of approaches to test the EH under weakly RE;

i) **Formal test of the VAR restriction**:  

Under the EH with a constant risk premium, the cross equation parameter restriction should hold. This non-linear parameter restriction can be tested by a Wald test. The VAR restrictions ensure that the expected profits, conditional on the limited information set \( H_t \), are zero. Under the RE assumption the errors in forecasting future changes in short rates are independent of the information set, \( H_t \), at time \( t \) or earlier, i.e. the orthogonality condition.

ii) **Graphical comparison of \( S_t^{(n,m)} \) and \( S_t^{(n,m)'} \)**

Movements over time of the actual spread and the theoretical spread should be very close, since, under the EH, only sampling errors are allowed in the VAR framework. Large differences between the actual spread and the theoretical spread would suggest a rejection of the theory.

iii) **Volatility ratio test statistic of the theoretical and actual spread**

\[
- \frac{\sigma(S_t^{(n,m)'})}{\sigma(S_t^{(n,m)})}
\]

iv) **Correlation Coefficient of the actual and theoretical spread**

\[
- (\text{Corr}(S_t^{(n,m)}, S_t^{(n,m)'}))
\]

The final two tests follow direct from Campbell and Shiller (1991), where the standard deviation and correlation of the actual and theoretical spread are compared.
Under the null hypothesis of the EH, the volatility test ratio and the correlation coefficient are equal to one. ⁵ If the EH plus weakly rational expectations holds then this should be reflected in the standard deviation and the correlation being close to unity (and the graphs of the 2 series moving together). If $\sigma(S_t^{(n,m)}) > \sigma(S_t^{(n,m)'})$ then there is 'excess volatility', that is the actual spread is more volatile than the optimal predictor of future short rates.

The VAR methodology has a number of advantages over single equation regressions for tests involving multi-period expectations. In order to test the theory, all that is required is to estimate the unrestricted coefficients in the VAR (Cuthbertson, 1996b). The Wald test on the parameter estimates of the VAR can be formulated for the general case for any $n$ (long maturity) and $m$ (short maturity), for which $k = n/m$, is an integer⁶. Using Monte Carlo experiments Hodrick (1992) has shown that the VAR based tests have better small sample properties, than the single equation testing procedure. Finally, Campbell and Shiller (1991) have emphasized the advantages of the VAR in exploring the sources of the deviations from the theory.

### 6.3 VAR Empirical Evidence

There is a great deal of evidence on the EH for the US based on the Campbell and Shiller (1991) VAR methodology for monthly data on spot rates. Using a wide variety of maturities from 1 to 12 months and for 2, 3, 4 ... 10-years they test the EH on monthly data from January 1952 to February 1987. Overall they find little support for the EH at the short end, but do find some support at the long end. Campbell and Shiller calculate the standard deviation ratio test and the correlation between the actual and the theoretical spread. Over the complete sample period the point estimates are relatively

---

⁴ The Wald tests will be used to test the non-linear VAR restrictions. The VAR estimation is carried out using a GMM correction on the variance covariance matrix.

⁵ The standard errors of $\sigma(S/S')$ and Corr$(S,S')$ are non-linear functions of the estimated A matrix from the VAR and can be computed as $[f(\gamma)' \Psi f(\gamma)]$ where $f(\gamma)$ are the statistics of interest and $\Psi$ is the (GMM) corrected variance-covariance matrix of the parameters $\gamma$.

⁶ This point is apparent from the GAUSS programme used to carry out the empirical in section 6.4. The programme, complete with procedures, is reported in Appendix 3.
low, not exceeding 0.68, while the correlation test results are more supportive with point estimates close to one. Shea (1992) using a similar data set to Campbell and Shiller (1991) carries out a formal test of the cross parameter restrictions. Shea (1992) finds that the EH holds at the long end of the maturity spectrum, but not at the short end, a similar finding to Campbell and Shiller.

Evidence using UK data is generally supportive of the EH. Cuthbertson (1996a) uses London Interbank (offer) rates (LIBOR) with maturities of 7 days, 1, 3, 6, and 12 month to test the EH at the short end of the maturity spectrum. The data set is sample weekly beginning on the 2\textsuperscript{nd} Thursday in January 1981 and ending on the 2\textsuperscript{nd} Thursday of February 1992. Cuthbertson finds evidence in favour of the EH using the Campbell-Shiller VAR methodology. The results are in marked contrast to the US findings by Campbell and Shiller (1991) who reject the EH at the short end. For all maturities there is a strong correlation between the actual and theoretical spread, however the standard deviation test ratios are more than two standard deviations from unity in 3 out of 8 cases. The Wald test results of the VAR restrictions are rejected in four out of the eight cases. In contrast to the formal VAR tests, the perfect foresight spread equations yield evidence in favour of the economic theory. In all cases except that for the 4-week/1-week spread Cuthbertson finds support of the EH + RE.

These findings are also in contrast to those of Campbell and Shiller (1991) for the US at the short end of the maturity spectrum. Cuthbertson (1996a) explains the contrasting results from the perfect foresight spread and the cross equation parameter VAR restriction, by the fact that different information sets are used for the two approaches to form expectations of future short term interest rates. The perfect foresight spread regression allows potential future events to influence expectations, whereas this is not possible in the VAR methodology.

Hurn, Moody and Muscatelli (1995) also test the term structure of interest rates at the short end of the maturity spectrum using UK LIBOR for 1, 3, 6, and 12 month maturities. In contrast to Cuthbertson (1996a), the authors use monthly data over a longer sample period covering January 1975 to December 1991. The authors find evidence in favour of the EH and the VAR restrictions cannot be rejected for any of
the maturity combinations, which is in contrast to the results from Cuthbertson\textsuperscript{7}. The findings from the perfect foresight spread regression are supportive of the EH, with slope coefficients ranging between 0.816 for the 12 and 3 months spread and 1.168 for the 3 and 1 month spread combination.

Finally, there have also been a number of studies using data other from other European countries. Kugler (1988) using US, German and Swiss monthly data on one and three month Euromarket deposit rates found support for the EH only on German data (for the period of March 1974 to August 1986). Similarly, Engsted (1994) using Danish money market rates and Engsted and Tanggaard (1993) for longer maturity bonds find considerable support for the EH when the variation in interest rates is relatively large, such as in the post-1992 ERM 'crisis period'. Using zero coupon bonds for 1 month to 15 years maturity and for a sample period January 1976 to December 1991, they find that the various correlation coefficients between the spread and the theoretical spread are very close to unity. The point estimates of the standard deviation test ratio's are also supportive of the EH. However the authors do not report the standard errors of the standard deviation tests and so it is not possible to say whether the ratios are statistically within a confidence interval of one. The authors also analyse two sub-samples, the first January 1976 to July 1985, a period during which the Danish central bank targeted monetary policy. The second sub-sample from August 1985 to December 1991, is characterised by controlling short-term interest rates.

\textsuperscript{7} Hurn, Moody and Muscatelli (1995) use non-overlapping data and test the EH for the following interest rate combinations of (n,m): S(3,1), S(12,1), S(6,3), S(12,6) and S(12,3) months.
6.4 Empirical Results:

6.4.1 The Data

The data set, which will be used in this chapter, consists of Irish money market rates (spot rates) with a term to maturity of 1, 3, and 6 months. These rates were kindly provided by a leading commercial bank in Ireland, Bank of Ireland from screen-quoted rates. The data set is from January 1984 to October 1997. It is from the same data set that has been analysed in the previous 2 chapters. The cointegration analysis, from the previous chapter, offers strong support for the EH in the long-run. Given that I have γ interest rates, I consistently found γ-1 cointegrating vectors. The data also satisfies the restrictions predicted by the EH. Although the interest rates move together in the 'long-run' there is also substantial movements in the interest rate spreads, as can seen in figures 6.1 to 6.3. The GAUSS programme used in the next section is given in Appendix 3.

6.4.2 The Theoretical Spread and the VAR Results

Given the tests carried out in previous chapters, the results indicate that the null hypothesis that changes in short-term interest rates (Δr_t) and the yield spread (S_t^{(n,m)}) are integrated of order zero, I(0).

Table 6.1 contains the results from the VAR models for S_t^{(n,m)} and Δr_t^{(m)}. The EH is tested on the following interest rate combinations S(6,3), S(6,1), S(3,1). The lag length is chosen to minimise the AIC and SBC, except for the rare occasions when additional lags are required to avoid any serial correlation in the residual. A weak test of the EH is that the spread Granger-causes changes in short-term interest rates and this is not rejected for all maturities (table 6.1, column 3). There is also Granger-
causality from $\Delta r_i^{(m)}$ to $S_t^{(n,m)}$ for the (6m,3m) case, indicating feedback in the VAR regression.

i) **Formal test of the VAR restriction:**

The Wald test results are reported in table 6.3. As can be seen the results are accepted in all cases, the only exception being the (3 month,1 month) case. The mixed Wald tests results are consistent with the results from previous studies. However a detailed explanation of the possible reasons for the rejection of the Wald test will be given in section 6.5.

ii) **Graphical comparison of $S_t^{(n,m)}$ and $S_t^{(n,m)}$'**

For illustrative purposes the graph of the actual spread $S_t$ and the theoretical spread $S_t'$ are shown for the interest rate combinations in figure 6.4 to 6.6. As can be seen there is a very close relationship for each of the interest rate combinations. The results of the regression of $S_t$ on $S_t'$ are also reported, table 6.2. Under the EH, the point estimates should be unity. The empirical results show that the point estimates of the slope coefficients are very close to unity. The intercept in these regressions are not statistically significant in each case.

iii) **Volatility ratio test statistic of the theoretical and actual spread**

\[
\sigma(S_t^{(n,m)}')/\sigma(S_t^{(n,m)})
\]

---

8 The VAR regression parameters and hence the Wald tests of the restrictions are not necessarily invariant to the frequency of the data set employed or on the way the non-linear restrictions are 'set-up' (Gregory and Veall, 1985). Also evidence in Shea (1992), based on the simulated par-bond yield data, suggests that the Wald test reject the too often when the EH is true. If the latter conclusion is applied to the term structure based on pure discount rates, then this may account for the rejection of the VAR restrictions.
v) Correlation Coefficient of the actual and theoretical spread

\[-(\text{Corr}(S_t^{(n,m)}, S_t^{(n,m)^{\prime}}))\]

Table 6.3 provides the metrics for the relationship between the actual spread $S_t$ and the theoretical spread $S'_t$. The results indicate that the VAR restrictions are not rejected. For all maturities there is a strong correlation (column 4) between the actual spread $S_t^{(n,m)}$ and the predicted (theoretical) spread $S_t^{(n,m)^{\prime}}$. The standard deviation ratio, $VR = \sigma(S_t^{(n,m)^{\prime}})/\sigma(S_t^{(n,m)})$ yields estimates (column 3) which are all within two standard errors of unity.

6.5 Interpretation of Results

In this section I analyse the results and compare them with those from other studies. On balance the results favour the EH. The standard deviation ratios $\sigma(S_t^{(n,m)^{\prime}})/\sigma(S_t^{(n,m)})$, and the coefficient of determination $\text{Corr}(S_t^{(n,m)}, S_t^{(n,m)^{\prime}})$ (table 6.3), are consistent with the EH and this may be contrasted with the rejection of the VAR cross-equation restrictions for the (3m,1m) combination. Therefore on balance the results would appear to support the EH, but how do we interpret the rejection of the VAR restrictions.

- Firstly, the VAR approach requires the explicit information set to be known by both agents and the econometrician. Hence, if the econometrician erroneously excludes variables affecting traders' perceptions, then the estimated VAR coefficients may be biased, resulting in rejection of the VAR cross-equation restrictions.

- Secondly, if agents actually do use the VAR methodology for forecasting, one would expect them to utilise almost minute by minute observations of $(S_t^{(n,m)}, \Delta t^{(m)})$; hence forecasts based on monthly data seem unlikely to adequately mimic such behaviour.
- Thirdly, Campbell and Shiller (1991) have argued that rejection of the cross-equation parameter restrictions although statistically significant may not constitute a major departure from the EH on economic grounds, as long as the theoretical spread closely tracks the actual spread. As can be clearly seen from figures 6.3 – 6.5 table 6.3 the actual spread and the theoretical spread move closely together over time.

- Finally, agents may use alternative (non-regression) forecasting schemes (e.g. Chartists, see Allen and Taylor, 1989), in which case the VAR methodology breaks down.

On balance the reported results would appear to support the EH. Campbell and Shiller find that the Corr($S_t^{(n,m)}$, $S_t^{(n,m)'}$) are relatively low being in the range 0-0.7 and the values the variance ratio are in the range 2-10 for maturities of less than 1 year. The authors do not directly test the VAR cross-equation restrictions but this has been done subsequently by Shea (1992) who in general finds they are rejected. The results reported in this chapter are consistent with those of Cuthbertson (1996a) using UK data at short maturities. Cuthbertson’s results from the VAR models for $S_t^{(n,m)}$ and $\Delta R_t^{(m)}$ indicate that $S_t^{(n,m)}$ Granger causes $\Delta R_t^{(m)}$: a weak test of the PEH. The author also finds that for all maturities there are strong correlations between the actual and theoretical spread, and that the variance ratio’s are close to unity.

---

9 For example, suppose theory suggests an elasticity of unity between 2 variables and the estimated equation is $\ln y = 0.99 \ln x$ with a standard error 0.001. While we strongly reject the null of a unit elasticity, the predicted values of $\ln y_t$ will closely mirror the actual values.

10 Consistent with the results found in our study, Cuthbertson (1996a) finds Granger causality from $\Delta R_t^{(m)}$ to $S_t^{(n,m)}$ indicating substantial feedback in the VAR regressions.
6.6 CONCLUSION

Using a number of short-term maturities on monthly Irish money market rates from 1982 to 1997, I perform a number of tests of the EH of the term structure of interest rates for Ireland. On balance the reported results would appear to lend support to the EH. They are consistent with recent findings for the UK. For the US, the poor performance of the EH appears to be as a result of a number of possibilities. First, if interest rate smoothing took place, then the long-short spread will have little predictive power on future changes in short rates (see, Mankiw and Miron, 1986). On the other hand, volatile interest rates may lead to sizeable time varying risk premia, which could invalidate the EH (see, Engle, Lilien and Robins (1987), Hall, Anderson and Granger (1992) and Tzavalis and Wickens (1995))\textsuperscript{11}. Finally, rejection of the EH, using US data, has recently been attributed to small sample bias, Bekaert, Hodrick and Marshall (1997a). The authors favour a pooling the data in a panel data approach, as it may address the bias and dispersion in the small sample distributions.

The standard deviations ratio and the correlation coefficients give results in favour of the EH, while the VAR cross-equation restrictions are rejected for only 1 case out of 3 (for the 3m,1m combination). These results are consistent with those from Cuthbertson (1996a), using UK data. I do provide a number of possible reasons why the cross-equation restrictions may be rejected. As has been mentioned, Campbell and Shiller (1991) have argued that rejection of the cross-equation parameter restrictions may not constitute a major departure form the EH on economic grounds, as long as the theoretical spread closely tracks the actual spread. Given the graphical evidence and the other reported metrics, I conclude that the data is consistent with EH.

\textsuperscript{11} Chapter 7 will focus on the VAR approach to testing the EH with a time varying term premium.
Table 6.1: VAR model for \((S_t^{(n,m)}, \Delta r_t^{(m)})\)

<table>
<thead>
<tr>
<th>Spread ((n,m))</th>
<th>Lag</th>
<th>Granger Tests Causality</th>
<th>Ljung-Box Q(26)</th>
<th>R(^2) Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6,3) Month</td>
<td>2</td>
<td>(S_t) on (\Delta r_t^{(m)}) (&lt;0.01)</td>
<td>13.3</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Delta r_t^{(m)}) on (S_t) (&lt;0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,1) Month</td>
<td>2</td>
<td>(S_t) on (\Delta r_t^{(m)}) (&lt;0.01)</td>
<td>11.3</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Delta r_t^{(m)}) on (S_t) 0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6,1) Month</td>
<td>2</td>
<td>(S_t) on (\Delta r_t^{(m)}) (&lt;0.01)</td>
<td>9.36</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\Delta r_t^{(m)}) on (S_t) 0.48</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
'Lag' denotes the lag length that minimises the AIC and the SBC. Where the latter (occasionally) results in an equation system with serial correlation, the AIC is overridden and extra lags added (back) until any residual serial correlation is eliminated. The critical value for Q(26) is 38.89 (5% significance level). In columns 3 and 4 we report the marginal significance levels for the Granger-causality tests of \(S_t^{(n,m)}\) on \(\Delta r_t^{(m)}\) and vice versa (statistics are calculated after applying the GMM correction for heteroscedasticity used in Campbell and Shiller, 1991). The final 2 columns give the \(R^2\) statistic for each equation. The regressions are estimated for the whole sample period, January 1984 to October 1997.
Table 6.2:
Regression of the Actual Spread $S_t$ on the Theoretical Spread $S'_t$

<table>
<thead>
<tr>
<th>Interest Rate Maturity</th>
<th>$\alpha$</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>$\beta$</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>$R^2$ Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6,1)</td>
<td>-0.01</td>
<td>0.02</td>
<td>0.90</td>
<td>0.06</td>
<td>0.87</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>(6,3)</td>
<td>0.02</td>
<td>0.01</td>
<td>0.81*</td>
<td>0.07</td>
<td>0.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,1)</td>
<td>0.02</td>
<td>0.03</td>
<td>0.83</td>
<td>0.13</td>
<td>0.80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
The regressions are estimated for the whole sample period, January 1984 to October 1997. The estimated regression is $S_t = \alpha + \beta S'_t + \epsilon$, which is estimated by GMM with heteroscedastic corrected errors. A star indicates the estimated coefficient is statistically different from that implied by the null hypothesis (at a 5% significance level), which for $\alpha$ is $H_0: \alpha = 0$ and for $\beta$ is $H_0: \beta = 1$. The theoretical spread $S'_t$ is obtained from the predictions from the VAR using $z = [S_t, \Delta r_t]$. 
Table 6.3:  
Tests of the EH using weakly rational expectations

<table>
<thead>
<tr>
<th>Spread (n,m)</th>
<th>Wald Test [.] = p-value</th>
<th>$\sigma(S_t^{(n,m)},/)\sigma(S_t^{(n,m)},)$</th>
<th>Corr($S_t^{(n,m)}, S_t^{(n,m),}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3,1) Month</td>
<td>12.19 [0.02]</td>
<td>0.97(0.23)</td>
<td>0.89(0.12)</td>
</tr>
<tr>
<td>(6,3) Month</td>
<td>3.09 [0.54]</td>
<td>1.05(0.24)</td>
<td>0.93(0.10)</td>
</tr>
<tr>
<td>(6,1) Month</td>
<td>4.91 [0.30]</td>
<td>1.08(0.19)</td>
<td>0.96(0.05)</td>
</tr>
</tbody>
</table>

Notes:  
The regressions are estimated for the whole sample period, January 1984 to October 1997.
Figure 6.3: 6 Month - 3 Month Interest Rate Spread
Figure 6.6: Actual and theoretical Spread (6m-3m)
CHAPTER 7

THE VAR APPROACH TO TESTING THE EXPECTATIONS HYPOTHESIS WITH A TIME VARYING RISK PREMIUM

7.1 INTRODUCTION

The expectations hypothesis (EH) of the term structure (with a constant or zero term premium) implies that the yield spread between the long rate and short rate is an optimal predictor of future changes in short rates, over the life of the 'long bond'. There is a great deal of empirical work using US data. The main conclusion is that for a wide variety of maturities from 1 to 12 months and for 2,3,4 ... 10-year maturities the empirical evidence does not support the EH. Although the spread predicts future changes in short rates in the right direction, actual movements in the spread are greater than that required under the null that the EH is the correct model. This is often referred to as the "over-reaction hypothesis" and is sometimes stated in terms of the actual spread not being an unbiased predictor of future changes in short rates. This is one of the main explanations for the failure EH (e.g. Mankiw, 1986, Campbell and Shiller, 1991 and Hardouvelis, 1994). A second possible explanation is that long rates not only contain information about future short-rates, but also about the risk premium (e.g. Fama, 1984, Mankiw, 1986, Tzavalis and Wickens, 1997).

Mankiw and Miron (1986), argue that the EH is likely to perform better empirically under a policy of monetary targeting, rather than interest rate smoothing.
Kugler (1988) using US, German and Swiss monthly data on one and three month Euromarket deposit rates found support for the EH only on German data (for the period of March 1974 to August 1986), which he interprets as broadly consistent with the Mankiw-Miron hypothesis. Similarly, both Engsted (1994) using Danish money market rates and for longer maturity bonds (Engsted and Tanggaard 1993) find considerable support for the EH providing the variation in interest rates is relatively large. (i.e. in the post-1992 ERM ‘crisis period”). This is to be expected given the analysis of Mankiw and Miron (1986), if interest rate stabilisation results in random walk behaviour for short rates, then the expected change in short rates is zero and the spread has no predictive power for future short rates, contrary to the EH (See also Rudebusch, 1995).

Although it is clear from Mankiw-Miron (1986) that econometric tests of the EH require sufficient variability in expected changes in short rates, it is also the case that very large (unpredictable) changes may increase agents perceptions of the riskiness in holding bonds (bills). This will also invalidate the EH because of the presence of a time-varying term premium (see Engle, Lilien and Robins 1987, Hall, Anderson and Granger 1992, Tzavalis and Wickens 1997).

Cuthbertson (1996a) using the Campbell-Shiller VAR methodology on data at the short end of the maturity spectrum (i.e. up to one year) finds reasonable support for the EH on UK data. However, Taylor (1992) focusing on longer maturities, 5, 10 and 15 years, finds strongly against the EH (see also MacDonald and Speight 1991). Taylor (1992) noted that the failure of the EH at the long end of the maturity spectrum may be due to the presence of a time varying (yet stationary) risk premium. Drawing on Tzavalis and Wickens (1998), Cuthbertson and Nitzsche (1998) model long maturity rates (2 years – 10 years) in the UK, with a 3-variable VAR which incorporates a time varying risk premium.

The main focus of this chapter, is testing the EH of the term structure for Irish rates at the long end of the maturity. This chapter represents an extension on that of the previous chapter. In the previous chapter I found broad support for the EH, results which were consistent with UK data, Cuthbertson (1996). Based on the results of
Taylor (1992), where the excess holding period yield is found to be time varying when using a single equation format, and Cuthbertson and Nitzsche (1998), I modify the standard 2-variable VAR to allow for a time varying risk premium.

The main insight here is the use of the excess holding period return to provide a proxy for a possible time varying term premium. Nearly all previous studies using the VAR methodology have used only the spread and the change in (short) rates and they have ignored the excess holding period return. The exception here is Tzavalis and Wickens (1998) who show using US data on 3, 6 and 12 month maturities that a 3 variable VAR including the holding period return provides useful incremental evidence on the importance of a time varying term premium. The authors note, that models using the spread to predict future changes in short rates will involve stationary variables and hence their estimates will be effected by omitted variable bias, as result of ignoring the (stationary) time varying term premium. Tzavalis and Wickens (1998) view this as the reason why to date long-run evidence has provided more reliable basis for testing the EH than the short-run evidence. Indeed, they find that the ‘over-reaction hypothesis’ is rejected when the excess holding period return is included in the analysis. This study also uses a high quality data set for spot rates and so avoids the application of the ‘par yield’ approximation for yields to maturity.

The remainder of the chapter is organised as follows. The theoretical model is outlined in detail in section 7.2, while section 7.3 introduces the various testable methods. In section 7.4 we present the results from previous studies in this area. The empirical results are reported in section 7.5. I conclude with a summary in section 7.6.

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1 As has already been noted in chapter 5, cointegration analysis of the returns will still yield superconsistent estimates of the long-run relationships, even if the term premia (stationary time varying) is omitted (Tzavalis and Wickens, 1998).
7.2 THEORETICAL MODEL

The EH states that, after adjusting for risk, the expected return from holding for one period a bond that has \( n \) periods to maturity is the same as the same as a certain return from a one period bond, i.e.,

\[
E[h(n, t+1)] \equiv E\{\ln P(n-1,t+1) - \ln P(n,t)\} = r_t + T(n,t) \tag{7.1}
\]

Where, \( h(n, t+1) \), equals the capital gain from holding an \( n \)-period bond for one period. From chapter 3, the long rate implies that the yield from holding a long bond to maturity equals the expected return from rolling over a series of one period bonds plus the average risk premium on an \( n \)-period bond until it matures;

\[
R^n_t = \frac{1}{n} \sum_{i=0}^{n-1} E_{t+i}r_{t+i} + E_t \Phi(n,t) \tag{7.2}
\]

Subtracting \( r_t \) from both sides and re-arranging yields :

\[
S_{t}^{(n)} = E_p S_{t}^{* (n)} + E_t \Phi(n,t) \tag{7.3}
\]

Where;

\[
S_{t}^{(n)} = R_{t}^{(n)} - r_t = actual \ spread \tag{7.3a}
\]

\[
S_{t}^{* (n)} = \sum_{i=1}^{n-1} [1-i/n] \Delta r_{t+i} = perfect \ foresight \ spread \tag{7.3b}
\]
\[ \Phi(n,t) = \frac{1}{n} \sum_{i=1}^{n-1} T(n-I) (t+i) = \text{'average' risk premium} \]  
(7.3c)

Equation (7.1) indicates that the expected excess holding period return \( E_h(n,t+1) - r_t \) reflects changes in the (one-period) term premium \( T(n,t) \). Equation (7.3) is the familiar ‘spread equation’ indicating that the actual spread \( S_t^{(a)} \) is an optimal predictor of expected future changes in short rates \( E_s S_t^{(a)} \) plus future changes in the average term premium \( E\Phi(n,t) \). \( S_t^{(a)} \) is the weighted change in short rates assuming investors have perfect foresight. Under the EH, the expectations of \( S_t^{(a)} \) equals the actual spread. \( E\Phi(n,t) \) is a rolling risk premium, and is the average of the expected future one-period term premium over the rest of the bonds life.

Assuming RE, Equation 7.2 can be used to decompose the innovations in the excess holding period return, \( E_h(n,t+1) - r_t \), into news about future short-term interest rates and future term premia. By substituting Equation 7.2 into \( E_h(n,t+1) \) gives

\[ \begin{align*}
\text{eh}(n,t+1) &= -(E_{t+1} - E_t) \sum_{i=1}^{n-1} r_{t+i} - (E_{t+1} - E_t) \sum_{i=1}^{n-1} T(n-i,t+i) \\
&= -(r_t - T(n,t)) \sum_{i=1}^{n-1} r_{t+i} - (E_{t+1} - E_t) \sum_{i=1}^{n-1} T(n-i,t+i)
\end{align*} 
\]  
(7.4)

where \( E_{eh}(n,t+1) = 0 \). The above can be written more compactly as;

\[ \text{eh}(n,t+1) = -\{ er(n,t+1) + eT(n,t+1) \} 
\]  
(7.5)

where;

---

2 Given that the innovations, \( eh(n,t+1) = h(n,t+1) - E_h(n,t+1) \) and using Equation 7.1, this equals to; \( eh(n,t+1) = h(n,t+1) - r_t - T(n,t) \). Using \( h(n,t+1) = nR(n,t) - (n-1)R(n-1,t+1) \) this reduces to Equation 7.4 (Tzavalis and Wickens, 1998).
\[
\begin{align*}
    eh(n, t+1) &= h(n, t+1) - E_t h(n, t+1) \\
    er(n, t+1) &= (E_{t+1} - E_t) \sum_{i=1}^{n-1} r(t+i) \\
    eT(n, t+1) &= (E_{t+1} - E_t) \sum_{i=1}^{n-1} T(n-i, t+1)
\end{align*}
\] (7.5a,b,c)

The term, \( er(n, t+1) \) is 'news' about future spot rates \( r(t+i) \), and \( eT(n, t+1) \) is 'news' about future term premia. Equation 7.5 is not a behavioural equation, but a dynamic accounting identity that imposes internal consistency on expectations, Campbell (1991). The intuition behind Equation 7.5 is as follows. For an \( n \)-period bond, if there is an unexpected rise in its one period return \( h(n, t+1) - E_t h(n, t+1) \) this must be due to an unexpected fall in long rates \( R_t^{(n)} \), which in turn must be due to an unexpected fall in current or future short rates (i.e. the \( er(n, t+1) \) term). Alternatively, the unexpected rise in \( h(n, t+1) \) could be caused by an unexpected fall in future risk premia (i.e. the term \( eT(n, t+1) \).

### 7.3 Testing the Model

The above analysis gives rise to a number of tests which can be implemented using the VAR methodology of Campbell-Shiller (1991). I assume throughout that the term premia \( T(n, t) \) is stationary (for a contrary view on US data see Evans and Lewis 1994)\(^3\). Consider the VAR system comprising

\(^3\) A possible reason for the difference between the conclusions from Evans and Lewis (1994), and those of Tzavalis and Wickens (1998), is that the former ignore the effects of the regime shift over the sample period.
If $Z^*$ consists of I(1) variables then Equations 7.1 and 7.2 imply that the system should contain 2 co-integrating vectors which we can interpret as the long-short spread, $R_t(\nu) - r_t$ and the excess holding period yield, $h(n,t+1) - r_t$. Note that the presence of a time varying I(0) term premium should not seriously bias tests of the number of cointegrating vectors. If the above cointegration relationships hold then the vector

$$Z = [S_t(\nu), \Delta r_t, h(n,t+1) - r_t]$$

contains stationary variables. Hence, there exists a trivariate Wold representation (Hannan 1970) which may be approximated by a VAR of order p, which in companion form is;

$$Z_{t+1} = AZ_t + \nu_{t+1}$$

The selection vectors are $e_1, e_2, e_3$ which are $3p \times 1$, with unity in the first, second and third rows respectively and zeros elsewhere. I can use the VAR to forecast $E_t h(n,t+1)-r_t$, and the future changes in short rates $\Delta r_{t+1}$ in Equation 7.3, and 'pick out' the actual spread $S_t = e_1'Z_t$.

Equation 7.1 implies that the expected excess return $E_t h(n,t+1)-r_t$ is a constant, only if the term premium is time invariant and in terms of the VAR this implies (since all variables are expressed as deviations from means):

$$Z^* = [R_t(\nu), r_t, h(n,t+1)]$$
Violation of this (linear) restriction indicates that a time varying term premium may be empirically important.

The forecast of future changes in short rates in Equation 7.3 is referred to as the theoretical spread and using the predictions from the VAR is given by;

$$S_t^{(n)} = e2'f(A)Z_t$$

(7.10)

where $f(A) = A[I-(1/n)(I-A^n)(I-A)^{-1}](I-A)^{-1}$

In the absence of a time varying term premium the forecast of $\Delta r_{t+1}$ from the VAR namely $S^{(v)}_t$ should 'track' the actual spread $S^{(o)}_t = e1'Z_t$ and hence we expect $S^{(a)}_t = S^{(v)}_t$. I can test the theory by focusing on a number of metrics, which have been discussed in detail in the previous chapter.

$$e1' - e2'f(A) = 0$$

(7.11)

$$S^{(a)}_t = a + \beta S^{(o)}_t + e_t$$

(7.12)

$$\sigma(S_t) / \sigma(S_t) = 1$$

(7.13)

$$\rho(S_t;S_t) = 1$$

(7.14)

The above non-linear cross equation restrictions imply $S^{(a)}_t = S^{(o)}_t$ and are tested using a Wald statistic. As in the previous chapter I graph $S^{(a)}_t$ and $S^{(o)}_t$, which provides an informal evaluation of the EH (with a constant term premium) while the tests in (7.11), (7.12) and (7.13) provide more formal measures of this association. Since $\beta = \rho.\sigma(S)/\sigma(S')^4$ a rejection of $\beta = 1$ can be apportioned between the over

4 The estimates of the coefficient spread are calculated using.
reaction hypothesis or the presence of a time varying term premium. If the standard deviation ratio is greater than 1, while the correlation is close to unity, this would imply that $\beta > 1$, and that although there is strong relationship between $S(n)_t$ and $S(n')_t$, the long term interest rate is over-reacting to current information about future short rates, i.e. the over-reaction hypothesis. On the other hand, if neither of the two are close to unity, although there is over-reaction, the $S(n)_t$ and $S(n')_t$ are not moving closely, and this is evidence in favour of a time varying term premium. The fact that there is over-reaction in this case may be purely as a result of the time varying term premium.

From the theoretical review in section 7.2, the variation in the ex-post excess holding period returns is as a result of 3 factors; fluctuations in the term premium, news about term premia and news about short rates. I can now use the VAR methodology to test their importance. From Equation 7.1 and 7.5 I obtain:

$$ h(n,t+1) - r(t) = T(n,t) - \epsilon_r(n,t+1) - \epsilon_T(n,t+1) $$ (7.15)

The explanatory power of the final equation in the VAR system will be a measure of the contributions of variations to the term premium. It also follows that the residuals of this final equation are an estimate of the combined contributions of $\epsilon_r(n,t+1)$ and $\epsilon_T(n,t+1)$. From Equation 7.5(b) a separate estimate of $\epsilon_r(n,t+1)$ can be obtained from the VAR errors;

$$ \hat{\beta}(n) = \rho(S_t, S'_t) \times \left[ \frac{\sigma(S_t)}{\sigma(S'_t)} \right] $$

i.e. the sample correlation from between the spread and the theoretical spread multiplied by the ratio between of their sample standard deviations. Hence for, $\hat{\beta}(n)$ to be close to unity, either both the correlation and the ratio of the standard deviations must be close to unity, or one of them must be approximately the inverse of the other.

This draws on a similar idea which has been used in Campbell and Shiller (1988) and Campbell (1991).
\[ er(n,t+1) = (E_{t+1} - E_t) \sum_{i=1}^{n-1} r(t+i) \]

\[ er(n,t+1) = (E_{t+1} - E_t) \left[ (n-1)r(t) + \sum_{i=1}^{n-1} \sum_{j=1}^{\infty} \Delta r(t+j) \right] \]

\[ er(n,t+1) = (E_{t+1} - E_t) \sum_{i=1}^{n-1} \sum_{j=1}^{\infty} \Delta r(t+j) \] (7.16)

From Equations (7.5a-c and 7.8) and the above, the 'surprises' are the residuals from the VAR:

\[ eT(n,t+1) = -er(n,t+1) - eh(n,t+1) \] (7.17)

\[ = e_2' \{(n-1)I + (n-2) A + (n-3) A^2 + \ldots \{n-(n-1)\} A^{n-2}\} v_{t+1} - v_{3t+1} \]

The first term is merely the weighted sum of the surprises in future short rates [i.e. \((E_{t+1} - E_t) \sum_{i=1}^{n-1} \sum_{j=1}^{\infty} \Delta r(t+j)\)] where \(e2'\) 'picks out' the second element in \(v_{t+1}\) which corresponds to the surprise in short rates. The A-matrices represent the degree of persistence in news about future short rates. The term \(v_{3,4t+1} = e3' v_{t+1}\) is the surprise in the excess holding period return \(h(t+1) - E_t h(t+1)\), the third element in the Z-vector of the VAR. If news about future term premia are very small (i.e. \(eT(n,t+1) \approx 0\)) then we expect the surprise in the one period return to wholly reflect 'news' about future short rates, hence \(eh(t+1) = -er(t+1)\). The metrics can tested as follows;

\[ \sigma(er) / \sigma(eh) = 1 \] (7.18a)

\[ \rho(er,eh) = -1 \] (7.18b)
Therefore one would expect the standard deviation ratio, Equation 7.18a, and the correlation coefficient, Equation 7.18b, to be close to unity. In addition, if \( eT(n, t+1) = 0 \), the ‘R-squared’ of the excess return equation in the VAR (i.e. the third equation) indicates the proportion of the excess holding period return that is due to news about short rates and ‘(1-R-squared)’ is the proportion attributable to news about the risk premium.

The VAR methodology as has already been mentioned in the previous chapter as having a number of advantages. Tests of the EH + RE can be carried out by estimating the unrestricted coefficients in the VAR and the Wald test can be formulated for the general case of any \( n \) or \( m \), Cuthbertson (1996b). Another advantage of the methodology is that it enables one to decompose the term premium biases into their component terms and analyse them rigorously, Tzavalis (1999).

### 7.4 Empirical Evidence using Long-Rates

The study by Taylor (1992) and recent work by Cuthbertson and Nitzsche (1998) provide a close comparison to the work being carried out in this chapter. Taylor (1992) uses UK data on bond maturities for 10, 15 and 20 years over the period January 1985 to November 1989. Taylor reports strong rejections of the Wald restrictions (p-values of 0.00), a rejection of the restriction that the variance ratios equal unity, the smallest value being 1.5 (standard error = 0.14). He does not report the correlation between \( S \) and \( S' \) but the graph of these variables (see Taylor 1992 - figure 3) for the 10 year-3 month spread indicates a very low positive correlation (or even a negative one).

Taylor uses a two variable VAR, where \( Z_t = (S_t^{(n)}, \Delta n) \) and hence does not allow the excess holding period return to provide a proxy for movements in the one-period expected term premium. However, Taylor does find that, in a single equation context, the excess holding period return is time varying and depends on the proportion
of debt at each maturity (i.e. the market segmentation hypothesis). This finding is not incorporated in the VAR analysis in Taylor’s study. Another possible drawback in Taylor’s study, is use of a VAR in the 13th difference of the short rate which will involve misspecification and biased parameters if the true model involves first differences.

Cuthbertson and Nitzsche (1998) use maturities from 2, 3, … 10 years from June 1982 to March 1995. The authors use continuously compounded spot rates from the Bank of England. Cuthbertson and Nitzsche (1998) results are in sharp contrast to Taylor’s (1992)6. The difference in results may be due to Taylor’s use of the yield to maturity rather than spot yields and the consequent approximation involved in the term structure relationship (which requires the yield to maturity to be close to the par yield over the whole data set, see Shiller 1979). Cuthbertson and Nitzsche (1998) avoid the par yield approximation by using spot rates. Cuthbertson and Nitzsche (1998) follow the modification (as suggested by Tzavalis and Wickens 1996) and use a 3-variable VAR with the excess holding period yield as a proxy for a time varying term premium7. The authors note that, as a result of the incorporation of the TVTP in the VAR analysis and this can ‘pick up’ variations in the one period term premium.

Cuthbertson and Nitzsche (1998) do find evidence in favour of a (stationary) time varying term premium which influences the one period excess return. However, the impact of this time varying term premium on a weighted average of future short rates is negligible compared to movements in the long-short spread. This is because the one period term premium is not persistent and hence has a relatively small impact on a weighted average of future short rates. The authors also find that surprises in one period excess returns are due to news about future short rates and not due to revisions

6 The formulation of the Wald restrictions on weekly data (e.g. Taylor, 1992) are different from those applicable for monthly data (e.g. Cuthbertson and Nitzsche, 1998) and as is well known non-linear restrictions can be very sensitive to the form of the non-linearity (Gregory and Veal 1985).

7 A number of earlier studies have accounted for time varying term premia, e.g. Fama (1984), Hamburger and Platt (1975), and Shiller, Campbell and Shoenholtz (1983). Although such studies have found statistical evidence in favour of a time varying term premia, what is important is an economic interpretation on the impact.
about future term premia. These results are supported by recent evidence on US Treasury bills by Tzavalis and Wickens (1998), which shows evidence of a TVTP\(^8\).

Tzavalis (1999) has extended the Tzavalis and Wickens (1998) study, by focusing on whether a time varying term premium can explain the failure of the EH across different monetary regimes, using US data. The full sample is from January 1947 to February 1991 and the author splits the data into four sub-samples\(^9\). Tzavalis (1999) finds evidence in favour of a time varying term premium and finds this is a consistent result, whether interest rates are allowed to fluctuate freely or are targeted by the authorities. The author does note that the term premium causes much more bias in the forecasting of future long rates than the forecasting of future short rates. This is due to the fact that the term premium enters the two spread models in different ways. The term premium will cause greater bias in the forecast of future long rates as it enters into both the long rate and the term spread. Tzavalis (1999) concludes that taking account of term premium effects can save the EH and so the term spread models conform with the theoretical predictions.

\(^8\) Rejection of the EH, using US data, has recently been attributed to small sample bias, Bekaet, Hodrick and Marshall (1997a). The authors favour a pooling the data in a panel data approach, as it may address the bias and dispersion in the small sample distributions. Bams and Wolff (1998) also adopt a similar approach. The authors (Bekaet, Hodrick and Marshall (1997b)) also study the possibilities that the anomalies in the US term structure may be due to a generalised peso problem.

CHAPTER 7: THE VAR APPROACH WITH A TIME VARYING RISK PREMIA

7.5 EMPIRICAL RESULTS:

7.5.1 THE DATA

The data used consists of spot rates for 5, 10 and 15 years and were kindly provided by Davy’s Stockbroking firm. The complete data set is sampled monthly (Wednesday, 4pm rates) beginning on the second Wednesday in January 1989 and ending on the second Wednesday of October 1997. The estimation is carried out using the 1 month rate as the representative short rate. Data on the 1 month rate versus the 10 year rate is graphed in figure 7.1. What is clear from the graph is that the two series move together in the long run and there is considerable variability in the spread.

7.5.2 UNIT ROOTS

Table 7.1 reports the unit root results. Using both the Dickey-Fuller and the Phillips and Perron tests there is no evidence against the null that the individual series \( R_t \) are all I(0), whilst we find that \( \Delta R_t \) and \( S_t \) are I(1). Previous empirical evidence has found that the spread is stationary, (see for instance Hall, Anderson and Granger (1993) for the US and Cuthbertson, Hayes and Nitzsche (1998) for the UK). Figure 7.2 shows the long-short spread for the 10 year and the 1 month interest rate combination. Given that the central assumption is that the term premium is stationary, I must also test its order of integration. The term premium can be tested for stationarity by using the above tests on the excess holding period returns (Tzavalis and Wickens, 1997). As can be seen from the test results in table 7.1, the values for both test statistics suggest the rejection of the null of a unit root in the excess returns (term premium), for all \( n \).
7.5.3 VAR ANALYSIS

Table 7.2 contains the results from the 3 variable VAR system. As has already been mentioned, the third equation in the system will provide an implicit estimate of the term premium since, \( E_h(n, t+1) - R(t) = T(n, t) \). The lag length is chosen according to the AIC and the SBC, except for the rare occasions when additional lags are required to avoid any serial correlation in the residuals. The summary statistics for the Ljung-Box Q statistic show the absence of residual serial correlation for each of the interest rate combinations at the 5% critical value. The restriction that the excess holding period return \( E_h(n, t+1) - r_t \) is not time varying, namely \( e_3' A = 0 \) can not be rejected for maturities \( n = 5, 10 \) and 15 years at better than a 5% level of significance (table 7.3). Given the result that the risk premium is not time varying, the results from the modified 3 variable VAR, should be quantitatively similar to the standard 2 variable VAR, with a constant risk premium.

For illustrative purposes, the graph of the actual spread \( S_t \) and the theoretical spread \( S_t' \) for the 10 year and the 1 month, shows a very close correspondence (figure 7.3). However, from table 7.4 the regression of \( S_t \) on \( S_t' \) shows that although the slope coefficients appear to be close to unity, they are statistically different from 1.

The results in table 7.5 which provide metrics for the relationship between the actual spread \( S_t \) and theoretical spread \( S_t \) show a mixed set of results. The Wald test and the standard deviation ratios show broad support for the theory\(^{11}\), however the correlation coefficients between the actual and theoretical spread are statistically different from unity in all of the cases examined.

\(^{10}\) A non-stationary term premium casts doubt on the ability of the REHTS to be a valid equilibrium model (see Baillie, 1989)

\(^{11}\) The exception here being the 5 year and 1 month combination.
7.5.4 INTERPRETATION

Given the result that the risk premium is not time varying, the results from the modified 3 variable VAR, should be quantitatively similar to the standard 2 variable VAR, with a constant risk premium. This is in fact the case, e.g. the correlation coefficients are 0.99 for all cases, and the standard deviation ratios range from 0.89 for the (5 year, 1 month) combination to 0.99 for the (15 year, 1 month) combination, using the 2 variable VAR.

As a comparison to previous studies, namely Tzavalis and Wickens (1998) and Cuthbertson and Nitzsche (1997), I also compare the time series behaviour of the unexpected return, \( \text{eh}(n,t+1) = h(n,t+1) - \text{Eh}(n,t+1) \), with 'news' about future changes in interest rates \( e_{r,t+1} \). Cuthbertson and Nitzsche (1997) found that although variations in the one-period term premium \( T(n,t+1) \) do have a pronounced influence on one period returns, the spread depends on the average of all future expectations of \( T(n,t+i) \) \( (i = 1,2...n) \) of which the current value \( T(n,t+1) \) only has a weight of \( (1/n) \). The authors suggest that there is no strong persistence in \( T(n,t+i) \). The reported results to date have found that the metrics comparing \( S_t \) and \( S_t' \) broadly support the EH.

The results in table 7.6 offer further support in favour of the EH. For all maturities the standard deviation ratio \( \sigma(\text{er}) / \sigma(\text{eh}) \) and the correlation coefficient \( \rho(\text{er,eh}) \) are very close to +1 and -1 respectively which indicates (see Equation 7.5) that most of the variation in \( \text{eh}(n,t+1) \) is due to news about future short rates \( er(n,t+1) \) and very little is due to 'news' about the future average risk premium.

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12 Tzavalis and Wickens (1998) also find similar results from their variance decomposition of the excess holding period return, using US treasury bills.
7.6 Conclusions

Testing the EH while allowing for a time varying risk premium requires a 3-variable VAR, which not only contains the spread and the change in short rates (as used in earlier work) but also includes the excess holding period return, where the latter variable captures movements in the term premium. The analysis is carried out using spot rates for 5, 10 and 15 years, sampled monthly (Wednesday, 4pm rates) beginning on the second Wednesday in January 1989 and ending on the second Wednesday of October 1997. The central assumption is that the term premium is stationary and like previous studies I find, using excess returns as the proxy, that this is the case (Tzavalis and Wickens, 1997). Unlike previous evidence, using UK long maturity data, I do not find a time varying term premium. I therefore report results which verifies that the modified 3 variable VAR gives quantitatively similar results to the 2 variable VAR.

The reported results are consistent with recent evidence for the UK, in that I cannot reject the EH. The results in table 7.6 offer further support in favour of the EH. For all maturities the standard deviation ratio $\sigma(\text{er}) / \sigma(\text{eh})$ and the correlation coefficient $\rho(\text{er},\text{eh})$ are very close to +1 and -1 respectively which indicates (see equation 7.5) that most of the variation in $\text{eh}(n,t+1)$ is due to news about future short rates $\text{er}(n,t+1)$ and very little is due to 'news' about the future average risk premium.
Table 7.1:
Unit Root Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Maturity</th>
<th>ADF</th>
<th>PP-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest Rate: R&lt;sub&gt;t&lt;/sub&gt;&lt;sup&gt;(a)&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Month</td>
<td></td>
<td>-2.56</td>
<td>-2.08</td>
</tr>
<tr>
<td>5 Year</td>
<td></td>
<td>-1.12</td>
<td>-0.80</td>
</tr>
<tr>
<td>10 Year</td>
<td></td>
<td>-0.96</td>
<td>-0.82</td>
</tr>
<tr>
<td>15 Year</td>
<td></td>
<td>-1.04</td>
<td>-1.04</td>
</tr>
<tr>
<td>Change in interest rate: ΔR&lt;sub&gt;t&lt;/sub&gt;&lt;sup&gt;(a)&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Month</td>
<td></td>
<td>-6.46</td>
<td>-7.49</td>
</tr>
<tr>
<td>Spread: S&lt;sub&gt;t&lt;/sub&gt;&lt;sup&gt;(n,m)&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Year, 1 Month</td>
<td></td>
<td>-3.27</td>
<td>-2.87</td>
</tr>
<tr>
<td>10 Year, 1 Month</td>
<td></td>
<td>-3.07</td>
<td>-2.66</td>
</tr>
<tr>
<td>15 Year, 1 Month</td>
<td></td>
<td>-3.05</td>
<td>-2.58</td>
</tr>
<tr>
<td>Excess Holding Period Returns H(n,t+1) - r&lt;sub&gt;t&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Year, 1 Month</td>
<td></td>
<td>-6.54</td>
<td>-9.04</td>
</tr>
<tr>
<td>10 Year, 1 Month</td>
<td></td>
<td>-6.68</td>
<td>-9.65</td>
</tr>
<tr>
<td>15 Year, 1 Month</td>
<td></td>
<td>-7.35</td>
<td>-10.01</td>
</tr>
</tbody>
</table>

Notes: The sample period is from January 1989 to October 1997. ADF is the augmented Dickey-Fuller statistic with 6 lags, which ensures the regressions are free of serial correlation. 'PP' is the Phillips-Perron (1988) statistic with correction for up to 5th order serial correlation. The critical value for both test statistics is -2.89 at the 5% significance level.
Table 7.2: Summary Statistics
VAR: \( Z = [ S_t, \Delta r_t, ( h(n,t+1) - r_t ) ] \)

<table>
<thead>
<tr>
<th>Interest Rate Maturity Combination</th>
<th>( S(t) ) – Equation</th>
<th>( \Delta r(t) ) – Equation</th>
<th>( h(n,t+1) - r(t) ) – Equation</th>
<th>( S(t) )</th>
<th>( \Delta r(t) )</th>
<th>( h(n,t+1) - r(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5 year, 1 month)</td>
<td>Q(1) 0.04 Q(6) 1.32 Q(12) 4.02</td>
<td>Q(1) 0.18 Q(6) 4.06 Q(12) 5.67</td>
<td>Q(1) 0.01 Q(6) 6.46 Q(12) 13.0</td>
<td>Q(1) 0.79</td>
<td>Q(6) 0.19</td>
<td>Q(12) 0.04</td>
</tr>
<tr>
<td>(10 year, 1 month)</td>
<td>Q(1) 0.01 Q(6) 1.92 Q(12) 4.55</td>
<td>Q(1) 0.01 Q(6) 4.23 Q(12) 5.92</td>
<td>Q(1) 0.01 Q(6) 3.27 Q(12) 15.4</td>
<td>Q(1) 0.82</td>
<td>Q(6) 0.17</td>
<td>Q(12) 0.02</td>
</tr>
<tr>
<td>(15 year, 1 month)</td>
<td>Q(1) 0.01 Q(6) 1.85 Q(12) 4.61</td>
<td>Q(1) 0.01 Q(6) 4.15 Q(12) 5.93</td>
<td>Q(1) 0.01 Q(6) 4.40 Q(12) 18.90</td>
<td>Q(1) 0.83</td>
<td>Q(6) 0.16</td>
<td>Q(12) 0.01</td>
</tr>
</tbody>
</table>

Notes:
The sample is from January 1989 to October 1997. The lag-length of the VAR, chosen using the Schwartz criteria, is one. The Ljung-Box Q-statistic are reported for lag lengths of 1, 6, and 12 with critical values (at 5% significance level) of 3.84, 12.59, and 21.03 respectively. A rejection of the null hypothesis of no serial correlation of the residuals at the 5% significance are indicated by a star.
### Table 7.3:
Are Excess One Period Returns Time Varying?

<table>
<thead>
<tr>
<th>Interest Rate Maturity</th>
<th>Wald Test $H_0: e_3'A = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
</tr>
<tr>
<td>(5 year, 1 month)</td>
<td>7.09</td>
</tr>
<tr>
<td>(10 year, 1 month)</td>
<td>3.59</td>
</tr>
<tr>
<td>(15 year, 1 month)</td>
<td>1.65</td>
</tr>
</tbody>
</table>

**Notes:**
The sample is from January 1989 to October 1997. The standard errors used in the Wald test are heteroscedastic-robust. The null hypothesis for a non-time varying (one period) term premia is $H_0: e_3'A = 0$. 
Table 7.4:
Regression of the Actual Spread $S_t$ on the Theoretical Spread $S'_t$

<table>
<thead>
<tr>
<th>Interest Rate Maturities</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$R^2$ - Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>coeff.</td>
<td>s.e.</td>
<td>Coeff.</td>
</tr>
<tr>
<td>(5 year, 1 month)</td>
<td>-0.000001</td>
<td>0.0001</td>
<td>0.90*</td>
</tr>
<tr>
<td>(10 year, 1 month)</td>
<td>-0.00001</td>
<td>0.00004</td>
<td>0.94*</td>
</tr>
<tr>
<td>(15 year, 1 month)</td>
<td>-0.00002</td>
<td>0.00004</td>
<td>0.99*</td>
</tr>
</tbody>
</table>

Notes:
The regressions are estimated for the whole sample period, January 1989 to October 1997. The estimated regression is $S_t = \alpha + \beta S'_t + \epsilon_t$ which is estimated by GMM with heteroscedastic corrected errors. A star indicates the estimated coefficient is statistically different from that implied by the null hypothesis (at a 5% significance level), which for $\alpha$ is $H_0: \alpha = 0$ and for $\beta$ is $H_0: \beta = 1$. The theoretical spread $S'_t$ is obtained from the predictions from the VAR using $z = [S_t, \Delta r_t, h_{t-1} - r]$. 
### Table 7.5:
**Actual Spread $S_t$ and Theoretical Spread $S_t'$**

<table>
<thead>
<tr>
<th>Interest Rate Maturities</th>
<th>Wald test $W_{H_0 : S_t = S_t'}$</th>
<th>$\sigma(S_t)/\sigma(S_t')$</th>
<th>$\rho(S_t, S_t')$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
<td>P-Value</td>
<td>Statistic</td>
</tr>
<tr>
<td>(5 year, 1 month)</td>
<td>6.80</td>
<td>0.078</td>
<td>0.90</td>
</tr>
<tr>
<td>(10 year, 1 month)</td>
<td>3.86</td>
<td>0.27</td>
<td>0.94</td>
</tr>
<tr>
<td>(15 year, 1 month)</td>
<td>1.41</td>
<td>0.70</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Notes:**
The regressions are estimated for the whole sample period, January 1989 to October 1997. The null that the actual spread $S_t$ is a sufficient statistic for future changes in short-rates (i.e. the theoretical spread, $S_t'$) is denoted $H_0 : S_t = S_t'$ and implies cross-equation restrictions on the $A$-matrix of the VAR which are tested using the Wald statistic. The variables in the VAR are $Z = [S_t, \Delta r_t, h_{t+1} - r_t]$. 

Table 7.6:

Variance Decomposition
‘News’ About Future Short Rates ($er_{t+1}$) and One Period Returns ($eh_{t+1}$)

<table>
<thead>
<tr>
<th>Interest Rate Maturities</th>
<th>$\rho(er_{t+1}, eh_{t+1})$</th>
<th>Statistic</th>
<th>Std. Error</th>
<th>$\sigma(er_{t+1})/ \sigma(eh_{t+1})$</th>
<th>Statistic</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5 year, 1 month)</td>
<td>-0.992</td>
<td>1.105</td>
<td>0.028</td>
<td>1.002</td>
<td>0.125</td>
<td></td>
</tr>
<tr>
<td>(10 year, 1 month)</td>
<td>-0.996</td>
<td>1.038</td>
<td>0.021</td>
<td>1.012</td>
<td>0.127</td>
<td></td>
</tr>
<tr>
<td>(15 year, 1 month)</td>
<td>-0.998</td>
<td>1.002</td>
<td>0.018</td>
<td>1.014</td>
<td>0.115</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
The sample size is for January 1989 to October 1997. The null hypothesis is that unexpected one period excess returns $eh = h_{t+1} - E_{t}h_{t+1}$ are solely due to news about future short rates ($er$) implying $\rho(er, eh) = -1$ and $\sigma(er)/ \sigma(eh) = 1$. The variables in the VAR are $Z = [S_t, \Delta \tau, h_{t+1} - r_t]$. 
Figure 7.1: Irish 1 Month and 5 Year Rate
Figure 7.4: 5 Year - 1 Month Interest Rate Spread
Figure 7.5: 10 Year - 1 Month Interest Rate Spread
Figure 7.6: 15 Year - 1 Month Interest Rate Spread
Figure 7.7: Actual and Theoretical Spread (5YR - 1M)
Figure 7.8: Actual and Theoretical Spread (10YR. - 1M)
Figure 7.9: Actual and Theoretical Spread (15YR. - 1M)
PART II
CHAPTER 8

MODELLING EXCHANGE RATE VOLATILITY
AND IRISH EXPORTS

8.1 INTRODUCTION

The volatility of exchange rate movements following the breakdown of the Bretton-Woods agreement has led to an intense interest in the impact that this volatility has on international trade. It has fuelled a debate as to the desirability of the forces of supply and demand determining the rate at which currencies are valued. Although laissez faire economists welcomed the transition to floating exchange rates, there have been a number of commentators who viewed the transition as having a negative impact on international trade. The argument of those who opposed the transition was, that risk averse exporters would reduce their output as a result of increased exchange rate volatility due to the move to floating exchange rates.

This negative relationship between exchange rate volatility and trade is what I will term the traditional view, and is the view adopted by the main policy makers. The main policy makers take the view that there is no debate in the literature and that higher exchange rate volatility leads to reduced trade. This view has been taken by the Group of Twenty-Four and the Group of Ten in studies tabled by the IMF. These report that a major flaw of the floating exchange rate regime has been that they have discouraged trade as a result of higher exchange rate volatility. This view has also been used as an argument for economic and monetary union (EMU) among European countries.

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1 See Group of Twenty-Four (1985) pp. 19.

See Group of Ten (1985) pp. 9: 'The Deputies have noted that short-term exchange rate variability has been substantial and has not shown any tendency to diminish over time. Although empirical studies conducted by the IMF have been unable to find a significant systematic link between short-term exchange rate volatility and the volume of international trade, concern has been expressed that
Although this may be the view taken by policy makers, recent studies prove that the relationship between exchange rate volatility and trade is uncertain. A number of recent studies do find support for the hypothesis that exchange rate volatility leads to a reduction in international trade. However, there have also been a number of studies, which find that higher exchange rate volatility in fact leads to greater levels of trade.

The remainder of this chapter, I focus on the theoretical and empirical contributions addressing the impact of exchange rate volatility on trade. In section 8.2, I discuss the theoretical contributions, which model the impact of exchange rate volatility on a hypothetical firm. Section 8.3 focuses on studies, which actually use trade data and investigate the empirical affect of exchange rate volatility on trade. The empirical evidence on Irish exports to date has not explicitly modelled the impact of exchange rate volatility. As a result, section 8.4 gives a brief summary of the empirical evidence on the determinants of Irish exports. Finally section 8.5 contains a summary.

8.2 THEORETICAL MODELS

The early research on the impact of exchange rate volatility found support for the traditional view, i.e. the negative impact on trade as a result of exchange rate volatility. This view is based on the fact that unexpected changes in the exchange rates impacted on the decisions made by risk averse firms, and so led to a reduction in output and trade (Artus (1983), Brodsky (1984)). However as will seen below there have been a number of recent studies that interpret higher exchange rate volatility as leading to greater trade, De Graauwe (1988).

Ether (1973) modelled the impact on a risk averse firm from exchange rate uncertainty (measured as standard deviation of the spot exchange rate) in terms of the volume of goods to be imported and the amount of exchange rate cover required. Given volatility may discourage investment and trade by adding to uncertainty and financial risks for investors and traders'.
that the price of imports is denominated in the foreign currency and that the firm knows in advance its level of profit for any given exchange rate value, then exchange rate uncertainty will have no effect on trade. However given that it is unlikely that a firm will hold such information, Ethier (1973) extends the model to deal with uncertainty facing the firm's position. When the model is extended the author finds that there is a negative effect on trade as a result exchange rate volatility. Similar results are found by Clark (1973).

However, the risk averse assumption is not required in order to find a negative relationship. Demers (1991) assumed a risk neutral firm, which faced uncertainty about demand, due to price uncertainty, which was due to exchange rate risk. Given this uncertainty, the author shows that the irreversibility of investment in physical capital leads to lower production and trade.

In recent years, a number of studies have focused on the possibility of a positive relationship. Evidence in favour of the positive relationship between exchange rate volatility and trade was reported by Franke (1991). The author considers the optimal strategy facing a firm that incurs costs of entering the foreign market. A firm will increase exports in response to increased exchange rate volatility if the present value of expected cash flows from exports exceeds the sum of entry and exit costs. The theoretical model attributes the likely positive association between exchange rate volatility and exports to goods market imperfections. Higher exchange rate volatility makes it more likely that price differences across countries will develop and hence an increase in international trade will ensue to arbitrage away these differences.

De Grauwe (1988) focuses on the decision by a firm to either sell in the domestic market or the foreign market. In the model the domestic and foreign prices are fixed and therefore the only source of risk to the firm is the local currency price of the exports. The effect of the higher exchange rate volatility (measured as the variance of the exchange rate) depends on the expected marginal utility of the export income, and

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2 Belongia (1992)
3 Exchange rate uncertainty will however effect the degree of forward cover required.
4 The author does note however, that the significant negative effect declines, the more speculative the firm.
whether it is a concave or convex function to the exchange rate. The author takes the view that risk averse producers will reduce their output and so exports, as a result of higher exchange rate volatility. This is due to the fact that higher exchange rate risk reduces the expected marginal utility of export revenues. The author also considers the case of the extremely risk averse producer, in this case they (*the exporter*) consider the worst possible outcome.

The author's thesis is that higher exchange rate risk will raise the

\[ \text{expected marginal utility of export revenue and, therefore, induce them [i.e., exporters] to increase their export activity} \] (p. 66).

Assuming a utility function with constant relative risk aversion, an increase in risk causes both a substitution and an income effect. The substitution effect shows how an increase in exchange rate volatility will lead to a fall in exports. The income effect works in the opposite direction. The lower expected export revenue means that trade will increase in order to offset the loss in revenue. In the case where the income effect is greater than the substitution effect, higher exchange rate volatility will lead to more exports.

An alternative channel that is consistent with a positive association between exchange rate volatility and exports is based on the idea that exchange rate movements are not just a source of risk but also create opportunities to make profits, because they affect the real opportunities of the firm. Assuming that firms make their production and export decisions once they have observed the exchange rate, higher exchange rate uncertainty may increase the average profit of the firm. De Grauwe (1994, pp. 64-65), assuming a profit-taking firm shows that, at a higher price (due to an exchange rate change), the firm enjoys higher profits per unit of output and so it expands its output. At a low price, the firm reduces its output and, hence, limits the reduction in its profits. Equivalently, in this analysis, exporting represents an option. At a favourable exchange rate the firm exercises the option to export. The opposite happens when the exchange rate moves in the other direction. Since the value of the option increases with the variability of the underlying asset, the firm is better off when exchange rate variability
increases. Under risk aversion, it is still possible that exchange rate volatility increases exports, provided that the increase in utility of the firm, from the increase in the average profit, offsets the decline in utility of the risk averse firm due to the greater uncertainty of profits.

### 8.3 Empirical Models

In this section I will focus on studies that have used actual trade data and compare the various measures of exchange rate volatility, the estimation techniques and the empirical results. Briefly, the impact of income and a competitiveness measure would appear to comply with the theory. Kenen and Rodrik (1986), Chowdhury (1993), Caporale and Chui (1995) and Arize (1995) support the predictions of the theory concerning income (positive relationship between income and trade) and relative prices (negative relationship between relative prices and trade). Exceptions are De Grauwe (1988), Pozo (1992) and Chowdhury (1993) who found mixed signs for relative prices.

The empirical evidence of the influence of volatility on exports is also mixed. De Grauwe (1987), Kenen and Rodrik (1986), Koray and Lastrapes (1988), Peree and Steinherr (1989), Pozo (1992), Chowdhury (1993), Arize (1995) and Holly (1995) all find evidence of a negative relationship between exchange rate volatility and trade. Asseery and Peel (1991) and IMF (1984) show evidence of a positive relationship between exchange rate volatility and trade, while Gotur (1985), Bailey, Tavlas and Ulan (1986), Peree and Steinherr (1989), and Gagnon (1993) were unable to find evidence of any significant effect of exchange rate volatility on trade. IMF (1984), Cote (1994) and McKenzie (1999) provide comprehensive reviews of the empirical literature. Rather than giving another review of the empirical literature, I will discuss a number of issues of importance in relation to the empirical literature. The topics which will be discussed below include; stationarity issues, the appropriate measure and estimation procedure
when modeling volatility, and the appropriate sample period. As will be seen these have important implications when modeling the impact of exchange rate volatility and trade.

8.3.1 Stationarity Issues

What is of interest in this part of the thesis is the analysis of the long-run relationship and the short-run interactions. Although most of the previous studies focus on the short-run, an important reason for focusing on the long-run, is the fact that short-run risk may be hedged against. Medium to long-run instruments are not frequently available and therefore international traders are exposed to unhedgable risk. From econometric theory, a group of non-stationary variables may be cointegrated if there exists a linear combination that is stationary. Therefore a long-run relationship exists. The issue of stationarity has important implications for our study. The trade variables covered in the study are likely to be non-stationary. However the measure of volatility may be stationary and therefore may not appear in a long-run relationship as a determinant of trade. As a result the issue of stationarity is of vital importance in determining which variables are included in the long-run relationship.

Early empirical studies disregarded the issue of nonstationarity of macroeconomic time series and used classical regression analysis. These studies, therefore, are subject to the “spurious regression” criticism (Granger and Newbold, 1974). Table 8.1 gives a comprehensive summary of the studies which test stationarity of the data. They include Gotur (1985), Kenen and Rodrik (1986), Koray and Lastrapes (1989), Peree and Steinherr (1989) and Pozo (1992). A number of recent studies test for stationarity of the relevant time series and, in some cases, employ cointegration

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5 The literature notes that the availability hedging instruments may have a negligible effect even in the short-run. This is due to the fact that hedging markets are incomplete, the timing of the cover and location offered and the high cost. A major limitation is the size of the contracts. There are generally large, and average more than one million dollars per contract. A further limitation is the requirement that customers must maintain minimum deposit balances and maturities are commonly in multiple of 30 days (Caporale and Doroodian, 1994).

6 A comprehensive discussion of the issue of cointegration is covered in chapter 2.
techniques, e.g., Lastrapes and Koray (1990), Asseery and Peel (1991), Chowdhury (1993), Arize (1995, 1997), Holly (1995) and Fountas and Aristotelous (1999). It is very obvious from table 8.1 that a large number of the studies in this area do not take account of non-stationary data, and even fewer have considered cointegration.

The importance of the issue of stationarity and the implications of not accounting for the non-stationary nature of economic data has been raised in a study by Asseery and Peel (1991). The authors attribute the mixed results found by early studies to the failure to account for the non-stationary nature of the data. The Augmented Dickey-Fuller (ADF) test is used to check the time series properties of the data. Using data for Australia, Japan, Germany, the UK, and the US and for a sample period from 1972-87, they model exports as a function of income, relative prices, the exchange rate and volatility. As can be seen from table 8.2, Asseery and Peel (1991) use the residuals from an ARIMA model applied to the log of the real exchange rate. The authors find, using a two-stage Engle-Granger error correction framework, that exchange rate volatility has a positive impact on exports.

A multivariate error correction model is also adopted by Chowdhury (1993), but the author finds a negative relationship between volatility and exports. Chowdhury (1993) also notes the importance of the time series characteristics of the data and that,

'It is quite possible that the surprisingly weak relationship between trade flows and exchange rate variability reported in several previous studies are due to insufficient attention to the stochastic properties of the relevant time series' (pp. 705)

The author focused on data from the OECD G-7 group of countries for the sample period 1973-1990. As can be seen from table 8.2, Chowdhury (1993) used the an eight period moving sample standard deviation of the growth rate of the real exchange rate as a measure of volatility. The author finds that volatility has a significantly negative impact on exports and so concludes that the findings are supportive of the traditional view. McIvor (1995) has also raised the issue of
nonstationary data in the literature and like Chowdhury (1993) finds in favour of the traditional view, i.e. that exchange rate volatility has a negative impact on trade.

As can be seen from the above discussion, issues of stationarity and cointegration have important implications for the literature. A large portion of the empirical evidence has not taken into account the non-stationary nature of economic data. As a result of this a number of recent studies have concluded that this may have led to the finding of a weak relationship between exchange rate volatility and trade, (Chowdhury, 1993).

8.3.2 Exchange Rate Volatility: Measurement and Estimation

Although economist would agree that it is uncertainty in the exchange rate, which constitutes exchange rate volatility, there is a vibrant debate as to the correct measure of exchange rate volatility. In table 8.2, I give a summary of the various measures that have been used to proxy exchange rate volatility in the trade literature.

Given that the exchange rate is determined by;

\[ e_t = \alpha + \beta X_t + \varepsilon_t \]  

(8.1)

where \( e_t \) is the spot exchange rate, \( X_t \) are explanatory variables representing exchange rate fundamentals, \( \varepsilon_t \) a stochastic error term, \( \alpha \) is constant and \( \beta \) is a vector of regressor coefficients. As can be seen from table 8.2 a number of different measures of volatility have been used in the literature. These include un/conditional variance (or standard deviation) of \( e_t, \varepsilon_t \) and \( e_t - \varepsilon_{t-1} \) or unanticipated changes in the exchange rate.
What is apparent from table 8.2, is that in recent years the literature has focused on two measures of volatility; the moving average (MA) of the standard deviation of the exchange rate and autoregressive conditional heteroscedasticity (ARCH) models. A number of recent studies have used ARCH models to generate volatility estimates, e.g. Pozo (1992), McKenzie and Brooks (1997), and McKenzie (1998). Caporale and Dorooodian (1994) adopted the multivariate framework (M-GARCH) in mean to test if real exchange rate volatility has affected US imports from Canada over the period 1974-92. Their results suggest that volatility has had a negative impact on trade. However, in the empirical part of this section I opt to use the MA of the standard deviation of the exchange rate. A number of recent studies have taken this approach, e.g. Bini-Smaghi (1991), Kumar and Dhawan (1991), Lastrapes and Koray (1990), Koray and Lastrapes (1989) and Chowdhury (1993). The principle reason for adopting this approach is that a large amount of the economic data used in the study is only available quarterly. Although ARCH are generally highly significant with daily and weekly data, both Diebold (1988) and Baillie and Bollerslev (1989) note that ARCH effects tend to be weakened with less frequently sample data. As a result I choose to adopt the MA approach.

Pozo (1992) uses both approaches to model the impact of exchange rate volatility on exports from Britain to the US from 1900 to 1940. The author finds a negative relationship between volatility and trade, using a GARCH and an MA measure. Chowdhury (1993) adopts the MA and in defence of this notes the finding from Pozo (1992); i.e. that the results are not sensitive to the particular measure of volatility.

Besides the appropriate measure of exchange rate volatility, there is also the decision whether to model real or nominal exchange rate volatility. In general most of the early work in the literature has focused on nominal exchange rate volatility. These studies include, Ethier (1973), Clark (1973), Baron (1976), Hooper and Kohlhagen

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7 The ARCH class of model was introduced by Engle (1982), in order to capture the fact that prices of speculative assets tend to cluster into periods of high volatility and periods of low volatility. Bollerslev, Chou and Kroner (1992) note that 'since the introduction of the ARCH model several hundred research papers applying this modelling strategy to financial time series data have already appeared' pp. 6.
9 For example the unit export values are only available quarterly.
(1978). Bini-Smaghi (1991) argues in favour of the use of nominal exchange rate volatility,

...risk should regard nominal rather than real exchange rate risk... (as the latter) depends in effect not only on the variance of the nominal exchange rate but also on that of relative prices' (pp. 933)

Akhtar and Spence-Hilton (1984) focus on bilateral trade between the US and Germany for the period 1974-1981, using quarterly data. The authors also opt for nominal exchange rate volatility. They argue that a

'measure of variability that partly reflects fluctuations in price levels does not allow for the distinctions between the risk due to exchange rate changes independent of price movements and the risk associated with all other factors which might affect inflation at home or abroad' (pp. 10)

The authors overall view is of a negative relationship between exchange rate volatility and trade. This negative effect can be seen as the direct effect; the fact that profit uncertainty reduces the volume traded, and the indirect effect; firms decisions are biased toward the domestic economy. In their empirical analysis Akhtar and Spence-Hilton (1984) proxy exchange rate volatility as the standard deviation of the nominal effective exchange rate. The authors modelled exports and imports as a function of income, relative prices, capacity utilisation, the exchange rate and exchange rate volatility. Their results indicate that exchange rate volatility did have a negative impact on German and US bilateral trade.

Gotur (1985) took issue with the Akhtar and Spence-Hilton (1984) study, commenting that over the medium term time horizon,

'... the real exchange rate is the more relevant measure because the effects of uncertainty on a firm's revenues and costs that arise from fluctuations in the nominal exchange rates are likely to be offset in large part by movements in costs and prices' (pp. 480)
Koray and Lastrapes (1989) agree with the view that real exchange rate volatility should be adopted. Gotur (1985) used the Akhtar and Spence-Hilton model, but extended the number of countries in the study from Germany and US, to also include France, Japan and the UK. Gotur (1985) finds no evidence to support the tradition view, but finds that volatility is either the 'wrong' sign or is statistically insignificant. Gotur (1985) interprets the results as;

'... it is difficult to interpret (these new results) as supportive of the hypothesis that exchange rate volatility has systematically undercut world trade' (pp. 496)

The Hooper and Kohlhagen (1978) study was updated by Cushman (1983). Cushman (1983) used real as opposed to nominal trade volumes, two extra years of data and a four quarter moving standard deviation of the change in the real exchange as a proxy for volatility. From the sixteen equations, the authors finds six negative and significant results, which although representing an improvement on the Hooper and Kohlhagen (1978) study, is not considered to be comprehensive support for the tradition view. This point is emphasised by studies undertaken by the IMF (1984) and Chan and Wong (1985), who both find no evidence in favour of the traditional view using the same methodology as Cushman (1983).

Bailey, Tavlas and Ulan (1987) analyse the effect of floating exchange rates by modelling the pre and post floating series and testing the impact of exchange rate volatility. The full sample period runs from 1962 to 1985 for the OECD Big Seven and four other countries. The authors use the an absolute percentage change and a log of the standard deviation measure of the exchange rate volatility for both nominal and real rates. Overall, the authors find very mixed results. For the pre-1973 period, they found 6 out of 7 equations had a positive relationship, 3 of which were statistically significant. For the post-1973 period, 35 equations were tested and only 3 reported a negative
A number of equations did report a positive relationship using both real and nominal exchange rate volatility.

In the last number of years the majority of the empirical studies have concluded that there is no substantial difference between using nominal and the real measure. Focusing on the period of floating exchange rates, Qian and Varangis (1994) provide evidence which shows that the real and nominal exchange rates have moved very closely over this period. As a result the authors argue that there is no substantial difference, whether the real or the nominal exchange rate is used in the analysis. Therefore an obvious test would be to compare the results from using nominal and the results from real exchange rates. Thursby and Thursby (1987) carry out such a study and find that there is no substantial difference between using nominal and the real measure.

In a recent study McKenzie and Brooks also find similar results. McKenzie and Brooks (1997) focus on US-German bilateral trade, but unlike the Akhtar and Spence-Hilton (1984), they extend the sample period from 1973 to 1993. Trade is modelled as a function of income, prices and exchange rate and volatility. The results suggest that there is a significant positive relationship between volatility and trade. The authors also conclude that there is no difference between real and nominal exchange rate volatility. McKenzie and Brooks (1997) use an ARCH model fitted to both nominal and real exchange rates and found no significant difference in the parameter results.

As can be seen from the above survey, the debate over real versus nominal measures of exchange rate volatility has fuelled a large amount of empirical work. However, the results from recent studies suggest that there is no significant difference between real and nominal measures of volatility.

Finally there is also the issue of the appropriate estimation procedure to adopt. As can be seen from table 8.1, the most popular estimation technique has been OLS. However, in recent years studies have adopted vector autoregression (VAR) with

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10 In the case where a negative and significant result was found, this was only so for real volatility.
increasing frequency\textsuperscript{11}. The VAR approach has two main advantages. Firstly, VAR estimation can accommodate dynamic relationships between variables. A number of authors have cited the lack of a dynamic specification as a possible reason for the findings of insignificant results, see Bini-Smaghi (1991) (pp. 932). A second advantage of the VAR approach is that it imposes no explicit theoretical restrictions on the variables in the system, Cuthbertson (1996).

A number of recent studies have adopted the VAR approach; Lastrapes and Koray (1990), Chowdhury (1993), Koray and Lastrapes (1989). Koray and Lastrapes (1989) focused on bilateral trade from the US to 5 industrialised countries over the sample period 1959-85 and found only a weak relationship, using the moving standard deviation approach. Lastrapes and Koray (1990) also used VAR estimation, US data and a moving standard deviation measure, but this time focused on multilateral trade. The authors also excluded the fixed exchange rate period (1959-1972) from this study. The authors do find a significant relationship, but note that it is likely to be quantitatively small.

\textbf{8.3.3 Data Sample Period}

The issue of the impact of exchange rate volatility has become important as a result of the move to floating exchange rates. An important issue in the literature is the need for a substantially large sample size, which consists of data covering the floating period\textsuperscript{12}. This is especially the case for the earlier studies. Clark and Haulk (1972) focused on Canada's experience of floating exchange rates prior to 1962, using the full sample period 1952-1970. Although the impact of exchange rate volatility was different in the two periods, it did not have a significant effect in either case\textsuperscript{13}. The period of

\textsuperscript{11} See chapter 2 for a detailed account of VAR's.

\textsuperscript{12} The issue of the appropriate sample will be considered further in relation to Irish trade with other European countries, in chapter 10.

\textsuperscript{13} The authors took the standard deviation of the daily exchange rates about the average for each quarter as the proxy for exchange rate volatility.
estimation was extended to the end of 1973, in a similar study by Makin (1976). The author was again unable to find a significant relationship. Akhtar and Spence-Hilton (1984) comment that their negative finding in their results is due to the fact that their study did not include observations from the fixed exchange rate regime.

Pozo (1992) examined bilateral exports from UK to the US for the sample period, 1900-1940. Over the sample there were 2 periods of floating exchange rates, 1920-1925 and 1932-1940. The proxy for volatility was the conditional variance of the real exchange rate modelled as a GARCH process and a moving standard deviation of the real exchange rate. The author found that exchange rate volatility has a consistent negative impact on trade. A number of studies have taken the approach of a comparison between fixed and floating exchange rate regimes. Bailey, Tavlas and Ulan (1987) is one such study. The authors found a positive relationship between volatility and trade in the fixed exchange rate regime, while positive and negative relationships were found for the floating period. Warner and Kreinin (1982) also compared a fixed and floating regime and found, not surprisingly, that the parameters did change between the two periods.

De Grauwe (1988) modelled bilateral exports as a function of income, prices, trade arrangements and exchange rate variability for 10 major industrial countries over a fixed (1960-1969) and floating period (1973-1984). The proxy for volatility was taken as the variability of the yearly percentage changes of the bilateral exchange rate around the mean in that period\textsuperscript{14}. The author found, using seemingly unrelated regression estimation (SURE), that there was a negative impact on trade.

As can be seen from table 8.1, there are a number of studies that analysed the difference of exchange rate volatility during fixed and floating periods. What is clear from the empirical evidence is that although the results from the fixed periods are different to those from floating period, they are not easily distinguishable from those studies which focus on the homogeneous sample (McKenzie, 1999).

\textsuperscript{14} DeGrauwe (1988) modelled both the impact of real and nominal exchange rate volatility.
8.4 Irish Evidence

Most of the studies that focus on the determinants of Irish exports were produced in the 1970s and 1980s and are subject to the "spurious regression" criticism. A second problem associated with these studies is that they do not explicitly model the impact of exchange rate volatility. They include O'Connell (1978), Browne (1982), Lynch (1983), and Flynn (1984).

O’Connell’s (1978) study is one of the first attempts to provide econometric evidence on the determinants of Irish exports. The author estimates a single-equation model that is unreliable as the assumption of infinitely elastic supply is unrealistic for an SOE. O’Connell (1978) also estimates the equilibrium and disequilibrium versions of a two-equation model and derives price elasticity’s of export demand and supply equal to -1.44 and 2.33, respectively. According to O’Connell, the price elasticity of demand estimate is small for a SOE, when compared to the Goldstein and Kahn (1978) results for Belgium and Netherlands. Browne (1982) estimates the SOE version of the Goldstein-Kahn model. He obtains estimates of the price elasticity of supply that are lower and demand that are higher than in O’Connell (1978) and, hence, more consistent with the SOE assumption. Lynch (1983) estimates a single-equation model for Irish manufacturing exports using quarterly data from 1963 to 1981. He includes both supply and demand side determinants in order to get a more complete picture of export demand and uses a 2SLS procedure to account for simultaneity between prices and quantities. He obtains estimates of the income elasticity of exports in the range 1.10 to 2.69 and estimates of the price elasticity of exports in the range -1.23 to -0.26.

Flynn (1984) analyses the determinants of both manufacturing and industrial exports. His approach differs from Lynch (1983) in his variable choice and the choice of a dynamic set up. Since exports of foreign multinationals (MNC’s) in Ireland accounted for 70% of total exports in 1980, Flynn (1984) drops the relative price variable from the estimated equation because of the way in which these firms make their decisions. Flynn’s (1984) estimates for the income elasticity are 0.59 and 0.49 for manufacturing and industrial exports, respectively, and are much smaller than those
obtained by O'Connell (1978) and Browne (1982). Flynn (1984) speculates that this is due to transfer pricing.

There are some recent studies that employ modern nonstationary time-series techniques using Irish data. Caporale and Chui (1995) pursue a multicountry time series study that includes Ireland. Using annual data for the period 1960-1992 the authors estimate income and price elasticity's of exports using cointegration techniques. Employing the Dynamic OLS (DOLS) procedure, the authors derive estimates of the income and relative price elasticity's equal to 2.97 and -0.34, respectively. Quite similar elasticity's are obtained for Belgium, another SOE. In a recent study, McGettigan and Nugent (1995) attempt to estimate short-run and long-run export functions using ECMs and cointegration techniques, respectively. Using quarterly data for the period 1975 to 1994, the authors obtain long-run income elasticity's in the range 1.78 (for merchandise exports) to 2.04 (for manufacturing exports). The estimates of the relative price elasticity are -4.33 and -7.58, respectively. The latter value appears to be very large.
8.5 CONCLUSIONS AND OUTLINE FOR PART II

The purpose of this chapter is to present both the theoretical and empirical work on the impact of exchange rate volatility on trade. As can be seen there has been an extensive range of studies on this area since the breakdown of the Bretton-Woods. The main conclusion would be that the move towards floating exchange rates, and higher exchange rate uncertainty, has had an ambiguous impact on international trade. From the theoretical viewpoint, researchers have developed models which show exchange rate volatility as having both a positive and a negative impact on trade. The empirical studies show consistent results with the world trade data, i.e. unable to find a significant relationship.

Recent studies have however made a number of advances. Important issues, such as the measure of volatility, the time series properties of the data, and the error correction framework have all led to greater success in establishing a statistically significant relationship between volatility and trade. Given that 60% of the world trade is undertaken by only 10 developed countries: Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Switzerland, UK and the US, DeGrauwe and Bellefroid (1986), most of the empirical research has focused on these countries. Given that the international trade performance of a small open economy (SOE), such as Ireland, is such an important part of the economic health of the country, it is natural that the impact of exchange rate volatility should be analysed. The share of Irish merchandise exports in Gross Domestic Product (GDP) has grown dramatically in recent years (from 43% in 1979 to a level of 82.8% in 1998), thus rendering the economy more open than before and more dependent on foreign markets. Therefore in chapter 9 and 10 the impact of exchange rate volatility will be analysed for Irish exports to the UK and the EU respectively.

In chapter 9, cointegration and error correction models will be used to estimate a long-run and short-run export demand functions for Ireland. I consider three determinants of exports: foreign income, relative prices and exchange rate volatility. In contrast to all previous studies on Irish exports, I include a measure of exchange rate
volatility to investigate the effects of such movements on exports. There are two reasons for focusing primarily on exports to the UK; firstly, the UK represents Ireland's major trading partner and secondly, since the break-up of the one-to-one link between the two currencies at the outset of the EMS, there has been an increase in the volatility of the (Sterling/Irish pound) exchange rate.

In *chapter 10*, I analyse the long-run and short-run relationship between merchandise export volume and its determinants, foreign income, relative prices and exchange rate variability, using the techniques of cointegration and error correction. The model will be estimated for Irish exports and sectoral exports SITC 0-4 and SITC 5-8 to the European Union using quarterly data for the period 1979-1992. The sectoral classification corresponds to the exports of mainly indigenous Irish firms and multinationals, respectively. International trade and the openness of the economy has played a key role in the recent unprecedented growth in the Irish economy. The fact that a large amount of Ireland's recent growth in international trade has been attributed to the multinational corporation (MNC) sector has meant that the economy is more and more dependent on foreign markets. Hence the impact of exchange rate volatility on sectors dominated by both indigenous industry and MNC's is important.
Table 8.1: Summary of Estimation Techniques and Results from Exchange Rate Volatility and Trade Literature

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Results summary

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| Positive Volatility Coefficients | 4 | 4 | 13 | 24 | 1 | 2 | 0 |
| Statistically Significant | 0 | 2 | 0 | 5 | 0 | 0 | 0 |
| Negative Volatility Coefficients | 4 | 2 | 15 | 18 | 4 | 22 | 28 |</p>
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Table 8.1 (cont.): Summary of Estimation Techniques and Results from Exchange Rate Volatility and Trade Literature

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Table 8.1 (cont.) : Summary of Estimation Techniques and Results from Exchange Rate Volatility and Trade Literature

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Table 8.1 (cont.): Summary of Estimation Techniques and Results from Exchange Rate Volatility and Trade Literature

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Notes:
Explanatory variables may be generalised into the following categories:
(1) = income
(2) = prices
(3) = capacity utilisation
(4) = exchange rates
(5) = exchange rate volatility
(6) = export earnings of oil producing nations
(7) = production costs
(8) = trade integration variable
(9) = money supply
(10) = interest rates
(11) = wages
(12) = tariff levels
(13) = transport costs
(14) = importer hedging variable
(15) = consumer tastes

IV refers to instrumental variable estimation
OLS refers to ordinary least squares
VAR refers to vector autoregression
SURE refers to seemingly unrelated regression equation
‘ ___ ’ not applicable or specified

Source: McKenzie (1999)
Table 8.2
Summary of Exchange Rate Volatility Measures

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<td>1) Absolute percentage change of the exchange rate, ( V_t = \frac{</td>
<td>(e_t - e_{t-1})</td>
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<td>Where ( e ) is the spot exchange rate and ( t ) refers to time</td>
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<td>2) Average absolute difference between the previous forward and the current spot rate, ( V_t = \sum_{i=1}^{n}</td>
<td>f_{t-1} - e_t</td>
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<td>Where ( f ) is the forward rate</td>
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<td>3) Variance of the spot exchange rate around its trend which is predicted from, ( \ln e_t = \theta_1 + \theta_2 t + \theta_3 t^2 + \epsilon_t )</td>
<td>Thursby and Thursby (1987)</td>
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<td>Where ( Z ) is the log relative price of foreign consumer goods in terms of US Consumer goods and ( m=12 ) (Koray and Lastrapes (1989))</td>
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<td>Measure of Exchange Rate Volatility</td>
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<tr>
<td>5) Long-Run exchange rate uncertainty, measured as;</td>
<td>Peree and Steinherr (1989)</td>
</tr>
<tr>
<td>[ V_t = (\max X_{t+k}^t - \min X_{t+k}^t)/\min X_{t+k}^t + [1 + \left(\frac{</td>
<td>X_t - X_{P,t}</td>
</tr>
<tr>
<td>Where ( X_t ) is the nominal exchange rate at time ( t ), ( \max X_{t+k}^t ) and ( \min X_{t+k}^t ) refer to maximum and minimum values of the nominal exchange rate over a given time interval of size ( k ) up to time ( t ), and ( X_{P,t} ) is the 'equilibrium' exchange rate.</td>
<td></td>
</tr>
</tbody>
</table>

*Source: McKenzie (1999)*
CHAPTER 9

EXCHANGE RATE VOLATILITY AND EXPORTS TO THE UK

9.1 INTRODUCTION

In recent years a significant volume of research has taken place in order to evaluate empirically the determinants of export demand in industrial countries. The literature can be generally divided up into papers that use conventional estimation procedures and those that recognise the nonstationary nature of real exports and its determinants. Studies which can be grouped into the former category include Kenen and Rodrik (1986), Pozo (1992), Bailey, Tavlas and Ulan (1986), while those included in the latter, Lastrapes and Koray (1990), Chowdhury (1993) and Arize (1995). Chowdhury (1993) and Arize (1995) also use cointegration and error correction models (ECM) to estimate a long-run and a short-run Irish export demand function.1

The motivation for estimating the Irish export demand function derives from the recent extraordinary growth in the share of Irish exports in GDP. Most empirical studies of the determinants of Irish exports have used traditional estimation techniques (e.g. O'Connell (1978), Browne (1982), Lynch (1983) and Flynn (1984)) and therefore, have not considered the integration properties of the time series involved in the analysis. Browne (1982) sees exports in a small open economy as being primarily driven by supply side considerations, while Lynch (1983) includes both supply and demand side factors in order to get a more complete picture of exports. The general

1 A comprehensive survey of the literature on exchange rate volatility and trade is given in chapter 8.
conclusion of the above papers is that factors such as world income and measures of competitiveness are major determinants of Ireland’s exports. Studies with more advanced estimation techniques include Caporale and Chui (1995) and McGettigan and Nugent (1995). As opposed to previous studies on Ireland, Caporale and Chui (1995) and McGettigan and Nugent (1995) recognise the non-stationary nature of exports and it's determinants.

In this chapter, I estimate the Irish export demand function and improve on previous studies in several ways. Firstly, it is recognised that exports and its determinants are potentially non-stationary variables, and so estimate a demand function for Irish exports to the UK, the most important market for Irish exports, since the launch of the EMS (March 1979). Second, in contrast to all other studies, a measure of exchange rate volatility is included, to investigate the effect of such movement on exports. This is justified by the increased volatility in the (sterling/Irish pound) exchange rate following the break up of the one-to-one link between the two currencies at the outset of the EMS. Finally, both long-run and short-run export demand functions are estimated, through the use of the econometric techniques of cointegration and ECM’s.

The chapter is organised as follows. In section 9.2 I outline the approach adopted to modelling the impact on Irish exports to the UK as a result of exchange rate volatility. The empirical results are covered in section 9.3. Finally a summary is given in section 9.4.
9.2 MODELLING EXPORTS AND EXCHANGE RATE VOLATILITY

The empirical literature on the estimation of export functions uses the following long-run export function (see, e.g., Arize, 1995 and 1997 and Chowdhury, 1993):

\[
\ln X_t = \beta_0 + \beta_1 \ln Y_t + \beta_2 \ln P_t + \beta_3 V_t
\]  

(9.1)

Where,  

\( X_t \) = real exports  

\( Y_t \) = real foreign income  

\( P_t \) = relative prices (a measure of competitiveness)  

\( V_t \) = exchange rate volatility

Equation 9.1 represents the basis of the modern empirical literature on the estimation of export functions (see, e.g., Chowdhury, 1993, and Arize, 1995, 1997). Economic theory would suggest that the income level of the trading partners of the domestic country and a measure of competitiveness between the domestic country and its major trading partners should be included in an export function. Finally, a measure of exchange rate volatility would also be included in an export function. As has been discussed, traditional trade theory would suggest that exchange rate volatility would depress trade because exporters would view it as an increase in the uncertainty of profits on international transactions. The theoretical arguments that exchange rate volatility actually might benefit trade are examined by De Grauwe (1988), Franke (1991), Giovannini (1988), Sereu and Vanhulle (1992) and Viaene and de Vries (1992). Hence, the sign of \( \beta_3 \) is ambiguous from a theoretical point of view.

As has been discussed in chapter 8, the mixed results obtained by most of the previous studies using classical regression analysis may be due to the non-stationarity
CHAPTER 9: EXCHANGE RATE AND EXPORTS TO THE UK

of real exports and its determinants. Variables such as real exports and real income are by their nature potentially nonstationary. In this study cointegration analysis is used to test for a long-run export function of Irish exports to the UK. Tests for cointegration require nonstationary time series of the same order of integration. Therefore, I first test for the presence of a unit root in both the level and the first difference of the four variables in Equation 9.3, using the Dickey-Fuller (DF), and the Augmented Dickey-Fuller (ADF) tests (Fuller, 1976 and Dickey and Fuller, 1979). I will also use the Phillips-Perron (PP) test (Phillips and Perron, 1988). The method used to test for cointegration is the Johansen procedure introduced in Johansen (1988) and extended in Johansen and Juselius (1990).

The choice is justified by Phillips (1991) who finds that the Johansen approach is optimal in terms of symmetry, unbiasedness and efficiency. While, a Monte Carlo study by Gonzalo (1994) supports the superior properties of the Johansen technique relative to several other single and multivariate techniques. In the Johansen framework, all variables, including exchange rate volatility, are treated as endogenous. Provided that cointegration exists among our variables, the cointegrating vector is normalised on exports to give the long-run income and relative price elasticity's for export demand.

I also estimate the short-run export equation using the ECM;

\[ \Delta \ln X_t = \alpha_0 + \alpha_1 R_{t-1} + \sum_{i=1}^{n} \gamma_i \Delta \ln X_{t-i} + \sum_{i=1}^{n} \delta_i \Delta \ln Y_{t-i} + \]

\[ + \sum_{i=1}^{n} \epsilon_i \Delta \ln P_{t-i} + \sum_{i=1}^{n} \zeta_i \Delta V_{t-i} + \epsilon_t \]  

(9.2)

If the variables are cointegrated, then the ECM will be of the above form, where \( R_{t-1} \) is the ECT (i.e., the lagged residual from the cointegration regression).
Finally, as a measure of time-varying exchange rate volatility I use the moving sample standard deviation of the growth rate of the real effective exchange rate:

\[ V_t = \left[ \frac{1}{m} \sum_{i=1}^{m} (\ln Z_{t+i-1} - \ln Z_{t+i-2})^2 \right]^{1/2} \]  

(9.3)

Where \( Z \) is the real effective exchange rate and \( m \), the order of the moving average, is set equal to 8. This measure of exchange rate volatility is adopted by Kenen and Rodrik (1986), Koray and Lastrapes (1989) and Chowdhury (1993).

### 9.3 Empirical Results

#### 9.3.1 Data

The sample covers the period 1979Q2 – 1993Q3. As can be seen from figure 9.1, the vast majority of Irish exports are to Europe. The UK is still Ireland's key trading partner, figure 9.2, therefore in this chapter, I will focus on Irish exports to the UK. I start the sample in the second quarter of 1979 since the objective is to estimate the long-run and the short-run demand function for Ireland's exports to the UK since the beginning of the EMS (March 1979) that coincided with increasing volatility in the (sterling/Irish pound) exchange rate. Figures 9.3 – 9.6 show the variables entering the cointegration system.

The export variable is taken from the Central Statistic Office (CSO) publication, and was divided by Ireland's unit export value to obtain the real exports figure. The first explanatory variable in the export demand function is foreign income. It is proxied

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2 The reported results in section 9.3 are robust for alternative choices of the lag length.
by the quarterly GDP data of the UK that was obtained the International Financial Statistics (IFS) tape, and was then converted to a common currency (Irish pound). The exchange rate was obtained from the Central Bank Bulletin of the Irish Central Bank. The second right hand side variable in Equation 9.1 is the measure of competitiveness. It is defined as the ratio of the Irish unit export value to that of the UK, denominated in Irish pounds. Data on the export unit value for both countries was again obtained from the IFS tape.

Finally, the measure of exchange rate volatility, as already has been discussed, is the moving standard deviation of the growth of the real exchange rate. The order of the moving average, $m$, is equal to 8. The real exchange rate is calculated by the ratio of the relative prices (unit export values) multiplied by the sterling exchange rate\(^3\). This measure of exchange rate volatility, is adopted by Kenen and Rodrik (1986), Koray and Lastrapes (1989) and Chowdhury (1993)\(^4\).

### 9.3.2 Unit Root Results

The first step in the analysis is to establish the order of integration of the variables in Equation 9.3. This is done using the ADF test, including up to 4 lagged differences. As has already been discussed in chapter 8, the issue of stationarity is of vital importance, especially in the case of the volatility measure. As a result, the Phillips-Perron (PP) test results are also reported. The unit root test results, both with and without a trend, are shown in table 9.1. As can be seen all variables are integrated of order one, I(1).

\(^3\) Appendix 1 gives a complete account of the data used in the study.

\(^4\) A complete discussion of the various exchange rate volatility measures and the empirical results is given in chapter 8.
9.3.3 Cointegration Results

Therefore, I can now proceed to the cointegration tests. The results of these tests are shown in table 9.2. The appropriate lag length in the VAR was chosen to minimise the Akaike Information Criteria (AIC) and the Swartz Bayesian Criterion (SBC). As can be seen from table 9.2, both the maximum eigenvalue and the trace test both confer that cointegration takes place. As the primary interest is the long-run relationship, the cointegration vector is taken and normalised on exports.

The cointegration vectors and likelihood ratio (LR) test statistics are reported below:

*Normalised Cointegrating Vector*

\[
\ln X_t = 5.75 \ln Y_t - 4.73 \ln P_t + 7.44 V_t
\]

*Likelihood Ratio Tests*

<table>
<thead>
<tr>
<th>(H_0:)</th>
<th>Statistic: (\chi^2(1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_1 = 0)</td>
<td>25.44*</td>
</tr>
<tr>
<td>(\beta_2 = 0)</td>
<td>24.98*</td>
</tr>
<tr>
<td>(\beta_3 = 0)</td>
<td>0.93</td>
</tr>
</tbody>
</table>

*Notes: The test for the null \(H_0: \beta_i = 0\) for the Equation \(\ln X_t = \beta_1 \ln Y_t + \beta_2 \ln P_t + \beta_3 V_t\) has a \(\chi^2(1)\) distribution under the null hypothesis. * Denotes significance at the 5% level.*

The cointegration coefficients can be interpreted as long-run export elasticity's. The relationship between Irish exports and foreign economic activity is positive, large and statistically significant. The figure for income elasticity also serve
to highlight the extent to which the economic health of a small open economy, such as Ireland, is positively dependent on economic growth of the UK. The competitiveness variable (price elasticity) is negative and significant for Irish exports.

As can be seen, the income and relative prices have the appropriate sign and are statistically significant, while the elasticity's are larger than what previous studies have found for overall Irish exports (see Caporale and Chui (1995) and McGettigan and Nugent (1995)). The higher elasticity's may be due to the fact that the UK is the most important trading partner for Ireland. Finally the volatility measure has a positive relationship with exports. This would satisfy the profile of an extreme risk averse trader, as depicted by DeGrauwe (1988) thesis. However the likelihood ratio test indicates that this coefficient is statistically insignificant.

9.3.4 Error Correction Models

The short-run export demand function was estimated using an ECM and the results are shown below;

<table>
<thead>
<tr>
<th>Lag</th>
<th>R(-1)</th>
<th>Δln X</th>
<th>Δln Y</th>
<th>Δln P</th>
<th>ΔV</th>
<th>Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.06 (3.13*)</td>
<td>-0.73 (-8.12*)</td>
<td>-0.55 (-2.58*)</td>
<td></td>
<td></td>
<td>R² = 0.60 AR = 1.74</td>
</tr>
<tr>
<td>2</td>
<td>0.28 (1.34)</td>
<td></td>
<td>-2.98 (-2.32)</td>
<td></td>
<td></td>
<td>ARCH = 1.05</td>
</tr>
</tbody>
</table>

Notes: Figures in parenthesis are the t-statistics. * Denotes significance at the 5% level.
Only the most significant lagged differenced terms are included in the model. Before I discuss the results, I give a brief account of the summary statistics. These tests indicate that the ECM’s are adequate for further analysis. The adjusted $R^2$ is 0.60, which compares well with those reported in other studies for regressions based on first differences in variables. The Breusch-Godfrey serial correlation (AR) test indicates the absence of serial correlation in the residuals of the estimated equations at the 5% level. Moreover, autoregressive conditional heteroskedasticity (ARCH) does not seem to be a problem according to the ARCH LM test.

The error correction term (ECT) has a positive sign, which indicates that exports do not restore the long-run equilibrium. A possible explanation for the positive sign is that multinational corporations (MNC’s), which make up a large part of Irish exports, are price setters and so the adjustment towards long-run equilibrium most likely takes place through the competitiveness measure and not exports. It has been shown by Murphy (1994) and Walsh (1996) that a large percentage of growth in output and exports may be traced to the activities of MNC’s. Both income and relative prices have the appropriate sign. The fact that the relative price coefficients is larger than the income coefficient indicates that Irish exports to the UK are more responsive to changes in relative prices than to changes in income in the UK.

Finally, and most important, I find that the measure of volatility has a negative sign and is statistically significant. This shows that in the short-run, exchange rate volatility has a negative effect on Irish exports to the UK. Again, as I did for the long-run results, I can draw on the actions of MNC’s, which make up a large part of Irish exports. I would not expect MNC’s to react to exchange rate volatility by engaging in market switching, in the sense of favouring the domestic market in times of increased exchange rate volatility. However, international plant switching is a viable option, when plants are not operating at full capacity. In other words, a short-run response to increased exchange rate volatility could result in increased servicing of a market from plants located in that country or in countries whose exchange rate exhibits less variability with respect to the currency of the destination country market.

5 The difference between the impact of exchange rate volatility on trade by MNC’s and indigenous industries will be the main focus in chapter 10.
This result carries some important policy implications. A European monetary system that would possibly include both Ireland and the UK would eliminate all uncertainty associated with bilateral exchange rate changes and therefore, boost Irish exports to the UK.

9.4 Conclusions

This chapter has focused on modelling short-run and long-run export demand functions for Irish exports. I concentrate on Irish exports to the UK as they represent the most important component of Irish export activity. The results suggest that exports are very sensitive to income and relative prices changes, and in particular in the long-run. With respect to the influence of exchange rate volatility on exports, the reported results indicate that in the long-run, the influence is insignificant. However in the short-run exchange rate volatility and the associated uncertainty has a negative effect on real exports.

From figure 9.1, it is clear that the UK is an important trading partner for Ireland. However, Ireland's trade is increasingly focusing on other EU countries. An interesting extension would be to analyse the impact of exchange rate volatility on Irish exports to its key European trading partners. A second extension is to consider the fact that the Irish export sector is dualistic, in that it is characterised by two types of firms; small scale indigenous enterprises exporting low-technology goods and subsidiaries of multinationals, whose exports are characterised by their high technology. The importance of MNC's has already been discussed in this chapter, as a possible reason for the reported long-run and short-run impact of exchange rate volatility. Therefore, in the next chapter I will estimate separate export functions for these two types of enterprises for Irish exports to its key EU trading partners. I will
also estimate the function for overall exports, as comparison to the study carried out in this chapter.
### Table 9.1

**Dickey-Fuller Tests**

<table>
<thead>
<tr>
<th></th>
<th>Levels differences</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_{\mu}$</td>
<td>$\tau_{\tau}$</td>
<td>$\tau_{\mu}$</td>
</tr>
<tr>
<td>E</td>
<td>-0.99</td>
<td>-2.27</td>
<td>-14.67*</td>
</tr>
<tr>
<td>Y</td>
<td>-2.27</td>
<td>-2.42</td>
<td>-6.32*</td>
</tr>
<tr>
<td>P</td>
<td>-2.09</td>
<td>-2.20</td>
<td>-6.59*</td>
</tr>
<tr>
<td>V</td>
<td>-2.39</td>
<td>-2.17</td>
<td>-6.80*</td>
</tr>
</tbody>
</table>

### Phillips-Perron Tests

<table>
<thead>
<tr>
<th></th>
<th>Levels differences</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_{\mu}$</td>
<td>$\tau_{\tau}$</td>
<td>$\tau_{\mu}$</td>
</tr>
<tr>
<td>E</td>
<td>-1.22</td>
<td>-2.38</td>
<td>-21.37*</td>
</tr>
<tr>
<td>Y</td>
<td>-2.29</td>
<td>-2.42</td>
<td>-8.85*</td>
</tr>
<tr>
<td>P</td>
<td>-2.06</td>
<td>-2.16</td>
<td>-5.72*</td>
</tr>
<tr>
<td>V</td>
<td>-2.32</td>
<td>-2.06</td>
<td>-9.79*</td>
</tr>
</tbody>
</table>

**Note:** The augmented Dickey-Fuller (ADF) is reported with and without a time trend, with lags equal to 4. The regressions are free of serial correlation. The Phillips-Perron (1988) statistic with correction for up to 4th order serial correlation using the Newey-West (1987) lag window is also reported. A * implies significance at 5%. The critical values for both test statistics are, for the no trend and trend models, -2.91 and -3.49, respectively.
Table 9.2

Cointegration Test Results

<table>
<thead>
<tr>
<th>H₀:</th>
<th>Maximum Eigenvalue Test</th>
<th>Trace Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>38.72*</td>
<td>54.21*</td>
</tr>
<tr>
<td>r = 1</td>
<td>11.22</td>
<td>15.50</td>
</tr>
<tr>
<td>r = 2</td>
<td>4.02</td>
<td>4.27</td>
</tr>
<tr>
<td>r = 3</td>
<td>0.26</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Notes: * Denotes significance at 5%.
Figure 9.1: Total Irish Exports

Annual Trade Data: 1979-1994
Source: CSO Trade Statistics
Figure 9.2: Ireland's Trade Weights

Annual Trade Data: 1979-1994
Source: CSO Trade Statistics
Figure 9.4: UK Real Income (in logarithms)
Figure 9.5: Competitiveness Measure
(Ratio of Irish Unit export value to that of the UK - in logarithms)
Figure 9.6: Volatility Measure
(Moving Standard Deviation of the Growth of the Real Exchange Rate)
CHAPTER 10

EXCHANGE RATE VOLATILITY AND EXPORTS TO THE EU

10.1 INTRODUCTION

The international trade performance of a small open economy (SOE), such as Ireland, plays a central role in the economic health of the country. The share of Irish merchandise exports in Gross Domestic Product (GDP) has grown dramatically in recent years (from 43% in 1979 to a level of 82.8% in 1998), thus rendering the economy more open than before and more dependent on foreign markets. Hence, policies designed to enhance export performance are of increasing importance to national economic welfare. Good policy decisions are assisted by having relevant information on the factors that determine the level of exports and imports. In this chapter, I examine long-run and short-run Irish export demand by the country's most important trading partners; that is to say, by the principal member states of the European Union (EU).

There have been different empirical studies of the determinants of Irish exports. A common feature of most of these studies is their use of traditional estimation methods; in other words, classical regression techniques (see, for example, O'Connell, 1978, Browne, 1982, Lynch, 1983, and Flynn, 1984). More recent studies by Caporale and Chui (1995) and McGettigan and Nugent (1995) adopted more advanced estimation techniques that recognise
the non-stationarity of economic variables. In this chapter I continue in the recent tradition by treating exports and their determinants as potentially non-stationary variables. In contrast to all previous studies, the effect of exchange rate volatility on exports is explicitly considered. This is of immense contemporary policy significance given the single European currency.

Theory does not help one to determine the effect of exchange rate variability on trade flows. Volatility can increase trade or reduce trade depending, among other things, on the degree of risk aversion displayed by exporters. Risk aversion depends on, inter-alia, the resources of an enterprise, its profit margin and its strategic options. The tighter a firm's profit margin and more limited its strategic options, the more likely it is that its behaviour will be characterised by risk aversion. The Irish export sector is dualistic, in that it is characterised by two types of firms; small scale indigenous enterprises exporting low-technology goods and subsidiaries of multinationals, whose exports are characterised by their high technology.

Given the dualistic character of the Irish export sector, it was considered appropriate to estimate separate export functions for these two types of enterprises (besides a general export function). In this, I was facilitated by the fact that sectors occupied by indigenous firms and multinationals are, in general, mutually exclusive. The determinants of exports often tend to have a lagged effect. This is taken into account, by estimating both long-run and short-run export functions using the techniques of cointegration and error-correction models (ECMs). The sample period commences with Ireland's entry into the European Monetary System (EMS), which resulted in a reduction in exchange rate variability between the Irish pound and other EMS currencies.

The chapter is organised as follows. In section 10.2, I outline the approach to modelling the impact on Irish exports to the EU as a result of

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1 See chapter 8 for a full discussion on the theoretical and empirical studies on the impact of exchange rate volatility on trade.
exchange rate volatility. The empirical results are covered in section 10.3. Finally section 10.4 concludes the chapter.

10.2 MODELLING EXPORTS TO THE EU

As has been discussed in the previous chapter, the empirical literature on the estimation of export functions uses the following long-run export function (see, e.g., Arize, 1995 and 1997 and Chowdhury, 1993):

\[ \ln X_t = \beta_0 + \beta_1 \ln Y_t + \beta_2 \ln P_t + \beta_3 V_t \]  

(10.1)

Where;  
\( X_t \) = real exports  
\( Y_t \) = real foreign income  
\( P_t \) = relative prices (a measure of competitiveness)  
\( V_t \) = exchange rate volatility

In this chapter, cointegration analysis is again used to test for a long-run export function of Irish exports to the EU. Tests for cointegration require nonstationary time series of the same order of integration. Therefore, I first test for the presence of a unit root in both the level and the first difference of the four variables in Equation 10.1, using the Augmented Dickey-Fuller (ADF) tests (Fuller, 1976 and Dickey and Fuller, 1979) and the Phillips-Perron (Phillips and Perron, 1988) approach. The method used to test for cointegration is the Johansen procedure introduced in Johansen (1988) and extended in Johansen and Juselius (1990). In the Johansen framework, all variables, including exchange rate volatility, are treated as endogenous. The treatment of volatility as an endogenous variable is particularly important in
the context of the EU where Central Banks have tried systematically to stabilise the nominal exchange rates against the DM and hence against the currencies of the other ERM-member countries. Provided that cointegration exists among our variables, the cointegrating vector is normalised on exports to give the long-run income and relative price elasticity’s for export demand.

As in the previous chapter I also estimate the short-run export equation using the ECM:

\[
\Delta \ln X_t = \alpha_0 + \alpha_1 R_{t-1} + \sum_{i=1}^{n} \gamma_i \Delta \ln X_{t-i} + \sum_{i=1}^{n} \delta_i \Delta \ln Y_{t-i} + \sum_{i=1}^{n} \epsilon_i \Delta \ln P_{t-i} + \sum_{i=1}^{n} \zeta_i \Delta V_{t-i} + \epsilon_t
\]

(10.2)

If the variables are cointegrated, then the ECM will be of the above form, where \( R_{t-1} \) is the ECT (i.e., the lagged residual from the cointegration regression).

Finally, as a measure of time-varying exchange rate volatility I use the moving sample standard deviation of the growth rate of the real effective exchange rate:

\[
V_t = \left[ \frac{1}{m} \sum_{i=1}^{m} (\ln Z_{t+i-1} - \ln Z_{t+i-2})^2 \right]^{1/2}
\]

(10.3)

Where \( Z \) is the real effective exchange rate and \( m \), the order of the moving average, is set equal to 8. This measure of exchange rate volatility is

\footnote{It is also the case in this study, that the main results are robust to alternative choices of lag length.}

10.3 EMPIRICAL RESULTS

10.3.1 Data

The sample period covers the period 1979Q2-1992Q4. As mentioned earlier, the main aim is to estimate the short-run and long-run function for Ireland's exports to the EU, since the launch of the EMS in March 1979. To accomplish this objective, the sample period starts in the second quarter of 1979 and the analysis covers Irish exports to the EU alone. As shown in figure 10.1, these exports make up the majority of Irish exports. The export variable is taken from the Trade Statistics Series of the CSO publication, and was divided by Ireland's unit export value to obtain the real exports figure. The aggregate figure of Ireland's exports to the EU is split up into SITC (Standard Industry Trade Classification) divisions 0-4 and 5-8. Division 5-8 is the standard definition of manufacturing exports. However, 5-8 is also the division where MNC's are very prominent.

Murphy (1994) and Walsh (1996) argue that a large percentage of Irish output and export growth may be traced to the activities of MNC's in three specific areas; computer and related areas, chemicals (including pharmaceuticals) and cola concentrates. I can therefore distinguish between exports of SITC 0-4 (dominated by indigenous industries) and exports of SITC 5-8 (dominated by the MNC's). I also employ the total figure SITC 0-8 in the empirical analysis.

The first explanatory variable in the export function is foreign income. This series is constructed by taking the weighted average of the GDP series of Ireland's five most important EU trading partners. The EU-5 are in order of

\[ \text{EU-5 are in order of} \]

\[ \text{Choosing the weighted average of the income levels of the most important trading partners is standard procedure in the literature (see Lynch, 1983 and Chowdhury, 1993).} \]
importance, UK, Germany, France, Netherlands and Italy. The trade weights are calculated by aggregating the export and import figure for each particular country and then dividing by the aggregate figure for exports and imports for all countries. These weights are given in figure 10.2. The quarterly GDP data were obtained from the International Financial Statistics (IFS) tape, and was then converted to a common currency (Irish pound). The exchange rate was obtained from the Central Bank Bulletin.

The second right-hand side variable in Equation 10.1 is a measure of competitiveness. It is defined as the ratio of the exchange rate-adjusted price of Irish exports to the price of exports of Ireland's major trading partners, as defined above. Hence, it is the ratio of the Irish unit export value to the weighted average of the unit export values of the EU-5, denominated in Irish pounds. The weights are identical to those used in the construction of the income variable. Data for the export unit value was again obtained from the IFS tape. Finally, as a measure of time-varying exchange rate volatility, I use the moving standard deviation of the growth rate of the real effective exchange rate, as was used in chapter 9. The real effective exchange rate is calculated by the weighted average of the exchange rate-adjusted relative prices (unit export values) where the trade weights are the ones used in creating foreign income and relative prices.

10.3.2 Unit Root Results

The first step in the analysis is to establish the order of integration of the variables in Equation 10.1. This is done using the ADF test, including four lagged differences and the PP test. The unit root test results, both with

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4 Unit export value data were not available in disaggregated form and, therefore, the same aggregate figure was used for both divisions 0-4 and 5-8.

5 Although we use real exchange rates to calculate our volatility measure, Thursby and Thursby (1987) and Lastrapes and Koray (1990) obtain similar results when using nominal exchange rates instead.

6 Appendix 1 gives a complete account of the data used in the study.
and without a trend, are shown in table 10.1. As can be seen all variables are integrated of order one, I(1). Therefore, I can now proceed to the cointegration tests.

10.3.3 Cointegration Results

The results of these tests are shown in table 10.2. The lag depth is chosen to minimise the AIC and the SBC. As shown in table 10.2, cointegration takes place for all three groups. The results conclusively suggest that there is 1 cointegrating vector. The cointegration vector is taken and normalised on exports.

The cointegration vectors and likelihood ratio (LR) test statistics are given in table 10.3. The cointegration coefficients can be interpreted as long-run export elasticities. The relationship between Irish exports and foreign economic activity is positive, large and statistically significant, especially on those sectors (SITC 5-8) dominated by MNC's. The latter is to be expected, as exports of foreign corporations located in Ireland are generally high-technology products, which tend to be highly income elastic. The figures for income elasticity also serve to highlight the extent to which the economic health of a small open economy, such as Ireland, is positively dependent on economic growth in our main EU partner countries.

The competitiveness variable (price elasticity) is negative and significant for Irish exports in general and, more particularly, for Irish exports in sectors dominated by multinationals (SITC 5-8). The relative price variable is insignificant in sectors where indigenous firms are prominent (SITC 0-4). These results are at first glance surprising, when one recalls that Flynn (1984) dropped relative prices from his estimation because of the way in which MNC's make decisions. However, if the Irish real exchange rate falls, then Ireland becomes a relatively lower cost location, which would make it more attractive as a site for MNC activity and vice versa. This would explain the long-run negative price elasticity in sectors 5-8 and for the
economy generally (given the prominence of MNC exports in overall exports). The insensitivity of indigenous exports to relative prices is harder to rationalise. The data sample covers a period characterised by poor performance by the indigenous sector as a whole (notwithstanding a depreciation of the Irish pound against sterling). This has been attributed to other factors, such as poor quality products and a lack of marketing expertise.

The volatility measure is positive and statistically significant for overall exports, for the exports of indigenous firms (SITC 0-4), and the exports of the MNC's (SITC 5-8). The results suggest that the firms are very risk averse. When firms are very risk averse, an increase in volatility should lead to an increase in exports. Irish indigenous firms tend to be relatively small and they operate on tight margins. They would satisfy the profile of very risk averse firms. MNC's that locate in Ireland do so to export to countries of the EU. A more plausible explanation for the positive and statistically significant findings in terms of MNC's, is that they are involved in international plant switching. Therefore, increased exchange rate volatility could result in increased servicing of a market from plants located in that country or in countries whose exchange rate exhibits less variability with respect to the currency of the destination country market.

10.3.4 Error Correction Model Results

The estimation of the ECM will give information on the short-run export function. The results are shown in table 10.4. To decide on the final form of the ECM, I initially started with four lagged differences of each variable and then deleted the insignificant lagged variables. Variables were not deleted if this introduced autocorrelation. Before I discuss the results, I give a brief account of the adequacy of the ECM's. For this reason, I performed a number of tests reported in the last column of table 10.3. These tests indicate that the ECM's are adequate for further analysis. The adjusted $R^2$ ranges from 0.65 to 0.74. Such values compare well with those reported in
other studies for regressions based on first differences in variables. The Breusch-Godfrey serial correlation (AR) test indicates the absence of serial correlation in the residuals of the estimated equations at the 5% level. Moreover, autoregressive conditional heteroskedasticity (ARCH) does not seem to be a problem according to the ARCH LM test.

The ECT shows the adjustment speed towards the elimination of disequilibrium and is expected to be negative. The coefficient is statistically significant in the case of overall exports (SITC 0-8), but has a positive sign. A possible explanation for the positive sign is that MNC's which make up a large part of exports in SITC 0-8 are price setters and so the adjustment towards the long-run equilibrium takes place through the competitiveness measure and not exports\(^7\).

The income and price elasticities for overall Irish exports are positive and negative, respectively, while the magnitudes are less in the short run compared to the long run. These results are as one would expect. The large sign on the income variable for each of the trade classification indicates the ability to respond to changes in demand. Relative prices have a negative impact on trade and are statistically significant in all cases, unlike the reported results for the long-run.

By contrast with the long-run results, the short-run volatility measure is negative and statistically significant for Irish exports from the indigenous sector and for the exports of the MNC dominated sectors, while it is statistically insignificant for overall exports\(^8\). I would not expect MNC's to react to exchange rate volatility by engaging in market switching, in the sense of favouring the domestic market in times of increased exchange rate volatility. However, international plant switching is a viable option, when plants are not operating at full capacity. In other words, a short-run response to increased exchange rate volatility could result in increased servicing of a market from plants located in that country or in countries whose exchange

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\(^7\) A similar finding was reported in chapter 9 for exports to the UK.

\(^8\) In the case of the SITC 0-4, the volatility variable is significant at the 10% level.
rate exhibits less variability with respect to the currency of the destination country market.

10.3.5 Interpretations and Further Results

The sample period analysed above, commences with Ireland's entry into the European Monetary System (EMS), which resulted in a reduction in exchange rate variability between the Irish pound and other EMS currencies. As an extension to the above analysis, I also report the results of estimating the model over a larger sample period. The motivation for this extension is to test whether the use of data outside the EMS period, which is characterised by greater exchange rate volatility, will effect the reported results. As a result the model is re-estimated using an extended sample; 1978Q3 – 1995Q4.

The cointegration test results are reported in table 10.5. The appropriate lag length in the VAR was again chosen on the basis of the AIC and the SBC. As can be seen from table 10.5, cointegration takes place for all three groups. Using the extended sample, the results conclusively suggest that there is 1 cointegrating vector.

The cointegration vectors and likelihood ratio (LR) test statistics are given in table 10.6. The relationship between Irish exports and foreign economic activity is again positive, large and statistically significant, especially on those sectors (SITC 5-8) dominated by MNC’s. This result is consistent with the previous smaller sample and is intuitive given that the exports of foreign corporations located in Ireland are generally high-technology products, which tend to be highly income elastic. The income elasticity for overall exports again exceeds the results from previous studies, e.g. Caporale and Chui (1995), reflecting the increasing income sensitivity of Irish exports to the EU and the increasing importance of MNC's in Irish exports. Of course, part of the difference is accounted for by the exclusive focus, in contrast to other studies, on exports to the EU alone.
CHAPTER 10: EXCHANGE RATE VOLATILITY AND EXPORTS TO THE EU

The competitiveness variable (price elasticity) is negative and significant for each of the trade classifications. This is not a surprising result especially when the MNC's are considered. If the Irish real exchange rate falls, then Ireland becomes a relatively lower cost location, which would make it more attractive as a site for MNC activity and vice versa. The earlier results, using the smaller sample, found that there was not a statistically significant relationship between relative prices and exports from the indigenous sector. It was argued that this was due to the poor performance by the indigenous sector as a whole over this period. With the extra 3 years, which have been characterised by dramatic economic growth, there has also been extensive restructuring of the industrial bodies, which promote international trade. For example the industrial development authority (IDA) has been split into two separate bodies, one which supports and promotes foreign direct investment, IDA Ireland, and the other which supports indigenous industry, Forbairt. As a result there has been intensive development of products and improved marketing techniques. The reported estimates of the relative price elasticity are also higher than those obtained by some previous studies (e.g. Caporale and Chui, 1995) but in broad agreement with Lynch (1983). However as has already been mentioned, the results are not directly comparable as, in contrast with these studies that looked at overall Irish exports, this study focuses attention on Irish exports to the EU.

The volatility variable is positive and significant for each of the trade classifications. These results are consistent with those found using the smaller data sample in the last section. The results indicate that the firms are very risk averse and an increase in volatility leads to an increase in exports. This would certainly satisfy Irish indigenous firms, which tend to be relatively small and operate on tight margins. A more plausible explanation for MNC's, is that they are involved in international plant switching. Therefore, increased exchange rate volatility could result in increased servicing of a market from plants located in that country or in countries whose exchange rate exhibits less variability with respect to the currency of the destination country market.
Using the cointegrating vectors normalised on exports, I again estimated the ECM’s that provide information on the short-run export functions. The results are shown in table 10.7. As previously, I provide a summary of the diagnostic tests and the model does not appear to suffer from mis-specification. The ECT shows the adjustment speed towards the elimination of disequilibrium and is expected to be negative. The coefficient has a negative sign in all cases, but is only statistically significant for SITC 0-4. The negative sign on the ECT for the indigenous classification verifies that the term is in fact equilibrium correcting. For the MNC and the overall classification, the term is negative, but statistically insignificant. A possible explanation for the insignificant coefficient is that MNC’s, which make up a large part of exports in SITC 5-8 (and, therefore, SITC 0-8), are price setters and so the adjustment towards the long-run equilibrium takes place through the competitiveness measure and not export volumes.

The income elasticity’s for overall Irish exports are positive while the magnitudes are less in the short run compared to the long run. Moreover, the short-run income elasticity of export demand is higher for the output of MNC’s than for the output of indigenous firms. Apart from the differing nature of the products in both classifications (which also explains the differences in long-run income elasticities), a larger short-run income elasticity for the MNC sector, compared to the indigenous sector, indicates a greater ability to respond to changes in demand. This would be consistent with the existence of horizontally integrated plants in different countries operating at undercapacity. The price elasticity tends to have no effect on the indigenous and MNC sector, but has a negatively significant effect on overall exports.

In contrast with the long-run results, the short-run volatility measure has a negative sign, but is statistically insignificant. One would not expect MNC’s to react to exchange rate volatility by engaging in market switching, in the sense of favouring the domestic market in times of increased exchange rate volatility. As a result, in the short-run there is no effect on MNC exports. The short-run exports of indigenous firms may be insensitive to exchange rate
variability because of (i) pre-existing contracts that have to be honoured and (ii) the hedging activities of such firms when they enter into such contracts.

10.4 CONCLUSIONS

In this chapter, I analyse the long-run and short-run relationship between export volume and its determinants; namely relative prices, foreign income and exchange rate variability, using the techniques of cointegration and error-correction methods. The model was estimated for Irish exports and sectoral exports SITC 0-4 and SITC 5-8 to the EU. The sectoral classification corresponds to the exports of mainly indigenous Irish firms and multinationals, respectively. The initial sample period extends from the second quarter of 1979 to the fourth quarter of 1992; from the launch of the EMS to its effective disintegration. The model is also estimated using an extended sample size to take into account the higher exchange rate volatility outside the EMS period, 1978Q3 – 1995Q4.

The empirical findings are interesting, in that exchange rate volatility is shown to have a negative effect on the exports of the multinational sector and to a lesser extent on the indigenous sector. In contrast, when using the larger sample, the volatility variables is not statistically significant in any of the trade classifications.

In the long-run, volatility has a positive effect on the exports from each of the trade classifications. Again focusing on the larger sample, the results are consistent. In all of the classifications, I find that volatility is both positive and statistically significant. Although the indigenous sector may be considered to be very risk averse, the MNC’s are viewed as adopting a programme of international plant switching.

These results allow one to tentatively conclude that the long-run export behaviour of multinationals will not change, with Ireland included in a single currency. The results conclusively conclude that in the long-run
Ireland's exports have not been adversely affected by exchange rate volatility.

In fact the results lend support to the view that volatility leads to greater trade.
Table 10.1

Dickey-Fuller Tests

<table>
<thead>
<tr>
<th></th>
<th>Levels ( \tau_\mu )</th>
<th>( \tau_\tau )</th>
<th>differences ( \tau_\mu )</th>
<th>( \tau_\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(0-4)</td>
<td>-0.44</td>
<td>-3.21</td>
<td>-3.43*</td>
<td>-3.74*</td>
</tr>
<tr>
<td>E(5-8)</td>
<td>-0.49</td>
<td>-2.22</td>
<td>-3.53*</td>
<td>-3.52*</td>
</tr>
<tr>
<td>E(0-8)</td>
<td>-0.82</td>
<td>-3.33</td>
<td>-18.70*</td>
<td>18.51*</td>
</tr>
<tr>
<td>Y</td>
<td>-1.72</td>
<td>-0.77</td>
<td>-4.65*</td>
<td>-4.78*</td>
</tr>
<tr>
<td>P</td>
<td>-2.44</td>
<td>-2.98</td>
<td>-5.94*</td>
<td>-5.90*</td>
</tr>
<tr>
<td>V</td>
<td>-2.79</td>
<td>-2.88</td>
<td>-3.79*</td>
<td>-3.82*</td>
</tr>
</tbody>
</table>

Phillips-Perron Tests

<table>
<thead>
<tr>
<th></th>
<th>Levels ( \tau_\mu )</th>
<th>( \tau_\tau )</th>
<th>differences ( \tau_\mu )</th>
<th>( \tau_\tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(0-4)</td>
<td>-0.58</td>
<td>-3.23</td>
<td>-5.91*</td>
<td>-5.07*</td>
</tr>
<tr>
<td>E(5-8)</td>
<td>-0.45</td>
<td>-2.44</td>
<td>-10.43*</td>
<td>-10.30*</td>
</tr>
<tr>
<td>E(0-8)</td>
<td>-0.83</td>
<td>-2.65</td>
<td>-30.42*</td>
<td>-31.45*</td>
</tr>
<tr>
<td>Y</td>
<td>-1.95</td>
<td>-0.56</td>
<td>-5.71*</td>
<td>-6.13*</td>
</tr>
<tr>
<td>P</td>
<td>-2.53</td>
<td>-3.40</td>
<td>-5.88*</td>
<td>-5.84*</td>
</tr>
<tr>
<td>V</td>
<td>-2.83</td>
<td>-3.34</td>
<td>-3.69*</td>
<td>-3.63*</td>
</tr>
</tbody>
</table>

Note: The augmented Dickey-Fuller (ADF) is reported with and without a time trend, with lags equal to 4. The regressions are free of serial correlation. The Phillips-Perron (1988) statistic with correction for up to 4th order serial correlation using the Newey-West (1987) lag window is also reported. A * implies significance at 5%. The critical values for both test statistics are, for the no trend and trend models, -2.91 and -3.49, respectively.

Table 10.2
Cointegration Test Results

<table>
<thead>
<tr>
<th></th>
<th>Maximum Eigenvalue Test</th>
<th>Trace Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stc 0-4</td>
<td>Stc 5-8</td>
</tr>
<tr>
<td>H₀: r = 0</td>
<td>r ≤ 1</td>
<td>r ≤ 2</td>
</tr>
<tr>
<td>SITC 0-4</td>
<td>38.10*</td>
<td>16.61</td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>28.59*</td>
<td>9.24</td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>28.73*</td>
<td>7.24</td>
</tr>
<tr>
<td></td>
<td>r ≤ 2</td>
<td>r ≤ 3</td>
</tr>
<tr>
<td>SITC 0-4</td>
<td></td>
<td>4.66</td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>7.44 4.57</td>
<td>4.57</td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>4.26 2.83</td>
<td>2.83</td>
</tr>
</tbody>
</table>

Note: * Denotes significance at 5 %.

### Table 10.3

**Cointegration Vectors and Likelihood Ratio Tests**

<table>
<thead>
<tr>
<th>Export Divisions</th>
<th>Normalised Cointegration Vectors</th>
<th>$H_0: \beta_1=0$</th>
<th>$H_0: \beta_2=0$</th>
<th>$H_0: \beta_3=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 0-4</td>
<td>$X_t=2.34Y_t - 0.22P_t + 15.36V_t$</td>
<td>8.20*</td>
<td>0.12</td>
<td>16.49*</td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>$X_t = 4.00Y_t - 1.24P_t + 8.48V_t$</td>
<td>11.77*</td>
<td>5.71*</td>
<td>10.44*</td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>$X_t = 3.86Y_t - 1.82P_t + 10.25V_t$</td>
<td>10.27*</td>
<td>12.59*</td>
<td>12.91*</td>
</tr>
</tbody>
</table>

**Note:** The test $H_0: \beta_r=0$ for the Equation

$X_t = \beta_0 + \beta_1Y_t + \beta_2P_t + \beta_3V_t$ has a $\chi^2(1)$ distribution under the null hypothesis.

* Denotes at the 5% level.

**Sample period 1979Q2 – 1992Q4**
Table 10.4

Regression Results for Error-Correction Models

<table>
<thead>
<tr>
<th>Export Divisions</th>
<th>lag</th>
<th>R(-1)</th>
<th>ΔX</th>
<th>ΔY</th>
<th>ΔP</th>
<th>ΔV</th>
<th>Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 0-4</td>
<td>1</td>
<td>-0.10</td>
<td>-0.64</td>
<td>-0.99</td>
<td></td>
<td></td>
<td>$\bar{R}^2 = 0.73$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.61)</td>
<td>(4.28)**</td>
<td>(1.66)*</td>
<td></td>
<td></td>
<td>AR=7.15 (0.55)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ARCH=2.72 (0.61)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.28</td>
<td></td>
<td></td>
<td></td>
<td>-4.27</td>
<td>(1.65)*</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.60</td>
<td>1.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>1</td>
<td>0.03</td>
<td>-0.72</td>
<td>0.81</td>
<td>-0.64</td>
<td>-2.30</td>
<td>$\bar{R}^2 = 0.83$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.42)</td>
<td>(5.36)**</td>
<td>(1.85)*</td>
<td>(2.63)**</td>
<td>(2.10)**</td>
<td>AR = 7.86 (0.10)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>-0.69</td>
<td></td>
<td></td>
<td></td>
<td>ARCH= 1.03 (0.31)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.63</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.27</td>
<td>0.79</td>
<td>-0.73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>1</td>
<td>0.21</td>
<td>-1.09</td>
<td>1.14</td>
<td>-1.09</td>
<td></td>
<td>$\bar{R}^2 = 0.84$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.43)**</td>
<td>(7.69)**</td>
<td>(2.36)*</td>
<td>(4.00)**</td>
<td></td>
<td>AR=1.94 (0.75)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.59</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
<td>ARCH=0.01 (0.76)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.46</td>
<td>0.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.25</td>
<td></td>
<td></td>
<td>-0.72</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Figures in parentheses are the absolute t-statistics. * and ** denote significance at the 10% and 5% level, respectively. The LM(4) test statistic for autocorrelation (AR) and the LM(4) test statistic for autoregressive conditional heteroskedasticity (ARCH) are reported. P-values are given in parentheses.

Table 10.5
Cointegration Test Results

Maximum Eigenvalue Test

<table>
<thead>
<tr>
<th>H₀:</th>
<th>r = 0</th>
<th>r ≤ 1</th>
<th>r ≤ 2</th>
<th>r ≤ 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 0-4</td>
<td>37.90*</td>
<td>12.93</td>
<td>9.24</td>
<td>1.06</td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>48.49*</td>
<td>13.70</td>
<td>7.33</td>
<td>0.17</td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>46.04*</td>
<td>13.11</td>
<td>7.98</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Trace Test

<table>
<thead>
<tr>
<th>H₀:</th>
<th>r = 0</th>
<th>r ≤ 1</th>
<th>r ≤ 2</th>
<th>r ≤ 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 0-4</td>
<td>61.13*</td>
<td>23.23</td>
<td>10.31</td>
<td>1.06</td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>69.69*</td>
<td>21.20</td>
<td>7.50</td>
<td>0.17</td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>67.25*</td>
<td>21.22</td>
<td>8.11</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Notes: * Denotes significance at 5 %.

### Table 10.6
Cointegration Vectors and Likelihood Ratio Tests

<table>
<thead>
<tr>
<th>Export Divisions</th>
<th>Normalised Cointegration Vectors</th>
<th>$H_0: \beta_1=0$</th>
<th>$H_0: \beta_2=0$</th>
<th>$H_0: \beta_3=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 0-4</td>
<td>$\ln X_t = 2.77 \ln Y_t - 0.88 \ln P_t + 8.15 V_t$</td>
<td>23.15*</td>
<td>5.10*</td>
<td>5.79*</td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>$\ln X_t = 3.98 \ln Y_t - 0.77 \ln P_t + 8.12 V_t$</td>
<td>31.38*</td>
<td>6.33*</td>
<td>8.14*</td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>$\ln X_t = 3.61 \ln Y_t - 0.84 \ln P_t + 7.85 V_t$</td>
<td>29.92*</td>
<td>8.26*</td>
<td>8.56*</td>
</tr>
</tbody>
</table>

**Note:** The test $H_0: \beta_i=0$ for the equation

$\ln X_t = \beta_0 + \beta_1 \ln Y_t + \beta_2 \ln P_t + \beta_3 V_t$ has a $\chi^2(1)$ distribution under the null hypothesis.

* Denote significance at the 5% level.

Table 10.7

Regression Results for Error-Correction Models

<table>
<thead>
<tr>
<th>Export Divisions</th>
<th>lag</th>
<th>R(-1)</th>
<th>ΔlnX</th>
<th>ΔlnY</th>
<th>ΔlnP</th>
<th>ΔV</th>
<th>Summary Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>SITC 0-4</td>
<td>1</td>
<td>-0.47 (2.42**)</td>
<td>-0.78 (4.46**)</td>
<td>-0.98 (1.38)</td>
<td>-5.44 (0.80)</td>
<td></td>
<td>$R^2 = 0.72$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AR = 3.05 (0.55)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ARCH = 6.58 (0.16)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.58 (3.26**)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.32 (2.67**)</td>
<td>1.64 (1.69*)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SITC 5-8</td>
<td>1</td>
<td>-0.03 (0.30)</td>
<td>-0.87 (6.98**)</td>
<td>-0.51 (1.87)</td>
<td>2.69 (0.83)</td>
<td></td>
<td>$R^2 = 0.65$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AR = 1.65 (0.80)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ARCH = 1.34 (0.85)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.73 (6.00**)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.52 (3.07**)</td>
<td>2.17 (3.80**)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SITC 0-8</td>
<td>1</td>
<td>-0.09 (0.75)</td>
<td>-0.98 (7.18**)</td>
<td>-0.84 (1.98**)</td>
<td>1.19 (0.33)</td>
<td></td>
<td>$R^2 = 0.69$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>AR = 0.60 (0.96)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ARCH = 0.98 (0.92)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-0.69 (4.58**)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-0.39 (3.53**)</td>
<td>2.32 (3.77**)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Figures in parentheses are the absolute t-statistics. * and ** denote significance at the 10% and 5% levels, respectively. The LM(4) test statistic for autocorrelation (AR) and the LM(4) test statistic for autoregressive conditional heteroskedasticity (ARCH) are reported. P-values are given in parentheses.

**Figure 10.1: Total Irish Exports**

- **ROW**: 20%
- **US**: 5%
- **EU**: 72%

Annual Trade Data: 1979-1994
Source: CSO Trade Statistics
Annual Trade Data: 1979-1994
Source: CSO Trade Statistics
Figure 10.3: Irelands Trade Weights (Revised)

Annual Trade Data: 1978-1995
Source: CSO Trade Statistics
CHAPTER 11

CONCLUSIONS

11.1 CONCLUSIONS OF PART I

In Part I of thesis I test the EH of the term structure of interest rates using a number of different methods. I use a high quality data set, modern econometric techniques and number a number of alternative methods to test whether the data is consistent with the theory. This study of the term structure of Irish interest rates represents the first known study using Irish data. Techniques used in Part I, include two and three variable VAR’s, cointegration-ECM’s, and Monte Carlo experiments.

A number of single equation methods are reported in chapter 4. The first part of this chapter concentrated on the perfect foresight spread equations. The unit root tests on the Irish interest rate series at the short end of the maturity spectrum are non-stationary, but integrated of order one. I have focused on tests of whether the spread predicts future changes in short rates at a number of interest rate maturities. In all cases we do not reject the null of $H_0 : \beta = 1$ or that information, available at time $t$ or earlier does not incrementally add to the predictions of future interest rates. This is the case for each of the chosen interest rate maturity combinations, at the short end of the spectrum. The results therefore do not reject the EH + RE.

This chapter has also looked at some alternative single equation tests of the EH. Focusing in particular on the 6 month and 3 month maturities, I initially test the model using OLS. However given the previous evidence, that OLS estimation of the actual
change in the short rate on the lagged yield spread plus a constant, results in coefficients which are often both the wrong size and sign (Mankiw and Miron, 1986), I also use IV and GMM estimation. Finally, I confirm the findings of other studies, using Monte Carlo experiments, that when the single equations are written as the regression of the change in the short-term rate on the lagged spread is prone to severe over-rejection of the EH. However I also show that tests of the spread on the first difference of the short-rate reject at the correct rate.

In chapter 5, I test the EH of the term structure of interest rates under the weak assumptions of an I(0) term premium and I(0) forecast errors (which need not necessarily result from agents using RE). The EH implies that given γ yields (they are assumed to be I(1)), then (γ-1) yield spreads should form the basis of the cointegrating space. This hypothesis is tested using short-term interest rate data from Ireland. Overall the results are extremely favourable. Using a number of different estimation techniques, the data would appear to be consistent with the theory. OLS estimation as well Phillips-Hansen fully modified approach were adopted and found weak evidence in favour of the EH. Johansen's systems based approach was also adopted. For γ interest rates, I found that the rank of the cointegrating space is (γ-1). The test of the joint null hypothesis that the set of (γ-1) form the basis of for the cointegration space is also not rejected. The results for the ECM are also supportive of the EH. On balance, one might conclude that as far as the behaviour of interest rates are concerned, the EH is not at variance with the data.

In chapter 6, I use a similar data set, but use the VAR methodology to examine the EH of the term structure of interest rates with a constant term premium. The standard deviations ratio and the correlation coefficients give results in favour of the EH, while the VAR cross-equation restrictions are rejected for only 1 case out of 3 (for the 3m,1m combination). The reported results appear to fit the data well and are
consistent with recent findings for the UK. For the US, the poor performance of the EH appears to be as a result of number possibilities. One possibility which was considered in chapter 7, is that volatile interest rates may lead to sizeable time varying risk premia, which could invalidate the EH. Recently rejection of the EH, using US data, has recently been attributed to small sample bias, Bekaert, Hodrick and Marshall (1997). The authors favour a pooling the data in a panel data approach, as it may address the bias and dispersion in the small sample distributions.

Chapter 7 augments the Campbell and Shiller VAR methodology to account for time varying term premia. The central assumption in the analysis is that the term premium is stationary and the reported results verify this is the case. However, unlike previous evidence using UK long maturity data, I do not find a time varying term premium. I therefore report results which verifies that the modified 3 variable VAR gives quantitatively similar results to the 2 variable VAR. The reported results are consistent with recent evidence for the UK, in that I cannot reject the EH.

11.2 Conclusions of Part II

In Part II of the thesis I focus on tests of the impact of exchange rate volatility on trade. The recent extraordinary growth in Irish exports plus the membership of the single European currency form the motivation for part II of the thesis. Irish exports have grown dramatically in recent years (from 43% (of GDP) in 1979 to a level of 82.8% in 1998), thus rendering the economy more open than before and more dependent on foreign markets. The important question is whether inclusion in a single European currency will lead to greater trade in the future, as a result of the end of the end of exchange rate volatility among European countries. However, before focusing on exports to Europe as a whole, I first analysed the case of exports to the UK, Ireland's major trading partner.
Chapter 9 focused on modelling short-run and long-run export demand functions for Irish exports to the UK. I use the techniques of cointegration and error-correction methods in order to test the impact of exchange rate volatility. In the short-run I find exchange rate volatility and the associated uncertainty has a negative effect on real exports. While volatility has an insignificant effect in the long-run. The reported results also suggest that exports are very sensitive to income and relative prices changes, and in particular in the long-run. Given that a large portion of Irish exports are from the MNC sectors may have important implications and as a result I extend the study to include other European countries and also to focus on the actions of the MNC and the indigenous sector.

Based on the results for exports to the UK, in chapter 10 I model the impact of exchange rate volatility on overall exports, but also sectoral exports SITC 0-4 and SITC 5-8 to the EU. The sectoral classification corresponds to the exports of mainly indigenous Irish firms and multinationals, respectively. I analyse the long-run and short-run relationship between export volume and its determinants; namely relative prices, foreign income and exchange rate variability. Two sample periods are analysed. The initial sample period extends from the second quarter of 1979 to the fourth quarter of 1992; from the launch of the EMS to its effective disintegration. The second sample takes into account the higher exchange rate volatility outside the EMS period, 1978Q3 – 1995Q4. Overall, exports are also shown to respond positively to exchange rate variability in the long run for both samples. Although the indigenous sector may be considered to be very risk averse, the MNC’s are viewed as adopting a programme of international plant switching. The reported results allow one to tentatively conclude that the long-run export behaviour of multinationals will not change, with Ireland included in a single currency.
APPENDIX 1

DATA DESCRIPTION

This data appendix describes the data sets which are used in this thesis.

PART I:

IRISH SHORT-TERM INTEREST RATES

The data for the Irish money market consist of unpublished rates, quoted from Bank of Ireland (main Irish commercial bank). The maturities are the following; 1 month, 3 month and the 6 month. The data set consists of monthly data, 15\textsuperscript{th} of each month.

\textbf{Sample Period:} January 1984 – October 1997

\textit{Data Source:} Bank of Ireland, Dublin
IRISH LONG-TERM INTEREST RATES

The data used consists of spot rates and were kindly provided by Davy’s Stockbroking firm (main Irish stockbroking firm). The maturites consist of 5, 10 and 15 years. The complete data set is sampled monthly (Wednesday, 4pm rates) beginning on the second Wednesday in January 1989 and ending on the second Wednesday of October 1997.

Sample Period: January 1989 – October 1997

Data Source: Davy’s, Dublin

PART II:

- Irish Exports to the UK
- Irish Exports to the EU (UK, Germany, Netherlands, France, Italy)
[also split into the Standard Industry Trade Classification (SITC) Divisions 0-4 and Divisions 5-8]
**Source:** Trade Statistics Series, Central Statistics Office (CSO), Dublin

- UK, German, Netherlands, France and Italy GDP Data
- Irish Unit Export Value
- UK, German, Netherlands, France and Italy Unit Export Value

**Source:** International Financial Statistics (IFS) tape

- UK, German, Netherlands, France and Italy Exchange Rates

**Source:** Irish Central Bank Bulletin

**Exchange Rate Volatility and Exports to the UK**

*Sample Period:* 1979Q2 – 1993Q3.

**Exchange Rate Volatility and Exports to the EU**


APPENDIX 2: MONTE CARLO PROGRAMME

/*;
**
:::-----------------------------------------------:
**
** Long rate, 2 period rate (6 month) and 1 period rate (3 month)
**
** File Title: MCIRL.PRG
**
** -----------------------------------------------
**
** Independent Random Normal Errors
**
** Testing the single equation tests
**
**
Model 1 : Expected Slope coefficient : 2

\[ \Delta r(t) = \alpha_1 + \beta_1 S(t-1) + \epsilon_{1t} \]

Model 2 : Expected Slope coefficient : 0.5

\[ S(t) = \alpha_2 + \beta_2 \Delta r_{t+1} + \epsilon_{2t} \]

:::---------------------------------------------------------:
*/;

create (u) 1 5000; @ switch for running in GX or GAUSS @;
@ new; @ cls;
library pgraph; graphset;
format /ml/rd 10,4;
outwidth 255;
output on;

/*;
** ================
** This program does not require any Data - only Starting values
**
** Setting starting values here (taken from TSP Program)
**
*/;

long = 2; @ Maturity of long rate @;
short = 1; @ Maturity of short rate @;

1_s = long/short;

b0 = 0.0088; b1 = 1.11; b2 = -0.47; b3 = 0.24;
c0 = 0.0077;
rej_eq1 = zeros(1,1); @ rejection frequencies for t-test for
Equation 1 @;
ApPENDIX 2: MONTE CARLO PROGRAMME

```
rej_eq2 = zeros(1,1);   % rejection frequencies for t-test for
                      % Equation 2 @;

nstart = 50 ;         % this will be start of estimation @;
nob = 550 ;          % this is T(100, 200, 500) + 50 @;
estend = nob-1;

draws = 1 ;            % this is the start of the outer loop @;
ndraws = 1000 ;        % this should be 1000 in final prg @;

/*;
** ========================================
** Gauss Vector for Me Results (TSIR model)
** ========================================
*/;
beta_eq1 = zeros(ndraws,2) ;   beta_eq2 = zeros(ndraws,2) ;
se_eq1 = zeros(ndraws,2) ;    se_eq2 = zeros(ndraws,2) ;
bias_eq1 = zeros(ndraws,1) ;   bias_eq2 = zeros(ndraws,1) ;

/*;
** Starting the LOOPS Starting the LOOPS
**
*/;
do until draws > ndraws ;
    r_sh = zeros(nob,1) ;
    R_lo = zeros(nob,1) ;
    r_sh[1:5,1] = I|1|1|1|1 ;
    rfit_sh = zeros(nob,1) ;
    e = rndn(nob,1) ; SIGE = E*0.011 ;
    u = rndn(nob,1) ; SIGU = U*0.012 ;

k = 5 ;
do until k > nob-1 ;
    r_sh[k+1,1] = b0 + b1*r_sh[k,1] + b2*r_sh[k-1,1] + b3*r_sh[k-2,1]
                  + SIGU[K+1,1] ;
    rfit_sh[k+1,1] = b0 + b1*r_sh[k,1] + b2*r_sh[k-1,1] + b3*r_sh[k-2,1];
    R_lo[k,1] = c0 + (1/l_s)*(r_sh[k,1] + rfit_sh[k+1,1]) + SIGE[K,1] ;
```
APPENDIX 2: MONTE CARLO PROGRAMME

\[ k = k + 1 \]
\[ \text{endo} ; \]

@ GaussX commands follow now (until DROP Command) @ ;
smpl 1 nob ;
store r_sh rfit_sh R_lo ;

\[
\begin{align*}
\text{genr } r_{shL1} &= \text{lag}(r_{sh},1) ; \\
\text{genr } dr_{sh} &= r_{sh} - r_{shL1} ; \\
\text{genr } dr_{shL1} &= \text{lag}(dr_{sh},1) ; \\
\text{genr } dr_{shL2} &= \text{lag}(dr_{sh},2) ; \\
\text{genr } dr_{shL3} &= \text{lag}(dr_{sh},3) ; \\
\text{genr } dr_{shP1} &= \text{lag}(dr_{sh},-1) ; \\
\end{align*}
\]
@ This is a lead variable @ ;

\[
\begin{align*}
\text{genr } sp21 &= R_{lo} - r_{sh} ; \\
\text{genr } sp21L1 &= \text{lag}(sp21,1) ; \\
\text{genr } sp21L2 &= \text{lag}(sp21,2) ; \\
\text{genr } sp21L3 &= \text{lag}(sp21,3) ; \\
\text{genr } sp21L4 &= \text{lag}(sp21,4) ; \\
\end{align*}
\]

/*;
** ****************************************
** Model 1 : Expected Slope coefficient : 2
** Dr(t) = a + b Sp(t-1)
** ****************************************
*/;
smpl nstart estend ;

param a0 a1 ;

frml eql dr_sh = a0 + a1*sp21L1;
gmm eql;
method = nr nr robust;
inst = c sp21L2 sp21L3 sp21L4 ;
coefs = coeff ;
serror = stderr;
vcovl = vcov;
r = { 0 1 };
f = r*coefs; " f = " f;
res = f - 2.00; " res = " res;
n = inv(r*vcovl*r'); " n = " n;
APPENDIX 2: MONTE CARLO PROGRAMME

\[ w = \text{res}'n*\text{res}; \quad \text{"w = " w;} \]
\[ p = \text{cdfchic}(w,1); \quad \text{"wald stat. = " w;} \]
\[ @\@ pl = p; \quad \text{"p value = " pl;} \]

\[
\text{if pl < 0.05; } \\
\text{rej_eql = rej_eql+1; } \\
\text{elseif pl > 0.05; } \\
\text{rej_eql = rej_eql; } \\
\text{endif; } \\
\]
\[ @\@ " \text{rej_eql = " rej_eql; } \]

\[
\text{beta_eql[draws,1] = coefs[1];} \quad \text{beta_eql[draws,2] = coefs[2];} \\
\text{se_eql[draws,1] = serror[1];} \quad \text{se_eql[draws,2] = serror[2];} \\
\]

\[
\text{smpl nstart estend; } \\
\]

\[
\text{param a0 a1; } \\
\text{frml eq2 sp2l = a0 + a1*dr_shPl;} \\
\text{gmm eq2; } \\
\quad \text{method = nr nr robust;} \\
\quad \text{inst = c dr_sh dr_shL1 dr_shL2 ;} \\
\quad \text{coefs = coeff ;} \\
\quad \text{serror = stderr ;} \\
\quad \text{vcovl = vcov; } \\
\]

\[
\text{r = \{ 0 1 \};} \\
\text{f = r*coefs;" f = " f;} \\
\text{res = f - 0.5;" res = " res;} \\
\]
n = inv(r*vcov1*r'); " n = " n;
w = res'*n*res; " w = " w;
p = cdfchic(w,1);
" Wald stat. = " w;
@@ p2 = p;
" p value = " p2;

/*;
** *******************************
** Calculating the Proportions that fail the t-test ;
** *******************************
*/;
if p2 < 0.05;
   rej_eq2 = rej_eq2+1;
elseif p2 > 0.05;
   rej_eq2 = rej_eq2;
endif;

@@ " rej_eq2 = " rej_eq2;

beta_eq2[draws,1] = coefs[1] ;
se_eq2[draws,1] = serror[1] ;
se_eq2[draws,2] = serror[2] ;

smpl 1 nob ;
drop r_sh rfit_sh R_l0 ;
draws = draws + 1 ;
deno ; @ this is the end of the outer loop @;

@@ " rej_eq1 = " rej_eq1;
@@ " rej_eq2 = " rej_eq2;

/*;
** *******************************
** Calculating the MC Statistics : Model 1
** *******************************
*/;
bias_eq1 = beta_eq1[.,2] - 2 ;
@ Important : set true parameter for bias here @ ;
mean_beta = meanc(beta_eq1) ;
mean_se = meanc(se_eq1) ;
mean_bias = meanc(bias_eq1) ;
MC_se = (ndraws^(-0.5))*((ndraws-1)^(-0.5))
   * (sumc((bias_eq1-mean_bias)^2)^0.5) ;
PRINTING THE SUMMARY STATISTICS OF THE MC PROGRAMME

Model 1: Expected Slope coefficient: 2

\[ \Delta r(t) = \alpha_1 + \beta_1 S(t-1) + \epsilon_t \]

Summary statistics have been calculated using a MC loop of "ndraws".
Size of generated data is ... "nob; GMM estimation is based on observations: ... "estend-nstart+1;

The MC mean of the Betas is ... "mean_beta; The MC mean of the std. errors is ... "mean_se;

The Monte Carlo Standard Error is ... "MC_se;

The Rejection Frequencies on the W-Test is ... "rej_eq1/ndraws;

Calculating the MC Statistics: Model 2

\[ \beta_{eq2}[.,2] = 0.5 \]

Important: set true parameter for bias here. 

mean_beta = meanc(beta_eq2);
mean_se = meanc(se_eq2);
mean_bias = meanc(bias_eq2);
MC_se = (ndraws^(-0.5))*((ndraws-1)^(-0.5))
*sumc((bias_eq1-mean_bias)^2)^(0.5));
Printing the Summary Statistics of the MC

Model 2: Expected Slope coefficient: 0.5

\[ S(t) = \alpha_2 + \beta_2 \Delta x_{t+1} + u_{2t} \]

Summary statistics have been calculated using a MC loop of \( \text{ndraws} \);
Size of generated data is ... \( \text{nob} \);
GMM estimation is based on observations: ... \( \text{estend-nstart+1} \);
The MC mean of the Betas is ... \( \text{mean\_beta} \);
The MC mean of the std. errors is ... \( \text{mean\_se} \);
The MC mean of the slope bias is ... \( \text{mean\_bias} \);
The Monte Carlo Standard Error is ... \( \text{MC\_se} \);
The Rejection Frequencies on the W-Test is ... \( \text{rej\_eq2/ndraws} \)
/*:
** The Programme estimates a 2 variable VAR
** File Title: ESTVAR.PRG
** D. Bredin October 1999
** 2 variable VAR system (Spread, change in short rates)
**
*/

new ; cls ;
output file = c:\windows\term\irl\prg\var.out;

format /ml/rd 12,6;
open ofile = var;

/*
* This part of the programme deals with the input of the data.
*/

start = 1 ;
endd = 166 ;

open f1=c:\windows\term\irl\amonth1 varindxi;
varbls=readr(f1,166);

S61 = varbls[.,1S61];
S63 = varbls[.,1S63];
S31 = varbls[.,1S31];
DR1 = varbls[.,1DR1];
DR3 = varbls[.,1DR3];

clear varbls ;

p = 2 ;

nt=endd+1-start ;

#LINESON ;
@ gives error when used with CTRL+X key @

#INCLUDE c:\windows\gauss\proc\powmat.prc ; @ GAUSS procedure: raises a square matrix
to N power @;
#INCLUDE c:\windows\gauss\proc\prtime.prc ; @ GAUSS procedure: prints current date and
time @;
#INCLUDE c:\windows\gauss\proc\lag.prc ; @ GAUSS procedure: will produce lags of a
vector or matrix @;

start = 1 ;
endd = 166 ;

open f1=c:\windows\term\irl\amonth1 varindxi;
varbls=readr(f1,166);

S61 = varbls[.,1S61];
S63 = varbls[.,1S63];
S31 = varbls[.,1S31];
DR1 = varbls[.,1DR1];
DR3 = varbls[.,1DR3];

clear varbls ;

p = 2 ;

nt=endd+1-start ;

@ this is the number of data points, this changes below @;
APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME (plus procedures) 159

/*; ..............................................................; */
** Now I construct a vector x of independent variables known at start of period. **
**
** y will be this vector led one period.
** note: for this data set p = 2.
** ..............................................................; */

runn=1; do until runn > 3;
if runn==1; nj=2; nn=6; mm=1; xO=S61-DR1;
elseif runn==2; nj=2; nn=6; mm=3; xO=S63-DR3;
elseif runn==3; nj=2; nn=3; mm=1; xO=S31-DR1;
endif;
x=xO;
i=2;
do until i>p;
x=x-lag(xO,i-1);
i=i+1;
endo;
x=xO;
i=2;
do until i>p;
x=x-x-lag(xO,i-1);
i=i+1;
endo;
y=y[p+1:nt-1, ]; x=x[p+1:nt-1, ];
yyu=yyu[p+1:nt-1, ]; xxu=xxu[p+1:nt-1, ]; xx0=xx0[p+1:nt-1, ];
nt=nt-p-1;

/*; ..............................................................; */
** IMPORTANT GLOBAL SPN FOR LATER PROCs **
*/;
y = y - (meanc(y)' ).* ones(nt,nj);
x = x - (meanc(x)' ).* ones(nt,nx);
APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME

plus procedures

spn=xxO(1:nt,1);  /* EXTRACT the actual spread - not devn from mean */
xx=moment(x,1);
clear xO;
dh=1e-6;

/* ---------------------------------------- */
/* LOADING THE PROCEDURES */
/* ---------------------------------------- */

#INCLUDE c:\windows\term\irl\prg\proc\evarstdn.prc ; /* Main calculation
procedure */
#INCLUDE c:\windows\term\irl\prg\proc\varols.prc;
#INCLUDE c:\windows\term\irl\prg\proc\wandcv.prc ;
#INCLUDE c:\windows\term\irl\prg\proc\compzm.prc ;
#INCLUDE c:\windows\term\irl\prg\proc\gradie.prc ;

#INCLUDE c:\windows\term\irl\prg\proc\waldf2a.prc ;   # gets wald stats
#INCLUDE c:\windows\term\irl\prg\proc\waldy2a.prc ;   # does wald test + se's

#INCLUDE c:\windows\term\irl\prg\proc\nlstat.prc ;   # requires spn as global
#INCLUDE c:\windows\term\irl\prg\proc\nlstatse.prc ;  # gets vector of ratios and
se's
#INCLUDE c:\windows\term\irl\prg\proc\dnsdash.prc ;   # produces S & S'
#INCLUDE c:\windows\term\irl\prg\proc\tablcl.prc ;   # o/p from VAR
#INCLUDE c:\windows\term\irl\prg\proc\tabw.prc ;     # o/p from wald test (waldy)

/* ---------------------------------------- */
/* EXECUTING THE PROCEDURES */
/* ---------------------------------------- */

{c, vc} = evarstdn(x, y, dh) ;
   # This is the main procedure where all estimation is carried out #;

{Rb} = waldf2a(c) ;
{wstat, degfw, pvw, seRb, tRb} = waldy2a(c, vc, Rb) ;
   # The Wald test reported in Table 6.3 #;

{statvec} = nlstat(c) ;
{stvecse} = nlstatse(c, vc) ;

/****:................................................:*:............................**
** This procedure produces the variance ratios for the Spread (S) **
** and the Theoretical spread (S') and also the correlation coefficients **
** between S and S' **
*/;
APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME (plus procedures)

/* ===========================*/
/* The Results Output */
/* ===========================*/

/* ----------------- */
/* OUTPUT STATISTICS */
/* ----------------- */
tabkcl(ofile, statvec, stvecse); /*The Output for the Metrics on S and S';
tabw(ofile, wstat, deqfw, pvw, Rb, seRb, tRb); /*The Output for the Wald Test;

/* ------------ */
/* OUTPUT DATA */
/* ------------ */
{zzz} = dnsdash(c); /* This procedure produces the series S and S';
runn=runn+1;
endo;
end ;
APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME (plus procedures)

/*;
**
*:---------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:
** PROC: evarstdn.proc October 1999
** This is the main procedure in the Programme
** This procedure estimates a VAR and computes some statistics.
**
** NOTE: The following must be compiled before this file:
** VAROLS.PROC (computes OLS coefficients)
** WANDCV.PROC (constructs some GMM matrices)
** COMPM2.PROC (computes a special GMM matrix)
** GRADIE.PROC (computes the gradient of the special matrix)
**
**
** INPUTS:
** X MT x NY matrix of regressors
** Y MT x NY matrix of LHS variables
** RHO 1 x 1 parameter of linearization
** DH 1 x 1 epsilon for derivative computation
**
** OUTPUTS:
** C NC x 1 coefficient estimates
** VC NC x NC covariance matrix of coefficients
**
*:---------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:-----------------------------------------------:
*/;

$LINESON;
proc(z) evarstdn(x, y, dh);
local c, w, se, g, s, nx, ny, nt, vc, xv ;
local a, v, e;
ny = cols(y);
x = cols(x);
nt = rows(y);
{a, v, e} varols(x, y, nt); @ OLS estimates @
{c, w} wandcv(x, v, e, a); @ GMM weighting matrix @
g = gradie(x, y, c, dh); @ GMM gradients @
vc = invpd(g'w * g); @ parameter covariance matrix VC has @
xv = vex(x); @ White heteroscedasticity correction @
retp(c, vc) ;
endp;
/*;
 ** PROC: varols.prc
 ** This procedure computes OLS estimates of a vector autoregression.
 ** INPUTS:
 ** \( X \) \( NT \times NX \) matrix of regressors for VAR
 ** \( Y \) \( NT \times NY \) matrix of dependent variables for VAR
 ** \( NT \) \( 1 \times 1 \) number of observations
 ** OUTPUTS:
 ** \( A \) \( NX \times NY \) regression coefficient estimates
 ** \( V \) \( NY \times NY \) residual covariance estimates
 ** \( E \) \( NT \times NY \) residual estimates
 */
#LINESON;
proc(3) = varols(x, y, nt);
local a,v,e;
a = y / x;
eps = y - x * a;
v = eps ./ nt;
retp(a, v, eps);
endp;

;/*
 ** PROC: wandcv.prc
 ** This procedure computes the weighting matrix for Generalized Method of
 ** Moments Estimation for a linear regression model. A byproduct is a
 ** convenient vector form of the input estimates.
 ** INPUTS:
 ** \( X \) \( NT \times NX \) matrix of regressors
 ** \( E \) \( NT \times NY \) matrix of estimated residuals
 ** \( V \) \( NY \times NY \) residual covariance estimates
 ** \( A \) \( NX \times NY \) regression coefficient estimates
 ** OUTPUTS:
 ** \( W \) \( NC \times NC \) GMM weighting matrix
 ** \( C \) \( NC \times 1 \) vector form of \( A \) and \( V \) parameters
 ***/
#LINESON;
proc(2) = wandcv(x, v, e, a);
local w, c, ny, f, i, j, k;
ny = cols(e);
f = e[,]^2 - v[1,1];
c = vec(a) | v[1,1];
i = 2;
j = 1;
k = 2;
do while i le (ny^2 + ny) / 2;
f = f - e[,] * e[,] - v[j,k];
c = c | v[j,k];
k = k + 1;
if k gt ny;
j = j + 1;
k = j;
endif;
i = i + 1;
end;
i = ny;
do while i ge 1;
f = e[,] * x - f;

/*
APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME (plus procedures)

i = i - 1;  // in the sample.
i = i - 1;
endo;
w = invpd(f'f);
retp(c, w);
endpi

/*-----------PROC: compzm.prc
** This procedure computes "ZM" term of the GMM objective function from data
** matrices and a coefficient vector.
** INPUTS:
** X    NT x NX  matrix of regressors
** Y    NT x NY  matrix of LHS variables
** C    NC x 1   vector of linear model parameters
** OUTPUTS:
** ZM   NC x 1   zm' * w * zm is GMM objective
***/
*/

#LINESON;
proc(1) = compzm(x, y, c);
local zm, ny, f, i, v, a, j, k, e, nc, nx;
nc = rows(c);
ny = cols(y);
ex = c[nx*ny+1:nc,1];  // Extract coefficients from C
a = reshape(c[1:nx*ny,,], ny, nx)';
e = y - x * a;
f = e[.,1]^2 - v[1,1];
i = 2;
j = 1;
k = 2;
do while i le (ny^2 + ny) / 2;
f = f - e[.,j] .* e[.,k] - v[1,1];
k = k + 1;
if k gt ny;
j = j + 1;
endif;
i = i + 1;
endo;
i = ny;
do while i ge 1;
f = e[.,i] .* x - f;
i = i - 1;
endo;
zm = sumc(f);
retp(zm);
endp;

//
** PROC: gradie.prc  
**  
** This procedure computes the gradient of the ZM term of the GMM objective function with respect to the model parameters. This will be used with the GMM weighting matrix to compute a covariance matrix for all the estimated parameters.  
**  
** INPUTS:  
**  
X | NT x NX  
Y | NT x NY  
C | NC x 1  
DH | 1 x 1  
**  
** OUTPUTS:  
**  
G | NC x NC  
**  
G(i,j) is ∂ZM(i)/∂C(j) derivative

```
** PROC: gradie(x, y, c, dh);  
local g, i, nc, cp, cm;  
nc = rows(c);  
g = zeros(nc, nc);  
i = 1;  
do while i le nc;  
   cp = c;  
   cm = c;  
   cp[i,1] = cp[i,1] + dh;  
   cm[i,1] = cm[i,1] - dh;  
   g[.,i] = (compzm(x, y, cp) - compzm(x, y, cm)) / (2 * dh);  
   i = i + 1;  
endo;  
retp(g);  
endp;  
```
local a,i,e1,e2,e3,Rb,Rb3,an,am;
/* ---------------------------------- */
/* extract the a matrix from c matrix */
/* ---------------------------------- */
a=zeros(nj,nx);
i=1;
do until i>nj;
a[i, . ]=(c[(i-1)*nx+1:i*nx, .] ');
i=i+1;
endo;
@
do the transformations needed to get the companion matrix @
if p>1;
a=a*I(eye((p-1)*nj)-zeros((p-1)*nj,nj));
endif;
e1=zeros(nj,1);
e2=zeros(nj,1);
el(!,.)=l;
e2[2, .]=1;
if p>1 ;
el=zeros(nj+(p-1)*nj,1)
e2=zeros(nj+(p-1)*nj,1);
endif ;
an = powmat(a,nn) ;
am = powmat(a,mm) ;
/* ================================================================ */
Rb = el'-e2'*a*( eye(nx)-(mm/nn)*( eye(nx)-an )*inv( eye(nx)-am ) )
*inv( eye(nx)-a) ;
/* ============================================================== */
Rb=Rb' ; /* for gradp the return must be a coln. vector */
retp(Rb) ;
endp;
/*;
**;*/

** PROC: WALD2A(c) **
** NOTE: The following must be compiled before this file **
** WALDF2A.PRC (gives restriction, Rb) **
** INPUTS **
** C NC x 1 Coeffs, a(i,j) and cov m. resids
** VC NC x NC Var-Cov mat. all coeffts
** Rb = F(a) NX by 1 non-linear function of a(i,j)
** OUTPUTS: **
** WALD 1 x 1 Wald statistic
** degfw 1 x 1 Deg of freedom
** pw 1 x 1 p-value of wald stat
** seRb NX x 1 SE of coefft restrictions
** tRb NX x 1 T-stat of restrictions
** GLOBALS **
** c,vc, nj,nx,p,**
*/;
proc(S) = waldy2a(c,vc,Rb);
llocal a,i,rowi,coli,vc_a,g ;
llocal wstat,degfw,pw,seRb,tRb,GVG,avec ;

APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME (plus procedures)

/* extract the a matrix from c matrix */
a=zeros(nj,nx);
i=1;
do until i>nj;
a{i,..}=(c[(i-1)*nx+1:i*nx,..])';
i=i+1;
endo;
avec=vec(a') ;  \( a' \) is NX x NJ ; \( \text{avec} \) is (NX x NJ) x 1
rowi=seqa(1,1,nj*nx) ;
coli=seqa(1,1,nj*nx) ;
vc_a=submat(vc, rowi, coli) ; \( \text{vcov} \) of a's only

GVG = g'vc_a*g';
seRb = sqrt(diag(GVG)) ;

trB=Rb./seRb;
wwstat = Rb'*inv(GVG)*Rb;  \( \text{wstat} \)
degfw=rows(Rb) ;

retp(wwstat,degfw,pvw,seRb,ttRb) ;
endp ;

PROC: Derivative. of constraint on beta

** PROC: NLSTAT(C) **
** Do VARIANCE RATIO(S,S'): CORR(S,S') etc. **
** INPUTS **
** C **
** OUTPUTS: **
** STATVEC **
** GLOBALS **
** nj,nx,p,mm,nn (strictly these should be entered in proc) **
** spn  \( \text{spn} \)  \( \text{time series of actual spread} \) **

/;  

#LINESON ;
proc(1) = nlstat(c);

local a,i,j,statvec;
local difflev,mdiff,mdiffpc,diffpc;
local ssdash,vspn,vsdash,cross,cor_s,mratio,relv,relv1,stderrn/stdsdash,relstd;
local relstd1,an,am,e1,e2 ;
local qqq ;

/* extract the a matrix from the coefficient vector - c (global) */
a=zeros(nj,nx);
i=1;
do until i>nj;
a{i,..}=(c[(i-1)*nx+1:i*nx,..])';
i=i+1;
endo;

/* extract the innovations variance-covariance matrix from the coefficient vector */
d=c(nx*nj+1:nc,..);

/* do the transformations needed to get the companion matrix */
if p>1;
    a=a1(eye((p-1)*nj)-zeros((p-1)*nj,nj));
endif;

an = powmat(a,nn);
am = powmat(a,mm);

e1=zeros(nj,1); e1(1,.)=1;
e2=zeros(nj,1); e2(2,.)=1;

if p>1;
    e1=zeros(nj+(p-1)*nj,1); e1(1,.)=1;
e2=zeros(nj+(p-1)*nj,1); e2(2,.)=1;
endif;

ssdash = a*( eye(nx)-(mm/nn)*( eye(nx)-an )*inv( eye(nx)-am ) )
  *inv( eye(nx)-a ) ;

ssdash= (e2'*ssdash) ;
ssdash=xxu*ssdash ;
/* ssdash=x*ssdash ; */

vspn= sumc( (spn-meanc(spn) )^2 )/ (rows(spn)-1) ;
stdspn=sqrt(vspn) ;

vsdash= sumc( (ssdash-meanc(ssdash) )^2 )/ (rows(spn)-1) ;
stdsdash=sqrt(vsdash) ;
cor_s=cross/( sqrt(vspn)*sqrt(vsdash)*(rows(spn)-1) ) ;
relv = vspn/vsdash ;
relvl= vsdash/vspn ;
relstd=stdspn/stdsdash ;
relstdl=stdsdash/stdspn ;
difflev=spn-sdash ;
mdiff = meanc(difflev) ;
diffpc=(spn-sdash)*100/spn ;
mdiffpc= meanc(diffpc) ;

statvec=relvl|cor_s|relvl|vsdash|mdiff|mdiffpc|relstd|relstdl ;
retp(statvec);

endp;
/*;
**
** PROC: NLSTATSE(C,VC)
**
** SE of VARIANCE RATIO(S,S'): CORR(S,S') etc.
** Trial using KC int. rate stuff
**
** NOTE: This file requires compilation of NLSTAT
** before this file
**
** INPUTS
** C  NC x 1  Coeffs, a(i,j) and sigma(i,j) of resid
** VC  NC x NC  Var-Cov mat. ALL coeffts (as above)
**
** OUTPUTS:
** STVECSE  8 x 1  Vector of ratios of f(A) etc.
**
** GLOBALS
** dh, nc, nj, nx, p, mm, nn  (strictly should be in proc i/p)
**
*/;

LINESON;

proc(l) = nlstatse(c,vc);
local a, i, d, stvec, nstvec, gstat, ct, statp, statm;
local stvecse;

{stvec} = nlstat(c);

nstvec = rows(stvec);
gstat = zeros(nc, nstvec);
i = 1;
do until i > nc;
   ct = c;
   ct[i,.] = ct[i,.] + dh;
   {statp} = nlstat(ct);
   ct[i,.] = ct[i,.] - dh;
   {statm} = nlstat(ct);
   gstat[i,.] = (statp - statm) / (2 * dh);
   i = i + 1;
endo;

stvecse = sqrt(diag(gstat' * vc * gstat));
retp(stvecse);
endp;

/;
**
**
** FILE: DNSDASH.PRC
** Obtaining the series S and S'
**
** INPUTS:
** C  NC x 1  ALL coeffts
**
** OUTPUT:
** zzzz  NT x 1  Actual and Theoretical Spread
**
** GLOBALS:
** p, nn, mm, nt, nj, nx, yyu, xxu
**
*/;

LINESON;

proc(l) = dnsdash(c);
local a, i, avec, kon, e1, e2, an, am, ssdash, zzzz;
APPENDIX 3: VECTOR AUTOREGRESSION PROGRAMME (plus procedures)

/* extract the a matrix from c matrix */

a=zeros(nj,nx);
i=1;
do until i>nj;
a[i,]=c[(i-1)*nx+1:i*nx,]' ;
i=i+1;
endo;

/* -- GET CONST. TERM IN THE REGN. ---- */
kon = meanc(ywu) - meanc(xwu*a') ;

/* do the transformations needed to get the companion matrix */
if p>1;
a=a[eye((p-1)*nj)-zeros((p-1)*nj,nj)];
endif;
e1=zeros(nj,1); e1[1,]=1;
e2=zeros(nj,1); e2[2,]=1;
e1=zeros(nj+(p-1)*nj,1) ;
e2=zeros(nj+(p-1)*nj,1) ;
kon=kon|zeros(nj+(p-1)*nj-2,1) ;
endif ;
an = powmat(a,nn) ;
am = powmat(a,mm) ;

ssdash = a*[ eye(nx)-(mm/nn)*(eye(nx)-an)*inv(eye(nx)-am) ]
*inv(eye(nx)-a) ;

ssdash = (e2'*ssdash) ;
zzzz = spn-ssdash ;
retp(zzzz) ;

@end ;

@ make it a coln. of a(i,j) 

DEFINE ASCII VARIABLES

@ DEFINE ASCII VARIABLES
REFERENCES


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