Abstract

With the rapid development of information systems and our increasing dependency on computer-based systems, ensuring their dependability becomes one the most important concerns during system development. This is especially true for the mission and safety critical systems on which we rely not to put significant resources and lives at risk.

Development of critical systems traditionally involves formal modelling as a fault prevention mechanism. At the same time, systems typically support fault tolerance mechanisms to mitigate runtime errors. However, fault tolerance modelling and, in particular, rigorous definitions of fault tolerance requirements, fault assumptions and system recovery have not been given enough attention during formal system development.

The main contribution of this research is in developing a method for top-down formal design of fault tolerant systems. The refinement-based method provides modelling guidelines presented in the following form:

- a set of modelling principles for systematic modelling of fault tolerance,
- a fault tolerance refinement strategy, and
- a library of generic modelling patterns assisting in disciplined integration of error detection and error recovery steps into models.

The method supports separation of normal and fault tolerant system behaviour during modelling. It provides an environment for explicit modelling of fault tolerance and modal aspects of system behaviour which ensure rigour of the proposed development process.

The method is supported by tools that are smoothly integrated into an industry-strength development environment.

The proposed method is demonstrated on two case studies. In particular, the evaluation is carried out using a medium-scale industrial case study from the aerospace domain.

The method is shown to provide support for explicit modelling of fault tolerance, to reduce the development efforts during modelling, to support reuse of fault tolerance modelling, and to facilitate adoption of formal methods.
I would like to express my gratitude to all those who have provided support during the years of this work. Firstly, I would like to thank my supervisor Alexander Romanovsky for his valuable guidance and generous feedback given throughout the research. I am also thankful to my thesis committee members John Fitzgerald, Aad van Moorsel, and Cristina Gacek for their useful and timely advice at different stages of work.

I am grateful to my colleagues at the School of Computing Science at Newcastle University for their helpful feedback on various aspects of my research. Special thanks go to Alexei Iliasov for his collaboration and fruitful discussions, and Anirban Bhattacharyya for his support.

During my studies, I had an opportunity to visit the Department of Information Technologies at the Abo Akademi University, Finland. I would like to thank Elena Troubitsyna, Yuliya Prokhorova, and Linas Laibinis for their productive collaboration and hospitality.

I want to thank all colleagues who has been working on the DEPLOY project. On top of useful collaborations mentioned above, the project provided extensive research and application material that served as a basis for my studies.

I am grateful to the DEPLOY project, the TrAmS platform grant, and the School of Computing Science at Newcastle University for providing financial support for conducting this research.

Special thanks goes to the thesis examiners Yamine Ait-Ameur and Tom Anderson for productive discussions and highly valuable feedback on this work.

Last but not least, I would like to thank my parents and my brother. This work would not have been done without their encouragement and support. I am especially grateful to my wife Julia for her understanding and patience during this long endeavour.
Declaration

I certify that no part of the material offered has been previously submitted by me for a degree or other qualification in this or any other University.

Published Work

Part of the work presented in this thesis has or will have appeared as follows:

1. The FT/Mode Views method presented in Chapter 3, the airlock case study used in Chapter 4, and initial ideas of pattern-based development method appeared in:


   The research was carried out by I. Lopatkin with guidance and under supervision of the co-authors.

2. A first version of the AOCS case study developed using the FT/Mode Views approach and presented in Chapter 5 was published in:


   The full version of the case study development appeared as a technical report in:

The research was carried out by I. Lopatkin with guidance and under supervision of the co-authors.

3. Patterns for modelling control systems used as a basis for Section 4.8 were published in:


The initial patterns were defined by I. Lopatkin and Y. Prokhorova under the supervision of the remaining co-authors. The current work identifies and reserves a development step for incorporating the FMEA patterns. See more details in Section 4.8.

4. A summary of the development method presented in the thesis appeared as:

# Contents

Declaration iii

Contents v

List of Figures ix

1 Introduction 1

1.1 Motivations ........................................... 1
1.2 Research Hypotheses ................................... 2
1.3 Research Methodology and Contributions ................. 2
1.4 Thesis Structure ....................................... 3

2 Background 4

2.1 Modelling and Formal Methods ........................ 4

2.1.1 Usage of formal methods today ..................... 4
2.1.2 Success stories and problems ....................... 6
2.1.3 System context ..................................... 7
2.1.4 Event-B ........................................... 8
2.1.5 Usage of Event-B in industrial and academic settings 10

2.2 Fault Tolerance ........................................ 11

2.2.1 Definitions and taxonomy .......................... 11
2.2.2 Realistic systems and fault tolerance ............... 12
2.2.3 Fault analysis and formal modelling of fault tolerance 15

2.3 Views .................................................. 18

2.4 Problem Statement ..................................... 20

2.5 Conclusions ........................................... 21

3 Modal and Fault Tolerance Views 22

3.1 Overview and Definitions .............................. 22
3.2 Views Construction .................................... 23
3.3 Views Refinement ..................................... 26

3.3.1 Mode refinement rules .............................. 26
4.8.3 Error detection pattern ........................................ 73
4.8.4 Control phase patterns ....................................... 73
4.8.5 Prediction phase pattern ..................................... 74
4.8.6 Application in Event-B ....................................... 75
4.9 Summary of Patterns ........................................... 79
4.10 Conclusions ..................................................... 80

5 Evaluation ......................................................... 81
5.1 Requirements for AOCS ........................................ 82
5.2 AOCS modelling .................................................. 85
  5.2.1 Functional model M0 ....................................... 86
  5.2.2 Safe stop at M1 ............................................. 87
  5.2.3 Functional refinement at M2 ................................ 88
  5.2.4 Functional refinement at M3 ................................ 90
  5.2.5 Fault tolerant component refinement at M4 .......... 93
  5.2.6 Behaviour restriction at M4 ................................ 95
5.3 Conclusions ..................................................... 96

6 Conclusions ....................................................... 98
  6.1 Discussions and Directions of Further Research ........ 98
  6.2 Summary and Contributions .................................. 100

References .......................................................... 103

Appendix A: Airlock Case Study Model .......................... 117
  A.1 Context C0 ..................................................... 117
  A.2 Machine M0 .................................................... 117
  A.3 Machine M1 .................................................... 120
  A.4 Context C2 ..................................................... 123
  A.5 Machine M2 .................................................... 124
  A.6 Machine M3 .................................................... 128
  A.7 Machine M4 .................................................... 133
  A.8 Context C5 ..................................................... 144
  A.9 Machine M5 .................................................... 144

Appendix B: AOCS Case Study Model ........................... 158
  B.1 Context C0 ..................................................... 158
  B.2 Machine M0 .................................................... 158
  B.3 Machine M1 .................................................... 160
  B.4 Context C2 ..................................................... 162
  B.5 Machine M2 .................................................... 162
List of Figures

3.1 Example of a modal view ........................................ 24
3.2 Modal views development chain ................................. 25
3.3 Example of a modal view refinement ........................... 26

4.1 Properties and behaviour ...................................... 37
4.2 The steps of the method ........................................ 43
4.3 The airlock system .............................................. 45
4.4 Two abstract classes of fault tolerant systems ............... 51
4.5 Modal view of a safe stop system .............................. 52
4.6 Modal view associated with M1 ................................. 55
4.7 Error split template ............................................ 59
4.8 Behavioural split template ..................................... 59
4.9 Graceful degradation template ................................. 60
4.10 Modal view of airlock M2 model .............................. 63
4.11 Modal view of airlock M3 model .............................. 67

5.1 AOCS system modes ............................................ 83
5.2 AOCS case study models ........................................ 85
5.3 Modal view of AOCS M2 model ................................. 90
5.4 Modal view of AOCS M3 model ................................. 92
Chapter 1. Introduction

This chapter initially describes the motivations behind the thesis and the main topics related to this work. Research questions and hypothesis that we validate in this thesis are formulated next. Finally, the research methodology, contributions and the thesis structure are stated.

1.1 Motivations

Computer-based critical systems dependability
Our society is becoming increasingly dependent on computer-based systems due to the falling costs and improving capabilities of computers. There is a class of systems called critical that operate with resources of the highest value. Defects in such systems, unlike commercial day-to-day products, can have a significant impact on the environment, assets, and human life. Critical systems have to be dependable \cite{Avi+04}, so that they can be justifiably trusted to provide the required services.

Adoption of formal methods
One of the prominent solutions to ensuring systems dependability by fault prevention and/or fault removal is the inclusion of formal modelling in the development process. Even though formal methods are not always used in developing industrial systems, their use in development of dependable systems is increasing and is proven to be cost-effective \cite{Woo+09}. Among the main current obstacles to adopting formal methods by industry are the lack of tools and engineers’ experience in formal development. The latter can be significantly improved by teaching of examples, development patterns, and modelling practices.

Modelling fault tolerance
It is well-known that one cannot produce a faultless system functioning in a perfect fault-free environment \cite{LA90}. This is due to many reasons including changing environmental conditions, hardware failures, and inevitable mistakes during development. In order to achieve sufficient levels of dependability, systems need to mitigate faults during execution by employing fault tolerance mechanisms. While it is theoretically
possible to formally produce a system free of bugs, developers cannot assume system
environment to be fault-free. Deterioration of physical components makes it neces-
sary for systems to tolerate low-level errors, such as sensor and actuator failures, to
provide an acceptable level of dependability.

There are a number of safety analysis techniques for modelling low-level errors
and error propagation paths and analysis of system-level effects. However, there is
limited support for high-level design of fault tolerant systems using formal methods.
Top-down development methods have to support fault tolerance modelling at higher
levels of abstraction, where the overall critical system behaviour inherently contains
error recovery procedures.

1.2 Research Hypotheses

The aim of this research is to validate the following hypotheses:

1. It is feasible to develop systems in which fault tolerance is correctly designed.

2. Design of fault tolerance can be integrated into a formal top-down development
   process.

3. It is possible to develop a combination of modelling techniques, refinement
   strategies and guidelines that facilitate the development of fault tolerance in
   a structured, reusable, and tool-supported way.

4. The refinement-based Event-B method supports formal development of fault
   tolerant systems.

1.3 Research Methodology and Contributions

The main approach taken in this research is to propose and evaluate a method for
a top-down rigorous development of fault tolerance in critical systems via formal
modelling of faults and system behaviour. The development method relies on two
major contributions:

• a formally introduced concept of fault tolerance (FT) modelling views, accom-
  company by guidelines for its practical application, and

• a set of principles and practices for modelling fault tolerance in state-based
  formal methods.

This work binds the two parts together into a single consistent method for modelling
fault tolerance. The approach is exemplified for the Event-B formal method. Two
case studies are developed to evaluate the method and demonstrate its applicability.

The research methodology relies on the following concepts:
• **Industrial applications and experience.** The method benefits from the analysis of a number of industrial requirements documents within a range of problem domains including automotive, aerospace, transportation, and business sectors. The method is targeting industrial scale application.

• **Tool support.** The method is tool-supported and integrated into an industry strength modelling environment. The case studies used in this work for evaluation are developed and proved in the Rodin toolset.

• **Conservative extension.** Modelling of fault tolerance does not alter the original top-down development method. The proposed method is built on existing formal semantics and tools used by industry.

• **Top-down development of fault tolerance.** The method addresses fault tolerance at all levels of abstraction and provides a hierarchical approach to modelling.

### 1.4 Thesis Structure

Chapter 2 contains an overview of the research areas relevant to this work. The concept of Modal and Fault Tolerance Views is described in Chapter 3 as a self-contained approach to modelling modal and fault tolerance aspects of systems. The method for refinement-based formal modelling of fault tolerant systems is proposed in Chapter 4. The method consists of a number of refinement-based modelling solutions which are used according to a refinement strategy, and a practical application of modal views. The method is evaluated in Chapter 5 by modelling a second case study from the aerospace domain. We draw conclusions and discuss limitations of the method in Chapter 6.
Chapter 2. Background

This chapter provides a thesis background and a current state of the art in relevant research areas. This thesis contributes to the three areas each described in its section: an overview of the relevant aspects of formal methods is given in Section 2.1, fault tolerance is addressed in Section 2.2, and the current state of research in multi-views development is discussed in Section 2.3. We identify the key problems that we intend to address by this study in Section 2.4 and we draw our conclusions in Section 2.5.

2.1 Modelling and Formal Methods

*Modelling* is a process of creating an abstraction of (some aspect of) a system with the purpose of gaining confidence and deeper understanding of the resulting behaviour of the system. Different modelling frameworks provide different means of such assurance. A number of XML-based frameworks such as UML [JBR99; Amb04] and AADL [AADL] today are widely used by industry engineers to represent the domain knowledge and system architecture. Known as *model-driven engineering* (MDE) [Amb04], such an approach increases the quality of the end products and the predictability of their behaviour which generally improves systems dependability. Although there are works on extending semantics and using external analysis tools for model validation [RSH11; HLV11], the original frameworks do not provide formal development facilities.

### 2.1.1 Usage of formal methods today

Formal methods provide high level of assurance by applying mathematical rigour during the modelling process. Various formal techniques are used at all stages of system development including requirements engineering [Eas+98; CDV98; Ham+95], software specification [BS03; Abr96; Abr10; WD96; Jon90], high-level architectural design [All97; AADL], software design [Jon90; Bac80], implementation [Ros95], and testing [HBH08]. The thesis focuses on using formal methods at the early phases of specification and design.

The purpose of a formal specification of the system is to arrive at a correct model shown to satisfy the requirements. The formal specification is then used at later
stages of development to ensure the correctness of the implemented system. Formal specification may be also used for certification or to validate the correctness of an already deployed system post factum. There are several types of formal approaches which differ in their ways of assuring the model correctness.

*Model checkers* [Cla08] ensure the correctness of a model by executing the behavioural part of the models and checking, at each state or a group of states, whether the required properties hold. Model checkers typically accept properties expressed in a temporal logic. This allows developers to check liveness properties of the models. Although various optimisations have provided major improvements in model checking capabilities, the approach lacks of scalability due to state space explosion on real-world problems [Pel09].

Another formal approach used in software engineering is *test case generation* [Bro+05]. It consists in comparison of the formal specification of a system and its executable implementation. The specification is used to derive test cases which are run against the implemented code. Thus, the implementation is guaranteed to conform to the specification with respect to the test case coverage. The derivation of test cases is typically automated, and the process of implementation can follow the test-driven development approach [Bec03].

Other approaches closely related to test case generation are assertions and design by contract. The *assertions* [Ros95] are properties expressed on the local state of the programming unit checked statically or at run-time. Some assertions are statements over factual parameters of methods. They represent assumptions about the parameters passed by another block of code. The assertions can be statically analysed (proven) given that the guarantees of the caller are also defined. Such pre- and post-conditions are used during *design by contract*. There are a number of libraries and programming languages supporting the design-by-contract approach such as: MS Code Contracts [MSCC], MS Spec# library [MSSP], Eiffel programming language [Int06], and Java Modelling Language [Cha+06]. In these languages, a formal specification of the system behaviour is essentially intertwined with the implementation and is expressed at the same level of abstraction. The mentioned libraries are language-specific, and are used in practice for finding common programming bugs.

*Theorem proving* is a rigorous approach to formal assurance of the intended system behaviour. In proof-based methods [Abr96, Abr10, WD96, Jon90], one specifies the behaviour of the system and a set of safety properties. A developer is responsible for showing that the model satisfies the properties by proving proof obligations generated by the methods. Thus, when the system is implemented according to the specification, it will maintain the properties during execution. No actual execution of the model is performed during the formal development. Automatic theorem provers [RV01] may be also used to prove (a part of) the generated proof obligations. By proving the
obligations one guarantees the full coverage of the model state space against the safety properties specified \cite{WD96, Abr96}. Typically, liveness properties can also be verified using an external model checker that supports the notation of the method being used \cite{LB08}.

### 2.1.2 Success stories and problems

A number of surveys \cite{Woo09, HB95, BH06} report on an industrial uptake of formal methods during the last 20 years with an increasing use at early phases of specification and design. The surveys show a generally positive effect of using formal methods on time and cost of development, and quality of the final product.

Although formal methods are not widely used for developing day-to-day commercial software, the necessity of their use for building highly dependable systems is evident \cite{Rus89, HG93, HBV10, HB99}. There are a number of successful industrial projects where formal methods were applied and the resulting systems are now in operation. In transportation sector, Siemens Transportation Systems \cite{STS} (formerly Matra Transport) heavily uses B as a high-level design language for specification and proving correctness of the control logic of train systems. Line 14 of the Paris underground metro \cite{Beh09} and a train shuttle for Roissy Charles de Gaulle airport \cite{BA05} were developed using the company’s established development process based on B. Notably, both systems are driverless. The B and Event-B methods have been also used for the development of train signalling systems in Brazil conducted by the Aes Group \cite{RL12, D15.5}.

In microchip design, validation plays a crucial role due to sheer complexity of microprocessors. Verification and theorem proving have been the major techniques in validating the instruction set specifications \cite{PJB99, Mur08, Hun89, Win94}. One example of a successful application of formal methods is a formal validation of the instruction set architecture of the XMOS XCore using Event-B \cite{Yua11}.

Some other examples of applications of formal methods include the design and verification of embedded medical devices \cite{GO11, QNX}, subsystems of satellites in the aerospace domain \cite{Ili10}, voting algorithms \cite{Bry11}, and distributed systems coordination \cite{ASAA09}.

The increasing complexity of critical systems make them an appropriate target for a top-down development approach. The success of applying refinement-based formal methods mainly comes from the ability to design a system incrementally starting from an abstract representation. By defining an abstraction, the top-down methods allow developers to capture the essential functions of the system without spreading the modeller’s attention on details. At each step, the model is formally refined thus introducing lower-level concepts and behaviour. However, refinement is also an engineering process where design mistakes are inevitable. The process of arriving at
a detailed model of a system can be described as a traversal of a tree of models. A modeller starts from the root, and by refining the initial model he/she traverses through the tree until he finds an acceptable detailed model. At some point while verifying the required properties, a modeller can realise that he has made a mistake or some abstract formal elements prevent from modelling the desired system behaviour. Then, he needs to rollback and make a change to an abstract model. This leads to changes in the rest of the already modelled refinements. Therefore, the modelling and proof efforts for redevelopment of abstract behaviour is generally higher than that of concrete models due to proofs associated with refinement. To minimise such costs, there is a need for an effective way to cut out those models and modelling decisions which are known to be unacceptable a priori.

Despite the success stories and increasing use of formal methods, they are not yet the rule for developing dependable systems. Among the main problems of adopting formal methods are a steep learning curve for engineers and a general lack of tool support [Woo+09]. An effective solution to the former can be a set of principles, practices, and patterns that teach engineers the right ways to model certain aspects of the systems within a particular domain or formal method. In object-oriented software engineering, such approach is now widely used and is known as design patterns [Gam+94].

### 2.1.3 System context

A formal method is a flexible tool for specifying what a system should do omitting the details of how it should be done. However, the specification of what a system must do is a complex task in itself due to a significant semantic gap between informal language of requirements and a formal language of specification.

The purpose of any artificial system is to bring about changes to its problem domain. The part of the problem domain that can be observed and changed by a system represents the system environment, or its context. It can include a part of the physical world or another technical system or both.

The idea of the system context and analysis of its phenomena is given attention in the Problem Frames requirements analysis approach [Jac01]. In Problem Frames, after defining the context of the system, one gradually decomposes both the system and the environment until a sufficient level of requirements granularity is achieved.

The HJJ approach [HJJ03] builds on the Problem Frames thinking and focuses on the interface between a control system and its environment. It shows that specifications of many systems may be derived from those which include the context and its physical phenomena. The process of defining system requirements and its context provides insights into its intended behaviour, and helps in identifying requirement ambiguities and inconsistencies [D1.1].
Even at the finest level of details, requirements are still informal and have to be formalised for a concrete specification language. One solution to requirements formalisation can be a user-defined explicit mapping of requirement terms into formal specification terms [JHL11]. Thus, formal reasoning may reveal mistakes and omissions in requirements during modelling. With such a solution, the separation of the system from its context remains informal.

Different formal methods may provide different means for modelling environments. The common issue here is a semantic gap between the language used to express an environment and the formal language for specifying a system behaviour. For example, a physical environment of a control system may have continuous-time nature that is expressed using differential equations, and the high-level logic of the system may require discrete-time modelling. A number of solutions can be used to bridge this semantic gap: the system model and the environment model may be expressed using different languages and used during co-simulation [Fit+10], or an abstraction of the environment can be defined in the target formal language used to specify the system behaviour [HH11]. In both approaches, the system model has to contain definitions representing the relevant part of the system context.

2.1.4 Event-B

The development method described in this thesis is exemplified on Event-B formal method [Abr10]. Event-B is a state-based formalism closely related to Classical B [Abr96] and Action Systems [BS89]. The step-wise refinement approach is the corner stone of the Event-B development method. A combination of model elaboration, atomicity refinement and data refinement helps to formally transition from high-level architectural models to detailed, executable specifications ready for code generation [EB].

An extensive tool support makes Event-B especially attractive. An integrated Eclipse-based development environment [ROD] is under active development now and is well-supported. It is open for extension using the Eclipse plug-in mechanism [ECL]. The main verification technique is theorem proving and the development is supported by a collection of theorem provers [ATB] while there is also a capable model checker [PROB].

An Event-B model is defined by a tuple \((c, s, P, v, I, R_t, E)\) where \(c\) and \(s\) are constants and sets known in the model; \(v\) is a vector of model variables; \(P(c, s)\) is a collection of axioms constraining \(c\) and \(s\); \(I\) is a model invariant limiting the possible states of \(v\): \(I(c, s, v)\). The combination of \(P\) and \(I\) should characterise a non-empty collection of suitable constants, sets and model states: \(\exists c, s, v . P(c, s) \land I(c, s, v)\). The purpose of an invariant is to express model safety properties. In Event-B an invariant is also used to deduce model variable types.
$R_I$ is an initialisation action computing initial values for the model variables; it is typically given in the form of a predicate constraining next values of model variables without, however, referring to previous values - $R_I(c, s, v')$.

$E$ is a set of model events. The general form of an event in Event-B notation is

\[
\text{name} = \text{any } p \text{ where } H(c, s, p, v) \text{ then } S(c, s, p, v, v') \text{ end}
\]

where $p$ is a vector of event parameters, $H(c, s, p, v)$ is an event guard, and $S(c, s, p, v, v')$ is an event action expressed as a before-after predicate. An event may fire as soon as the condition of its guard is satisfied and no other event executes at the same time. In case there is more than one enabled event at a certain state, the demonic choice semantics is applied. The result of an event execution is some new model state $v'$.

The semantics of an Event-B model is usually given in the form of proof semantics, based on Dijkstra’s work on weakest precondition. A collection of proof obligations is generated from the definition of the model and these must be discharged in order to demonstrate that the model is correct. For an abstract model (a model that is not a refinement of another model) two such proof obligations are the invariant satisfaction and event feasibility. A new state produced by an event must satisfy the model invariant:

\[
I(c, s, v) \land P(c, s) \land H(c, s, p, v) \land S(c, s, p, v, v') \Rightarrow I(c, s, v')
\]

An event must also be feasible, in a sense that an appropriate new state $v'$ must exist for some given current state $v$:

\[
I(c, s, v) \land P(c, s) \land H(c, s, p, v) \Rightarrow \exists v'. S(c, s, p, v, v')
\]

There are also proof obligations to establish deadlock freeness, enabledness conditions and a collection of proof obligations for demonstrating Event-B forward simulation refinement [MAV05].

The traces of Event-B machine $M$ are defined as follows. Let us denote the universe of machine states as $\Omega$. Then $\Omega$ is the set of all safe states of a machine: $\Omega = \{v \mid I(v)\}$. For each machine event $e \in E$ consider a relational interpretation $[e]_R \subseteq \Omega \times \Omega$:

\[
[e]_R = \{v \mapsto v' \mid \exists p \cdot (H(v, p) \land S(v, p, v'))\}
\]

where $H$, $S$ and $p$ are, respectively, the guard, the body and parameters of event $e$. There are two special cases of relational interpretations. The relational form of initialisation is $[\text{INIT}]_R = \text{id}(\text{init})$ where $\text{init} \subseteq \Omega$ is the set of initial states for a machine. The relational form of $\text{skip}$ (a stuttering step event of a machine) is
\[ \text{skip} \] = \text{id}(\Omega).

Now, consider set \( Q \) of finite sequences of event identifiers, \( Q = \mathbb{P}(\text{seq}(E)) \). Using relational forms of events, one can convert a sequence \( q \in Q \) into a relation \( \psi(q) \subseteq \Omega \times \Omega \). Let \( \langle \rangle \), \( \langle e \rangle \) and \( q \bowtie t \) denote, correspondingly, an empty sequence, a sequence containing event \( e \) and a sequence concatenation of \( q \) and \( t \); \( \psi(q) \) can be then obtained using the following procedure:

\[
\psi(\langle e \rangle) = [e]_R \\
\psi(q \bowtie t) = \psi(q); \psi(t), \quad q \neq \emptyset \land t \neq \emptyset
\]

where \( ; \) is the relations composition operator: \( (f; g)(x) = g(f(x)) \). Let us consider now the sequences contained in set \( Q \). Some of them initiate with an event other than initialisation, and we need to reject such sequences. Also, some sequences may represent an empty relation, an event ordering that cannot be realised due to restrictions expressed in event guards. We define the traces of machine \( M \) as those event sequences that start with initialisation and represent non-empty relations:

\[
\text{TR}(M) = \{ q \mid q \in Q \land \exists t \cdot t \in Q \land q = \langle \text{INIT} \bowtie t \rangle \land \psi(q) \neq \emptyset \}
\]

2.1.5 Usage of Event-B in industrial and academic settings

This thesis is mainly based on results of the FP7 DEPLOY research project [DEP]. The overall aim of the project is to improve the development process for dependable systems by using formal methods. One of the project outcomes relevant to this study is a number of pilot developments modelled by four industrial partners [D1.1, D2.1, D3.1, D4.1]. During the pilot developments, Event-B has been used for achieving high system dependability by applying it in a number of different ways: it has been used as a development method with heavy use of functional refinement, as a specification language, and as a requirements engineering tool. Based on that experience, this work focuses on improving the usage of Event-B as a refinement-based specification language.

Event-B has a flexible notation which allows developers to express and refine system behaviour in various ways. Researchers and industrial practitioners have proposed a number of approaches to modelling in Event-B depending on the goal of modelling and the target domain. The original approach in J.-R. Abrial’s models [Abr10] mostly follows a top-down development of reactive systems, and heavily uses data refinement.

Another refinement technique that is given attention mainly in industry is atomicity refinement. In atomicity refinement [FBR12], the abstract event is refined by a group of concrete events. While all the concrete events represent the abstract state transition, only one concrete event formally refines the abstract event. The group of events thus represents a series of transitions which refines the abstract atomic action,
hence the name. At the concrete level the system becomes sequentially decomposed which limits the expression of system-level safety properties. The sequential decomposition of the model has a major influence on further refinements which is discussed in Chapter 4.

The problem of the semantic gap between the formal expressions of the system and its context (see Section 2.1.3) also impacts the modelling practices in Event-B. The properties that the modellers are typically interested in declare relationships between the system and its environment. Thus, the event guards have to reference system variables in order to re-establish the invariants. If an event represents the environment, such a reference in its guards would mean that the environment is aware of the system state. This is used in [SB11] to model the “tick” event which represents the flow of time. The event ensures that the system properties hold before advancing the time. In [But12] the same situation holds for events that represent the environment. The events representing the physical context refer to the controller state in their guards. This ensures that the environment changes its state only when the controller has finished the current control cycle. Such techniques should only be used under explicitly stated assumptions about the environment and the implementation context. Otherwise, they may implicitly introduce such assumptions through modelling the system behaviour. In particular, fault assumptions are essential for specifying system fault tolerant behaviour which is discussed in the next section.

2.2 Fault Tolerance

Critical systems’ complexity grows along with the societal demand for such systems. Many systems operate on resources of highest value such as health, lives, time, and money. We rely on critical systems and thus require developers to apply appropriate development techniques to ensure safety and efficiency [Kni02]. This essentially means minimisation of faults contained in the final system.

2.2.1 Definitions and taxonomy

In this work, we follow the terminology and taxonomy of dependable computing [Avi+04]. Fault is an internal flaw (or an external cause) of a system which resides dormant until certain circumstances arise. When the fault becomes active, it causes an error, a runtime deviation from a correct system state. An error which reaches the external interface of the system is considered as a failure of the system to provide its service. The concepts of error and failure are relative to the hierarchy under consideration: a failure of an internal component of a system can be considered as an error within the system.

There are a number of techniques to enhance the system dependability. Fault
**prevention** and **fault removal** techniques reduce the amount of faults in the product through enhancing the development process. Rigorous specifications, formal methods, software verification and testing all target the goal of producing an ideal system that does not fail.

However, any computer-based system also contains an interface with the physical world. That interface cannot be ideal due to physical deterioration. Even under the assumption of running a fault-free software (and especially without such assumption), any system eventually suffers from malfunctioning hardware or unforeseen circumstances. More generally, any non-deterministic part of the system context may introduce errors which are out of the system control, such as: physical environment, human operator mistakes, operating system exceptions, behaviour of the off-the-shelf components. To mitigate such situations at runtime, developers must introduce redundancy into the system, and **fault tolerance** [LA90].

Fault tolerance is a term for system mechanisms that are introduced during development and are used by the system during runtime to avoid system failure in presence of faults. Under certain conditions the system cannot provide a full service, and fault tolerance mechanisms can be used to gracefully degrade system functionality. Fault tolerance generally includes three phases:

- **Error detection.** The system must be able to detect that its state or the behaviour of its components is abnormal.

- **Error recovery** (or compensation). When the deviation of the behaviour is detected, the system performs some action to return to its normal operation.

- **Fault treatment.** To avoid repetition of the same error, the system can treat the fault if the cause is found.

The fault tolerance phases and their relationship with the concepts of faults, errors, and failures are described in [Avi+04].

### 2.2.2 Realistic systems and fault tolerance

The ultimate purpose of any system is to perform its function. As the complexity increases and additional constraints are enforced by requirements, non-functional properties become as important as the functional ones. In critical systems, safety and other dependability properties are major concerns. To improve dependability, the context of a system has to become wider and include physical phenomena, hardware and other component failures, operator behaviour etc. Any critical system has to specifically deal with undesired situations to perform its desired function. The undesired situations constitute an abnormal part of the system behaviour. However, the concept of “normality” is vague and specific to the system. The border between
normal and abnormal behaviour is important, however, it is not always feasible to fully differentiate between the two. For example, a degraded behaviour of a system is functional but the actual process of degradation is a reaction to abnormal situations which is a kind of fault tolerance.

Realistic critical systems may contain up to 35-40% of requirements devoted to fault tolerance. This is supported by our study of the requirements descriptions produced by deployment partners for the pilot and mini-pilot studies in the DEPLOY project. The detailed requirements documents for the case studies are largely confidential, but descriptions of the pilots are provided in public deliverables for the deployment workpackages \( D1.1; D2.1; D3.1; D4.1 \). Our study of the requirement documents shows that the major source of faults considered in these systems is the environment, including sensors, external networks and human operators. Dealing with software design faults is never stated as a requirement, and only rarely do requirements define hardware faults (e.g. node crashes in a distributed application) and state how these need to be addressed.

System requirements normally include description of degraded functionality, the most typical example being system safe stop. More generally, we observe that the requirements predominantly include information about how general system behaviour is affected by various abnormal situations. Unfortunately, this information is rarely explicitly stated as a priority requirement (sometimes, we needed to deduce this information from other requirements).

It has been found that many system requirements use the concept of operational modes \( \text{[Dot+09; IRD09]} \) to refer to different operational conditions resulting in different functionalities provided by the system. We observe that the description of system modes and mode transitions is often intertwined with error recovery. For example, at the system level, modes may represent fault handling through system degradation.

A final observation is that requirements related to error recovery are often not structured in a way that makes it easy for modellers to work with these issues. The relevant requirements are typically scattered over the whole requirements document and refer to issues related to different levels of abstraction. For example, none of the documents reviewed had a table of fault assumptions.

Dependability of critical systems is indeed a primary concern and significant efforts are being spent on analysis and improvement of reliability and safety. Nevertheless, such systems do fail and their failures often lead to major losses. There are several well-known examples of critical systems’ failures such as: the Ariane 5 launch failure \( \text{[Age96]} \), the losses of the Mars Polar Lander \( \text{[Lab10]} \) and of the Mars Climate Orbiter \( \text{[AA99]} \), and the US and Canada Northeast blackout 2003 \( \text{[For04]} \). It is not always possible to identify a single cause of failure in such cases, it typically represents a combination of engineering and management omissions. For example, it is well-known
that the initial cause of the Ariane 5 failure was a software bug: one of the software components, the Inertial Reference System (IRS), produced an exception which led to termination of an important piece of control software. However, the IRS software was reused from Ariane 4 for which it was tested to work correctly. The impact of the change of both physical and software environment on the operation of IRS was not checked rigorously in the new system. This is a clear example of poor reasoning about environment assumptions. The final trigger for the failure was the error recovery action that shut down both the main IRS component and its duplicate due to exceptions. The primary cause of such an omission was an incomplete definition of fault assumptions: the error recovery procedures focused mainly on hardware failures, and the IRS component was treated as a piece of hardware. The implicit assumption in this case was that the IRS control software always produces correct output, and hot-swapping is a sufficient recovery action. Absence of design faults in software which is developed using traditional methods is an unrealistic fault assumption. The fault tolerance mechanism based on such an invalid assumption led to propagation of the error and eventually to a system failure.

Various fault tolerance techniques are used nowadays in highly dependable systems at all levels of operation. In hardware, many techniques are based on hardware redundancy for fault masking. That is, critical subsystems are built using a number of spare components and a voting mechanism that together provide a single function. The well-known example is a Triple Modular Redundancy (TMR) \[LV62\] which is built from three replicated active components and ensures fault masking by a voter. A more general design is called N-modular redundancy (NMR) which can tolerate \((N - 1)/2\) module faults during the majority voting. These approaches are considered as static redundancy; no action is performed upon detecting an error as the error is masked before reaching any other component. A complementary class of techniques includes dynamic hardware redundancy. These are used primarily in applications that can operate while receiving temporary erroneous results from hardware components. For example, duplication with comparison is an error detection mechanism; it uses two identical modules and a comparison mechanism. It always produces an output from one of the modules, be it correct or not, and a result of comparing the outputs of the two modules. The comparison result is then used by a higher-level logic for further recovery. There are also techniques that involve local reconfiguration of a component such as hot standby sparing, cold standby sparing, and pair-and-a-spare. With dynamic redundancy approaches, the reconfiguration process usually takes some time during which the component is not available or produces erroneous output. These static and dynamic redundancy techniques are often composed to achieve certain levels of reliability cost-effectively.

The primary causes of hardware faults are physical deterioration and external
interference, whereas the primary faults in software are design faults due to design complexity [Kni12]. In software, fault tolerance is present to some degree in every application written using a modern programming language. Exception handling and the principle of defensive programming form a common practice today. Nevertheless, it is not sufficient in many safety- and mission-critical applications. A well-known technique of *N-version programming (NVP)* is used in complex and/or critical applications to tackle design faults. In NVP, a number of different teams of developers are given the same specification to implement their “version” of software [Avi85]. All of these versions are then deployed in a single component of the system in a way similar to hardware modules. That is, they run simultaneously and vote on the output (NVP) or active sparing techniques could be used (*recovery blocks*). Most often, software is built using existing libraries, so called *off-the-shelf components (COTS)*. Additional measures are typically used to ensure the overall dependability of the critical software when using COTS. *COTS wrappers* [Pop+01] is the most popular fault tolerance technique that is given significant attention in research. In the distributed computing area, which includes business applications and high-throughput computing, there are solutions for tolerating *byzantine* faults.

Software fault tolerance has been traditionally an iterative engineering process. It is usually developed using the same principles as is the software performing the main functionality of systems. This means that it is susceptible to the same types of mistakes and, therefore, may contain faults. There is a need for methods and tools for development of highly dependable systems that would facilitate rigorous development of fault tolerance and help identify and eliminate faults during design.

### 2.2.3 Fault analysis and formal modelling of fault tolerance

To adequately handle the abnormal situations and improve dependability properties of a system, possible faults and failures must be specified and taken into account during design. There are a number of fault analysis techniques that are used by engineers in industry to achieve this.

Failure Modes and Effect Analysis (FMEA) is an inductive technique for safety analysis [FMIC; FMTR; Sto96]. It is a development procedure for analysis of potential failure modes of a system by listing their severity, probabilities, and effects. *Failure mode* is a general term for capturing possible faults, errors, and system failures. The technique is informal, it represents a part of the development process which helps engineers organise their expectations of the system failure modes and effects based on their previous experience. The technique mainly targets failure modes of individual components and their impact on the system behaviour.

Fault Tree Analysis (FTA) is another technique used in safety and reliability engineering [Ves81]. It is a deductive top down method in which a system failure or
another abnormal state is analysed into its low-level causes by using boolean logic. Given the probabilities of low-level errors (e.g. component failures), the likelihood of a target system failure can be estimated. Fault trees are used at various stages of development process from design to maintenance. The process of creating a fault tree is also informal and is based on engineer’s experience in a specific domain. FTA targets system and component failures at higher levels and allows for analysis into lower-level component errors.

In both FMEA and FTA it is an engineer who informally chooses which system failures need to be analysed. Such information can be also synthesised from the domain knowledge and a model of how system components are interrelated and communicate with the system context [McK+05; LGP11]. Hierarchically Performed Hazard Origin and Propagation Studies (HiPHOPS) [Pap+11] is an example of a model-based method for semi-automatic safety and reliability analysis. It allows developers to generate fault trees and FMEA tables based on a model of system architecture expressed in terms of components and material and data transfers. Such models provide useful information for the identification of error propagation paths that lead to system failures, and play an important role in the system design process. Cecilia OCAS [Bie+04] is another example of a model-based safety assessment framework that is capable of generating FMEA tables and fault trees from an architectural model. It is based on the AltaRica [Alt] language and is being used at an industrial level for architectural safety assessment of avionics systems.

Safety and reliability analysis is necessary for making design decisions during system development. As a part of specification, design, and possibly implementation, formal model of a system typically represents the decisions which were made based on safety and reliability analysis. Both during fault analysis and formal modelling, fault assumptions play the key role in defining the resulting behaviour and system properties [LA90; HJJ03]. A system is designed to perform its function within a certain environment. Thus, the estimation of the system dependability relies on the understanding of the environment and assumptions about uncontrolled phenomena. Wrong assumptions can lead to malfunctioning and unsafe systems which are still formally correct. Therefore, it is crucial to explicitly define fault assumptions upon which fault tolerance is modelled and then implemented in the system.

There are a number of studies on formal modelling of fault tolerance. Some research is done on extending original semantics of formal methods with additional fault tolerance modelling constructs. An example of such an approach is an extension of the Lustre data flow language for modelling faults and error propagation paths [JH07]. The extended LustreFM language allows developers to specify possible faults of a component and different aspects of fault activation such as triggers, durations, conditional activations, and error propagation rules. The authors envision the process of
safety analysis by using libraries of domain-specific fault model components that can be specialised for a particular system fault model. A similar approach to specifying error causality on top of functional models is taken in works on FPTN [FM92] and FSAP/NuSMV-SA [BV07]. A notable work on extending normal system behaviour with fault tolerance is described in [Jef+09]. Authors introduce a notion of partial refinement defined for state machines and use it to formally connect the normal behaviour of a system with the fault tolerant one.

There are numerous works on extending process-based formal methods such as CSP with additional formal constructs for modelling fault tolerant behaviour. One example could be the Peleska’s method for verification of fault tolerant systems with CSP [Pel91]. It provides algebraic and assertional techniques and is used in parallel with top-down design of FT systems. Other studies on extending CSP with FT-oriented semantics include an improved failures model [BR85] and message recovery techniques for CSP [Jal89].

Another class of FT modelling techniques include patterns and modelling styles for modelling fault tolerance in specific formalisms without extending their semantics. An example of such an approach is [LT04b] which provides a guidance to modelling fault tolerant control system in B. The authors focus on failsafe systems which can be safely stopped at any moment of time. The approach starts with an abstract general specification which is applicable to any failsafe control system. During refinement, system components are introduced and their failures are associated with the abstract safe stop. Error detection is paid significant attention as this is the phase where actual difference between normal and abnormal states is defined. The approach follows a typical control system design by modelling a control cycle consisting of the sensing, control, and acting phases. During sensing, errors can be detected and are classified into recoverable and non-recoverable types. In case when an error leads to a failure, the control operation is skipped and the system is stopped. A recoverable error is masked by one of the redundancy techniques, and the control operation continues. Thus, the functionality of the system is always provided under the assumption of fault-free components. The assumptions used in the approach limit its applicability. The approach is adequate for modelling low-level component failures that may be masked, but it is not designed for specification of graceful degradation and system-level recovery procedures.

A pattern for modelling fault tolerance in B is proposed in [LT04a]. The paper introduces a general formal specification pattern to be applied in development of dependable systems with a layered architecture. The pattern adds exception handling mechanism to each system layer and organizes communication between components within a hierarchical structure by means of exceptions. The layered exception hierarchy pattern is based on top-down refinement. The pattern follows the idea of idealised
fault tolerant component (IFTC) introduced in [LA90]. The IFTC is a generic component which explicitly differentiates between its normal and abnormal operation, and specifies the conditions under which it switches between the two. The system is thus constructed as hierarchical layers of IFTCs. Each component can handle certain exceptions, and it propagates the unhandled exceptions to the abnormal part of its higher-level component. The idea of IFTC implies sequential composition of component executions, and its application may undermine the ability to express system-level safety properties for some proof-based methods. Although the present work focuses on refinement-based development of reactive systems, we borrow the ideas of top-down system structuring and explicitness of system abnormal operation.

2.3 Views

Upon deployment, a computer-based system is required to perform its function, provide a certain level of availability and reliability, operate safely, and maintain other properties. The necessary properties of the system are defined by its requirements during early stages of development. Requirements typically cross-cut the system functionality, thus, developers need to create a solution which satisfies all of them. Some aspects of the system can be “kept in mind” through informal notes and experience, e.g. maintainability and performance requirements to a software product are typically met through an engineering effort by architects and software developers who have experience in low-level programming. When building critical systems, dependability aspects become the most important properties of the final system, and “keeping in mind” is insufficient to achieve high levels of safety and reliability. Besides, the development process and/or the final system can be required by a certification body to pass strict tests and comply to safety standards.

To address the problem of cross-cutting requirements and development issues, there are numerous works on their separation at different development phases. IEEE 1471 standard [S1471] describes a general framework for architectural description of software-intensive systems. In the standard, different aspects of development and system requirements are called concerns. Each concern is a reflection of interests of a particular stakeholder. A concern is represented in architecture by a viewpoint which can be related to other viewpoints and has a specific impact on the overall system architecture. A view is an instance of a viewpoint for a specific system under construction. The standard advocates an explicit separation of concerns through a set of documents (views) to enable specialists (stakeholders) to concentrate on specific problems in their area of knowledge. The ViewPoints approach [FKG90] is similar in its ideas of using multiple viewpoints at all stages of software development. The ViewPoints tool maintains the viewpoints consistency using distributed graph trans-
formations \cite{Goe+00}.

A similar approach can be found in architecture description languages. Authors of \cite{DR+10} describe a framework for semantic extension and MDE-based customisation of architecture description languages (ADLs) to address concerns defined for a particular project. Another example is the widely-used AADL \cite{AADL}. It contains a language for architectural description of functionality and an additional error model annex (viewpoint) that supports fault/reliability modelling and hazard analysis.

UML \cite{Amb04} offers facilities to model various aspects of a software product as separate diagrams. Each diagram is specifically designed to represent a certain concern of a developer. For example, use-case diagrams represent the specification of a system from the point of view of a user. Activity and state machine diagrams are behavioural descriptions. Class diagrams are tailored to object-oriented design of the system. Deployment diagram reflects the concern of hardware configurations during software deployment, etc. The main benefit of having separate diagrams in UML comes from their explicitness. They are used mainly for documentation and information exchange within the development team. Some of the diagrams related to object-oriented design and software behaviour can be used for code generation.

In formal methods, the separation of concerns is also given significant attention. An example of a formal approach to the separation of concerns is shown in the Rosetta framework \cite{AKS01, KA03}. The authors show how to formally accommodate and develop different views (facets) of the same system expressed using different computational models in a consistent manner. Another work on model views \cite{Jac95} gives a formal technique for partial specifications in Z. The work encourages multiple representations of the program state for separating different aspects of functionality. The views are then composed into a single model through cross-view invariants and common operations. Authors of \cite{DW06} propose a solution to the consistency problem between multiple view via model transformations. The paper introduces a proof technique that allows a developer to reason at a view level about cross-view model transformations. There are also some works on separation between functional and error models. A work on LustreFM framework \cite{JH07} offers a solution for separating the fault model from the functional model. The fault model is used for safety analysis and is composed at later stage with the nominal one. A similar goal of error modelling is achieved at the architecture level using the AADL error model annex mentioned previously \cite{AADL}.

The discussed multi-view development approaches, especially the ones designed for architecture and design levels, are widely used by industry. This highlights the claimed benefits of incorporating multiple viewpoints in a development process. Namely, separate viewpoints may provide explicitness and means for separation of responsibilities, improve documentation, and increase the overall quality of the products.
2.4 Problem Statement

As we showed throughout the chapter, the formal methods today are being successfully used for developing dependable systems, and there is significant research being done to improve the applicability of the methods. A major obstacle to wider adoption of formal methods remains the conservatism of engineering practices. The main arguments against using formal methods today are the lack of formal modelling training offered to industry users, and hence insufficient experience, and substantial efforts during formal modelling. Both of the problems can be addressed by providing guidelines to top-down development of critical systems. The engineers need support during the modelling process as well as during refinement planning. The effective guidelines must cover both of these activities: they shall provide practical modelling solutions that give specific design recipes, modelling patterns, and guide during refinement planning which is crucial for minimising proof efforts. We believe it is possible to define such guidelines, this is reflected by Hypothesis 3 in Section 1.2.

The problem of a semantic gap between specifications of a system and its environment needs to be taken into account during modelling. It is possible to introduce implicit assumptions about an environment while developing its abstraction. Since a system specification obtained via formal modelling is only correct with respect to the stated assumptions, there is a need for additional guidelines to adequately represent environments in system models.

Another problem of some of the current modelling solutions is that they may hinder the expression of the system-level safety properties. To overcome this, engineers need a set of explicit modelling principles that are oriented on using the original strength of formal methods without harnessing its applicability. In the context of fault tolerant systems, it means that the guidelines for modelling fault tolerance must maintain the original applicability of the formal method to modelling of functionality. We state this with respect to Event-B in Hypothesis 4.

We showed in this chapter that ensuring dependability properties of critical systems is given a high priority in research. Although many analysis techniques exist for revealing faults and error propagation paths in hardware architectures, formal top-down development of high-level fault tolerance is given little attention in practice. The techniques discussed do not provide means for smooth integration of fault tolerance and functional behaviour during formal refinement. Specifically, such fault tolerance mechanisms as graceful degradation and system-level recovery procedures comprise an inherent part of an abstract system behaviour. Existing fault tolerance modelling approaches treat such kind of fault tolerance as a system-specific functionality and leave the modelling to users. A solution to effective fault tolerance modelling may be a dedicated language integrated with functional modelling. However, current
approaches to multi-view specifications are typically informal and/or are designed for an iterative code-and-test development process. This makes their application to well-established refinement-based formal methods difficult in regard to fault tolerance modelling. A dedicated formal language for modelling of fault tolerance would validate our Hypotheses 1 and 2 stated in Section 1.2.

2.5 Conclusions

In this chapter, we provided a background on the three major areas or research that are relevant to the topic of the thesis. We showed that despite the increasing usage of formal methods, there are still major obstacles such as the lack of experience in formal modelling by engineers and significant efforts during modelling. Another topics that we covered include existing approaches to modelling fault tolerance and current research on multi-view development.

Finally, we have identified the key state-of-art problems that we address in this study by proposing a top-down method for developing critical systems which seamlessly integrates formal modelling of functional and fault tolerant behaviour.
Chapter 3. Modal and Fault Tolerance Views

This chapter presents a modelling environment for constructing modal and fault tolerant behaviour of systems. The environment provides facilities for creating formally defined views on Event-B models and provides consistency conditions between views and models.

We give an overview and basic definitions of the modelling language in Section 3.1. Then we introduce the rules of construction and top-down development of views in Sections 3.2 and 3.3 correspondingly. We present the formal consistency conditions between the views and Event-B in Section 3.4. Then we conclude and discuss some limitations of the approach in Section 3.5.

A practical application of the views is demonstrated throughout Chapter 4 as a part of the proposed method: Section 4.5.4 introduces a number of modelling templates and Section 4.6.3 shows the usage of the proposed Modal Views approach at one of the method steps.

3.1 Overview and Definitions

During early stages of the DEPLOY project [DEP] it was recognised [IRD09; Dot+09] that many challenging developments deal with dynamic system reconfiguration. Such models typically describe several “stable” phases of system behaviour and some activities that lead from one phase to another. In requirements, such phases, or modes, are often used to describe system behaviour in regard to environmental conditions, component errors, and system fault tolerance [D1.1; D2.1; D3.1; D4.1]. Such an observation has led to the design and implementation of the Modal and Fault Tolerance Views modelling language [WIFT; Dot+09; IRD09; LIR10]. The language provides an additional viewpoint introduced into the formal development process that is used to define modal and fault tolerant system behaviour.

The approach presented in this thesis builds on the initial idea of modal specifications of systems [Dot+09]. We provide a practical application and implementation of the idea, and extend the original approach to modelling fault tolerant behaviour of systems.

The language extends the Event-B modelling notation with a superstructure describing system modes and transitions between modes. It employs a simple visual
notation based on modecharts [JM94, MR98]. The graphical document, called a modal/fault tolerance view of a system, co-exists with an Event-B machine; the two define differing viewpoints on the same design. A modal view has a formal semantics (proof semantics and operational semantics [Dot+09]) from which a set of consistency and refinement conditions are derived. These demonstrate that a modal view and the corresponding Event-B machine are in a formal agreement. The approach is supported by a Modal and Fault Tolerance Views plug-in [WIFT] for the Rodin development environment [ROD].

A mode in a modal view is an island of relatively stable system behaviour. Within a mode a system still evolves but within far stronger limits than the safety invariant of an Event-B machine. Such limits are defined by a pair of assumption and guarantee predicates. The assumption predicate is normally interpreted as a set of conditions under which a system is able to stay in the mode; the guarantee describes what the system is doing while in the mode. The guarantee is a before-after predicate: it references both current and next states. Modes are related via mode transitions; these are also characterised formally and, in this respect, are similar to Event-B events. There are three types of modal transitions:

- **Normal** transitions represent functional reconfigurations of a system.
- **Error** transitions define changes of system behaviour in response to errors.
- **Recovery** transitions finalise the system recovery by returning it to normal operation.

Error and recovery transitions are special kinds of fault tolerance transitions. There are additional structural and refinement constraints on the usage of FT transitions.

The state model of a view is borrowed from an Event-B machine. The central feature of the method is a step-wise refinement of modal views along with the process of Event-B refinement. When a machine is refined, a developer also needs to refine the modal view to reflect the changes in the machine (or state view). There are a number of refinement laws describing possible ways of refining a modal view; in practice, these give rise to a number of templates offered to a developer.

### 3.2 Views Construction

The building blocks of a modal view are modes and transitions. A view must contain a single start mode and one or more regular modes. There is no explicit stop mode defined in the modal views language. In the method, we represent stop as a normal mode with stuttering behaviour, i.e. a livelock (self-loop) which does not change the system state. A system switches between modes via directed mode transitions.
Transitions are of three types as described previously. The initial transitions from the start mode must be normal. All modes must be reachable from the start mode through some transition path. These construction rules represent syntax-level consistency conditions that are checked by the tool during modelling.

An example of a modal view is shown in Figure 3.1. The start mode is represented by a circle, and regular modes are represented by named rectangles. The solid arrows depict the normal transitions (e.g., the initial transition and \textit{trans1}), the dashed arrows are the error transitions (e.g. \textit{error1}, \textit{error2}, and \textit{error3}), and dashed arrows starting with a filled circle represent the recovery transitions (e.g. \textit{recovery1}).

The special fault tolerance types of modal transitions define different types of modes. We differentiate modes by the types of transitions which are linked to them. There are three types of modes that we define:

- **Normal** modes typically depict a stable functioning of a system when it is fully operational.

- A **degraded** mode is used to describe the system behaviour under certain unrecoverable errors when the system can still perform some part of its functionality. There must be an error transition leading to this type of mode, but no outgoing recovery transition.

- A **recovery** mode represents the means of a system to recover from particular errors. A recovery mode is different from a degraded mode in that it has at least one outgoing recovery transition which returns the system to its normal operation.

For example, mode \textit{Rec} from Figure 3.1 is considered to be a recovery mode because it can recover the system from an error represented by \textit{error1}. Modes \textit{Mode1} and \textit{Mode2} are normal modes, they are connected by a normal transition. Modes \textit{Deg2} and \textit{Deg3} are degraded: only errors lead to these modes, and recovery back to normal operation is not possible. Note that mode \textit{Deg2} has two roles: it is degraded relative to error transition \textit{error2}, and it is normal relative to transition \textit{error3}.

We use the fault tolerance types of transitions to enforce fault tolerance related constraints on the construction of views. We require that at each level of modelling
there must be no cycles made of error transitions. More specifically, for each mode, if there is an error leading to some other mode and there exists a path back to the initial mode, then that path must contain a recovery transition. Thus, one should be able to differentiate normal from abnormal modes relative to a specific mode. E.g., a hypothetical transition \textit{error4} from mode \textit{Deg3} to mode \textit{Mode2} would make the view in Figure 3.1 invalid as there would be a cycle fully made of error transitions (\textit{error2-error3-error4}). That would mean that the system could switch between the three modes upon detection of certain errors indefinitely. Such a behaviour could not be given a comprehensible meaning.

Besides the structural rules given in this section, there are also formal consistency conditions described in Section 3.4.

### 3.3 Views Refinement

Once a modal view is in place for a particular Event-B model, it can be further refined to represent changes made in successive Event-B refinement. The modal views development process is a tree of documents much like Event-B development. There should be no confusion between two types of refinement: a mode can only refine another mode, and an Event-B model can only refine another Event-B model. The view refinement process forms a tree of modal views associated with a tree of Event-B machines (Figure 3.2).

![Figure 3.2: Modal views development chain](image)

A view can refine at most one abstract view, and can be associated with at most one model. However, the same Event-B model can be linked with any number of views, and will have proof obligations generated for each of the linked views. Therefore, there can be more than one view trees attached to the main Event-B development. A
modal view could also be abstract enough to represent a number of different systems. The models of the systems would then be associated with equivalent views. On the other hand, it is not mandatory to create a view for each model: a view is a modal representation of the system, and mode refinement can be skipped for those models that do not refine the system modal behaviour. We informally describe the view refinement process in the next two subsections devoted to mode refinement and transition refinement correspondingly.

### 3.3.1 Mode refinement rules

Each concrete mode must refine an abstract mode from an abstract view. Each abstract mode must be refined by at least one concrete mode; it can be a one-to-one mapping, however, the concrete mode must be expressed using the refined model variables (see Section 3.4).

Figure 3.3 shows an example of a refinement step performed over the modal view shown in Figure 3.1. For simplicity, only two modes *Mode*1 and *Deg*2 are refined. Abstract mode *Mode*1 (shown in a dashed rectangle) is refined into three sub-modes connected by normal transitions. Abstract degraded mode *Deg*2 is refined into two sub-modes.

![Figure 3.3: Example of a modal view refinement](image)

Whenever an abstract mode is refined into two or more concrete modes, each of the concrete modes depicts a more detailed and typically more deterministic part of functionality. This is ensured by the formal conditions of refinement described in Section 3.4.

### 3.3.2 Transition refinement rules

Each new transition has to refine either an abstract transition or an internal behaviour of an abstract mode. In the first case, a transition has to connect two concrete modes which refine two different abstract modes. The transition has to have the same
direction as the abstract transition. In this case, a concrete transition directly refines an abstract transition. In Figure 3.3, error transitions error2_1 and error2_2 directly refine abstract transition error2: they connect the concrete versions of abstract modes Mode2 and Deg2 and maintain the direction of transition. Another example is error3 which essentially depicts two transitions from Deg2_1 and Deg2_2, and is shown to originate from the group of the two concrete modes.

In the second refinement case, a transition can connect two concrete modes which refine the same abstract mode. Such a transition did not exist on the abstract level and is a refinement of the internal behaviour of an abstract mode. As an example, abstract mode Mode1 is refined into three sub-modes. There are three (unnamed) transitions which refine the internal behaviour of the abstract mode and become the three explicit transitions between the concrete modes.

It is a requirement that the concrete transitions belong to either one of the two types. We express such a requirement by the following rule: when concrete modes are projected onto their abstract counterparts, every transition must either project onto the internal behaviour of a mode or onto an explicitly defined transition with the same direction.

A fault tolerance transition can be only refined by a fault tolerance transition of the same type. This is demonstrated in the example by transitions error2_1 and error2_2. A regular mode transition can be refined by a more specific fault tolerance transition, but the opposite (generalisation) is not allowed.

### 3.4 Formalisation

The intention for the modal viewpoint is to offer a modelling assistant environment to Event-B developers. For the approach to be useful, there needs to be a formal relationship between a view and an Event-B model establishing that a model agrees with a view. Thus, a modal view alone would be enough to grasp the design of modal and fault tolerant behaviour in a model.

The formalisation approach is based on a more general notion of formal modal systems [IRD09]. There is a study on linking modal views and Event-B [Dot+09]. The consistency conditions discussed in this section maintain the original modal semantics and provide developers with a practical set of proof obligations.

#### 3.4.1 Well-definedness conditions

Mode is a general characterisation of a system behaviour. To match this notion in terms of Event-B models, all modes and transitions are mapped into event groups.

For a stronger notion of a view - model relationship, we consider an FT view as a set of modes providing different functionality under differing operating conditions. We
use the terms assumption to denote the different operating conditions and guarantee to denote the functionality ensured by the system under the corresponding assumption. With assumption and guarantee of a mode being predicates expressed on the same variables as an Event-B machine, we are able to impose restrictions on the way modes and transitions are mapped into model events and thus cross-check design decisions in either part.

Formally, a mode is characterised by a pair \( A/G \) where:

- \( A(v) \) is an assumption - a predicate over the current system state;
- \( G(v, v') \) is the guarantee, a relation over the current and next states of the system; and
- vector \( v \) is the set of model variables.

It is required to show that the assumptions exhaust the invariant and thus cover all the safe system states:

\[
I(v) \Rightarrow A_1 \vee A_2 \vee \cdots \vee A_n \quad \text{(COVER)}
\]

Here \( I(v) \) is a model invariant characterising valid states of \( v \).

One other important property of a mode is that it is possible for some state transition to take place within a mode. We do not need here to give a precise definition of such transition because this information would be later filled in by an Event-B machine. It is only necessary to show that there exists at least one such transition and thus mode characterisation makes sense:

\[
\exists v, v' \cdot I(v) \wedge A(v) \Rightarrow G(v, v') \quad \text{(FIS)}
\]

Thus, \( G \) can never be false everywhere while, under certain circumstances, this would be allowed for \( A \). Note that from above it follows that a mode assumption is satisfiable: \( \exists v \cdot A(v) \).

Each internal state transition must also preserve the machine invariant. For all the events, such condition is satisfied by Event-B proof obligations. Being a generalisation over a particular part of the machine behaviour, each mode has to preserve the invariant as well:

\[
I(v) \wedge A(v) \wedge G(v, v') \Rightarrow I(v') \quad \text{(INV)}
\]

In addition to modes, a view also includes transitions. Their purpose is to define
possible mode changes. A system switches from one mode into another through a
mode transition that non-deterministically updates the state of $v$ in such a way that
the assumption of the source mode becomes false while the assumption of the target
mode becomes true. Let us consider two modes, $i$ and $j$. A mode transition is
required to establish a new state $v'$ such that $\neg A_i(v')$ and $A_j(v')$, and it is not under
the obligation to respect $G_i(v, v')$.

It is required that all the modes are reachable. Although we could give a formal
test for this property, there is no need in additional proof obligation - such condition
is checked by the tool at the syntax level as mentioned in Section 3.2.

3.4.2 Event-B consistency conditions

With the basic formal framework of modes in place, it is possible to define consistency
conditions for a modal view and an Event-B machine. The core principle is seeing the
view as a source of further proof obligations for a machine. We do not translate modes
into Event-B or Event-B into modes. Instead, we add additional proof obligations to
a machine that establish the consistency with a given FT view. Formally, it does
not matter where the proof obligations are added - we could prove that a machine
is consistent with a view by adding theorems to views. It is, however, more natural
to deal with additional constraints in a machine, and the intuition is that a simpler
view should lead the development of a machine. As mentioned in Section 3.3, one can
prove that the same machine is consistent with more than one view.

One can also treat the resulting Event-B model as a composition of an original
Event-B model and a view. In this respect, it is related to Event-B $A$, $B$, and inter-
face types of decomposition [Hoa+11]. With Event-B (de)composition techniques,
one horizontally splits a model into two or more models to reduce proof efforts and
improve collaborative modelling. The current approach is different from existing de-
composition techniques in that it provides orthogonal models (views) that represent
the same system-level behaviour from different “angles” as opposed to component-
wise decomposition of behaviour. Thus, a model and a view “contain” each other
through formal consistency.

The first step to formally establishing such consistency is to relate modes and
transitions to machine elements. A view is linked with an Event-B model by attribut-
ing a list of Event-B model events to each mode and each transition:
\[ \begin{align*}
A_1/G_1 & \mapsto E_1 \\
A_2/G_2 & \mapsto E_2 \\
& \quad \ldots \\
A_n/G_n & \mapsto E_n \\
Trans_1 & \mapsto E_{n+1} \\
& \quad \ldots \\
Trans_k & \mapsto E_{n+k}
\end{align*} \]

The events which are mapped to a mode represent the internal state transitions that may occur while the system is in that mode. Each such event must preserve the mode guarantee and re-establish the mode assumption. The events mapped to a transition represent different ways in which the transition may happen. Unlike modes, the transitions are instantaneous. Thus, only one of the associated events fires while making the transition. Therefore, every event of a transition must establish the target mode assumption and falsify the source mode assumption as discussed earlier. Since the same event may be associated with both a mode and more than one outgoing transitions, the following proof obligation contains a disjunction of both conditions in its goal:

\[ I(v) \land A(v) \land H(v) \land S(v, v') \implies [A(v') \land G(v, v')] \lor \neg A(v') \land (A_1(v') \lor \ldots A_n(v')) \]  
(EVT_G)

where \( H(v) \) is an event guard, \( S(v, v') \) is an event action, \( A(v')/G(v, v') \) is the source mode, \( A_1(v') \ldots A_n(v') \) are the target modes of a transition. The first disjunct is only present in the obligation if the event is mapped to a mode. The second disjunct enables one of the target modes assumptions if the event is mapped to a transition.

The ability to map an event to many modes and transitions is the difference from the original approach to modal specifications \cite{Dot+09}. Although this distinction can dramatically weaken the proof obligations if overused, we allow such mapping for practical purposes. It allows for modelling modal features of systems at abstract levels where models typically contain non-deterministic events. Nevertheless, we assume and suggest that system models become more deterministic during refinement, and proof obligations generated from FT views at later refinement steps become stronger.

The next proof obligation that we describe states that the execution cannot progress if there is no suitable enabled event for a mode or a transition. It follows that the partitioning of the events into modes and transitions must be in agreement with the event guards. When an event is enabled then the assumption of its mode must hold. The same applies to a transition: the assumption of a source mode must hold when a transition event is enabled. Since an event can be associated with multiple
modes and transitions, the disjunction of all the relevant assumptions must hold:

\[ I(v) \land H(v) \implies A_1(v) \lor \cdots \lor A_k(v) \lor A_{tr1} \lor \cdots \lor A_{trn} \]  

(EVT\_A)

where \( A_1 - A_k \) are assumptions of the modes associated with the event, \( A_{tr1} - A_{trn} \) are assumptions of the modes of which outgoing transitions are associated with the event.

The requirement of execution progress also implies that at least one of the events associated with a mode must be enabled when the system is in that mode:

\[ I(v) \land A(v) \implies H_1(v) \lor \cdots \lor H_k(v) \]  

(ENBL)

where \( H_1(v) \ldots H_k(v) \) are the guards of the associated events.

### 3.4.3 Modal views refinement conditions

Refinement rules discussed previously in Section 3.3 are complemented with additional formal requirements. There are two proof obligations for each mode at each refinement step. The first obligation states that the assumptions of the concrete modes must be weaker than the abstract one:

\[ J(v,u) \land A(v) \implies A_1(u) \land \cdots \land A_k(u) \]  

(REF\_A)

where \( J(v,u) \) is the gluing invariant containing the relation between the abstract and concrete state variables \( v \) and \( u \) correspondingly, \( A(v) \) is the assumption of the abstract mode, \( A_1(u) \ldots A_k(u) \) are the assumptions of the concrete modes.

The second obligation requires that the guarantees of the concrete modes must be stronger than the abstract one:

\[ J(v,u) \land (G_1(u,u') \lor \cdots \lor G_k(u,u')) \implies G(v,v') \]  

(REF\_G)

where \( J(v,u) \) is the gluing invariant, \( G_1(u,u') \ldots G_k(u,u') \) are the guarantees of the concrete modes, \( G(v,v') \) is the guarantee of the abstract mode.

The understanding behind the proof obligations is that during refinement we widen the system operating conditions by weakening the assumptions, and make its behaviour more deterministic by strengthening the guarantees [Dot+09].
3.5 Conclusions and Limitations

The modal views approach is implemented as a plug-in for the industry-strength development environment Rodin [ROD] which is based on an open extension platform Eclipse [ECL]. The tool provides a view editor and seamless integration with the Event-B development extensions such as the project explorer and the proof obligation generator. The tool automatically generates verification conditions that are necessary to ensure that a given modal view is sound and consistent with an Event-B machine. A number of case studies have been developed using the tool to ascertain the scalability of the method and the implementation [WIFT; LIR10]. This work includes two industrial case studies described in the next chapters of this work.

One outcome of the experience with this environment (and the supporting tools) is that it is generally beneficial to stratify a design into aspects, or views. This permits a far more focused analysis and discussion of properties pertinent to a given view and a more explicit connection to any requirements about modal and/or fault tolerance properties.

The modal viewpoint brings benefits of explicitness and simplicity to non-formal engineers in industry. It provides an additional type of documents which can be used by requirements analysts and system engineers.

One of the main benefits of using the modal viewpoint is extra assurance in the final system behaviour. The views represent an additional source of behaviour definitions (along with behavioural part of the system model and safety properties) which helps in eliminating possible mistakes and omissions. The modal views approach follows the top-down formal refinement development thus supporting the main formal development. The views are a flexible solution for expressing orthogonal aspects of a system which are formally consistent.

Being a positive outcome of modal views, formal consistency also requires additional proof efforts from developers. On average, the usage of modal views doubles the number of proof obligations for those models which have one associated view. The proportion of automatically proved obligations is approximately equal to that of the original Event-B obligations. The complexity of those proof obligations that require interactive proof is also similar to the original Event-B proof obligations. However, complex views which have modes associated with large overlapping sets of events can generate complex EVT_A and ENBL proof obligations. This may be an indirect indication that the modal views are used at a very detailed level and should be simplified (abstracted).

The possible applications of modal views are limited by their semantics. A mode represents a possibly non-terminating behaviour, and therefore, liveness properties cannot be expressed with a modal view. Also since a single view covers the whole sys-
tem behaviour, it is unreasonable to create modal views for a sequentially decomposed system. The proposed method described in the next chapter benefits from using the modal viewpoint for modelling reactive behaviour of systems and leaves the sequential decomposition until later steps.

From an engineering perspective, the modal viewpoint is a flexible solution which requires engineering decisions and experience. However, it lacks modelling guidelines and usage patterns. The development method proposed in this thesis uses modal views as additional restrictions to the main modelling process. We provide a number of templates and define a development step for using the modal language.
Chapter 4. Development Method

In this chapter, we describe a method for top-down development of fault tolerant systems with a focus on abstract levels of modelling. The method addresses a number of issues of the state-of-the-art approaches that are described in Chapter 2 and is designed to fill the existing gap in modelling and verification of abstract fault tolerant behaviour.

The development method includes the following three constituents:

- the **modelling principles** stating the key points for modelling fault tolerant systems in refinement-based methods,
- the **refinement strategy** defining a sequence of refinement steps that need to be performed to arrive at a meaningful model of a fault tolerant system, and
- a set of **modelling patterns** and **modal view templates** that provide a reuse mechanism during modelling.

The three constituents together represent modelling guidelines for building fault tolerant systems in refinement-based formal methods in a systematic way.

The method is designed for modelling labelled transition systems. In our method, a system is composed of a set of states, a set of initial states, and a set of labelled transitions between states. Each transition is a partial relation over the set of states. The domain of a transition is defined by a domain restriction presented as a predicate. The set of states is partitioned using variables. The method specifies particular steps that need to be performed over the state transition system to adequately represent the system environment and correctly model the system fault tolerant behaviour. The guidelines proposed in this chapter can be applied during modelling in any state-based formal method with interleaving semantics such as Action Systems [BS89], B [Abr96], Event-B [Abr10], Z [WD96], VDM [Jon90], and TLA [Lam94].

The method consists of two parts. The first part is applicable to top-down development of fault tolerant systems in any problem domain. It produces a reactive system model [Ace+07] satisfying the required safety properties, and focuses on abstract modelling of fault tolerant behaviour as an inherent part of system functionality. The second part of the method is applicable to control systems. It contains guidelines
for modelling hardware units and implementation-level constructs such as a control
loop. This part prepares the reactive system model for implementation.

The description of the method is organised as follows. We start with stating the
basic assumptions and principles of the development method in Section 4.1. The
principles constitute a significant part of the method by underlying the key points
to be adhered during the refinement-based modelling of fault tolerant systems. In
Section 4.2 we provide a refinement strategy for modelling fault tolerant systems. We
introduce the first case study in Section 4.3 that is used throughout the chapter as a
running example to demonstrate the proposed development steps. The development
steps introduced in Section 4.2 are described in the rest of the chapter (namely, in
Sections 4.4, 4.5, 4.6, 4.7 and 4.8) and exemplified on the case study. We provide a
full list of the proposed modelling patterns and modal view templates in Section 4.9.
We finalise the chapter with conclusions in Section 4.10.

4.1 Assumptions and Principles

The development method is based on a number of assumptions and principles. We
start with an assumption that the system requirements were elicited prior to mod-
elling [HS99]. Changes to requirements during modelling can lead to (partial or com-
plete) redevelopment as it is the case with a waterfall development process [Sca02].

The next assumption highlights the intended outcome of the modelling process:

The ultimate purpose of modelling is to create implementable behaviour sat-
isfying the desired properties.

To satisfy the purpose, properties must be expressed in the model. These can be
safety properties expressed as invariants and proved by a theorem prover or liveness
properties to be verified by a model checker. To help developers in expressing the
desired properties, they must be easily expressible. The model should also be imple-
mentable. It should represent a behaviour that is sensible and can be implemented
and deployed in a real system. This assumption leads to a number of modelling
principles that are described in the following subsections.
4.1.1 Multi-view development

One of the key features of the proposed method is the usage of an additional viewpoint \[FKG90\] that contributes to the correctness criterion for the models. The additional viewpoint fits into state-based semantics of the target formal methods, references the model elements, and generates a number of extra proof obligations. The proof obligations represent an additional behaviour coverage that adds rigour to the development process. We treat the views as collections of diagrammatically represented properties that otherwise could be difficult to express in the model, or can be missed.

We use the Modal / Fault Tolerance Views approach for expressing modal and fault tolerant behaviour in separate views (see Chapter 3). A usage of the orthogonal modal viewpoint provides three major benefits:

- The activity of system design in terms of modes, errors, and recovery transitions gives additional understanding of the system and its modal and fault tolerant behaviour.

- In requirements engineering, additional viewpoint represents a means for tracing certain kinds of requirements, such as descriptions of system degradation, error conditions, and system-level error handlers, into the formal development process.

- For engineers who are not involved in formal modelling, the diagrammatic part of views is easier to understand than a formal model.

4.1.2 Co-refinement and restricted modelling

Refinement is a formal technique for adding details into a system model and arriving at a model sufficiently detailed for further development steps such as implementation or code generation. Application of formal methods, and in particular refinement, is an engineering task. Refinement does not prescribe the exact way to model a system. The same system can be modelled in various ways due to expressiveness of formal notations. The criterion for a model’s usefulness is whether the model contains the intended behaviour which satisfies the desired properties. Consider Figure 4.1. The system behaviour must always ensure that the desired properties hold. In this regard, properties represent the safe states of a system, but there can be more than one behaviour satisfying the same properties. That is, the currently defined behaviour may deviate from the intended one. By restricting the modeller’s choices, the proposed method helps the modeller to focus on the desired behaviour and properties of the model.

With the development process based on refinement, one starts from an abstract model of a system that is typically non-deterministic. By making a number of steps,
a deterministic implementable model is produced. One unveils system details through state refinement and removes non-determinism by restricting the behaviour. Removing non-determinism is analogous to cutting off the traces of behaviour that are not desired or irrelevant to the properties under investigation. The proposed method uses a concept of additional views on the system model as a means to introduce restrictions into the modelling process. The additional restrictions remove the behaviour that is not specified on the views thus forcing developers to make models sufficiently deterministic.

In the method, models and views are refined in parallel and when necessary. Refinement of views ensures that the modal behaviour of the system becomes more deterministic. By putting additional restrictions on the model, view refinement also ensures that the model becomes more deterministic and still complies with the modal and fault tolerant behaviour described in the views.

4.1.3 Behaviour restriction

In the development method proposed in this study, we see the system model as a transition system that is “composed” of two parts: an unconstrained behaviour and a set of functional and fault tolerance constraints. An unconstrained behaviour contains all system states and all transitions; it is merely a declaration of the system structure using variables. A model without constraints has a non-deterministic behaviour as it can go from any state to any other state. Although theoretically valid, such a model rarely has a practical application. In order for system to “behave”, i.e. realise certain properties, we introduce constraints that define valid sets of states and transitions.
The constraints define the part of a transition system that is safe and sensible in the problem domain.

We consider a transition system development as a sequence of two kinds of steps: *state refinement* and *behaviour restriction*. A state refinement functionally redefines a state (or a set of states) by its more detailed version that typically contains more states. A state refinement can also add new states thus expanding the state space. It is a top-down process of adding details to the system structure by declaring new variables and relating them to the previously defined variables. The second type of step, behaviour restriction, introduces new constraints and rejects previously acceptable states and transitions. By reducing the number of transitions originating at the same states we reduce the system non-determinism.

For example, in Event-B, state refinement is conventionally called “data refinement”\cite{Abr10} and is conducted during a vertical refinement step. Behaviour restriction does not have a conventional name in Event-B and is regarded as a part of functional refinement. Here, we stress the importance of separating the two aspects of refinement for better understanding of system behaviour and dependencies between system components.

The method advocates modelling the abstract functionality together with fault tolerance in its *unrestricted form* first. Such a form contains elements of behaviour that cover all possible system state changes without constraints. Additional constraints are introduced during behaviour restriction steps to satisfy requirements. Fault tolerance requirements represent the constraints over the system behaviour similarly to the functional requirements.

### 4.1.4 System environment

This principle is induced by our choice to use state-based formalisms for modelling high-level system logic. The semantics that we choose for our method is interleaving with atomic actions. It can differ from the language that is conventionally used for expressing the system environment. For example, a physical environment of a system would typically be of a continuous-time nature that might involve stochastic processes. We state that an environment has to be properly abstracted in order to create a correct model of a system:

```
The system environment must be adequately represented in the system model.
```

To represent the environment, one has to adequately model a state-based abstraction of the environment and define assumptions about the environment. The adequacy of the model is always relative to the assumptions that we state and the purpose of
modelling, that is the properties that we intend to demonstrate in our model. Developers may also need to refine the abstraction when using refinement-based formal methods. Starting from the top level of development, the model inevitably contains a certain form of abstract system environment. During refinement, both the system and the relevant part of environment have to be specified in details.

This leads to a requirement for the developer: he/she must understand the nature of the variables in the model, and their future implementation in the real system. The variables may represent the logical state of the system under control, the user, another technical system, or the system physical environment.

### 4.1.5 Implementable causality

In our method, we assume that a system observes some part of its environment and reacts to its changes. By reaction we mean any state change in the system: this includes changes in its internal behaviour and external actions towards the environment.

All state transitions that occur during system execution need to satisfy the causality rule:

A cause should not be dependent on a reaction.

Typically, one needs to define a context of a system by explicitly separating the system from its environment. The assumptions about the environment can be defined explicitly as properties or implicitly using the language semantics. The causality rule warns about a possible “trap” of modelling an environment that “waits” for system execution without explicitly stating that as an assumption. It is acceptable when an environment consists of another technical system that is designed to wait for the target system. However, in many cases the system reaction must follow the environmental cause, but not the opposite.

For example, let us define a property specifying a simple relationship between a cause, and a reaction of a system:

\[
\text{cause\_variable} = \text{CAUSE} \Rightarrow \text{reaction\_variable} = \text{REACTION} \quad (4.1)
\]

We assume that the cause is a part of the environment that we cannot control. To maintain this property, both variables \( \text{cause\_variable} \) and \( \text{reaction\_variable} \) need to be changed at the same time. We focus on modelling the system behaviour, and, thus, our abstract representation of the environment (such as reading a sensor) contains non-deterministic updates of variables. If a transition changes \( \text{cause\_variable} \) non-deterministically and does not update \( \text{reaction\_variable} \), it cannot be shown to re-establish the property due to weak hypotheses. To overcome this, the domain
of the transition may be restricted to certain values of \textit{reaction\_variable}. Such a constraint would mean that in order to change variable \textit{cause\_variable}, that represents an external phenomenon, the environment would have to wait until variable \textit{reaction\_variable}, that represents the system state, has a certain value. This breaks the causality principle.

Exemplified in Event-B, this principle means that the Event-B events representing environment must not refer to system variables in their guards. If they do, the justification for such a design has to be given explicitly as an assumption about the environment. Otherwise, environment events may represent false assumptions about the real world and become unimplementable. As a result, the modelling and proving effort becomes a mere mathematical exercise without real application.

A possible solution to this problem may include modelling the relevant part of the environment \cite{HHJJ03,HH11} and expressing environment phenomena as a part of system properties. However, our method focuses on modelling the system behaviour, and only stresses the necessity to use the implementable causality principle in regard to system environment.

4.1.6 Reactive systems and property coverage

Invariants at the current abstraction level can only cover state relationships within a single atomic change.

By an atomic change, we understand a transition that changes all (or a part of) the variables referenced in the property in such a way that preserves the property.

Let us consider the property defined in 4.1. If the system reaction consists of a sequence of steps, the original property cannot be proved as it does not hold until all steps are executed. Every step has to be covered by a finer-grained invariant; this means that the relationship between the cause and the final system state cannot be expressed as a single safety property at this particular level of abstraction in case of a sequential behaviour.

Note that the atomicity refinement can still be applied, and the system-level safety properties can be preserved during further refinement steps. However, the formal connection between the steps of a sequence cannot be captured in safety properties defined at the current level. Some formalisms provide means for expression of liveness properties as transition convergence (for example, Event-B \textit{variants}). However, this approach does not allow developers to explicitly define complex properties, and, in practice, is mostly used for demonstration of system-level termination. To allow developers express the necessary safety properties, we advocate the usage of abstrac-
tions and reactive style of modelling at higher levels and postponing the sequential decomposition until lower levels:

For high-level modelling, it is desirable to have as few blocks of atomicity as possible, one being an ideal case.

By reactive models we mean such definitions of behaviour that allow developers to express high-level properties in a form $cause \implies reaction$. The method proposed in this work facilitates development of reactive models. The step for modelling a classical control cycle of a control system is done at later stage when the reactive functionality of the system is modelled.

### 4.1.7 Error modelling

Faults and the resulting errors are inevitable phenomena of the final systems [LA90]. They need to be modelled appropriately so that the system behaviour stays deterministic in hazardous situations.

The proposed method focuses on modelling and refinement of abnormal system behaviour and its environmental causes. Abnormal situations impact the system behaviour in the same way as functional parameters do. However, such situations are usually associated with hazardous states and severe consequences.

The method adopts error modelling from the early stages of development, and facilitates tracing errors into requirements. Abstract errors are refined into more specific ones; this refinement corresponds and is formally related to the appropriate refinement of functional behaviour. For each functional level where a fault-tolerant behaviour exists, there must be an appropriate abstraction of errors.

Errors must be abstractly modelled from the early modelling steps where functionality depends on the environmental conditions.

The error modelling principle is closely related to fault injection techniques [Avr+96]. Fault injection approaches provide means for including faults to an existing model as external entities and for checking that the system behaviour remains satisfactory. In the proposed method, all faults originate in the system environment, and a system observes manifestations of faults through error detection. Thus, error modelling activity is a kind of fault injection, and compliance to the error modelling principle guarantees the formal correctness of the system behaviour in presence of injected faults.
4.1.8 Refinement planning

The result of a refinement-based modelling is a chain, sometimes a tree, of models. Each level of the chain has to be in the refinement relation with the previous one. The modeller arrives at the resulting model by exploring a tree of possible models. As each step is associated with a proof effort, the cost of redevelopment of the abstract levels is generally higher than that of the lower ones. To reduce the cost of redevelopment, one needs to plan ahead by considering options for modelling certain phenomena [GIL12]. As this method focuses on fault tolerance modelling, we provide a number of modelling practices and patterns, and formulate some modelling principles that need to be understood when planning the refinement chain.

The system components represent various parts of the system and form a tool for hierarchical abstraction. For example, a car engine is a component of a car and also is a system itself with subcomponents such as valves and various sensors. During the specification construction, components need to be defined in a top-down manner. This is because subsystems are designed to provide certain functions to support the main functionality of the system. For example, the process of creating a car specification does not start with specifying the sensors’ sensitivity. The necessary and sufficient level of sensor sensitivity is unknown until the desired properties of the engine are defined. On the other hand, the developers need to know possible low-level solutions in order to make decisions about high-level architecture. Such two-directional dependency between levels of abstraction is an essential property in many engineering fields including modelling. This includes refinement-based top-down formal modelling where one needs to know possible and acceptable solutions of low-level modelling in order to construct an abstract model.

Starting from the very abstract, models should represent system components at appropriate abstraction levels. At the first level, the system is a single component as a whole. The next level should introduce components that represent a decomposition of the system, and so on. Such components as sensors, actuators, and communication channels should be introduced at lower levels and linked formally with the abstract components and overall system behaviour.

The same principle applies to error modelling and fault tolerance. As the desired high-level properties of critical systems may include fault tolerance, low-level errors need to be abstracted and included in high-level components’ behaviour. This leads to the necessity of a pre-modelling hazard and failure analysis, an environment analysis, and a possibly informal description of a system architecture. Only based on such analysis, one is able to construct failure and error abstractions, and make high-level decisions about the system behaviour that includes fault tolerance. Levels of abstraction at which fault tolerance is modelled need to correspond to those for functionality as both types of behaviour are expressed in terms of the same components.
4.2 Refinement Strategy

The development method prescribes a number of steps that need to be performed to arrive at a meaningful model of a fault tolerant system. At each step, certain development actions are taken such as editing or refining a system state model or refining a modal view. The schematic procedure of the development method is shown in Figure 4.2. The development method is divided into two parts: the first part contains steps for a generic development of reactive fault tolerant systems and is applicable in any problem domain; the second part focuses on control systems and facilitates modelling of low-level components with an intention of further implementation.

Figure 4.2: The steps of the method
Abstract modelling of a reactive fault tolerant system starts with defining a failure-free functionality of the system. By failure-free functionality we mean the abstract behaviour only restricted by functional requirements. The abstract behaviour is then refined and strengthened to satisfy fault tolerance requirements. At the first abstract level where fault tolerance requirements impact the system model, a designer has to choose an abstract fault tolerance class of the system. We define two such classes by answering the question whether the system can mask all component errors. If the system can eventually stop due to unrecoverable errors, then the safe stop step is applied as a starting point for further refinement of fault tolerant behaviour.

The system functional behaviour is refined until it reaches a level of component granularity used in fault tolerance requirements. We assume that fault tolerance requirements enumerate component errors and describe error recovery procedures. At this level, the system becomes aware of possible component errors and contains appropriate reactions. The system stopped state is then refined by a combination of component errors. At subsequent steps, component error states are further refined by its sub-component errors etc. Thus, system functionality and component error states stay at the same level of abstraction at all refinement steps. The functional and error states related to a single component together comprise a fault tolerant component in this work. Once the error states are defined for all components at the current refinement step, the system behaviour must be restricted to contain the appropriate reactions, i.e. fault tolerance. The modelled error states restrict the possible state transitions of the system; this is done at the fault tolerance behaviour restriction step.

The process of alternating the two steps, functional refinement and refinement of fault tolerance with subsequent restriction, continues until all the required properties of the reactive system behaviour are expressed and verified.

The second part of the method refines the reactive model into a model of a control system. The two steps performed are inclusion of low-level hardware units (sensors and actuators) and realisation of a control cycle.

We describe each of the refinement steps in the rest of the chapter as follows. The steps for functional development are project-specific and are left to modellers. The discussion of the abstract fault tolerance classes of systems including the safe stop modelling is contained in Section 4.4. The step of fault tolerance component refinement and behaviour restriction takes the central place in the method and is split into Section 4.5 and Section 4.6 accordingly. The refinement of system components with sensors and actuators is described in Section 4.7. The step of implementing the control loop of a control system is given in Section 4.8.
4.3 Airlock Case Study

This section contains a description of a case study that is used in the rest of this chapter as a running example of the proposed ideas. We formulate the requirements for the system as a series of statements. Each statement has an identifier, such as ENV0, and an informal definition of a requirement itself. The prefix ENV identifies assumptions about the system environment, statements starting with FUN describe the desired functionality of the system, SAF define the safety properties, and FT define the fault tolerant behaviour of the system. Some requirements are explicitly traced into models where stated (most of SAF and FT) while others provide assumptions that are used implicitly (such as ENV). The identifiers serve as references that we use throughout this chapter to link the elements of modelling with requirements.

The case study is an airlock system. The function of the airlock is to separate two areas with different air pressures and allow users to pass safely between the areas (Figure 4.3).

![Airlock System Diagram](image)

Figure 4.3: The airlock system

For clarity let us call the two conjoining areas as external (the left area) and internal (to the right). Let us also assume that the pressure outside is lower than inside. We can describe these assumptions about the environment of the system as the following statements:

| ENV1. The airlock system separates two different environments. The pressure of the external environment is lower than that of the internal one. The internal environment is considered to be natural to humans. |
ENV2. In order to maintain different pressures, the two environments must be physically separated.

The primary function of the system can be expressed in the following form:

FUN1. When in operation, the airlock system must be able to let users pass safely between the two environments via the airlock.

Considering ENV1-2, the system implements its function FUN1 using a number of physical components:

ENV3. The system has two doors and a chamber. Each door when closed separates the chamber from the appropriate environment.

ENV3 already describes a part of the solution to the problem as it defines the components that are used to implement the desired function. Such solutions come from domain experts and engineers. The domain knowledge should be expressed explicitly in requirements so that they can be formalised and traced back. It is not always practical to formally establish all properties (e.g. FUN1 is a liveness property and can be difficult to express in proof-based methods), however, the system architecture and abstract components such as described in ENV3 is necessary for top-down refinement modelling. Safety requirements can be (and have to be) expressed as invariants in the models.

Safety properties described in this section do not completely cover all safety concerns that would arise for a real system. For example, a user would be required to wear special equipment while in the chamber in order to survive the change of pressure. We implicitly assume that this and other possible safety requirements are satisfied. We only focus on a particular part of system properties described in this section to limit the context of the case study.

The first safety requirement SAF1 limits the allowed range of pressure that the system cannot exceed. Such a requirement is implied by ENV1 and ENV3:

SAF1. The pressure in the chamber must always be between the lower external pressure and the higher internal one

Following ENV2, we can also state that it is unsafe to let a door open when the conjoining areas have different pressures, therefore:
SAF2. A door can only be opened if the pressure values in the chamber and the conjoined environment are equal

The other two safety requirements can be inferred from SAF2 and ENV1-3. Stating them explicitly facilitates the formal modelling of ENV1 and ENV2 as well as it can help to informally validate the requirements:

SAF3. Only one door is allowed to be opened at any moment of time

SAF4. The pressure in the chamber shall not be changed unless both doors are closed

In order to allow a user to pass from inside through the airlock into the external area, the system needs to perform the following steps:

1. equalise the chamber pressure to that of the internal environment,
2. open the second door to allow the user in the chamber,
3. close the second door,
4. equalize the pressure in the airlock to that of the external environment,
5. open the first door to allow the user out,

and vice versa for the opposite direction.

The components that perform these steps are described abstractly. The engineers need to define the physical means for performing these actions such as sensors and actuators:

ENV4. Each door is equipped with three positioning sensors and a two-way motor. The sensors consist of two boolean sensors representing the fully closed (SNS_CLOSED) and opened (SNS_OPENED) door states, and a range-value position sensor (SNS_POS) that returns values in a range between the fully closed and the fully opened states inclusively. The two-way motor (ACT_MOTOR) is the actuator that can open and close the door within its physical range of movement.

ENV5. There is a pressure sensor in each of the areas, three in total (SNS_PRESSURE_OUT, SNS_PRESSURE_CHAMBER, SNS_PRESSURE_IN).
ENV6. The pressure in the chamber can be changed by the pump actuator (ACT_PUMP).

On top of the functional requirements to the system, we also introduce a “fragile” environment where the physical components of the system may fail. We state this as the following fault assumption regarding the system low-level components:

ENV7. Any of the sensors and actuators may fail to provide a correct function.

In case of critical systems, ENV7 raises another type of requirements that concerns fault tolerance and system behaviour in a fragile environment. It is already mentioned in FUN1 that the system performs its function “when in operation”. Under the assumption of a fragile environment, such statement needs to be more elaborate. The system reaction to errors has to be specified:

FT1. A system must be able to tolerate internal errors where possible and continue its operation at an acceptable level

FT2. When errors cannot be tolerated, or it becomes dangerous to continue operation, the system must stop in a safe state (that is already ensured by the four safety conditions)

The system can only tolerate errors that affect redundant components. In this case study, such redundancy is provided for the door positioning sensors:

FT3. The boolean state sensors SNS_CLOSED and SNS_OPENED form a pair of devices that could be used as a hot spare for the more accurate positioning sensor SNS_POS.

When one of sensors SNS_CLOSED and SNS_OPENED fails, the door still remains operational using the positioning sensor SNS_POS, and vice versa. However, we consider such operation dangerous in the long term, and expect the system to gracefully degrade and finally stop for maintenance. The exact system behaviour under such conditions needs to be specified, and this is an analysis task where decisions must be made. To reason about the intended fault tolerant behaviour of the system, we need to explicitly specify the possible hazardous situations and system reactions.
Such information can be obtained by applying the Failure Modes and Effects Analysis (FMTR). In our case study, we specify the following reactions of the system to hardware component failures:

**FT4.** The system should disallow opening a door if one of the corresponding redundant sensors failed (see FT3)

**FT5.** The system should stop if one of the non-redundant sensors or actuators failed

**FT6.** If both doors have redundant sensors failed, the system should stop as soon as it is safe. It is considered to be safe to stop when there is no user trapped in the chamber. If there is a user in the chamber, the system should allow opening the internal door

In order to implement the fault-tolerant behaviour of the system, we decided to add an additional component:

**ENV8.** There is a boolean-valued sensor (SNS_USER) that indicates the presence of the user in the chamber

For our purpose of modelling, i.e. proof of high-level safety properties in presence of errors, we can assume that sensor failures and appropriate system state changes happen negligibly quick. In a method with interleaving semantics, such an assumption allows the model to react to one error at a time. This assumption simplifies modelling while still allows us to demonstrate the method.

It should also be noted that the method does not specify the technique of requirement elicitation and elaboration, it only stresses the necessity of having one. Problem Frames [Jac01] can be one way of such reasoning about requirements.

### 4.4 Abstract System Fault Tolerance Classes

According to the proposed method, the first decision a developer has to make is to choose an abstract class of a system. We define two abstract classes of systems from the fault tolerance modelling perspective: a class of failure-free systems and a class of safe stop systems. The differentiation between the two classes is an outcome of a study of existing models, and is a result of a defined refinement process using the Modal and Fault Tolerance Views’ templates (see Section 3.1 and Section 4.5.4). The
first class of systems is failure-free at the abstract level. It can mask all internal errors and operates indefinitely. The second class cannot tolerate certain errors and can eventually stop.

If the system is failure-free, its model should follow a general style for modelling reactive systems in the target formal method. There can be a number of refinement steps that refine abstract error-free functionality. The functionality refined at these first steps should not involve error conditions of system components or any other environmental state that can influence the system behaviour. Typically, control systems do not have such failure-free abstractions and their modelling should follow the safe stop approach.

The second class of systems may stop under certain conditions that no longer support safe system execution. The errors that can cause a system stop are called unrecoverable and will have to be specified at later refinement steps. We do not model the whole phenomena of the system environment and, therefore, are unable to simulate consequences of system failures. We only focus on an implementable reaction of a system, and consider safe stop systems. Safe stop systems can be stopped at any moment of time to ensure safety under undesired operating conditions.

The purpose of this step is to “reserve” an abstract representation of the overall system fault tolerant behaviour for further refinements. This step is only necessary if the stop behaviour is refined by component failures at subsequent levels. Although safe stop is usually the top abstraction for such systems, nothing prevents the developer from making several steps of refinement of failure-free functionality before this step.

### 4.4.1 Safe stop pattern

If the safe stop abstraction is chosen for the system, the modeller has to apply the safe stop pattern early during modelling to satisfy the error modelling principle. The actual level for application is a designer choice. However, we suggest to introduce the safe stop when the failure-free behaviour is specified.

Define a single variable representing the operational state of the system (stopped). One of its values shall represent the stopped state (e.g., stopped = TRUE). Separate the functional behaviour from the stopped state by using the declared variable. Define a transition that switches the system to the stopped state.

Safe stop is a special case of a more general error state variable pattern (see Section [4.5.1]). It applies to the most abstract level of fault tolerant behaviour. At this level, we treat the whole system as a single component from the fault tolerance perspective. The pattern assumes that the functional behaviour of the system is
defined on previous levels, and the system will be structurally decomposed on further
levels. The pattern explicitly separates the operational system behaviour from the
stopped state. The stop transition depicts an abstraction of unrecoverable errors.
The actual errors will be added during subsequent refinement steps.

4.4.2 Abstract modal views

Views are built in a step-wise manner, starting from the most primitive case and
introducing details along with model refinement. There are just two possible initial
views, defining the two system classes from the modelling perspective. The first class
does not have an unrecoverable error: all errors are recoverable and, at a sufficiently
abstract level, there are no errors at all. In the other case, there are errors that
cannot be masked and system necessarily transitions into a differing mode after an
error occurrence. What is considered to be an error is a design choice: the same
functionality may be implemented by either system class. Figure 4.4 illustrates the
two possible initial views.

Figure 4.4: Two abstract classes of fault tolerant systems

In the first view, the most abstract system is a normal mode. Further refinements
of the view may introduce only maskable errors. In the second view, in addition to
the normal mode there is an error leading to a degraded mode. Both normal and
degraded modes may be explained in further details by introducing new maskable
errors. The error originally present in the initial view may also be explained in terms
of a number of new errors.

The abstract view of the system corresponds to the chosen fault tolerance class. As
shown in Figure 4.4, the first view represents a system with a single mode of operation.
The assumption/guarantee (A/G) pair of the mode is trivial: FALSE/TRUE. It is
valid to use such view as the first abstract depiction of the system behaviour, however,
it can be skipped in practice as it does not contribute any proofs (they are all trivially
true).

The second view is a system that can eventually stop due to unrecoverable errors.
The stop transition is an abstraction of all unrecoverable errors, and has to be refined
later. The view elements use the variables and a transition created by the safe stop
pattern. The modes shall have their A/G specified as shown in Figure 4.5.

The proposed step of modelling an abstract fault tolerance class of systems is
exemplified using our airlock example.
4.4.3 Application in Event-B

We demonstrate the application of the step proposed above to modelling the airlock system in Event-B. In the first two models of the case study we model an abstract failure-free functionality of the system and its abstract class of fault tolerance.

In the initial model M0 we define an environment and a functional behaviour of the airlock system at the abstract level. Snippet 4.1 contains definitions and invariants that represent requirements discussed in Section 4.3. We represent the two environments described in ENV1 by their pressure values LOWPRESSURE and HIGHPRESSURE that we assume to be constant. The abstract components from ENV3 are represented by variables door1 and door2 for the two doors correspondingly, and variable pressure depicts the pressure in the chamber. The rest of the invariants correspond to the safety requirements. Namely, inv9 ensures that the pressure in the chamber stays within the allowed range as required by SAF1. Requirement SAF2 is split into two invariants inv4 and inv5 stating that each of the doors can only be opened when the chamber pressure is equal to that of the conjoining environment. Requirement SAF3 is ensured by inv6 requiring that at least one of the doors is closed at all times. Invariants inv7 and inv8 together represent SAF4 by ensuring that the chamber pressure is only changed when both doors are closed.

The behavioural part of model M0 contains system functionality free of errors. There are five events for each door that ensure the safe traversal of the corresponding door through its set of possible states (DOORSTATE at axm1), and two events for increasing and decreasing the level of pressure in the chamber correspondingly. For brevity, we show one event of a door behaviour (open1) and one pressure event (pump_up) on Snippet 4.1. Event open1 starts opening the first door if it is either closed or stopped at some intermediate position, event pump_up increments the chamber pressure value. The full model can be found in Appendix A.

The next step is to define the fault tolerance class of the system and apply an appropriate type of abstraction. The system as described in Section 4.3 is a safe stop system. It is aware of the fragile environment in which hardware system components can produce errors. Under certain conditions, the system may stop functioning, and this has to be appropriately represented in the model. The actual conditions are irrelevant at this abstraction level, and we only define the system reaction.
axioms

axm1: partition(\text{DOOR\_STATE}, \{\text{OPENED}\}, \{\text{CLOSED}\}, \{\text{OPENING}\},
\{\text{CLOSING}\}, \{\text{STOPPED}\})

axm2: \text{LOW\_PRESSURE} = 0

axm3: \text{HIGH\_PRESSURE} = 2

invariants

inv1: \text{door1} \in \text{DOOR\_STATE}

inv2: \text{door2} \in \text{DOOR\_STATE}

inv3: \text{pressure} \in \mathbb{N}

inv4: \text{door1} \neq \text{CLOSED} \Rightarrow \text{pressure} = \text{LOW\_PRESSURE}

inv5: \text{door2} \neq \text{CLOSED} \Rightarrow \text{pressure} = \text{HIGH\_PRESSURE}

inv6: \text{door1} = \text{CLOSED} \lor \text{door2} = \text{CLOSED}

inv7: \text{pressure} > \text{LOW\_PRESSURE} \Rightarrow \text{door1} = \text{CLOSED}

inv8: \text{pressure} < \text{HIGH\_PRESSURE} \Rightarrow \text{door2} = \text{CLOSED}

inv9: \text{pressure} \geq \text{LOW\_PRESSURE} \land \text{pressure} \leq \text{HIGH\_PRESSURE}

events

event open1 ≜

when

\text{grd1}: \text{door1} = \text{CLOSED} \lor \text{door1} = \text{STOPPED}

\text{grd2}: \text{pressure} = \text{LOW\_PRESSURE}

\text{grd3}: \text{door2} = \text{CLOSED}

then

\text{act1}: \text{door1} := \text{OPENING}

end

event pump_up ≜

when

\text{grd1}: \text{door1} = \text{CLOSED} \land \text{door2} = \text{CLOSED}

\text{grd2}: \text{pressure} < \text{HIGH\_PRESSURE}

then

\text{act1}: \text{pressure} := \text{pressure} + 1

end

Snippet 4.1: Definitions, invariants, and behaviour of M0
We refine model M0 by applying the safe stop pattern. We apply the pattern to the Event-B modelling in the following way. We define a single boolean variable \( \text{stopped} \in \text{BOOL} \). The value of the variable represents the operational availability of the system and separates the system normal behaviour from the stopped state. When \( \text{stopped} = \text{FALSE} \), the system operates as defined at M0. This is ensured by extending all functional events defined at M0 with guard \( \text{stopped} = \text{FALSE} \). When \( \text{stopped} = \text{TRUE} \), the system is considered to be in the stopped state. The new event \text{stop} puts the system in the stopped state, and represents an abstraction of all unrecoverable errors. The new event \text{stopped} models the system behaviour in the stopped state and has an empty action list. Because of the extended guards, the set of functional events and event \text{stopped} do not compete and are mutually exclusive. Thus, event \text{stopped} represents a system deadlock and guarantees that the system will stay in the stopped state; this is sufficient for modelling the system stop [LT04b]. Snippet 4.2 shows the two new events introduced by applying the pattern and an extension of event \text{open1}. All the 12 functional events defined at M0 are extended similarly to \text{open1}.

<table>
<thead>
<tr>
<th>invariants</th>
<th>inv1: ( \text{stopped} \in \text{BOOL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>events</td>
<td></td>
</tr>
<tr>
<td>event open1 ≜ extends open1</td>
<td></td>
</tr>
<tr>
<td>when</td>
<td></td>
</tr>
<tr>
<td>( \text{grd}_\text{stopped} ): ( \text{stopped} = \text{FALSE} )</td>
<td></td>
</tr>
<tr>
<td>event stop ≜</td>
<td></td>
</tr>
<tr>
<td>when</td>
<td></td>
</tr>
<tr>
<td>( \text{grd}_\text{stopped} ): ( \text{stopped} = \text{FALSE} )</td>
<td></td>
</tr>
<tr>
<td>then</td>
<td></td>
</tr>
<tr>
<td>( \text{act}_\text{stopped} ): ( \text{stopped} := \text{TRUE} )</td>
<td></td>
</tr>
<tr>
<td>end</td>
<td></td>
</tr>
<tr>
<td>event stopped ≜</td>
<td></td>
</tr>
<tr>
<td>when</td>
<td></td>
</tr>
<tr>
<td>( \text{grd} ): ( \text{stopped} = \text{TRUE} )</td>
<td></td>
</tr>
<tr>
<td>then</td>
<td></td>
</tr>
<tr>
<td>( \text{skip} )</td>
<td></td>
</tr>
<tr>
<td>end</td>
<td></td>
</tr>
</tbody>
</table>

Snippet 4.2: The safe stop pattern applied to model M1

Model M1 is associated with a modal view that ensures that the separation between operational and stopped modes is correct in the model. The modal view with
its assumption/guarantee pair and associated events is shown in Figure 4.6. It corresponds to the safe stop class of fault tolerant systems.

4.5 Fault Tolerant Component Refinement

A Fault tolerant component is a structural system unit that is described by its functional and error states. In this work, we assume that the fault tolerant system under development can be represented as a layered hierarchy of fault tolerant components. Each layer represents a level of abstraction with the system being the root of the hierarchy. The method traverses the hierarchy starting from its root by modelling component functional and error states at each layer. At each step, the set of components is decomposed into its subcomponents via state refinement.

The functional refinement of components is well-known by practitioners and typically includes both data and behaviour refinements. During functional refinement, the state representations of the higher-level components are refined by their more elaborate lower-level counterparts. The behaviour of the system is refined accordingly by specifying the allowed transitions.

In this section, we describe a refinement step for representing and refining the component error states. The step consists in application of three modelling patterns each described in its subsection.

4.5.1 Error state variable pattern

Each component at a current abstraction level has a functional state. For example, a door can be opened, closed, or in an intermediate position, a boolean sensor value can return true or false. These are the logical states of components that are necessary for modelling the functional behaviour. To model the fault tolerant behaviour, the system must also be aware of the availability, or error state, of its components. Examples of such component states are “operational” and “broken”. Error states can carry different meanings depending on the nature of the system context. In a physical environment, error state is an abstraction over the physical state. E.g., a sensor can be stuck, broken, or low on power supply. Such an abstraction can be derived from the hardware specifications and/or results of reliability analysis. When the system
context is another technical system, error state abstraction can be derived from the specification of the external system. However, the principles used in this and following patterns are general and are applicable to any nature of system environment.

For each component, specify a variable depicting the error state of the component at the current abstraction level.

This pattern prescribes to define a set of variables corresponding to error states of the components. In the simplest case, the variables can be boolean depicting whether the component is operational or not. They can also be defined over a set enumerating all possible error states of a particular component.

The safe stop pattern described in Section 4.4.1 is a special case of the error state variable pattern. At the most abstract level, the whole system is considered as a single component and its error state is represented by a single boolean variable. The variables introduced by the error state variable pattern are formally related with their abstract counterparts by the invariant pattern described next.

4.5.2 Error state invariant pattern

At the functional refinement step, the logical state of the component is refined into the logical states of its subcomponents, and the behaviour is refined accordingly. For the refinement relation to hold, one has to refine the error states of components as well. Abstract error representation needs to be refined each time a component is refined into its subcomponents. The error state refinement can be done together with functional refinement, or at a subsequent step. To ensure the correctness of the error state refinement, we introduce the error state invariant pattern:

Define a relation between the concrete and abstract error states.

Given a set of components at the concrete level, their error states need to be related to those at the abstract level. The pattern is applicable to any type of component hierarchy. For example, two abstract components can share a subcomponent in such a way that a subcomponent failure can lead to changes in error states of both components. The error states of abstract components can also be dependant on the functional state of the concrete ones. Therefore, the relation created by applying the error state invariant pattern can also refer to functional state. This pattern has to be applied each time the error state variable pattern is applied.

If the system is a safe stop system to which the safe stop pattern was applied, then a more specific invariant pattern can be used at the next consecutive level:
Define a relation between the abstract stopped state and concrete component error states. The suggested form is as follows: \( \text{stopped} = \text{TRUE} \Leftrightarrow \text{Predicate} \). Variable \( \text{stopped} \) is defined at the abstract level by the safe stop pattern, \( \text{Predicate} \) is a project-specific predicate expressed in terms of the concrete component error state variables.

The error state invariant patterns are necessary for formally establishing the relation between two subsequent layers of abstraction. Typically, refinement-based methods provide a means for expressing such a relation. In B and Event-B, this type of relation is expressed using gluing invariants that we demonstrate later in Section 4.5.5.

4.5.3 Fault tolerant behaviour pattern

The error state variables represent the knowledge of the system about its components conditions. At the lowest level, this knowledge comes from comparison of the current sensor readings against the system expectations about the environment that is based on assumptions. At all higher levels, there must be a behavioural abstraction of the system error detection mechanisms. We represent the abstract error detection by a number of transitions each of which corresponds to an occurrence of a certain error or a class of errors at the current abstraction level. All errors at the current abstraction level must be covered to satisfy the error modelling principle.

A fault tolerant system follows a detected error with an appropriate reaction. In order to conform to the reactive systems principle, we model error detection as a refinement of appropriate system reactions:

Every component error transition must refine the corresponding reaction transition of the system. All relevant functional states have to be covered by outgoing error transitions.

The same error may lead to different reactions depending on the current state. Both functional and error states are used when choosing an appropriate reaction. Therefore, this pattern applies not only to errors but to relevant changes in the functional part of the environment as well.

The fault tolerant behaviour pattern must also conform to the causality principle. If a model has a functional state that forbids components to fail, then its behaviour violates the causality principle. Therefore, it is mandatory to cover all possible system conditions under which the error can be detected. In the model, this means that given a set of errors and a set of possible system states, the resulting set of state transitions...
will be a Cartesian product of the former two. A part of the resulting set of state transitions are modelled as error detection transitions. Those detection transitions that lead to a system reaction specified at the abstract level should refine the appropriate reaction transition. The detection transitions that represent the maskable errors should be modelled as new transitions (they only change the state of new variables). Some part of error state transitions can be omitted in models if there are explicitly defined assumptions about such transitions, e.g., certain system operations can take negligible time during which the system does not react to environmental stimuli.

After applying this pattern, the model contains all relevant detection transitions that compete with functional transitions, both types of transitions originate at the same states. We believe this is sufficient to adequately represent the environment in a state machine for verifying safety properties.

4.5.4 Modal views

During the fault tolerant component refinement step, abstract components and their errors are refined by a number of concrete subcomponents and errors. The system behaviour is redefined in terms of subcomponent states. This allows developers to introduce new details into the behavioural model. This is also reflected by refining modes and splitting transitions defined on modal views.

Modes are abstractions over the overall system behaviour and not over separate system components. Therefore, the component decomposition of the model allows a more detailed representation of the overall system behaviour in terms of modes and mode transitions. The refinement of modes involves both the functional and fault tolerant system behaviour. The exact way of mode refinement is only limited by the modal refinement conditions, and is generally project-specific.

To assist in construction of modal views, we offer two templates for modal refinement. The first template is concerned with refinement of error transitions in a view. The idea here is to replace an abstract depiction of an error with two or more concrete errors. This process may be repeated as many times as needed and the result is a family of errors derived from a single abstract error.

As shown in Figure 4.7, there are two versions of this template. One for the case when an error leads to a degraded mode and another when there is also a subsequent recovery. This distinction is due to the fact that an obligation of successful recovery must be preserved during refinement. In the second version of the template at least one of the recovery modes must provide a recovery transition. Thus, either one of the recovery transitions on the second template at Figure 4.7 must remain while the other one can be removed. For example, we can remove the recovery transition from mode Recovery2 and still preserve the abstract recovery transition during refinement through Recovery1.
In the template, error transitions are labelled with the according events that represent the transitions in the model. During refinement, the abstract error detection event is refined by a number of lower-level component errors using the three fault tolerant component refinement patterns. If the definition of concrete-level errors allows developers to refine the system modal behaviour, then the modal view can follow this template to represent the more detailed reactions of the system to errors. As shown on the template, the mode transitions on the two consecutive levels should contain references to the appropriate events of the system reactions such as \textit{abstract\_error}, \textit{concrete\_error1} and \textit{concrete\_error2}.

The second template is a behavioural template, it splits a system mode into a chain of two (or possibly more) consecutive modes. This template can be used to model intermediate operations that the system needs to perform to arrive at a supposedly stable mode. In Figure 4.8, mode \textit{B} is refined into two consecutive modes \textit{B1} and \textit{B2}. The two new transitions \textit{A} \rightarrow \textit{B1} and \textit{A} \rightarrow \textit{B2} refine the abstract transition whereas transition \textit{B1} \rightarrow \textit{B2} is internal and typically represents a change in a functional state of the system. There are different applications of the template possible, however we focus on a particular application of a functional refinement. Modes \textit{A} and \textit{B2} are assumed to be important modes in which the system is supposed to stay for long
periods of time, mode $B1$ is a temporary mode in which the system stays until some functional condition is set to arrive at $B2$. Note that mode $A$ could be refined in a similar way. This template supports evolution of a system modal state over time, however, the modal views language does not guarantee termination of modes.

A particular application of the behavioural template can be used for specifying the system graceful degradation. As shown in Figure 4.9, a system may have one-way transitions between a set of operational modes such as Normal and Degraded. The chain of degradation can involve several modes and more than one path. The system eventually stops when it encounters the conditions under which its function is not safe. The transition to Stop can happen from any of the operational modes because the safe stop action does not depend on the system state.

### 4.5.5 Application in Event-B

At M0 and M1, the system functional behaviour is defined in terms of its components. Namely, the two doors and the airlock chamber are represented by their relevant functional state variables. However, the system error states are defined by a single variable stopped depicting the overall system error state.

We refine M1 by applying the fault tolerant component refinement step. Firstly, we apply the error state variable pattern and define error states of every system component participating in fault tolerance requirements. We introduce two error state variables corresponding to the two doors of the airlock: $door1\_cond$ and $door2\_cond$ as shown on Snippet 4.3. The possible door error states are \{BROKEN, DEGRADED, OK\}. The error states are abstracted from fault tolerance requirements FT3-FT6 given in Section 4.3.

<table>
<thead>
<tr>
<th>Axioms</th>
<th>Invariants</th>
</tr>
</thead>
<tbody>
<tr>
<td>axm1: $DOOR_CONDITION = {BROKEN, DEGRADED, OK}$</td>
<td>inv1: $door1_cond \in DOOR_CONDITION$</td>
</tr>
</tbody>
</table>
inv2: door2\_cond ∈ DOOR\_CONDITION
inv4: door1\_cond = BROKEN ∨ door2\_cond = BROKEN ∨
   (door1\_cond = DEGRADED ∧ door2\_cond = DEGRADED ∧
    obj\_presence = FALSE) ⇔ stopped = TRUE

events

event break ≡ extends stop
  any
  d1c d2c
where
  grd2.0: d1c ∈ DOOR\_CONDITION ∧ d1c ≤ door1\_cond
  grd2.1: d2c ∈ DOOR\_CONDITION ∧ d2c ≤ door2\_cond
  grd2.2: d1c = BROKEN ∨ d2c = BROKEN
  grd2.3: door1\_cond ≠ BROKEN ∧ door2\_cond ≠ BROKEN
  grd2.4: door1\_cond = DEGRADED ∧ door2\_cond = DEGRADED
    ⇒ obj\_presence = TRUE
  then
  act2.0: door1\_cond, door2\_cond := d1c, d2c
end

event degrade ≡
  any
  d1c d2c
where
  grd0: stopped = FALSE
  grd1: d1c ∈ DOOR\_CONDITION ∧ d1c ∈ {OK, DEGRADED} ∧
    d1c ≤ door1\_cond
  grd2: d2c ∈ DOOR\_CONDITION ∧ d2c ∈ {OK, DEGRADED} ∧
    d2c ≤ door2\_cond
  grd3: (door1\_cond = OK ∧ d1c = DEGRADED ∧ door2\_cond = d2c) ∨
    (door2\_cond = OK ∧ d2c = DEGRADED ∧ door1\_cond = d1c)
  grd4: d1c ≠ DEGRADED ∨ d2c ≠ DEGRADED ∨ obj\_presence = TRUE
  then
  act1: door1\_cond := d1c
  act2: door2\_cond := d2c
end

event stop_on_degrade ≡ extends stop
  when
    grd1: door1\_cond = DEGRADED ∨ door2\_cond = DEGRADED
    grd2: obj\_presence = FALSE
    grd4: door1\_cond = OK ∨ door2\_cond = OK
  then
    act1: door1\_cond := DEGRADED
Snippet 4.3: Fault tolerant component refinement applied at M2

Next, we apply the error state invariant pattern. We “glue” abstract variable `stopped` with the newly defined door error state variables by invariant `inv4` shown on Snippet 4.3. The example follows the safe stop invariant pattern. Note that the gluing invariant also refers to functional variable `obj_presence` to meet the requirements described in FT6. Such reference highlights the point that the fault tolerant properties of the system are inevitably tied to the functional state and both need to be taken into account during refinement.

To satisfy the gluing invariant that describes error states of components, we also refine the behavioural part of the model. We apply the fault tolerant behaviour pattern and define the system reaction to component errors. Snippet 4.3 shows three events `break`, `degrade` and `stop_on_degrade` changing the door error states in three different situations. Events `break` and `stop_on_degrade` extend abstract event `stop` with actions putting the doors into degraded and broken states to satisfy gluing invariant `inv4`. This shows how an abstract fault tolerant reaction is refined into more specific component failures. Event `degrade` is new at M2, it changes the door error states and continues the system operation. It represents the tolerance of the system to certain errors. The three presented events represent a part of requirements FT5 and FT6. Two events `degrade` and `stop_on_degrade` depict the same abstract detection of a door failure, but they lead to different reactions, and the choice depends on the current system state as required by FT6. We have to have both events in the model to cover all relevant system states at the moment of component failure to satisfy the causality rule (see Section 4.1.5).

An example of breaking the causality rule can be removal of event `degrade`. This event represents a situation when both doors get degraded while the user is still present in the airlock chamber. Without event `degrade` such situation can never arise and the underlying hypothetical assumption could then be formulated as follows:

| The door does not degrade while the user is present in the chamber |

Such an assumption is unrealistic and breaks the implementable causality rule: the door is a part of a physical environment and it cannot be forbidden to degrade or break. Although such an assumption is unimplementable, the formal behaviour that satisfies the assumption can be modelled and proved to be correct. This highlights the need in additional modelling principles and patterns that add rigour to the formal development process.
As an example of system modal and fault tolerant behaviour refinement, we show the modal view that is associated with M2 in Figure 4.10. It represents an application of the graceful degradation template. The operation of the system is split into four modes: the normal operation mode and three degraded modes. The system stays in mode Door1 when the first door is degraded and the second door is fully operational, and vice versa for mode Door2. When both doors are degraded and there is a user present in the chamber, the system stays in mode Trapped until the user leaves the chamber. The new mode Normal together with the three degraded modes formally refine abstract mode Normal as shown by a dashed area.

The abstract system failure transition is refined by four concrete transitions depicting the sources of failure. Transitions Stop on degrade and User leaves initiate at the new modes. Transition Break can initiate at any of the four modes within the dashed area.

![Diagram](image)

Figure 4.10: Modal view of airlock M2 model

The degraded modes on the view represent different sets of available components and the associated subsets of system behaviour. The assumptions of the modes split the possible combinations of the components error states into disjoint sets as shown on Snippet 4.4. The mode assumptions cover all the system states as specified in the model invariants. This is ensured by the well-definedness proof obligation COVER (see Section 3.4).
4.4 Behaviour Restriction

The behaviour restriction step follows the fault tolerant component refinement step described in the previous section. During this step, refined component error states and modal views are used to restrict the functional behaviour. When applied to the model, the step implements those fault tolerance requirements that define the changes in system behaviour under different operating conditions. The behaviour restriction step consists of a behaviour restriction pattern and an application of modal views.

4.6.1 Behaviour restriction pattern

The behaviour restriction pattern uses the component error states introduced by applying the error state variable pattern to restrict the functional behaviour.

Restrict the functional transitions that are not allowed with respect to the error state variables defined earlier: use the error state variables to strengthen the domain restriction predicates of transitions.

The error state variables represent the operational availability of components. Under different operational conditions, system functionality may provide different functions. For example, safety requirements might only permit a partial operation of certain components after errors have been detected. Moreover, fault tolerance requirements may contain specifications of different system behaviours under different operational conditions, e.g., graceful degradation. The behaviour restriction pattern ensures that the behavioural model satisfies such types of requirements by disallowing functional transitions that are not valid.

The pattern should be used together with modal refinement. The modal views provide consistency conditions that can be used to identify invalid functional transitions. Restriction predicates of relevant transitions are being strengthened using
this pattern to satisfy the associated modal views. In the simplest case, the conjuncts added to domain restriction predicates of transitions take the following form: \( \text{Component}\_\text{condition} \in \text{Possible}\_\text{conditions} \), where \( \text{Component}\_\text{condition} \) is the error state variable for the component, and \( \text{Possible}\_\text{conditions} \) is a set of error states allowing the functional transition to happen. The actual restriction predicates are project-specific. They may involve more than one component and refer to functional variables.

### 4.6.2 Modes for functionality and fault tolerance

Modal views provide a tool for specifying modal behaviour. A modal view may contain a mixture of fault tolerant and functional behaviour as it is not always possible to separate the two. Although a distinction between fault tolerance and functionality can be made for a specific system, we treat the specification of modal views in a generic way. The assumption/guarantee predicates of a mode define the nature of a specific part of the system modal behaviour, and relationship between the system and its environment, as well as between subsystems. Specifying assumption/guarantee predicates is a modelling task and is specific to the system and the properties of interest. However, we provide a general rule for specifying modes that is based on our distinction between functional variables and error state variables.

We regard the assumption predicate as an abstraction of the system environment that should refer to error state variables. The assumptions therefore state the combinations of available components that allow the system to provide its subsets of functionality. The functionalities should refer to the logical state variables and are contained in the guarantee predicates. Such a way of specifying the assumptions and guarantees can be treated as a specification of the system fault tolerant behaviour.

The modal principle of providing certain functionality under the stated conditions can be used for specifying the purely functional part of behaviour. For example, modes can specify system operation that depends on the current logical state of one of its components. Mode assumptions may also refer to a functional decision (represented by a state variable) that is made by a higher-level management component. In such cases, the views represent the functional behaviour of the system.

Although the two kinds of modal specifications can coexist in a single refinement chain, they are usually mixed as the choice of system mode typically depends on both component error and functional states.

### 4.6.3 Behaviour restriction by modal views

The modal views provide additional proof obligations. Certain model elements can be inferred from an associated view. Such inferred elements can be offered to a user
as a suggestion during modelling. The specific element that is inferred in the method is the fault tolerance part of the transition domain restriction predicate.

This step is applicable to state-based methods that allow definitions of safety properties. The properties in combination with modal views are used to help users to define correct behavioural model. The modal views must also support the target formal method and use the safety properties as part of their proof obligations. In the description of this step, we use invariants as safety properties, and refer to modal consistency conditions defined for Event-B in Section 3.4.2.

We assume that the view is used to model the fault tolerant behaviour of the system in a way described by the method: the mode assumptions refer to the error state conditions of the components depicting the environment, and the guarantee describes the functionality provided by the system under the stated environment condition.

For a particular state transition, the EVTₐ proof obligation of the modal views can be used to infer the domain restriction predicate. The proof obligation has the following simplified form:

$$I(v) \land H(v) \Rightarrow A_d(v)$$

where $I(v)$ is a model invariant, $H(v)$ is a transition domain restriction, and $A_d(v)$ represents the disjunction of all relevant assumptions (see EVTₐ in Section 3.4). We assume the restriction predicate consists of two parts:

$$H(v) = H_f(v) \land H_t(v)$$

where $H_f(v)$ is the functional part already specified by the developer, and $H_t(v)$ is the fault tolerance part that is to be inferred from the modal view.

In order to satisfy proof obligation (4.2), the fault tolerance part of the restriction predicate would be $H_t = A_d$. However, $A_d$ may also contain assumptions about functionality, and generally is a disjunction of several modes. We assume that invariants define a relationship between functional and error state variables that can be used to simplify $A_d$. For such simplification, existing provers can be used to apply inference rules to the following:

$$I(v) \land H_f(v) \Rightarrow A_d(v)$$

Invariant $I(v)$ combined with the functional part of predicate $H_f(v)$ can help in eliminating a substantial part of $A_d(v)$ arriving at a simplified predicate:
\[ I(v) \land H_f(v) \implies A'_d(v) \] (4.5)

The resulting predicate \( A'_d(v) \) can be suggested to the user as a possible fault tolerance part of restriction predicate \( H_f(v) \) that will satisfy EVT_A.

It is possible that the suggested predicate can make the whole restriction predicate \( H(v) \) false. Such a case signifies a contradiction between the behavioural model and the view. Proof obligation ENBL of the modal views becomes unprovable which may serve as an indication of the contradiction.

4.6.4 Application in Event-B

We show the behaviour restriction step applied at step M3 of the airlock case study. The M3 modal view of the airlock describes the modal behaviour of the system under differing error and logical conditions. The modal view is shown in Figure 4.11. We apply the behavioural split template to modes Door1, Door2 and Trapped: we split each of the modes into two new modes. For example, mode Door2_closing restricts the system to only operate with the second door, and only contains events that close or stop the door but do not open it. Upon the door closure, the system switches to mode Door2 that guarantees that the pressure is set to low and the door is closed, and thus only allows operating the first door.

![Figure 4.11: Modal view of airlock M3 model](image-url)

67
Abstract mode *Stop* is split into two modes *Broken* and *Degraded* using the error split template. The two modes differentiate between two different situations: the first situation arises when the system stops upon the detection of a complete door failure, the second situation is the safe stop due to the degraded state of both doors. The two stop modes can also contain failure-free behaviour such as two modes of a safety alarm although this is not modelled in the case study.

Let us focus on one of the events of the model and the modes in which it participates. Event *open1* initiates opening of the first door. It is associated with two modes *Normal* and *Door2*. The system is in mode *Normal* when all components are fully operational and, thus, the first door can be opened. The mode *Door2* depicts a situation when the second door is degraded and closed, and, thus, only the first door can be opened. The first door cannot be opened in all other modes as it is unsafe to do so. The A/G predicates of the two modes mentioned are shown on Snippet 4.5.

<table>
<thead>
<tr>
<th>mode Normal  ≜</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumption: door1_cond = OK ∧ door2_cond = OK</td>
</tr>
<tr>
<td>Guarantee: TRUE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>mode Door2  ≜</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumption: door2_cond = DEGRADED ∧ door1_cond = OK ∧ pressure = LOW_PRESSURE</td>
</tr>
<tr>
<td>Guarantee: pressure\’ = LOW_PRESSURE ∧ door2\’ = CLOSED</td>
</tr>
</tbody>
</table>

Snippet 4.5: The modes of M3 that can open the first door

On the previous levels, the guard of event *open1* only contained functional conditions of M0 and a fault tolerance condition of M1 that states that the system must be running in order to open and close its doors. At M3, the event is restricted by error state conditions of components derived from the modal view.

Let us apply the general form of proof obligation *EVT_A* to event *open1* (we omit the full invariant and the guard for brevity):

\[
I \land H \implies (\text{door1}\_\text{cond} = \text{OK} \land \text{door2}\_\text{cond} = \text{OK}) \lor \\
(\text{door2}\_\text{cond} = \text{DEGRADED} \land \text{door1}\_\text{cond} = \text{OK} \land \\
\land \text{pressure} = \text{LOW\_PRESSURE})
\]  

(4.6)

From the functional guard of the event defined at M0 we know that

\[
\text{pressure} = \text{LOW\_PRESSURE}
\]

(4.7)

this simplifies the goal of (4.6):
\[ I \land H \implies \text{door}_1\text{cond} = \text{OK}\land \\
(\text{door}_2\text{cond} = \text{OK} \lor \text{door}_2\text{cond} = \text{DEGRADED}) \] (4.8)

From gluing invariant \textit{inv4} at M2 we know the following:

\[ \text{door}_1\text{cond} = \text{BROKEN} \lor \text{door}_2\text{cond} = \text{BROKEN} \lor \\
(\text{door}_1\text{cond} = \text{DEGRADED} \land \text{door}_2\text{cond} = \text{DEGRADED} \land \\
\text{obj}\text{presence} = \text{FALSE}) \iff \text{stopped} = \text{TRUE} \] (4.9)

By negating (4.9) and applying it to (4.8), we arrive at:

\[ I \land H \implies \text{door}_1\text{cond} = \text{OK} \] (4.10)

Thus, using the previously defined invariants and information available from the modal view, we can suggest extending the guard of event \textit{open1} with \text{door}_1\text{cond} = \text{OK} to satisfy the modal view. Such suggestion is inferred for every event in model M3 given that the association between events and modes is consistent and satisfies other proof obligations of modal views. The suggestions may contain both error state and functional conditions. At the M3 step, we extend each of the functional events that operate with the two doors and the chamber pump by additional guards inferred from the associated modal view.

The current and all the previous steps facilitate the development of reactive system behaviour that is proven to maintain the required safety properties. The following two steps refine the abstract system model with implementation-level details. The two steps follow the abstract reactive modelling and finalise the modelling with sequential decomposition of the system.

4.7 Hardware

The component refinement and the behaviour restriction steps are performed in a top-down manner until the reactive model of the system contains the required safety properties. If the system interacts with the physical environment, then at the lowest level the system must contain a means for such interaction. The functional and fault tolerant behaviour obtained using the previous two steps can be further refined by introducing hardware units such as sensors and actuators. This step is a special case of the fault tolerant component refinement step that is applied at the lowest level of abstraction. This step only includes the refinement of the fault tolerant part of the system components. The functional refinement is done at the next step.
4.7.1 Application of fault tolerant component refinement

The components defined during the previous steps are logical and may consist of a number of hardware units. Each of the hardware units can fail and thus the system needs to be aware of their error states. To properly represent the hardware at the lowest level of abstraction, we apply the error state variable pattern:

For each hardware unit (sensors, actuators, or higher level hardware component), specify a variable depicting the error state of the unit.

The hardware error state variables have to be linked to the abstract system components. This is done by applying the error state invariant pattern:

Define a relation between the hardware error states and abstract component error states.

The actual gluing invariants between the error states of the abstract components and those of the hardware units are project-specific. It is possible for the error state of a hardware unit to contribute to a number of abstract components. The level of abstraction used for hardware components is also project-specific. The system behaviour can rely on complex hardware subsystems. In this case, levels of abstraction for functionality and errors can be different.

4.7.2 Application in Event-B

At the M4 step of the airlock development, we refine one of the doors into its hardware units. As required by ENV4, each door is equipped with four such units. We model the error states of each of the sensors and the actuator by a boolean variable as shown on Snippet 4.6. The error states of the abstract door component need to be refined by error states of the hardware units. Gluing invariant \textit{inv\_door1\_sensors\_conditions\_BROKEN} links the abstract BROKEN state with appropriate combinations of the motor and the sensors. Likewise, other two states OK and DEGRADED are refined and can be found in Appendix A. The events that had their guards or actions referencing the door error state variables have to be refined in terms of sensor and actuator error states. As the functional behaviour has been restricted previously by the the behaviour restriction pattern, most of the functional and all the detection events are refined.

The refinement of the second door component by its four hardware units is equivalent to that of the first door and is omitted for brevity.
invariants

\begin{align*}
\text{inv} \_ \text{door1} \_ \text{pos} \_ \text{cond}: & \quad \text{door1} \_ \text{pos} \_ \text{cond} \in \text{BOOL} \\
\text{inv} \_ \text{door1} \_ \text{closed} \_ \text{cond}: & \quad \text{door1} \_ \text{closed} \_ \text{cond} \in \text{BOOL} \\
\text{inv} \_ \text{door1} \_ \text{opened} \_ \text{cond}: & \quad \text{door1} \_ \text{opened} \_ \text{cond} \in \text{BOOL} \\
\text{inv} \_ \text{door1} \_ \text{motor} \_ \text{cond}: & \quad \text{door1} \_ \text{motor} \_ \text{cond} \in \text{BOOL} \\
\text{inv} \_ \text{door1} \_ \text{sensor} \_ \text{conditions} \_ \text{BROKEN}: & \quad \text{door1} \_ \text{cond} = \text{BROKEN} \iff \\
& \quad \text{door1} \_ \text{motor} \_ \text{cond} = \text{FALSE} \lor (\text{door1} \_ \text{pos} \_ \text{cond} = \text{FALSE} \land \\
& \quad (\text{door1} \_ \text{opened} \_ \text{cond} = \text{FALSE} \lor \text{door1} \_ \text{closed} \_ \text{cond} = \text{FALSE}))
\end{align*}

Snippet 4.6: Error states of sensors and actuators at M4

4.8 Control Cycle

A typical control system operates in a cycle with a certain (fixed or dynamic) frequency. During its operation, the control system reads the values of the sensors, performs the control algorithm, and produces commands for the actuators in order to control the environment. The functional state of the sensors and actuators is also a part of the system and represents the knowledge of the system about the physical phenomena. The actual values of the physical phenomena are unknown to the system and can only be inferred from the sensed data.

Up to this step, the system development followed the reactive style of modelling. As a result of applying the previous steps of the method, the system consists of high-level functional components and their error states represented by the low-level sensors and actuators. The functional connection between the high-level logic and low-level values of sensors and actuators cannot be expressed using data refinement and reactive modelling as discussed in Section 4.1.6. This step offers a modelling solution for extending the previously developed reactive system with a sequential control cycle. The reactive part of the system takes place during the control algorithm step of the cycle. Sequential decomposition of the reactive model is a part of implementation - the step does not help to specify what the system does but it specifies how the system operates.

The control cycle step is partly derived from our previous work on incorporating FMEA into Event-B specifications [Lop+11]. We model four phases of the cycle and provide patterns for filling each of the phases. The major difference from our previous work lies in the level of abstraction at which we regard it as reasonable to apply such a sequential decomposition. In the original work, we model the control cycle at the abstract level and leave the control logic for further refinements. Here, we advocate a top-down approach to modelling the functionality and extend it later with implementation-level restrictions such as a control cycle. Thus, the control cycle extension links with the previous steps and does not clutter the reactive model of the
system. The patterns described in this section can also accommodate the outcomes of the FMEA analysis, and thus the results of [Lop+11] can be reused for this work.

4.8.1 Control cycle pattern

Define a variable \( \text{phase} \in \text{PHASE} \) representing the current phase of the control cycle. Define a set of four phase states that include the sensing phase, the error detection phase, the control phase, and the prediction phase. Define four transitions that represent the behaviour during phases correspondingly.

The control cycle pattern splits the operation of the system into four sequential steps. During the sensing phase, the system reads the sensor values. During the second phase, it applies the low-level error detection mechanisms. The third phase contains the control algorithm, which is the reactive model of the system that was developed previously. The control algorithm produces new logical components’ states and a set of signals for the actuators. The new states and the actuator signals are used at the final prediction phase to predict the sensor readings at the next iteration of the cycle. The results produced by the prediction step are used by the detection mechanisms during the next iteration.

The actual transitions for each of the phases are defined by the following four patterns.

4.8.2 Sensing pattern

The error state variables of the system hardware units and their usage were already defined during the previous development step (see Section 4.7). This pattern defines the functional states of the hardware units:

Define a variable for each of the hardware units that would represent the functional state of the unit. Non-deterministically assign a new value to the variable during the sensing phase.

The values of the sensor variables represent the state of the sensors at the current iteration after the sensing phase. We represent the system environment by assigning non-deterministic values to the variables. The environment can be refined further during later steps if necessary. We do not formally specify the environmental phenomena such as laws of physics.
4.8.3 Error detection pattern

The next phase of the control cycle compares the sensor readings, which were obtained from the sensing phase, with the allowed range of values. The detection phase returns the detected hardware error states that are then used by the control algorithm.

The first step to implementing the detection mechanism is defining the variables necessary for correct refinement of the abstract model:

Define a variable for each of the hardware units that would represent the error state as detected by the error detection phase. Assign new values to the variables based on the sensor readings introduced by the sensing pattern, last known sensor error states, and sensor values produced by the prediction pattern during the previous iteration.

The detected error state variables are a copy of the sensor error state variables defined by the hardware pattern (see Section 4.7). The new error state variables are required for the error detection phase to ensure the correct refinement of the abstract reactive model.

The newly defined variables are assigned new values by detection transition cycle.detect. The assignments represent the detection mechanisms used by the system. We define several types of low-level error detection mechanisms based on the sources of information for the detection:

- the sensor reading may be compared against the statically defined acceptable range of values;
- the system may expect a certain value from the sensor reading that is based on the previous iteration of the cycle;
- the sensor reading may be compared to the active redundant sensors.

There may be other types of error checks depending on the system, we do not intend to cover all possible error detection mechanisms. Error detection at this level of abstraction is done sequentially as opposed to the abstract error detection described in Section 4.5.3.

4.8.4 Control phase patterns

We represent the control phase by the reactive behaviour of the system that is developed using the previous method steps. The reactive model of the system contains transitions that change the hardware error states and represent the system fault tolerance. The results of the detection phase of the control cycle are used here to feed
the obtained hardware error states to the reactive behaviour. Similarly, the sensor readings are used by the functional part of the system to ensure that the sensor values match the abstract logical state of the components.

The *functional control phase pattern*:

Restrict the functional transitions with additional domain restriction that specifies the sensor values necessary for the execution of the transition. The domain restriction has to be of the following form: $\text{sensor\_cond} = \text{TRUE} \Rightarrow P(\text{sensor\_value})$

where $\text{sensor\_cond}$ is the hardware error state variable, and $P(\text{sensor\_value})$ is a predicate over the acceptable sensor values. The functional control phase pattern ensures that the value of the sensor is only used when the sensor is in a working condition hence the implication. The added restriction predicate models the relation between the concrete-level hardware units and the abstract component states.

The *fault tolerant control phase pattern* ensures that the abstract error detection is refined to use the information obtained from the detection phase:

Restrict the domain of error detection transitions to include the previously detected hardware error states. Use the detected states as new values for abstract error states.

At the abstract level, error detection transitions represented the environment as a non-deterministic choice between different errors. This pattern restricts the detection transitions by using the detected error states, thereby providing a place for implementing the detection mechanisms.

During the control algorithm, the reactive behaviour of the system uses the sensor readings and the error state variables to change its logical state and produce signals for the actuators. The actuators are assumed to receive the signals instantly.

### 4.8.5 Prediction phase pattern

The last phase of the control cycle produces the expectations of the system about the sensor states at the next iteration.

Define a prediction variable for each of the sensor values. Assign the predicted value at the last phase of the control cycle.

Thus, the variables defined are a copy of the sensor value variables with the same types. The prediction represents the assumptions about the environmental phenomena, and the expectation of the system about the next state of its components. The
prediction variables are then used at the detection phase by the error detection pattern.

This is the last step of the method. After this step, the system is sufficiently detailed and can be used for implementation. The implementation can be manual or automated for a particular family of models that incorporate the control system patterns. The resulting formal specification can also be used for test-case generation to ensure correctness of the final code.

4.8.6 Application in Event-B

We exemplify the five patterns described in this section in Event-B and demonstrate their application on the airlock case study. Firstly, we instantiate the control cycle pattern by defining the four events of a control cycle as shown on Snippet 4.7. The events represent the four control cycle phases as described in Section 4.8.1. This is ensured by guards \textit{grd\_phase} and actions \textit{act\_phase}.

\textbf{axioms}

\textit{axm1: partition(\textit{PHASE}, \{\textit{ENV}\}, \{\textit{DET}\}, \{\textit{CONT}\}, \{\textit{PRED}\})}

\textbf{invariants}

\textit{inv\_phase: phase \in PHASE}
\textit{inv1: door1\_pos \in Z}
\textit{inv2: door1\_opened \in BOOL}
\textit{inv3: door1\_closed \in BOOL}
\textit{inv4: door1\_pos\_predicted \in Z}
\textit{inv5: door1\_opened\_predicted \in BOOL}
\textit{inv6: door1\_closed\_predicted \in BOOL}
\textit{inv7: door1\_motor \in \{-1, 0, 1\}}
\textit{inv10: door1\_pos\_cond\_detected \in BOOL}

\textbf{events}

\textit{event cycle\_sense =}

\textit{when}
\textit{grd\_phase: phase = ENV}
\textit{grd\_stopped: stopped = FALSE}
\textit{then}
\textit{act\_phase: phase := DET}
\textit{act5\_1: door1\_pos \in Z}
\textit{act5\_2: door1\_opened \in BOOL}
\textit{act5\_3: door1\_closed \in BOOL}
\textit{end}

\textit{event cycle\_detect =}
when
  grd_phase: phase = DET
  grd_stopped: stopped = FALSE
then
  act_phase: phase := CONT
  act5_1: door1_pos_cond_detected := bool(door1_pos_cond = TRUE ∧
    (min_door ≤ door1_pos ∧ door1_pos ≤ max_door) ∧
    (door1_opened_cond = TRUE ∧ door1_opened = TRUE ⇒
      door1_pos = max_door) ∧
    (door1_closed_cond = TRUE ∧ door1_closed = TRUE ⇒
      door1_pos = min_door) ∧
    (door1_pos = door1_pos_predicted))
end

event open1 ≜ extends open1
when
  grd_phase: phase = CONT
  grd_pos: door1_pos_cond = TRUE ⇒ door1_pos < max_door ∧
    (door1_pos = min_door ⇒ door1 = CLOSED) ∧
    (door1_pos > min_door ⇒ door1 = STOPPED)
  grd_closed: door1_closed_cond = TRUE ⇒
    (door1_closed = TRUE ⇒ door1 = CLOSED)
  grd_opened: door1_opened_cond = TRUE ⇒ door1_opened = FALSE
then
  act_phase: phase := PRED
  act_motor: door1_motor := 1
end

event degrade_door1 ≜ refines degrade_door1
when
  grd_phase: phase = CONT
  grd_degradation: (door1_pos_cond_detected = FALSE ∧ door1_pos_cond = TRUE) ∨
    (door1_closed_cond_detected = FALSE ∧ door1_closed_cond = TRUE) ∨
    (door1_opened_cond_detected = FALSE ∧ door1_opened_cond = TRUE) ∨
    (door1_motor_cond_detected = FALSE ∧ door1_motor_cond = TRUE)
  grd_stopped: stopped = FALSE
  grd1: door1_pos_cond = TRUE ∧ door1_closed_cond = TRUE ∧
    door1_opened_cond = TRUE ∧ door1_motor_cond = TRUE
  grd7: door2_cond = DEGRADED ⇒ obj_presence = TRUE
  grd_glue: door1_motor_cond_detected = TRUE ∧
    (door1_pos_cond_detected = FALSE ∧ door1_opened_cond_detected = TRUE ∧
    door1_closed_cond_detected = TRUE) ∨
(door1\_pos\_cond\_detected = TRUE ∧ \\
(\text{door}1\_\text{opened}\_\text{cond}\_\text{detected} = FALSE ∨ \text{door}1\_\text{closed}\_\text{cond}\_\text{detected} = \\
FALSE)))

with

\begin{align*}
\text{pos}\_\text{cond} & : \text{pos}\_\text{cond} = \text{door}1\_\text{pos}\_\text{cond}\_\text{detected} \\
\text{opened}\_\text{cond} & : \text{opened}\_\text{cond} = \text{door}1\_\text{opened}\_\text{cond}\_\text{detected} \\
\text{closed}\_\text{cond} & : \text{closed}\_\text{cond} = \text{door}1\_\text{closed}\_\text{cond}\_\text{detected} \\
\text{motor}\_\text{cond} & : \text{motor}\_\text{cond} = \text{door}1\_\text{motor}\_\text{cond}\_\text{detected}
\end{align*}

then

\begin{align*}
\text{act}\_\text{phase} & : \text{phase} := \text{PRED} \\
\text{act}5\_0 & : \text{door}1\_\text{pos}\_\text{cond} := \text{door}1\_\text{pos}\_\text{cond}\_\text{detected} \\
\text{act}5\_1 & : \text{door}1\_\text{closed}\_\text{cond} := \text{door}1\_\text{closed}\_\text{cond}\_\text{detected} \\
\text{act}5\_2 & : \text{door}1\_\text{opened}\_\text{cond} := \text{door}1\_\text{opened}\_\text{cond}\_\text{detected} \\
\text{act}5\_3 & : \text{door}1\_\text{motor}\_\text{cond} := \text{door}1\_\text{motor}\_\text{cond}\_\text{detected}
\end{align*}

end

event cycle\_predict =

when

\begin{align*}
\text{grd}\_\text{phase} & : \text{phase} = \text{PRED} \\
\text{grd}\_\text{stopped} & : \text{stopped} = \text{FALSE}
\end{align*}

then

\begin{align*}
\text{act}\_\text{phase} & : \text{phase} := \text{ENV} \\
\text{act}5\_1 & : \text{door}1\_\text{pos}\_\text{predicted} := \text{door}1\_\text{pos} + \text{door}1\_\text{motor} \\
\text{act}5\_2 & : \text{door}1\_\text{opened}\_\text{predicted} := \text{bool}( \\
& (\text{door}1\_\text{motor} = 1 \land \text{door}1\_\text{pos} = \text{max}\_\text{door} - 1) \lor \\
& (\text{door}1\_\text{motor} = 0 \land \text{door}1\_\text{opened} = \text{TRUE})) \\
\text{act}5\_3 & : \text{door}1\_\text{closed}\_\text{predicted} := \text{bool}( \\
& (\text{door}1\_\text{motor} = -1 \land \text{door}1\_\text{pos} = \text{min}\_\text{door} + 1) \lor \\
& (\text{door}1\_\text{motor} = 0 \land \text{door}1\_\text{closed} = \text{TRUE}))
\end{align*}

end

Snippet 4.7: Modelling the control cycle at M5

We apply the sensing pattern and define four variables representing the sensed values of the hardware units. We only model the hardware units of the first door as required by ENV4. Sensor door1\_pos returns the position of the door that is represented as an integer value (see inv1). Sensors door1\_opened and door1\_closed are boolean sensors at the two extreme positions of the door (inv2 and inv3 correspondingly). The signal for actuator door1\_motor represents the direction of the door movement: −1 for closing, 1 for opening, and 0 for stable position (inv7). The sensing phase of the control cycle is represented by event cycle\_sense. It assigns ar-
bitrary values to the sensor readings as we do not formally model the environmental phenomena.

At the next phase, the system executes its error detection mechanisms represented by event cycle\_detect. Snippet 4.7 shows an example of error detection for the positioning sensor of the first door. The error detection for the sensor includes the three types of detection mechanisms, and ensures that once the sensor failure is detected, the value will be kept during all subsequent iterations.

Once the hardware functional and error states are obtained, they are used during control phase to execute the system functional and fault tolerant behaviour. For example, we apply the functional control phase pattern and extend functional event open\_1 with additional guards (grd\_pos, grd\_closed, and grd\_opened) stating the expected functional and error states. The event now also produces a signal to the motor actuator by its action act\_motor. An example of applying the fault tolerant control phase pattern is a refinement of detection event degrade\_door\_1 shown on Snippet 4.7. The event now contains a number of guards that use the values obtained during the error detection phase to refine the previously non-deterministic error detection. Event actions directly assign the detected values to the error state variables defined during reactive modelling. The refinement relation is formally ensured by providing new detected values as event witnesses.

The last phase of the control cycle is the prediction phase. On Snippet 4.7 we show the predictions of the next expected states of the three sensors in event cycle\_predict. For example, sensor door\_1\_pos is expected to change its value according to the signal given to the motor, and return door\_1\_pos + door\_1\_motor during the next iteration.

The application of the control cycle step finalises the modelling of the airlock system, and provides placeholders in the model for further implementation.
4.9 Summary of Patterns

We provide a summary of modelling patterns and modal view templates that are used by the method in Table 4.1. The patterns are described in details in corresponding sections of this chapter as provided in the table.

<table>
<thead>
<tr>
<th>Name</th>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe stop pattern</td>
<td>4.4.1</td>
<td>A starting point for modelling fault tolerant behaviour. Introduced model elements are refined at later steps</td>
</tr>
<tr>
<td>Error state variable pattern</td>
<td>4.5.1</td>
<td>Defines representation of components error state</td>
</tr>
<tr>
<td>Error state invariant pattern</td>
<td>4.5.2</td>
<td>Links components error state at the current level with the abstract component</td>
</tr>
<tr>
<td>Fault tolerant behaviour pattern</td>
<td>4.5.3</td>
<td>Defines behaviour of the environment and ensures the appropriate system reaction</td>
</tr>
<tr>
<td>Behaviour restriction pattern</td>
<td>4.6.1</td>
<td>Restricts the functional behaviour with fault tolerance constraints</td>
</tr>
<tr>
<td>Control cycle pattern</td>
<td>4.8.1</td>
<td>Introduces a control loop by sequentially decomposing the model</td>
</tr>
<tr>
<td>Sensing pattern</td>
<td>4.8.2</td>
<td>Defines functional states of sensors and actuators, and the sensing phase of the control loop</td>
</tr>
<tr>
<td>Error detection pattern</td>
<td>4.8.3</td>
<td>Defines the error detection phase of the control loop and variables that represent the detected error states of the components</td>
</tr>
<tr>
<td>Functional control phase pattern</td>
<td>4.8.4</td>
<td>Links the reactive functionality with the sensor values obtained during the sensing phase</td>
</tr>
<tr>
<td>Fault tolerant control phase pattern</td>
<td>4.8.4</td>
<td>Refines the abstract representation of errors by values that were obtained during the detection phase</td>
</tr>
<tr>
<td>Prediction phase pattern</td>
<td>4.8.5</td>
<td>Defines the prediction phase of the control loop and variables that represent expectations of the system about the future functional states of components</td>
</tr>
<tr>
<td>Error split template</td>
<td>4.5.4</td>
<td>Splits an abstract error transition (and associated recovery and degraded modes) into two or more concrete errors</td>
</tr>
<tr>
<td>Behavioural split template</td>
<td>4.5.4</td>
<td>Refines an abstract mode by a sequence of two modes, introduces an intermediate mode</td>
</tr>
<tr>
<td>Graceful degradation template</td>
<td>4.5.4</td>
<td>Extends a system operation mode with a sequence of degraded modes</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of proposed patterns
4.10 Conclusions

In this chapter, we proposed a method for top-down development of fault tolerant systems. The method focuses on a notion of fault tolerant component and provides a modelling solution to connecting the refinement of component errors with system functionality through behaviour restriction.

We described the modelling principles and assumptions that should be adhered to during modelling. Based on the principles, we proposed a refinement strategy for building reactive models of systems and further sequential decomposition leading to implementation of control systems. We identified a number of patterns that support the refinement strategy throughout the chapter, and provided a summary of all the modelling patterns and modal view templates.

We demonstrated the proposed method on a medium-scale case study. The air-lock system has been modelled using the Rodin development environment and the associated plug-ins for Event-B model transformations and modal views modelling. To evaluate the method and the tools, we model another case study from a different domain that we describe in the next chapter.
Chapter 5. Evaluation

The aim of this chapter is to report on evaluation of the method proposed in Chapter 4. The evaluation process consists in applying the proposed steps of the method to developing an industrial case study. By developing the system, we validate the research hypotheses which we reprise in the following statements:

- The method supports and encourages top-down specification of fault tolerant system behaviour with the purpose of correct design of system fault tolerance in a refinement-based formal method
- The method provides a general refinement strategy which specifies the steps that need to be taken to arrive at a correct model of a fault tolerant system
- The refinement strategy is derived from modelling principles and is supported by modelling patterns
- The method provides facilities for explicit modelling of fault tolerance
- The method maintains the applicability of the formal method to modelling the system functionality, and allows for smooth integration of functionality with fault tolerant behaviour

To support the evaluation statements, we model the Attitude and Orbit Control System (AOCS) case study. This is a medium-scale control system which was independently developed by several industrial companies from the aerospace domain. One of them is Space Systems Finland (SSF) which was one of four industrial partners of the DEPLOY project. We define our simplified requirements for the system based on project deliverable [D3.1] and associated internal requirements documents.

The chapter is organised as follows. We introduce the requirements for the AOCS system in Section 5.1, then we provide the relevant parts of the models and describe the steps performed during modelling in Section 5.2, and we summarise our evaluation in Section 5.3.
5.1 Requirements for AOCS

In this section, we present the requirements for the AOCS system that we use during the modelling process.

The Attitude and Orbit Control System (AOCS) \[D3.1\] is a generic component of a satellite onboard software operating in the following environment:

ENV1. The environment of the AOCS consists of the physical phenomena of Newtonian dynamics applied to the object in orbit around the Earth, and other phenomena measured by the payload instrument.

The main function of the AOCS is defined as follows:

FUN1. The AOCS is a control system which ensures the desired attitude and orbit of a satellite. The control is necessary for the payload instrument to fulfil its mission.

Due to the tendency of a satellite to change its orientation because of disturbances from the environment, the attitude needs to be continuously monitored and adjusted. An optimal attitude is required to support the needs of payload instruments. For example, attitude control may ensure that an optical system of the spacecraft will continuously cover the required area on the ground. To fulfil the mission the AOCS is controlling a number of hardware units of the satellite:

ENV2. The AOCS is equipped with three hardware units: the Earth sensor (ES) gives the distance and the orientation relative to the Earth, the GPS sensor provides the GPS coordinates of the satellite, and the Payload instrument (PLI) fulfils the mission of the satellite by reading the relevant information from the environment.

The AOCS uses the ES and GPS hardware units to keep the satellite on a desired trajectory. The operation of the AOCS is mode-based which is described by FUN2:

FUN2. The AOCS can operate in a number of system modes. Each system mode is associated with a set of required unit modes. The ultimate function of the AOCS is to acquire the best possible system mode where the payload instrument is gathering its data.
In this case study, a satellite can be in three operational modes: Off, Nominal, and Science. The satellite is in the Off mode from the moment separation from the launcher is achieved. In this mode the AOCS is not operational; this mode is regarded as a preparatory phase. From the Off mode, the AOCS progresses to modes where more sensors are involved. The next mode to switch to is Nominal; in this mode the ES and GPS units are switched on, and the satellite acquires and preserves a stable attitude. The overall aim is to enter and stay in the Science mode where fine GPS positioning is achieved and scientific instruments (PLI units) are reporting readings. The described mode switching behaviour is a requirement that can be captured as a diagram shown in Figure 5.1.

Similar to the airlock example, the AOCS also operates in a fragile environment:

**ENV3.** Any of the satellite units can fail to provide its service.

To tolerate such faults, the system hardware is provided with redundancy:

**ENV4.** Each unit is supported by a redundant cold spare which can be used when the main unit fails.

The AOCS is expected to handle the control algorithm related errors (such as attitude computation errors) and the unit errors (including all errors related to loss of accuracy, invalid data, etc.). Such errors may happen at any moment of time including the transition phase between modes.

**FT1.** Upon the detection of any unit error, the AOCS must degrade to a lower mode in which the failed unit is not used. The failed unit must be replaced with
its spare. When the spare unit fails, the AOCS is no longer permitted to operate in those modes which use the failed type of unit.

For example, if both PLI units fail, the satellite can no longer operate in the Science mode, and is forced to stay in the Nominal mode. At the moment of failure, the system must degrade to a lower mode which is shown by backward transitions in Figure 5.1.

The requirements for the AOCS case study are derived from one of the DEPLOY project deliverables in space sector [D3.1]. The deliverable and related documents contain an industrial description of the AOCS system. The industrial requirements are simplified for clarity of the present work. Namely, the five modes of the original system are reduced to three in the case study due to their generic representation in the model. Attitude errors of the original system are not present in the case study because they are treated in our method as a type of transition errors. We further simplify the requirements by removing the concepts of mode, unit, and FDIR managers. We believe that the mentioned concepts represent software components, i.e. implementation, and should not be considered by our method.

Although AOCS is a control system, we do not intend to model the actual control algorithm. We focus on high-level logic of modal and fault tolerant behaviour of the system for evaluation purposes. The proposed case study was chosen and is adequate for evaluation of the method for the following reasons:

• It contains realistic requirements derived from industrial documents

• AOCS is a critical system and, therefore, needs to tolerate faults during its operation. Requirements include fault tolerance in a form of graceful degradation which cross-cuts the functionality

• As shown later, all the method steps for modelling reactive fault tolerant systems can be exemplified during modelling AOCS

• The system is mode-based, which allows us to evaluate the applicability of Mode/FT Views

The case study is limited to modelling a particular component of a real system specification. There may be certain relevant behaviour of the rest of the system that is not modelled here. For example, we focus on the abstract development of reactive modal behaviour of the AOCS and do not model the actual control algorithm. Also, the case study does not fully demonstrate the usefulness of event guard suggestions described in Section 4.6.3. This is due to inherently modal nature of the case study.
Similar to the airlock case study, we assume that error detection procedures and system state changes happen negligibly quick. Although in such a form, this assumption does not typically hold in real systems, we believe it is sufficient in this case for the purpose of demonstration of the proposed method.

5.2 AOCS modelling

During the development of the AOCS system, the steps and the patterns described in Chapter 4 are used. Before modelling, we identify the refinement steps that need to be performed based on the specified requirements.

As required by FUN2, we need to model system modes and associated unit modes. The prominent solution would be to define modes and refine the more “abstract” modal behaviour by units operation. What might seem to be the right way of refinement, however, would lead to complications at further levels during development. Each of the system modes would need to be refined by each of the participating units, e.g. both Nominal and Science modes would need to be refined by operation of the GPS unit, which means there would be two artificial events representing essentially the same unit behaviour. In terms of our method, such a refinement step would represent a state refinement which is hardly applicable for relating system modes with unit modes. Instead, we treat the system modes as a behaviour restriction over units operation. That is, units represent the abstract behaviour of the system which can be restricted, and the phenomenon of system modes is a horizontal state expansion which restricts the units’ operation. Therefore, we model units at an earlier step than

![Diagram of AOCS case study models](image-url)
we do modes. We conduct the same planning for the fault tolerant part of the system behaviour: we further restrict the units operation by detecting and acting upon unit errors. Following the method steps, unit errors need to refine the abstract system failure, therefore, we need to introduce safe stop before we model unit errors. This example shows the necessity of ( informal) refinement planning.

Using our reasoning about the system components and behaviour which need to be modelled, we define the refinement steps which are shown in Figure 5.2. The model of the AOCS case study includes five Event-B machines and four modal views. We start with modelling the abstract unit functionality at M0 which is described in Section 5.2.1. Then we introduce a system safe stop at M1 which is briefly discussed in Section 5.2.2. We conduct functional refinement at levels M2 and M3 and introduce modes and unit reconfiguration which is discussed in Section 5.2.3. Model M4 refines the AOCS safe stop into unit errors, and uses them to restrict the functional behaviour. We split the description of M4 into two sections: the fault tolerant component refinement is discussed in Section 5.2.5 and the behaviour restriction step is discussed in Section 5.2.6.

5.2.1 Functional model M0

The first model of the AOCS case study contains the abstract functionality of the system. It corresponds to the failure-free step of the proposed development method. The model defines the functional states of the three system units. The functionality of each unit is abstractly modelled by two events: event work corresponds to the unit operation, and event switch represents the switch of the unit between functional states. Snippet 5.1 shows the two events for the ES unit. The units GPS and PLI are represented similarly.

We omit modelling the actual control algorithm because such modelling should be either postponed until later steps or modelled using continuous-time notations. Therefore, events work for the three units contain no actions, they represent a mere fact that the units perform something useful when in operation.

At this stage, the model only represents the ability of the units to perform their function and switch between their modes which is required by ENV2. The model corresponds to the failure-free functionality step of the method. The units operation and switching between unit modes is the most abstract unrestricted functionality that covers any possible combination of units. All further refinements introduce restrictions in a form of disallowing certain units to operate or switch as a result of functional and environmental causes.
axioms
axm1: UNIT_OFF = 0
axm2: UNIT_ON = 1
axm3: GPS_COARSE = 1
axm4: GPS_FINE = 2

invariants
inv1: unitES ∈ \{UNIT_ON, UNIT_OFF\}
inv2: unitGPS ∈ \{UNIT_OFF, GPS_COARSE, GPS_FINE\}
inv3: unitPLI ∈ \{UNIT_OFF, UNIT_ON\}

events

event ES_work \艰巨
  when
  grd1: unitES = UNIT_ON
  then
  skip
  end

event ES_switch \艰巨
  any
  newState
  where
  grd1: newState ∈ \{UNIT_ON, UNIT_OFF\}
  grd2: newState ≠ unitES
  then
  act1: unitES := newState
  end

Snippet 5.1: AOCS model M0

5.2.2 Safe stop at M1

AOCS is a safe stop system in terms of the controlled units. When a unit fails in such a way that the system can no longer operate, it switches to the stop mode which depicts a safe mode where none of the three units are required.

At the M1 modelling step we apply the safe stop step described in Section 4.4. The Event-B model is extended according to the safe stop pattern, and the modal view of the system follows the safe stop template and contains two modes: Operation and Stop. This step is required because we plan to refine the safe stop by unit errors at one of the next refinement levels.
5.2.3 Functional refinement at M2

At the next two steps M2 and M3 we restrict the unit switching events by modelling the AOCS system modes of operation. First, we introduce the system modes and transitions, and restrict the units operation by modal requirements. On the next level we restrict the modes reconfiguration process by ensuring that the reconfiguration takes place until all units switch to their appropriate modes. In terms of our method, M2 contains a state refinement and a behaviour restriction step, and M3 contains a behaviour restriction step towards system reconfiguration. The refinement steps M2 and M3 also demonstrate how the modal views are applied to modelling of functional behaviour.

<table>
<thead>
<tr>
<th>invariants</th>
</tr>
</thead>
<tbody>
<tr>
<td>inv1: stable ∈ BOOL</td>
</tr>
<tr>
<td>inv2: mode ∈ MODE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>events</th>
</tr>
</thead>
<tbody>
<tr>
<td>event goAdvance ≡</td>
</tr>
<tr>
<td>when</td>
</tr>
<tr>
<td>grd_stopped: stopped = FALSE</td>
</tr>
<tr>
<td>grd1: stable = TRUE</td>
</tr>
<tr>
<td>grd2: mode &lt; SCIENCE</td>
</tr>
<tr>
<td>then</td>
</tr>
<tr>
<td>act1: mode := mode + 1</td>
</tr>
<tr>
<td>act2: stable := FALSE</td>
</tr>
<tr>
<td>end</td>
</tr>
</tbody>
</table>

| event downgrade ≡ |
| any |
| newMode |
| where |
| grd_stopped: stopped = FALSE |
| grd_par: newMode ∈ MODE |
| grd1: mode > OFF |
| grd2: newMode < mode |
| then |
| act1: mode := newMode |
| act2: stable := FALSE |
| end |

Snippet 5.2: Introducing AOCS system modes at M2

At M2 we introduce the system modes and the process of system reconfiguration.
As shown on Snippet 5.2, we define two new events `goAdvance` and `downgrade` that represent the initiation of the reconfiguration process. State variables `mode` and `stable` describe the current mode of operation. When `stable = FALSE`, variable `mode` contains the target mode for the reconfiguration. When `stable = TRUE`, the system invokes the units functionality as required by a particular mode. So far, the only functional property that M2 provides is the behavioural ability to switch between modes. However, system modes do not define any formal relationship with unit modes - the reconfiguration process is modelled in its unrestricted form.

In order to meet the requirement FUN2, we need to associate system modes with appropriate subsets of units. We apply the `behaviour restriction` step to the functional behaviour and restrict units operation to allowed subsets of modes. For example, unit ES is required to switch on and work in the Nominal system mode and should be switched off in any other mode (Snippet 5.3). This is a direct application of the `behaviour restriction` pattern where instead of error state variables we use a part of the functional behaviour (system modes).

```plaintext
events

event ES_work ≜ extends ES_work
    when
    grd_mode: mode = NOMINAL
    then
    skip
end

event ES_switch_on ≜ refines ES_switch
    when
    grdStopped: stopped = FALSE
    grd1: unitES = UNIT_OFF
    grd2: mode = NOMINAL
    with
    newState: newState = UNIT_ON
    then
    act1: unitES := UNIT_ON
end
```

Snippet 5.3: Adding functional modes at M2

The modal view of M2 reflects the introduced modes of operation as shown in Figure 5.3. Three modes `Off`, `Nominal` and `Science` refine abstract mode `Operation`. The assumptions of the modes are of form `mode = <MODE>`, where `<MODE>` is substituted for specific mode constants accordingly. The modes refer to the appropriate unit events. For example, mode `Nominal` only allows operation for the ES and
GPS units but disallows the PLI unit to work or switch on (Snippet 5.4).

Each of the events of mode Nominal have to satisfy the appropriate proof obligations for that mode. Namely, the event guard needs to satisfy the mode assumption as required by the EVT-A proof obligation. The modal view in Figure 5.3 is used for behavioural restriction of units’ operation, and we strengthen the guards of the unit events as described in Section 4.6.3.

Note that the system normal operation on the view is equivalent to the diagram from Figure 5.1 which represents the requirement. Thus, we show that an informal requirement described in a simple diagram may be formalised using modal views and incorporated into a formal (Event-B) modelling process.

5.2.4 Functional refinement at M3

This step refines the unrestricted reconfiguration process. Currently, the system can initiate and finalise the reconfiguration arbitrarily, without considering the unit modes. At this level we apply a behaviour restriction step to system modes. We strengthen the events responsible for switching between system modes to ensure that the reconfiguration initiates and finalises correctly.

We introduce a mapping between the modes and configurations of units. Specifically, we add invariants which map each of the modes to its associated units as shown...
on Snippet 5.5. Event `reconf_finish` is extended with a guard which satisfies the invariant and ensures that after reconfiguration the unit modes correspond to the system mode. The guards of the switching events are extended to ensure that the units can only switch on and off during the reconfiguration process (exemplified by event `ES_switch_on` on the snippet).

**invariants**

- `inv_modes1`: `mode = OFF ∧ stable = TRUE` =>
  
  `unitES = UNIT_OFF ∧ unitGPS = UNIT_OFF ∧ unitPLI = UNIT_OFF`

- `inv_modes2`: `mode = NOMINAL ∧ stable = TRUE` =>
  
  `unitES = UNIT_ON ∧ unitGPS = GPS_COARSE ∧ unitPLI = UNIT_OFF`

- `inv_modes3`: `mode = SCIENCE ∧ stable = TRUE` =>
  
  `unitES = UNIT_OFF ∧ unitGPS = GPS_FINE ∧ unitPLI = UNIT_OFF`

- `inv_modes4`: `unitES = UNIT_ON ∨ unitGPS ∈ {GPS_FINE, GPS_COARSE} ∨ unitPLI = UNIT_ON` =>
  
  `¬(stable = TRUE ∧ mode = OFF)`

**events**

- `event ES_switch_on` extends `ES_switch_on`
  
  when
  
  `grd2_0: stable = FALSE`

- `event reconf_finish` extends `reconf_finish`
  
  when
  
  `grd_mode_off: mode = OFF` => `unitES = UNIT_OFF ∧ unitGPS = UNIT_OFF ∧ unitPLI = UNIT_OFF`

- `grd_mode_nominal: mode = NOMINAL` => `unitES = UNIT_ON ∧ unitGPS = GPS_COARSE ∧ unitPLI = UNIT_OFF`

- `grd_mode_science: mode = SCIENCE` => `unitES = UNIT_OFF ∧ unitGPS = GPS_FINE ∧ unitPLI = UNIT_ON`

- `event stop` extends `stop`
  
  when
  
  `grd_units: unitES = UNIT_ON ∨ unitGPS ∈ {GPS_FINE, GPS_COARSE} ∨ unitPLI = UNIT_ON`

Snippet 5.5: Introducing the unit configurations of AOCS modes at M3

We double-check the reconfiguration behaviour by constructing an elaborate modal view of the system. The modal view shown in Figure 5.4 is refined using the behavioural split template introduced in Section 4.5.4. Each of the operational modes is split into two modes: the stable mode ensures the unit configuration and provides the
unit functionality, and the preceding mode provides the appropriate reconfiguration functionality. Snippet [5.6] lists the two new modes which refine the abstract Nominal mode. The new Nominal mode includes the work events of the ES and GPS units and does not take part in reconfiguration. It guarantees that in this mode the units ES and GPS are operational and the PLI is not used. Mode toNominal is responsible for enabling the units as required by Nominal, however, it does not guarantee the exact unit configuration until mode Nominal is achieved.

Figure 5.4: Modal view of AOCS M3 model

<table>
<thead>
<tr>
<th>mode toNominal</th>
<th>(\hat{=})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumption:</td>
<td>(mode = NOMINAL \land stable = FALSE \land stopped = FALSE)</td>
</tr>
<tr>
<td>Guarantee:</td>
<td>(stopped' = FALSE)</td>
</tr>
<tr>
<td>Events:</td>
<td>(GPS_switch_off, PLI_switch_off, ES_switch_on, GPS_switch_on,)</td>
</tr>
<tr>
<td></td>
<td>(ES_work, GPS_work)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>mode Nominal</th>
<th>(\hat{=})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumption:</td>
<td>(mode = NOMINAL \land stable = TRUE \land stopped = FALSE)</td>
</tr>
<tr>
<td>Guarantee:</td>
<td>(unitES' = UNIT_ON \land unitGPS' = GPS_COARSE \land)</td>
</tr>
<tr>
<td></td>
<td>(unitPLI' = UNIT_OFF)</td>
</tr>
<tr>
<td>Events:</td>
<td>(ES_work, GPS_work)</td>
</tr>
</tbody>
</table>

Snippet 5.6: The behavioural split of the Nominal mode at AOCS M3

This step partly covers the requirement FT1. The model contains the necessary behaviour for downgrading when errors are detected. However, the actual errors are not yet defined. To do that, we apply the fault tolerant component refinement step at the last refinement level M4.
5.2.5 Fault tolerant component refinement at M4

At steps M0-M3 the system represents a single component from the fault tolerance perspective. Variable *stopped* depicts the state of the fault-tolerant system and has to be refined further along with the refinement of functionality which is done at steps M2 and M3.

The M4 step of the AOCS case study refines the abstract fault tolerant component into its subcomponents. We apply the error state variable pattern and define an integer variable with suffix _cond for each of the three units as shown on Snippet 5.7. These variables represent the unit error states in range \{0, 1, 2\} which contains a number of available units of a particular type. Initially, the system has two units of each type (one active and one cold spare), and upon detecting an error the variables are decremented. When a unit error state becomes zero, the system can no longer use that unit.

To map the newly defined unit error state variables to the abstract error state, we apply the error state invariant pattern. The AOCS system is considered to be operational when there are at least one ES unit and one GPS unit available. This is captured by invariant *inv.glue*.

The model now contains the representation of the unit error state, and we define the errors which can cause the units to fail. We introduce abstract error detection events by applying the fault tolerant behaviour pattern. The fault tolerant behaviour of the system consists of two events: *stop* and *degrade*. Failures of the three units of the system must refine one of the two events. Failures of the primary units are all tolerable and therefore they refine event *degrade*. On the other hand, failures of the secondary ES and GPS units lead to the system stop and, thus, they refine event *stop*. Failure of the secondary PLI unit does not stop the system and, thus, it refines event *degrade*. The two detection events for the ES unit are shown on Snippet 5.7.

This refinement step demonstrates abstract modelling and refinement of errors, and ensures the *error refinement* principle of the method described in Section 4.1.7. In the method, the system errors can be seen as meaningful annotations which are attached to appropriate system reactions. By refining reactions into errors we ensure that the system model is reactive and we are capable of expressing rich functional and fault tolerance properties (the principle described in Section 4.1.6). This step also ensures the *implementable causality* principle (Section 4.1.5). Indeed, the guards of the events which represent unit errors do not “block” the environment from activating the errors. They only contain sensible conditions for limiting the error activation such as that a unit error cannot happen if the unit is switched off. The other modelling principles are ensured by the second part of the M4 refinement step which is given in the next section.
invariants

\[ \text{inv}_{\text{ES} \_ \text{cond}}: \text{unitES} \_ \text{cond} \in \{0, 1, 2\} \]
\[ \text{inv}_{\text{GPS} \_ \text{cond}}: \text{unitGPS} \_ \text{cond} \in \{0, 1, 2\} \]
\[ \text{inv}_{\text{PLI} \_ \text{cond}}: \text{unitPLI} \_ \text{cond} \in \{0, 1, 2\} \]
\[ \text{inv}_{\text{glue}}: \text{stopped} = \text{TRUE} \Leftrightarrow \text{unitES} \_ \text{cond} = 0 \lor \text{unitGPS} \_ \text{cond} = 0 \]

events

event \text{ES} \_ \text{break} \triangleq \text{refines} \ \text{stop}

when
\[ \text{grd1}: \text{unitES} \_ \text{cond} = 1 \]
\[ \text{grd2}: \text{unitES} = \text{UNIT \_ ON} \]
\[ \text{grd3}: \text{unitGPS} \_ \text{cond} > 0 \]
then
\[ \text{act1}: \text{unitES} \_ \text{cond} := 0 \]
\[ \text{act2}: \text{stopped} := \text{TRUE} \]
end

event \text{ES} \_ \text{downgrade} \triangleq \text{refines} \ \text{downgrade}

when
\[ \text{grd1}: \text{mode} > \text{OFF} \]
\[ \text{grd2}: \text{unitES} = \text{UNIT \_ ON} \]
\[ \text{grd3}: \text{unitES} \_ \text{cond} = 2 \]
\[ \text{grd4}: \text{unitGPS} \_ \text{cond} > 0 \]
with
\[ \text{newMode}: \text{newMode} = \text{OFF} \]
then
\[ \text{act1}: \text{mode} := \text{OFF} \]
\[ \text{act2}: \text{stable} := \text{FALSE} \]
\[ \text{act3}: \text{unitES} \_ \text{cond} := 1 \]
end

event \text{ES} \_ \text{work} \triangleq \text{extends} \ \text{ES} \_ \text{work}

when
\[ \text{grd3}: \text{0}: \text{unitES} \_ \text{cond} > 0 \]
end

Snippet 5.7: Unit error states at M4
5.2.6 Behaviour restriction at M4

We use the error state variables which are introduced in the previous section for behaviour restriction by a modal view. The modal view of M4 is equal to that of M3 in terms of modes and transitions; no new modes appear during the model refinement. However, the mode assumptions are refined to include the new unit error state variables. As shown on Snippet 5.8, the part of the abstract assumptions \( \text{stopped} = \text{FALSE} \) is refined by unit error state conditions. Such refinement is also supported by the gluing invariant that was added in the previous section via the error state invariant pattern.

\[
\text{mode toNominal} \equiv
\]

Assumption: \( \text{mode} = \text{NOMINAL} \wedge \text{stable} = \text{FALSE} \wedge \text{unitES}_{\text{cond}} > 0 \wedge \text{unitGPS}_{\text{cond}} > 0 \)

Guarantee: \( \text{stopped}' = \text{FALSE} \)

Events: \( \text{GPS}_{\text{switch off}}, \text{PLI}_{\text{switch off}}, \text{ES}_{\text{switch on}}, \text{GPS}_{\text{switch on}}, \text{ES}_{\text{work}}, \text{GPS}_{\text{work}} \)

\[
\text{mode Nominal} \equiv
\]

Assumption: \( \text{mode} = \text{NOMINAL} \wedge \text{stable} = \text{TRUE} \wedge \text{unitES}_{\text{cond}} > 0 \wedge \text{unitGPS}_{\text{cond}} > 0 \)

Guarantee: \( \text{unitES}' = \text{UNIT ON} \wedge \text{unitGPS}' = \text{GPS COARSE} \wedge \text{unitPLI}' = \text{UNIT OFF} \)

Events: \( \text{ES}_{\text{work}}, \text{GPS}_{\text{work}} \)

Snippet 5.8: The assumption refinement at M4 FT view

The new mode assumptions force the participating events to refine their guards and actions to satisfy the modal view proof obligations. For example, we extend event \( \text{ES}_{\text{work}} \) with a guard \( \text{unitES}_{\text{cond}} > 0 \) to ensure that the unit is used only when at least one of the ES units is available (Snippet 5.7). Event \( \text{ES}_{\text{break}} \) is also extended and now contains the following among its guards: \( \text{unitGPS}_{\text{cond}} > 0 \). This might seem as a violation of the causality rule, however, such a guard is necessary for satisfying the EVT.A proof obligation. It does not violate any of the method principles, and states that an error in the secondary ES unit may only happen when at least one GPS unit is available. Indeed, with two previously failed GPS units the system would have already stopped and \( \text{ES}_{\text{break}} \) would not be enabled.

This step supports the multi-view development and restricted modelling principles. By using an additional view on the model, we add rigour to the development process. Without such a view, it would be possible to remove some of the error events from the model and still prove the correctness. The reason behind such an omission is
that the formal method does not guarantee the coverage of error transitions which, in fact, only contain informal meaning. An additional view focuses attention on abstract errors and system modal behaviour, and formally ensures that the model implements the former.

5.3 Conclusions

Prior to modelling the AOCS system, we conducted a refinement planning activity and identified the refinement steps for this particular system. The steps followed the refinement strategy proposed in Chapter 4 and were adequate for the AOCS requirements. During modelling, we have used all the modelling patterns for building a reactive system, and one of the view templates of the method. Namely, we used

- the safe stop pattern from Section 4.4.1 at M1,
- the error state variable and invariant patterns from Section 4.5.1 and Section 4.5.2 correspondingly at M4,
- the fault tolerant behaviour pattern from Section 4.5.3 at M4,
- the behaviour restriction pattern from Section 4.6.1 at M4,
- the behaviour split template from Section 4.5.4 at M3.

The patterns and the refinement strategy are derived by applying the method principles. Thus, we argue that the refinement strategy is adequate for reactive system modelling, is supported by a set of patterns, and can be used for creating verified models of fault tolerant systems.

The AOCS development includes a number of modal views where we explicitly defined the system modal behaviour from both the functional and fault tolerance perspectives. Therefore, the method supports explicit modelling of fault tolerance as a specific case of modal specifications.

By following the top-down strategy of the method, we arrived at a model of the AOCS system which meets both functional and fault tolerance requirements. The behaviour restriction step ensures that the functional behaviour and properties can be modelled in full prior to introducing the environmental restrictions which is shown at steps M0, M2 and M3. Thus, the method maintains the applicability of the formal method to modelling the functionality. It advocates and facilitates early modelling of fault tolerance which is smoothly integrated with functional behaviour.

The method provides facilities for top-down specification of fault tolerant systems in a form of a set of principles, a refinement strategy which is supported by patterns, and an additional modal and fault tolerance viewpoint on the system. As shown
throughout the evaluation chapter, these facilities can be effectively used to correctly design fault tolerant systems in a refinement-based formal method.
Chapter 6. Conclusions

This section summarises the contributions of the thesis, discusses the strengths and weaknesses of the proposed method, and shows some of the possible directions of future research.

6.1 Discussions and Directions of Further Research

The principles and modelling solutions of the proposed method are based on a number of assumptions. One of the assumptions is the availability of requirements for the system prior to modelling (see Section 4.1). The proposed method supports a top-down development process, and we expect to have a requirements document in order to proceed with the modelling. However, the current engineering practices are mostly iterative; many significant parts of requirements cannot be initially given with enough details and are refined during development. This is especially true for such aspects of systems as dependability and, in particular, fault tolerance. In order to estimate reliability and other dependability properties and, thus, define fault tolerance requirements, one needs to obtain a specification and at least a high-level design of a system. This implies that the proposed method shall be used iteratively in conjunction with other currently used model-based design approaches (UML, AADL), reliability analysis techniques (FMEA, FTA), and traditional software development steps (prototyping, testing). Generally, iterative process requires additional re-modelling efforts associated with the changes in requirements. This holds for the proposed method as well; the step-wise process of formal refinement used in the method can only preserve modelling and proof efforts associated with unchanged levels of abstraction. For example, changes in low-level behaviour of the system will require re-modelling at low levels of abstraction whereas abstract models would be preserved. In such cases, the correctness of abstract refinement steps is maintained without extra efforts, and the assumption about the defined requirements is guaranteed to hold at those steps.

A truly top-down application of the method may only be realised in the context of verifying an existing design, i.e. produced using traditional (iterative or top-down) methods for which requirements at this stage are fully defined.

Further issue related to the definition of assumptions is a classical notion of a model
adequacy and abstraction of environment. As stated in the principle of environment modelling in Section 4.1.4, a model must contain an adequate representation of the system environment. However, the continuous, real-time and stochastic aspects of many systems may have a significant impact on the modelling and verification process as it is not always possible to define an adequate discrete state-based abstraction of the underlying phenomena. Thus, the proposed method is limited to modelling and verification of those properties and behaviours for which adequate state-based abstractions can be defined. In the context of fault tolerance modelling, this means that important properties of many fault tolerance mechanisms cannot be verified using the proposed method and, more generally, using non-stochastic state-based formal methods. For example, the essential property of N-modular redundancy, i.e. the reduced failure rate of the system, cannot be captured in the method. The model would necessarily contain a discrete form of such a technique. At the level of abstraction supported by the method the system with and the system without NMR would be equivalent from the fault tolerance perspective [TTL09].

We envision a number of possible extensions of the method that could widen its applicability. One possible extension of the approach is to use formalisms with stochastic choice semantics [HT10; Bal01] as opposed to the currently used demonic choice semantics. This would allow developers to express rich fault tolerance properties that involve failure rates and other probabilistic estimations. These can be based on fault assumptions derived directly from reliability analysis techniques. For example, fault trees could be used for deriving formal relationship between subsequent levels of abstraction when applying the error state invariant pattern.

Currently, the method focuses on modelling the system behaviour in reactive style and postpones sequential decomposition. This ensures expressiveness of safety properties. We regard using the Use Case (Flow) language [Ili11; Ili12] as a potentially promising direction of work for verifying local properties carried through sequences of steps. This could add flexibility to the method in expressing sequential behaviour and introduce additional type of views bringing benefits associated with multi-view development.

At the architectural level, the method supports architectural structuring during functional refinement. Such a structuring could be also derived from an architectural description described in an existing well-established architecture description language such as AADL [AADL] and Wright [All97], or in a model-based design language such as UML [JBR99] and SysML [Gro].

The second part of the method that focuses on control system modelling introduces implementation-level constructs such as a control loop and explicit hardware read/write operations (see Section 4.8). Models containing these low-level constructs can be used to obtain verified code using existing code generators such as [EB; MS11].
The mentioned solutions could make a strong link between the proposed method and existing engineering tools and constitute a complete development process.

An important aspect of the proposed development method is the reuse mechanism. It is currently based on generic modelling patterns that are domain-independent (see Section 4.9). We envision two possible directions of improvement in this area. The first solution targets at facilitating reuse within a particular domain. The generic patterns could be used to instantiate formal models from those expressed in domain-specific languages (DSL). The main purpose for such an instantiation would be verification, and the main system design could be done using a DSL.

The second solution is domain-independent and could provide an additional benefit of proof reuse. A formal technique that can improve reuse of modelling decisions and proofs can be based on generalisation and further instantiation of parts or complete models. For example, the airlock case study can be generalised to an N-door “airlock” representing an access control system. The system would provide access to N actors according to the (externally) defined rules. Possible instantiations of such a system could follow rules for a single reader / single writer access, mutual exclusion, or for N out of M accessors. We believe that such an approach would be particularly useful during the architectural development process where architects typically seek for and use existing solutions in the form of patterns. However, the effort required for proofs in such an approach might be bigger than that for domain-specific models; this is due to the generality of models and properties being verified.

The proposed method is a top-down development method that is applicable to a wide range of problem domains. It is especially beneficial for modelling systems with a single source of control such as embedded systems. The method does not provide means for model decomposition which is necessary when modelling distributed systems, business applications, and communication protocols. Such systems are typically composed of communicating subsystems with distributed logic. In these domains, the method can be applied at the system and subsystem levels separately. To formally connect the two levels one can use existing decomposition techniques such as shared variables, shared events, or modularisation for Event-B [Hoa+11].

6.2 Summary and Contributions

This thesis proposes a development method for formal modelling of fault tolerant systems. The method focuses on top-down modelling of abstract fault tolerant behaviour with the purpose of verifying fault tolerance requirements.

The method provides modelling guidelines for building fault tolerant systems. The guidelines consist of three constituents:

- *Modelling principles* postulate rules which must be obeyed during modelling
• *Refinement strategy* specifies a sequence of refinement steps for arriving at a correct model

• *Modelling patterns* support specific refinement steps and provide a reuse mechanism for building fault tolerant systems

The method allows developers to further refine an abstract fault tolerant system into a control system.

We demonstrate the applicability of the method by instantiating patterns and templates in Event-B and modelling two medium-scale case studies. The thesis contributes to the following areas:

**Adoption of formal methods**
The proposed development method and its contributions mentioned above further the adoption of formal methods. The method provides formal developers with a set of modelling guidelines that cover three levels of application. The modelling principles facilitate general understanding of using refinement-based formal methods with interleaving semantics for modelling faults and fault tolerance. The refinement strategy introduces a well-defined sequence of steps that need to be performed to arrive at a meaningful model. The modelling patterns and the modal view templates support engineers with generic modelling solutions and represent a reuse mechanism.

**Modelling fault tolerance for systems dependability**
The method facilitates modelling fault tolerance formally from the early stages of development. Critical systems typically employ fault tolerance as an indistinguishable part of their behaviour. The early consideration of fault tolerance in refinement-based methods reduces the modelling efforts, and helps to ensure the overall dependability of such systems.

**Multi-view development**
The method includes additional views on the models that capture fault tolerance and modal features of the systems. The modal viewpoint adds rigour to the formal development process through additional proof obligations. It also contributes to readability of formal models by engineers. The views shorten the gap between requirements and formal models by allowing modal and fault tolerance requirements to be expressed diagrammatically.

**Rigorous development**
The necessity of having formal consistency between views and models brings extra proof efforts. According to our estimations, application of modal views doubles the...
proof efforts for both case studies described in this work. The proving process gives assurance in system behaviour, therefore, the approach adds rigour to the development process.

**Tool support**
The method is tool supported. The modal viewpoint is implemented as a plug-in for the Rodin environment which includes a diagram editor and a smooth integration with prover facilities [WIFT]. The patterns of the method can be reused for a particular domain or application by using the Transformation Patterns plug-in [WIPT]. The patterns plug-in provides facilities for writing model transformations in a simple object-oriented language, and executing them on demand. Tool support further facilitates the adoption of the approach and formal methods for building highly dependable systems.
References


[Alt] The Altarica language. URL: https://altarica.labri.fr/forge


Atelier B, the Industrial Tool to Efficiently Deploy the B Method. URL: http://www.atelierb.eu


[FMIC] Info Centre for FMEA. URL: [http://www.fmeainfocentre.com](http://www.fmeainfocentre.com).


E. Gamma, R. Helm, R. Johnson, and J. Vlissides. Design Patterns: Elements of Reusable Object-Oriented Software. 1st ed. Addison-Wesley Professional, Nov. 1994. ISBN: 0201633612.


<table>
<thead>
<tr>
<th>Citation</th>
<th>Title</th>
<th>Location</th>
<th>Authors</th>
<th>Date</th>
<th>ISBN</th>
</tr>
</thead>
</table>


Appendix A: Airlock Case Study Model

This appendix contains full Event-B models for the airlock system used as a case study throughout Chapter 4. The development produced a total of 417 proof obligations, 356 of which were proven automatically, and 61 required interactive proof. A Rodin project containing the proved models can be found in the Examples section at [WIFT].

A.1 Context C0

context c0
sets
  DOOR_STATE
constants
  OPENED CLOSED OPENING CLOSING STOPPED
  LOW_PRESSURE HIGH_PRESSURE
axioms
  axm1: partition(DOOR_STATE, {OPENED}, {CLOSED},
               {OPENING}, {CLOSING}, {STOPPED})
  axm2: LOW_PRESSURE = 0
  axm3: HIGH_PRESSURE = 2
end

A.2 Machine M0

machine m0 sees c0
variables
  door1  door2  pressure
invariants
  inv1: door1 ∈ DOOR_STATE
  inv2: door2 ∈ DOOR_STATE
  inv3: pressure ∈ N
inv4: \( \text{door1} \neq \text{CLOSED} \Rightarrow \text{pressure} = \text{LOW\_PRESSURE} \)

inv5: \( \text{door2} \neq \text{CLOSED} \Rightarrow \text{pressure} = \text{HIGH\_PRESSURE} \)

inv6: \( \text{door1} = \text{CLOSED} \lor \text{door2} = \text{CLOSED} \)

inv7: \( \text{pressure} > \text{LOW\_PRESSURE} \Rightarrow \text{door1} = \text{CLOSED} \)

inv8: \( \text{pressure} < \text{HIGH\_PRESSURE} \Rightarrow \text{door2} = \text{CLOSED} \)

inv9: \( \text{pressure} \geq \text{LOW\_PRESSURE} \land \text{pressure} \leq \text{HIGH\_PRESSURE} \)

events

event INITIALISATION

then
  act1: \( \text{door1} := \text{CLOSED} \)
  act2: \( \text{door2} := \text{CLOSED} \)
  act3: \( \text{pressure} := \text{HIGH\_PRESSURE} \)
end

event open1 ≜

when
  grd1: \( \text{door1} = \text{CLOSED} \lor \text{door1} = \text{STOPPED} \)
  grd2: \( \text{pressure} = \text{LOW\_PRESSURE} \)
  grd3: \( \text{door2} = \text{CLOSED} \)
then
  act1: \( \text{door1} := \text{OPENING} \)
end

event opened1 ≜

when
  grd3: \( \text{door1} = \text{OPENING} \)
then
  act1: \( \text{door1} := \text{OPENED} \)
end

event close1 ≜

when
  grd1: \( \text{door1} = \text{OPENED} \lor \text{door1} = \text{STOPPED} \)
then
  act1: \( \text{door1} := \text{CLOSING} \)
end

event closed1 ≜

when
  grd1: \( \text{door1} = \text{CLOSING} \)
then
  act1: \( \text{door1} := \text{CLOSED} \)
end
event open2 ≜
   when
   grd1: door2 = CLOSED ∨ door2 = STOPPED
   grd2: pressure = HIGH_PRESSURE
   grd3: door1 = CLOSED
   then
   act1: door2 := OPENING
   end

event opened2 ≜
   when
   grd1: door2 = OPENING
   then
   act1: door2 := OPENED
   end

event close2 ≜
   when
   grd1: door2 = OPENED ∨ door2 = STOPPED
   then
   act1: door2 := CLOSING
   end

event closed2 ≜
   when
   grd1: door2 = CLOSING
   then
   act1: door2 := CLOSED
   end

event pump_up ≜
   when
   grd1: door1 = CLOSED ∧ door2 = CLOSED
   grd2: pressure < HIGH_PRESSURE
   then
   act1: pressure := pressure + 1
   end

event pump_down ≜
   when
   grd1: door1 = CLOSED ∧ door2 = CLOSED
   grd2: pressure > LOW_PRESSURE
   then
   act1: pressure := pressure − 1

119
end

event stop1 ≜ 
  when 
    grd1: door1 = OPENING ∨ door1 = CLOSING 
  then 
    act1: door1 := STOPPED
end

event stop2 ≜ 
  when 
    grd1: door2 = OPENING ∨ door2 = CLOSING 
  then 
    act1: door2 := STOPPED
end

end

A.3 Machine M1

machine m1 refines m0 sees c0
variables
  door1 door2 pressure stopped
invariants
  inv1: stopped ∈ BOOL
events
event INITIALISATION
  extended
  then 
    act1: door1 := CLOSED
    act2: door2 := CLOSED
    act3: pressure := HIGH_PRESSURE
    act4: stopped := FALSE
  end

event open1 ≜  extends open1
  when 
    grd1: door1 = CLOSED ∨ door1 = STOPPED
    grd2: pressure = LOW_PRESSURE
    grd3: door2 = CLOSED
    grd_stopped: stopped = FALSE
then
  act1: door1 := OPENING
end

event opened1 \equiv \text{extends opened1}
  when
    grd1: door1 = OPENING
    grd\_stopped: stopped = FALSE
  then
    act1: door1 := OPENED
end

event close1 \equiv \text{extends close1}
  when
    grd1: door1 = OPENED \lor door1 = STOPPED
    grd\_stopped: stopped = FALSE
  then
    act1: door1 := CLOSING
end

event closed1 \equiv \text{extends closed1}
  when
    grd1: door1 = CLOSING
    grd\_stopped: stopped = FALSE
  then
    act1: door1 := CLOSED
end

event open2 \equiv \text{extends open2}
  when
    grd1: door2 = CLOSED \lor door2 = STOPPED
    grd2: pressure = HIGH\_PRESSURE
    grd3: door1 = CLOSED
    grd\_stopped: stopped = FALSE
  then
    act1: door2 := OPENING
end

event opened2 \equiv \text{extends opened2}
  when
    grd1: door2 = OPENING
    grd\_stopped: stopped = FALSE
  then
    act1: door2 := OPENED

121
event close2 \overset{\hat{\text{extends}}}{=} \text{close2}

\text{when}
\begin{align*}
\text{grd1:} & \quad door2 = \text{OPENED} \lor door2 = \text{STOPPED} \\
\text{grd\_stopped:} & \quad stopped = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
\text{act1:} & \quad door2 := \text{CLOSING}
\end{align*}
\text{end}

event closed2 \overset{\hat{\text{extends}}}{=} \text{closed2}

\text{when}
\begin{align*}
\text{grd1:} & \quad door2 = \text{CLOSING} \\
\text{grd\_stopped:} & \quad stopped = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
\text{act1:} & \quad door2 := \text{CLOSED}
\end{align*}
\text{end}

event pump\_up \overset{\hat{\text{extends}}}{=} \text{pump\_up}

\text{when}
\begin{align*}
\text{grd1:} & \quad door1 = \text{CLOSED} \land door2 = \text{CLOSED} \\
\text{grd2:} & \quad pressure < \text{HIGH\_PRESSURE} \\
\text{grd\_stopped:} & \quad stopped = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
\text{act1:} & \quad pressure := pressure + 1
\end{align*}
\text{end}

event pump\_down \overset{\hat{\text{extends}}}{=} \text{pump\_down}

\text{when}
\begin{align*}
\text{grd1:} & \quad door1 = \text{CLOSED} \land door2 = \text{CLOSED} \\
\text{grd2:} & \quad pressure > \text{LOW\_PRESSURE} \\
\text{grd\_stopped:} & \quad stopped = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
\text{act1:} & \quad pressure := pressure - 1
\end{align*}
\text{end}

event stop \overset{\hat{\text{}}}{=}

\text{when}
\text{grd\_stopped:} \quad stopped = \text{FALSE}
\text{then}
\begin{align*}
\text{act\_stopped:} & \quad stopped := \text{TRUE}
\end{align*}
\text{end}

event stop1 \overset{\hat{\text{extends}}}{=} \text{stop1}

\text{when}
grd1: door1 = OPENING \lor door1 = CLOSING
grd\_stopped: stopped = FALSE

then
act1: door1 := STOPPED
end

event stop2 \triangleq extends stop2
  when
  grd1: door2 = OPENING \lor door2 = CLOSING
  grd\_stopped: stopped = FALSE
  then
  act1: door2 := STOPPED
  end

end

A.4 Context C2

context c2 extends c0

constants
OK DEGRADED BROKEN DOOR\_CONDITION

axioms
axm1: DOOR\_CONDITION = \{0, 1, 2\}
axm2: BROKEN = 0
axm3: DEGRADED = 1
axm4: OK = 2
axm5: DOOR\_CONDITION \subseteq \mathbb{N}

end
A.5 Machine M2

machine m2 refines m1 sees c2

variables

   door1 door2 pressure stopped door1_cond door2_cond obj_presence

invariants

   inv1: door1_cond ∈ DOOR_CONDITION
   inv2: door2_cond ∈ DOOR_CONDITION
   inv3: obj_presence ∈ BOOL
   inv4: door1_cond = BROKEN ∨ door2_cond = BROKEN ∨
   (door1_cond = DEGRADED ∧ door2_cond = DEGRADED ∧
    obj_presence = FALSE) ⇔ stopped = TRUE

events

event INITIALISATION

extended

   then
   act1: door1 := CLOSED
   act2: door2 := CLOSED
   act3: pressure := HIGH_PRESSURE
   act4: stopped := FALSE
   act2.0: door1_cond := OK
   act2.1: door2_cond := OK
   act2.2: obj_presence := FALSE
   end

event open1 ≜ extends open1

when

   grd1: door1 = CLOSED ∨ door1 = STOPPED
   grd2: pressure = LOW_PRESSURE
   grd3: door2 = CLOSED
   grd_stopped: stopped = FALSE

then

   act1: door1 := OPENING

end

event opened1 ≜ extends opened1

when

   grd1: door1 = OPENING
   grd_stopped: stopped = FALSE

then

   act1: door1 := OPENED

end
event close1 ∧ extends close1
   when
     grd1: door1 = OPENED ∨ door1 = STOPPED
     grd_stopped: stopped = FALSE
   then
     act1: door1 := CLOSING
   end

event closed1 ∧ extends closed1
   when
     grd1: door1 = CLOSING
     grd_stopped: stopped = FALSE
   then
     act1: door1 := CLOSED
   end

event open2 ∧ extends open2
   when
     grd1: door2 = CLOSED ∨ door2 = STOPPED
     grd2: pressure = HIGH_PRESSURE
     grd3: door1 = CLOSED
     grd_stopped: stopped = FALSE
   then
     act1: door2 := OPENING
   end

event opened2 ∧ extends opened2
   when
     grd1: door2 = OPENING
     grd_stopped: stopped = FALSE
   then
     act1: door2 := OPENED
   end

event close2 ∧ extends close2
   when
     grd1: door2 = OPENED ∨ door2 = STOPPED
     grd_stopped: stopped = FALSE
   then
     act1: door2 := CLOSING
   end

event closed2 ∧ extends closed2
   when
     grd1: door2 = CLOSING

125
event pump_up \Rightarrow extends pump_up
\begin{align*}
\text{when} & \quad \text{grd1: door} = \textit{CLOSED} \land \text{door}2 = \textit{CLOSED} \\
& \quad \text{grd2: pressure} < \text{HIGH\_PRESSURE} \\
& \quad \text{grd\_stopped: stopped} = \textit{FALSE} \\
\text{then} & \quad \text{act1: pressure := pressure + 1}
\end{align*}

event pump_down \Rightarrow extends pump_down
\begin{align*}
\text{when} & \quad \text{grd1: door} = \textit{CLOSED} \land \text{door}2 = \textit{CLOSED} \\
& \quad \text{grd2: pressure} > \text{LOW\_PRESSURE} \\
& \quad \text{grd\_stopped: stopped} = \textit{FALSE} \\
\text{then} & \quad \text{act1: pressure := pressure - 1}
\end{align*}

event break \Rightarrow extends stop
\begin{align*}
\text{any} & \quad d_{1c} \; d_{2c} \\
\text{where} & \quad \text{grd\_stopped: stopped} = \textit{FALSE} \\
& \quad \text{grd\_2\_0: } d_{1c} \in \textit{DOOR\_CONDITION} \land d_{1c} \leq \text{door}1\_cond \\
& \quad \text{grd\_2\_1: } d_{2c} \in \textit{DOOR\_CONDITION} \land d_{2c} \leq \text{door}2\_cond \\
& \quad \text{grd\_2\_2: } d_{1c} = \textit{BROKEN} \lor d_{2c} = \textit{BROKEN} \\
& \quad \text{grd\_2\_3: } \text{door}1\_cond \neq \textit{BROKEN} \land \text{door}2\_cond \neq \textit{BROKEN} \\
& \quad \text{grd\_2\_4: } \text{door}1\_cond = \textit{DEGRADED} \land \text{door}2\_cond = \textit{DEGRADED} \Rightarrow \text{obj\_presence} = \textit{TRUE} \\
\text{then} & \quad \text{act\_stopped: stopped := TRUE} \\
& \quad \text{act\_2\_0: } \text{door}1\_cond, \text{door}2\_cond := d_{1c}, d_{2c}
\end{align*}

event stop1 \Rightarrow extends stop1
\begin{align*}
\text{when} & \quad \text{grd1: door} = \textit{OPENING} \lor \text{door}1 = \textit{CLOSING} \\
& \quad \text{grd\_stopped: stopped} = \textit{FALSE} \\
\text{then} & \quad ...
\end{align*}
act1: door1 := STOPPED
end

event stop2 ≡ extends stop2
when
  grd1: door2 = OPENING ∨ door2 = CLOSING
  grd_stopped: stopped = FALSE
then
  act1: door2 := STOPPED
end

event detect ≡
when
  grd0: door1_cond ∈ {OK, DEGRADED}
  grd1: door2_cond ∈ {OK, DEGRADED}
  grd2: door1_cond = OK ∨ door2_cond = OK
  grd5: stopped = FALSE
then
  act1: obj_presence ∈ BOOL
end

event degrade ≡
any
  d1c  d2c
where
  grd0: stopped = FALSE
  grd1: d1c ∈ DOOR_CONDITION ∧ d1c ∈ {OK, DEGRADED} ∧ d1c ≤ door1_cond
  grd2: d2c ∈ DOOR_CONDITION ∧ d2c ∈ {OK, DEGRADED} ∧ d2c ≤ door2_cond
  grd3: (door1_cond = OK ∧ d1c = DEGRADED ∧ door2_cond = d2c) ∨
       (door2_cond = OK ∧ d2c = DEGRADED ∧ door1_cond = d1c)
  grd4: d1c ≠ DEGRADED ∨ d2c ≠ DEGRADED ∨ obj_presence = TRUE
then
  act1: door1_cond := d1c
  act2: door2_cond := d2c
end

event stopped ≡ extends stopped
when
  grd: stopped = TRUE
then
  skip
end

event object_leave ≡ extends stop
when
grd\_stopped: stopped = FALSE
grd1: door1\_cond = DEGRADED \& door2\_cond = DEGRADED \&
\hspace{1cm} obj\_presence = TRUE

then
act\_stopped: stopped := TRUE
act1: obj\_presence := FALSE

end

end

event stop\_on\_degrade \equiv \text{extends stop}
when
grd\_stopped: stopped = FALSE
grd1: door1\_cond = DEGRADED \lor door2\_cond = DEGRADED
grd2: obj\_presence = FALSE
grd4: door1\_cond = OK \lor door2\_cond = OK
then
act\_stopped: stopped := TRUE
act1: door1\_cond := DEGRADED
act2: door2\_cond := DEGRADED
end

end

A.6 Machine M3

machine m3 refines m2 sees c2
variables
\hspace{1cm} door1 door2 pressure stopped door1\_cond door2\_cond obj\_presence

events

event open1 \equiv \text{extends open1}
when
grd1: door1 = CLOSED \lor door1 = STOPPED
grd2: pressure = LOW\_PRESSURE
grd3: door2 = CLOSED
grd\_stopped: stopped = FALSE
grd2\_1: door1\_cond = OK
then
act1: door1 := OPENING
end

event opened1 \equiv \text{extends opened1}
when
grd1: $\text{door1} = \text{OPENING}$

grd_stopped: $\text{stopped} = \text{FALSE}$

grd2_0: $\text{door1}_\text{cond} \in \{\text{OK, DEGRADED}\}$

grd2_1: $\text{door2}_\text{cond} \in \{\text{OK, DEGRADED}\}$

grd2_2: $\text{door1}_\text{cond} = \text{DEGRADED} \land \text{door2}_\text{cond} = \text{DEGRADED} \Rightarrow \text{obj.p} = \text{TRUE}$

then

act1: $\text{door1} := \text{OPENED}$

end

event close1 $\bowtie$ extends close1

when

grd1: $\text{door1} = \text{OPENED} \lor \text{door1} = \text{STOPPED}$

grd_stopped: $\text{stopped} = \text{FALSE}$

grd2_0: $\text{door1}_\text{cond} \in \{\text{OK, DEGRADED}\}$

grd2_1: $\text{door2}_\text{cond} \in \{\text{OK, DEGRADED}\}$

grd2_2: $\text{door1}_\text{cond} = \text{DEGRADED} \land \text{door2}_\text{cond} = \text{DEGRADED} \Rightarrow \text{obj.p} = \text{TRUE}$

then

act1: $\text{door1} := \text{CLOSING}$

end

event closed1 $\bowtie$ extends closed1

when

grd1: $\text{door1} = \text{CLOSING}$

grd_stopped: $\text{stopped} = \text{FALSE}$

grd2_0: $\text{door1}_\text{cond} \in \{\text{OK, DEGRADED}\}$

grd2_1: $\text{door2}_\text{cond} \in \{\text{OK, DEGRADED}\}$

grd2_2: $\text{door1}_\text{cond} = \text{DEGRADED} \land \text{door2}_\text{cond} = \text{DEGRADED} \Rightarrow \text{obj.p} = \text{TRUE}$

then

act1: $\text{door1} := \text{CLOSED}$

end

event open2 $\bowtie$ extends open2

when

grd1: $\text{door2} = \text{CLOSED} \lor \text{door2} = \text{STOPPED}$

grd2: $\text{pressure} = \text{HIGH PRESSURE}$

grd3: $\text{door1} = \text{CLOSED}$

grd_stopped: $\text{stopped} = \text{FALSE}$

grd2_0: $\text{door2}_\text{cond} = \text{OK} \lor (\text{door1}_\text{cond} = \text{DEGRADED} \land \text{door2}_\text{cond} = \text{DEGRADED} \land \text{obj.p} = \text{TRUE})$

then

act1: $\text{door2} := \text{OPENING}$
end

event opened2  ≡  extends opened2 when
grd1: door2 = OPENING
grd_stopped: stopped = FALSE
grd2_0: door1-cond ∈ \{OK, DEGRADED\}
grd2_1: door2-cond ∈ \{OK, DEGRADED\}
grd2_2: door1-cond = DEGRADED ∧ door2-cond = DEGRADED ⇒ obj\_presence = TRUE
then
act1: door2 := OPENED
end

event close2  ≡  extends close2 when
grd1: door2 = OPENED ∨ door2 = STOPPED
grd_stopped: stopped = FALSE
grd2_0: door1-cond ∈ \{OK, DEGRADED\}
grd2_1: door2-cond ∈ \{OK, DEGRADED\}
grd2_2: door1-cond = DEGRADED ∧ door2-cond = DEGRADED ⇒ obj\_presence = TRUE
then
act1: door2 := CLOSING
end

event closed2  ≡  extends closed2 when
grd1: door2 = CLOSING
grd_stopped: stopped = FALSE
grd2_0: door1-cond ∈ \{OK, DEGRADED\}
grd2_1: door2-cond ∈ \{OK, DEGRADED\}
grd2_2: door1-cond = DEGRADED ∧ door2-cond = DEGRADED ⇒ obj\_presence = TRUE
then
act1: door2 := CLOSED
end

event pump_up  ≡  extends pump_up when
grd1: door1 = CLOSED ∧ door2 = CLOSED
grd2: pressure < HIGH\_PRESSURE
grd_stopped: stopped = FALSE
grd2_0: door1-cond ∈ \{OK, DEGRADED\}
grd2_1: door2\_cond = OK \lor (door1\_cond = DEGRADED \land 
\hspace{1cm} door2\_cond = DEGRADED \land \text{obj\_presence} = TRUE) 
then 
  \hspace{1cm} act1: pressure := pressure + 1 
end 
event pump\_down \triangleq \text{extends pump\_down} 
when 
  \hspace{1cm} grd1: door1 = CLOSED \land door2 = CLOSED 
  \hspace{1cm} grd2: pressure > LOW\_PRESSURE 
  \hspace{1cm} grd\_stopped: stopped = FALSE 
  \hspace{1cm} grd2\_0: door1\_cond = OK 
then 
  \hspace{1cm} act1: pressure := pressure - 1 
end 
event INITIALISATION 
extended 
then 
  \hspace{1cm} act1: door1 := CLOSED 
  \hspace{1cm} act2: door2 := CLOSED 
  \hspace{1cm} act3: pressure := HIGH\_PRESSURE 
  \hspace{1cm} act4: stopped := FALSE 
  \hspace{1cm} act2\_0: door1\_cond := OK 
  \hspace{1cm} act2\_1: door2\_cond := OK 
  \hspace{1cm} act2\_2: \text{obj\_presence} := FALSE 
end 
event break \triangleq \text{extends break} 
any 
  \hspace{1cm} d1c, d2c 
where 
  \hspace{1cm} grd\_stopped: stopped = FALSE 
  \hspace{1cm} grd2\_0: d1c \in DOOR\_CONDITION \land d1c \leq door1\_cond 
  \hspace{1cm} grd2\_1: d2c \in DOOR\_CONDITION \land d2c \leq door2\_cond 
  \hspace{1cm} grd2\_2: d1c = BROKEN \lor d2c = BROKEN 
  \hspace{1cm} grd2\_3: door1\_cond \neq BROKEN \land door2\_cond \neq BROKEN 
  \hspace{1cm} grd2\_4: door1\_cond = DEGRADED \land door2\_cond = DEGRADED \Rightarrow 
  \hspace{1cm} \text{obj\_presence} = TRUE 
then 
  \hspace{1cm} act\_stopped: stopped := TRUE 
  \hspace{1cm} act2\_0: door1\_cond, door2\_cond := d1c, d2c 
end 
event stop1 \triangleq \text{extends stop1}
when
grd1: door1 = OPENING \lor door1 = CLOSING
grdStopped: stopped = FALSE
then
act1: door1 := STOPPED
end

event stop2 \triangleq extends stop2
when
grd1: door2 = OPENING \lor door2 = CLOSING
grdStopped: stopped = FALSE
then
act1: door2 := STOPPED
end

event degrade \triangleq extends degrade
any
d1c d2c
where
grd0: stopped = FALSE
grd1: d1c \in DOOR\_CONDITION \land d1c \in \{OK, DEGRADED\} \land d1c \leq door1\_cond
grd2: d2c \in DOOR\_CONDITION \land d2c \in \{OK, DEGRADED\} \land d2c \leq door2\_cond
grd3: (door1\_cond = OK \land d1c = DEGRADED \land door2\_cond = d2c) \lor
( door2\_cond = OK \land d2c = DEGRADED \land door1\_cond = d1c)
grd4: d1c \neq DEGRADED \lor d2c \neq DEGRADED \lor obj\_presence = TRUE
then
act1: door1\_cond := d1c
act2: door2\_cond := d2c
end

event detect \triangleq extends detect
when
grd0: door1\_cond \in \{OK, DEGRADED\}
grd1: door2\_cond \in \{OK, DEGRADED\}
grd2: door1\_cond = OK \lor door2\_cond = OK
grd5: stopped = FALSE
then
act1: obj\_presence \in BOOL
end

event stopped \triangleq extends stopped
when
grd: stopped = TRUE
then
skip
end

event object_leave \triangleq extends object_leave
when
grd\_stopped: stopped = FALSE
grd1: door1\_cond = DEGRADED \land door2\_cond = DEGRADED \land
obj\_presence = TRUE
then
act\_stopped: stopped := TRUE
act1: obj\_presence := FALSE
end

event stop\_on\_degrade \triangleq extends stop\_on\_degrade
when
grd\_stopped: stopped = FALSE
grd1: door1\_cond = DEGRADED \lor door2\_cond = DEGRADED
grd2: obj\_presence = FALSE
grd4: door1\_cond = OK \lor door2\_cond = OK
then
act\_stopped: stopped := TRUE
act1: door1\_cond := DEGRADED
act2: door2\_cond := DEGRADED
end
end

A.7 Machine M4

machine m4 refines m3 sees c2
variables
door1 door2 pressure stopped door2\_cond obj\_presence
door1\_pos\_cond door1\_closed\_cond door1\_opened\_cond door1\_motor\_cond
invariants
inv\_door1\_pos\_cond: door1\_pos\_cond \in BOOL
inv\_door1\_closed\_cond: door1\_closed\_cond \in BOOL
inv\_door1\_opened\_cond: door1\_opened\_cond \in BOOL
inv\_door1\_motor\_cond: door1\_motor\_cond \in BOOL
inv\_door1\_sensors\_conditions\_OK: door1\_cond = OK \iff
door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land
door1\_opened\_cond = TRUE \land door1\_motor\_cond = TRUE

133
inv\_door1\_sensors\_conditions\_DEGRADED: \(door1\_cond = DEGRADED \iff\)
\[door1\_motor\_cond = TRUE \land\]
\[(door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land\]
\[door1\_closed\_cond = TRUE) \lor\]
\[(door1\_pos\_cond = TRUE \land\]
\[(door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))\]

inv\_door1\_sensors\_conditions\_BROKEN: \(door1\_cond = BROKEN \iff\)
\[door1\_motor\_cond = FALSE \lor (door1\_pos\_cond = FALSE \land\]
\[(door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))\]

events

event INITIALISATION

then
act1: \(door1 := CLOSED\)
act2: \(door2 := CLOSED\)
act3: \(pressure := HIGH\_PRESSURE\)
act4: \(stopped := FALSE\)
act2_1: \(door2\_cond := OK\)
act2_2: \(obj\_presence := FALSE\)
act4_2: \(door1\_pos\_cond := TRUE\)
act4_3: \(door1\_closed\_cond := TRUE\)
act4_4: \(door1\_opened\_cond := TRUE\)
act4_5: \(door1\_motor\_cond := TRUE\)

end

event open1 \(\equiv\) refines open1

when
grd1: \(door1 = CLOSED \lor door1 = STOPPED\)
grd2: \(pressure = LOW\_PRESSURE\)
grd3: \(door2 = CLOSED\)
grd\_stopped: \(stopped = FALSE\)
grd\_cond: \(door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land\]
\[door1\_opened\_cond = TRUE \land door1\_motor\_cond = TRUE\]

then
act1: \(door1 := OPENING\)

end

event opened1 \(\equiv\) refines opened1

when
grd1: \(door1 = OPENING\)
grd\_stopped: \(stopped = FALSE\)
grd4_0: \(door1\_motor\_cond = TRUE\)
grd4_1: (\(door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land\]
\[door1\_opened\_cond = TRUE\)) \lor
(door\_1\_pos\_cond = FALSE \land door\_1\_opened\_cond = TRUE \land 
\hspace{1cm} door\_1\_closed\_cond = TRUE) \lor 

(door\_1\_pos\_cond = TRUE \land 
\hspace{1cm} (door\_1\_opened\_cond = FALSE \lor door\_1\_closed\_cond = TRUE))

grd4\_2: door2\_cond \in \{OK, DEGRADED\}

grd4\_3: door1\_motor\_cond = TRUE \land 
\hspace{1cm} ((door\_1\_pos\_cond = FALSE \land door\_1\_opened\_cond = TRUE \land 
\hspace{3cm} door\_1\_closed\_cond = TRUE) \lor 

(door\_1\_pos\_cond = TRUE \land 
\hspace{3cm} (door\_1\_opened\_cond = FALSE \lor door\_1\_closed\_cond = FALSE)) \land 

\hspace{1cm} door2\_cond = DEGRADED \Rightarrow \text{obj.presence} = TRUE 

then

act1: door1 := OPENED

end

event closed1 \cong \text{refines closed1}

when

grd1: door1 = OPENED \lor door1 = STOPPED

grd\_stopped: stopped = FALSE

grd4\_0: door1\_motor\_cond = TRUE

grd4\_1: (door\_1\_pos\_cond = TRUE \land door\_1\_closed\_cond = TRUE \land 
\hspace{2cm} door\_1\_opened\_cond = TRUE) \lor 

(door\_1\_pos\_cond = FALSE \land door\_1\_opened\_cond = TRUE \land 
\hspace{3cm} door\_1\_closed\_cond = TRUE) \lor 

(door\_1\_pos\_cond = TRUE \land 
\hspace{2cm} (door\_1\_opened\_cond = FALSE \lor door\_1\_closed\_cond = FALSE)) \land 

\hspace{1cm} door2\_cond = DEGRADED \Rightarrow \text{obj.presence} = TRUE 

then

act1: door1 := CLOSING

end

event closed1 \cong \text{refines closed1}

when

grd1: door1 = CLOSING

grd\_stopped: stopped = FALSE

grd4\_0: door1\_motor\_cond = TRUE

grd4\_1: (door\_1\_pos\_cond = TRUE \land door\_1\_closed\_cond = TRUE \land 
\hspace{2cm} door\_1\_opened\_cond = TRUE) \lor 

(door\_1\_pos\_cond = TRUE \land 
\hspace{2cm} (door\_1\_opened\_cond = FALSE \lor door\_1\_closed\_cond = FALSE)) \land 

\hspace{1cm} door2\_cond = DEGRADED \Rightarrow \text{obj.presence} = TRUE
(door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧ 
   door1_closedCond = TRUE) ∨ 

(door1_posCond = TRUE ∧ 
   (door1_openedCond = FALSE ∨ door1_closedCond = FALSE))

grd4.2: door2Cond ∈ {OK, DEGRADED}

grd4.3: door1_motorCond = TRUE ∧ 
   ((door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧ 
     door1_closedCond = TRUE) ∨ 
    (door1_posCond = TRUE ∧ 
     (door1_openedCond = FALSE ∨ door1_closedCond = FALSE))) ∧ 
    door2Cond = DEGRADED ⇒ obj_presence = TRUE

then
  act1: door1 := CLOSED
end

event open2 ≡ refines open2

when
  grd1: door2 = CLOSED ∨ door2 = STOPPED
  grd2: pressure = HIGH_PRESSURE
  grd3: door1 = CLOSED
  grd_stopped: stopped = FALSE
  grd2.0: door2Cond = OK ∨ ((door1_motorCond = TRUE ∧ 
    ((door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧ 
      door1_closedCond = TRUE) ∨ 
     (door1_posCond = TRUE ∧ 
      (door1_openedCond = FALSE ∨ door1_closedCond = FALSE))) ∧ 
     door2Cond = DEGRADED ∧ obj_presence = TRUE)

then
  act1: door2 := OPENING
end

event opened2 ≡ refines opened2

when
  grd1: door2 = OPENING
  grd_stopped: stopped = FALSE
  grd4.0: door1_motorCond = TRUE
  grd4.1: (door1_posCond = TRUE ∧ door1_closedCond = TRUE ∧ 
     door1_openedCond = TRUE) ∨ 
   (door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧ 
    door1_closedCond = TRUE) ∨ 
   (door1_posCond = TRUE ∧ 
    (door1_openedCond = FALSE ∨ door1_closedCond = FALSE))
  grd4.2: door2Cond ∈ {OK, DEGRADED}
grd4_3: door1\_motor\_cond = TRUE \land \\
((door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
door1\_closed\_cond = TRUE) \lor \\
(door1\_pos\_cond = TRUE \land \\
(door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))) \land \\
door2\_cond = \text{DEGRADED} \Rightarrow \text{obj\_presence} = \text{TRUE}

then

act1: door2 := OPENED
end

event close2 \triangleq \text{refines close2}
when

grd1: door2 = OPENED \lor door2 = STOPPED

grd\_stopped: stopped = FALSE

grd4_0: door1\_motor\_cond = TRUE

grd4_1: (door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \\
door1\_opened\_cond = TRUE) \lor \\
(door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
door1\_closed\_cond = TRUE) \lor \\
(door1\_pos\_cond = TRUE \land \\
(door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))) \land \\
door2\_cond = \text{DEGRADED} \Rightarrow \text{obj\_presence} = \text{TRUE}
then

act1: door2 := CLOSING
end

event closed2 \triangleq \text{refines closed2}
when

grd1: door2 = CLOSING

grd\_stopped: stopped = FALSE

grd4_0: door1\_motor\_cond = TRUE

grd4_1: (door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \\
door1\_opened\_cond = TRUE) \lor \\
(door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
door1\_closed\_cond = TRUE) \lor \\
(door1\_pos\_cond = TRUE \land \\
(door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))) \land \\
door2\_cond = \text{DEGRADED} \Rightarrow \text{obj\_presence} = \text{TRUE}

then

act1: door2 := STOPPED
end
grd4.3: door1\_motor\_cond = TRUE \land \\
( (door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
door1\_closed\_cond = TRUE)) \lor \\
( door1\_pos\_cond = TRUE \land \\
( door1\_opened\_cond = FALSE \lor \\
door1\_closed\_cond = FALSE)) \land \\
door2\_cond = DEGRADED \Rightarrow \text{obj\_presence} = TRUE
then
act1: door2 := CLOSED
end

event pump\_up \cong \text{refines pump\_up}
when
grd1: door1 = CLOSED \land door2 = CLOSED
grd2: pressure < HIGH\_PRESSURE
grd\_stopped: stopped = FALSE
grd4.0: door1\_motor\_cond = TRUE
grd4.1: (door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \\
door1\_opened\_cond = TRUE) \lor \\
( door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
door1\_closed\_cond = TRUE) \lor \\
( door1\_pos\_cond = TRUE \land \\
( door1\_opened\_cond = FALSE \lor \\
door1\_closed\_cond = FALSE))
grd4.2: door2\_cond = OK \lor (( door1\_motor\_cond = TRUE \land \\
( door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
door1\_closed\_cond = TRUE)) \lor \\
( door1\_pos\_cond = TRUE \land \\
( door1\_opened\_cond = FALSE \lor \\
door1\_closed\_cond = FALSE))) \land \\
door2\_cond = DEGRADED \land \text{obj\_presence} = TRUE
then
act1: pressure := pressure + 1
end

event pump\_down \cong \text{refines pump\_down}
when
grd1: door1 = CLOSED \land door2 = CLOSED
grd2: pressure > LOW\_PRESSURE
grd\_stopped: stopped = FALSE
grd4.0: door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \\
door1\_opened\_cond = TRUE \land door1\_motor\_cond = TRUE
then
act1: pressure := pressure - 1
end

event break1 \cong \text{refines break}
any
pos\_cond \ closed\_cond \ opened\_cond \ motor\_cond

where

grd\_types: pos\_cond \in BOOL \land \ closed\_cond \in BOOL \land \ opened\_cond \in BOOL \land \ motor\_cond \in BOOL

grd\_degradation: (pos\_cond = FALSE \land door1\_pos\_cond = TRUE) \lor
(closed\_cond = FALSE \land door1\_closed\_cond = TRUE) \lor
(opened\_cond = FALSE \land door1\_opened\_cond = TRUE) \lor
(motor\_cond = FALSE \land door1\_motor\_cond = TRUE)

grd\_4\_0: door2\_cond \neq BROKEN

grd\_4\_1: \neg (door1\_motor\_cond = FALSE \lor (door1\_pos\_cond = FALSE \land
(door1\_open\_cond = FALSE \lor door1\_closed\_cond = FALSE)))

grd\_4\_2: motor\_cond = FALSE \lor (pos\_cond = FALSE \land
(opened\_cond = FALSE \lor closed\_cond = FALSE))

grd\_4\_3: (door1\_motor\_cond = TRUE \land
((door1\_pos\_cond = FALSE \land door1\_open\_cond = TRUE \land
door1\_closed\_cond = TRUE) \lor
(door1\_pos\_cond = TRUE \land
(door1\_open\_cond = FALSE \lor door1\_closed\_cond = FALSE)))) \land

\land door2\_cond = DEGRADED \Rightarrow obj\_presence = TRUE

grd\_5: stopped = FALSE

with

d2c : d2c = door2\_cond

d1c : d1c = BROKEN

then

act\_4\_0: door1\_pos\_cond := pos\_cond

act\_4\_1: door1\_closed\_cond := closed\_cond

act\_4\_2: door1\_opened\_cond := opened\_cond

act\_4\_3: door1\_motor\_cond := motor\_cond

act\_4\_4: stopped := TRUE

end

event stop1 \triangleq extends stop1

when

grd\_1: door1 = OPENING \lor door1 = CLOSING

grd\_stopped: stopped = FALSE

then

act\_1: door1 := STOPPED

end

event stop2 \triangleq extends stop2

when

grd\_1: door2 = OPENING \lor door2 = CLOSING
grd\_stopped: \(stopped = FALSE\)
then
act1: \(door2 := STOPPED\)
end

event degrade\_door1 \(\sqsupseteq\) refines degrade
any
\(pos\_cond\) \(\text{closed\_cond}\) \(\text{opened\_cond}\) \(\text{motor\_cond}\)
where
grd\_types: \(pos\_cond \in BOOL \land \text{closed\_cond} \in BOOL \land \text{opened\_cond} \in BOOL \land \text{motor\_cond} \in BOOL\)
grd\_degradation: \((pos\_cond = FALSE \land door1\_pos\_cond = TRUE) \lor\)
\((\text{closed\_cond} = FALSE \land door1\_closed\_cond = TRUE) \lor\)
\((\text{opened\_cond} = FALSE \land door1\_opened\_cond = TRUE) \lor\)
\((\text{motor\_cond} = FALSE \land door1\_motor\_cond = TRUE)\)
grd\_stopped: \(stopped = FALSE\)
grd1: \(door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \text{door1\_opened\_cond} = TRUE \land door1\_motor\_cond = TRUE\)
grd7: \(\text{door2\_cond} = DEGRADED \Rightarrow \text{obj\_presence} = TRUE\)
grd\_glue: \(\text{motor\_cond} = TRUE \land\)
\((\text{pos\_cond} = FALSE \land \text{opened\_cond} = TRUE \land\)
\(\text{closed\_cond} = TRUE) \lor\)
\((\text{pos\_cond} = TRUE \land\)
\(\text{closed\_cond} = FALSE) \lor\)
\((\text{pos\_cond} = TRUE \land\)
\(\text{opened\_cond} = FALSE \land \text{closed\_cond} = FALSE))\)
with
\(d2c : d2c = door2\_cond\)
\(d1c : d1c = DEGRADED\)
then
act4\_0: \(door1\_pos\_cond := pos\_cond\)
act4\_1: \(door1\_closed\_cond := \text{closed\_cond}\)
act4\_2: \(door1\_opened\_cond := \text{opened\_cond}\)
act4\_3: \(door1\_motor\_cond := \text{motor\_cond}\)
end

event degrade\_door2 \(\sqsupseteq\) refines degrade
when
grd\_stopped: \(stopped = FALSE\)
grd4\_0: \(\text{door1\_motor\_cond} = TRUE\)
grd4\_1: \((door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land\)
\(\text{door1\_opened\_cond} = TRUE) \lor\)
\((door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land\)
\(\text{door1\_closed\_cond} = TRUE) \lor\)
\((\text{door1\_pos\_cond} = TRUE \land\)
\((\text{door1\_opened\_cond} = FALSE \lor \text{door1\_closed\_cond} = FALSE))\)

140
grd4_2: $(\text{door1\_motor\_cond} = \text{TRUE} \land$
\begin{align*}
& (\text{door1\_pos\_cond} = \text{FALSE} \land 
\text{door1\_opened\_cond} = \text{TRUE} \land 
\text{door1\_closed\_cond} = \text{TRUE}) \lor \\
& (\text{door1\_pos\_cond} = \text{TRUE} \land 
\text{door1\_opened\_cond} = \text{FALSE} \lor 
\text{door1\_closed\_cond} = \text{FALSE})) \Rightarrow \\
& \text{obj\_presence} = \text{TRUE}
\end{align*}
grd4_3: $(\text{door2\_cond} = \text{OK})$

with
\begin{align*}
\text{d2c: } & (\text{d2c} = \text{DEGRADED}) \\
\text{d1c: } & (\text{d1c} = \text{OK} \iff 
\text{door1\_pos\_cond} = \text{TRUE} \land 
\text{door1\_closed\_cond} = \text{TRUE} \land 
\text{door1\_opened\_cond} = \text{TRUE} \land 
\text{door1\_motor\_cond} = \text{TRUE}) \land \\
& (\text{d1c} = \text{DEGRADED} \iff 
\text{door1\_motor\_cond} = \text{TRUE} \land 
((\text{door1\_pos\_cond} = \text{FALSE} \land 
\text{door1\_opened\_cond} = \text{TRUE} \land 
\text{door1\_closed\_cond} = \text{TRUE}) \lor \\
\text{door1\_pos\_cond} = \text{TRUE} \land 
\text{door1\_opened\_cond} = \text{FALSE} \lor 
\text{door1\_closed\_cond} = \text{FALSE})))
\end{align*}

then
\begin{align*}
\text{act: } & \text{door2\_cond} := \text{DEGRADED}
\end{align*}

end

\textbf{event stop\_on\_degrade\_door1} \equiv \textbf{refines stop\_on\_degrade any}
\textbf{pos\_cond closed\_cond opened\_cond motor\_cond}

\textbf{where}
\begin{align*}
\text{grd\_types: } & \text{pos\_cond} \in \text{BOOL} \land 
\text{closed\_cond} \in \text{BOOL} \land 
\text{opened\_cond} \in \text{BOOL} \land 
\text{motor\_cond} \in \text{BOOL} \\
\text{grd\_degradation: } & (\text{pos\_cond} = \text{FALSE} \land 
\text{door1\_pos\_cond} = \text{TRUE}) \lor \\
& (\text{closed\_cond} = \text{FALSE} \land 
\text{door1\_closed\_cond} = \text{TRUE}) \lor \\
& (\text{opened\_cond} = \text{FALSE} \land 
\text{door1\_opened\_cond} = \text{TRUE}) \lor \\
& (\text{motor\_cond} = \text{FALSE} \land 
\text{door1\_motor\_cond} = \text{TRUE}) \\
\text{grd\_stopped: } & \text{stopped} = \text{FALSE} \\
\text{grd1: } & \text{door2\_cond} = \text{DEGRADED} \\
\text{grd2: } & \text{obj\_presence} = \text{FALSE} \\
\text{grd\_glue: } & \text{motor\_cond} = \text{TRUE} \land \\
& ((\text{pos\_cond} = \text{FALSE} \land 
\text{opened\_cond} = \text{TRUE} \land 
\text{closed\_cond} = \text{TRUE}) \lor \\
\text{pos\_cond} = \text{TRUE} \land \\
\text{opened\_cond} = \text{FALSE} \lor 
\text{closed\_cond} = \text{FALSE}))
\end{align*}

then
\begin{align*}
\text{act4\_0: } & \text{door1\_pos\_cond} := \text{pos\_cond} \\
\text{act4\_1: } & \text{door1\_closed\_cond} := \text{closed\_cond} \\
\text{act4\_2: } & \text{door1\_opened\_cond} := \text{opened\_cond} \\
\text{act4\_3: } & \text{door1\_motor\_cond} := \text{motor\_cond}
\end{align*}
act2: stopped := TRUE
end

event stop_on_degrade_door2 ⊨ refines stop_on_degrade
  when
  grd_stopped: stopped = FALSE
  grd1: door2_cond = OK
  grd2: obj_presence = FALSE
  grd3: door1_motor_cond = TRUE ∧
        ((door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
          door1_closed_cond = TRUE) ∨
         (door1_pos_cond = TRUE ∧
          (door1_opened_cond = FALSE ∨ door1_closed_cond = FALSE))))
  then
    act1: door2_cond := DEGRADED
    act2: stopped := TRUE
  end

event detect ⊨ refines detect
  when
  grd4_0: door1_motor_cond = TRUE
  grd4_1: (door1_pos_cond = TRUE ∧ door1_opened_cond = TRUE ∧
           door1_closed_cond = TRUE) ∨
        (door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
           door1_closed_cond = TRUE) ∨
        (door1_pos_cond = TRUE ∧
         (door1_opened_cond = FALSE ∨ door1_closed_cond = FALSE))
  grd1: door2_cond ∈ {OK, DEGRADED}
  grd2: (door1_pos_cond = TRUE ∧ door1_closed_cond = TRUE ∧
        door1_opened_cond = TRUE ∧ door1_motor_cond = TRUE) ∨
        door2_cond = OK
  grd5: stopped = FALSE
  then
    act1: obj_presence ∈ BOOL
  end

event stopped ⊨ extends stopped
  when
  grd: stopped = TRUE
  then
    skip
  end

event object_leave ⊨ refines object_leave
  when
grd\_stopped: \( stopped = FALSE \)
grd1: \( \text{door1\_motor\_cond} = TRUE \land \)
\((\text{door1\_pos\_cond} = FALSE \land \text{door1\_opened\_cond} = TRUE \land \)
\( \text{door1\_closed\_cond} = TRUE) \lor \)
\((\text{door1\_pos\_cond} = TRUE \land \)
\( \text{door1\_opened\_cond} = FALSE \lor \text{door1\_closed\_cond} = FALSE)))
grd2: \( \text{door2\_cond} = \text{DEGRADED} \)
grd3: \( \text{obj\_presence} = TRUE \)

then
act1: \( \text{obj\_presence} := FALSE \)
act\_stopped: \( stopped := TRUE \)

end

\textbf{event break2} \overset{\Delta}{=} \textbf{refines} \textbf{break}

\begin{align*}
\text{when} & \quad \text{break4}_0: \text{door2\_cond} \neq \text{BROKEN} \\
\text{break4}_1: & \quad \neg (\text{door1\_motor\_cond} = \text{FALSE} \lor (\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \\
\text{door1\_closed\_cond} = \text{TRUE}) \lor \)
\((\text{door1\_pos\_cond} = \text{TRUE} \land \)
\( \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE}))
\text{break4}_3: \quad \text{door1\_motor\_cond} = \text{TRUE} \land \\
\((\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \\
\text{door1\_closed\_cond} = \text{TRUE}) \lor \)
\((\text{door1\_pos\_cond} = \text{TRUE} \land \)
\( \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE}))) \land \\
\text{door2\_cond} = \text{DEGRADED} \Rightarrow \text{obj\_presence} = \text{TRUE} \\
\text{break4}_5: \quad \text{stopped} = \text{FALSE}
\end{align*}

\textbf{with}
\begin{align*}
d2c & : \text{d2c} = \text{BROKEN} \\
d1c & : \quad (\text{d1c} = \text{OK} \Leftrightarrow \text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE} \land \\
\text{door1\_opened\_cond} = \text{TRUE} \land \text{door1\_motor\_cond} = \text{TRUE}) \land \\
\text{d1c} = \text{DEGRADED} \Leftrightarrow (\text{door1\_motor\_cond} = \text{TRUE} \land \\
((\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \\
\text{door1\_closed\_cond} = \text{TRUE}) \lor \\
\text{door1\_pos\_cond} = \text{TRUE} \land \)
\( \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE})))
\end{align*}

then
act4\_1: \( \text{door2\_cond} := \text{BROKEN} \)
act4\_4: \( \text{stopped} := \text{TRUE} \)

end

end
A.8 Context C5

context c5 extends c2
sets

PHASE

constants

ENV DET CONT PRED min_door max_door

axioms

axm1: partition(PHASE, {ENV}, {DET}, {CONT}, {PRED})
axm2: min_door < max_door
axm3: min_door ∈ N ∧ max_door ∈ N
axm4: max_door − min_door > 1

end

A.9 Machine M5

machine m5 refines m4 sees c5
variables

door1 door2 pressure stopped obj_presence door2Cond

door1_posCond door1_closedCond door1_openedCond door1_motorCond

phase door1_motor door1_motorCond_detected

door1_pos door1_pos_predicted door1_posCond_detected

door1_opened door1_opened_predicted door1_openedCond_detected

door1_closed door1_closed_predicted door1_closedCond_detected

invariants

inv_phase: phase ∈ PHASE
inv1: door1_pos ∈ Z
inv2: door1_opened ∈ BOOL
inv3: door1_closed ∈ BOOL
inv4: door1_pos_predicted ∈ Z
inv5: door1_opened_predicted ∈ BOOL
inv6: door1_closed_predicted ∈ BOOL
inv7: door1_motor ∈ \{-1, 0, 1\}
inv10: door1_posCond_detected ∈ BOOL
inv11: door1_openedCond_detected ∈ BOOL
inv12: door1_closedCond_detected ∈ BOOL
inv13: door1_motorCond_detected ∈ BOOL

events
event INITIALISATION
extended
then
act1: door1 := CLOSED
act2: door2 := CLOSED
act3: pressure := HIGH_PRESSURE
act4: stopped := FALSE
act2.1: door2_cond := OK
act2.2: obj_presence := FALSE
act4.2: door1_pos_cond := TRUE
act4.3: door1_closed_cond := TRUE
act4.4: door1Opened_cond := TRUE
act4.5: door1_motor_cond := TRUE
act_phase: phase := ENV
act5.1: door1_pos := 0
act5.2: door1_pos_predicted := 0
act5.3: door1_opened := FALSE
act5.4: door1_opened_predicted := FALSE
act5.5: door1_closed := TRUE
act5.6: door1_closed_predicted := FALSE
act5.7: door1_motor := 0
end

event cycle_sense \equiv
when
  grd_phase: phase = ENV
  grd_stopped: stopped = FALSE
then
  act_phase: phase := DET
  act5.1: door1_pos \in \mathbb{Z}
  act5.2: door1_opened \in BOOL
  act5.3: door1_closed \in BOOL
end

event cycle_detect \equiv
when
  grd_phase: phase = DET
  grd_stopped: stopped = FALSE
then
  act_phase: phase := CONT
  act5.1: door1_pos_cond_detected := bool(door1_pos_cond = TRUE \land
      (min\_door \leq door1_pos \land door1_pos \leq max\_door) \land
      (door1\_opened_cond = TRUE \land door1\_opened = TRUE \Rightarrow
        door1_pos = max\_door)) \land
\( (\text{door1\_closed\_cond} = \text{TRUE} \land \text{door1\_closed} = \text{TRUE} \Rightarrow \text{door1\_pos} = \text{min\_door}) \land \)
\( (\text{door1\_pos} = \text{door1\_pos\_predicted}) \)

\text{act5\_2: door1\_closed\_cond\_detected} :=
\text{bool}(\text{door1\_closed\_cond} = \text{TRUE} \land (\text{door1\_closed} = \text{door1\_closed\_predicted}))

\text{act5\_3: door1\_opened\_cond\_detected} :=
\text{bool}(\text{door1\_opened\_cond} = \text{TRUE} \land (\text{door1\_opened} = \text{door1\_opened\_predicted}))

\text{act5\_4: door1\_motor\_cond\_detected} := \text{bool}(\text{door1\_motor\_cond} = \text{TRUE} \land
((\text{door1\_pos} = \text{door1\_pos\_predicted}) \lor (\text{door1\_closed} = \text{door1\_closed\_predicted}))

\text{end}

\text{event cycle\_predict} \triangleq 
\text{when}
\text{grd\_phase: phase} = \text{PRED}
\text{grd\_stopped: stopped} = \text{FALSE}
\text{then}
\text{act\_phase: phase} := \text{ENV}
\text{act5\_1: door1\_pos\_predicted} := \text{door1\_pos} + \text{door1\_motor}
\text{act5\_2: door1\_opened\_predicted} := \text{bool}(\text{door1\_motor} = 1 \land \text{door1\_pos} = \text{max\_door} - 1) \lor
(\text{door1\_motor} = 0 \land \text{door1\_opened} = \text{TRUE})
\text{act5\_3: door1\_closed\_predicted} := \text{bool}(\text{door1\_motor} = -1 \land \text{door1\_pos} = \text{min\_door} + 1) \lor
(\text{door1\_motor} = 0 \land \text{door1\_closed} = \text{TRUE})
\text{end}

\text{event open1} \triangleq \text{extends open1}
\text{when}
\text{grd1: door1} = \text{CLOSED} \lor \text{door1} = \text{STOPPED}
\text{grd2: pressure} = \text{LOW\_PRESSURE}
\text{grd3: door2} = \text{CLOSED}
\text{grd\_stopped: stopped} = \text{FALSE}
\text{grd\_cond: door1\_pos\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE} \land
\text{door1\_opened\_cond} = \text{TRUE} \land \text{door1\_motor\_cond} = \text{TRUE}
\text{grd\_phase: phase} = \text{CONT}
\text{grd\_pos: door1\_pos\_cond} = \text{TRUE} \Rightarrow \text{door1\_pos} < \text{max\_door} \land
(\text{door1\_pos} = \text{min\_door} \Rightarrow \text{door1} = \text{CLOSED}) \land
(\text{door1\_pos} > \text{min\_door} \Rightarrow \text{door1} = \text{STOPPED})
\text{grd\_closed: door1\_closed\_cond} = \text{TRUE} \Rightarrow
(\text{door1\_closed} = \text{TRUE} \Rightarrow \text{door1} = \text{CLOSED})
\text{grd\_opened: door1\_opened\_cond} = \text{TRUE} \Rightarrow \text{door1\_opened} = \text{FALSE}
\text{then}
act1: door1 := OPENING
act_phase: phase := PRED
act_motor: door1_motor := 1
end

event opened1 := extends opened1

when
grd1: door1 = OPENING
grd_stopped: stopped = FALSE
grd4_0: door1_motor_cond = TRUE
grd4_1: (door1_pos_cond = TRUE ∧ door1_closed_cond = TRUE ∧
        door1_opened_cond = TRUE) ∨
        (door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
        door1_closed_cond = TRUE) ∨
        (door1_pos_cond = TRUE ∧
        (door1_opened_cond = FALSE ∨ door1_closed_cond = FALSE))
grd4_2: door2_cond ∈ {OK, DEGRADED}
grd4_3: door1_motor_cond = TRUE ∧
        ((door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
        door1_closed_cond = TRUE) ∨
        (door1_pos_cond = TRUE ∧
        (door1_opened_cond = FALSE ∨ door1_closed_cond = FALSE)))) ∧
        door2_cond = DEGRADED ⇒ obj_presence = TRUE

grd_phase: phase = CONT
grd_pos: door1_pos_cond = TRUE ⇒ door1_pos = max_door
grd_closed: door1_closed_cond = TRUE ⇒ door1_closed = FALSE
grd_opened: door1_opened_cond = TRUE ⇒ door1_opened = TRUE
then
act1: door1 := OPENED
act_phase: phase := PRED
act_motor: door1_motor := 0
end

event close1 := extends close1

when
grd1: door1 = OPENED ∨ door1 = STOPPED
grd_stopped: stopped = FALSE
grd4_0: door1_motor_cond = TRUE
grd4_1: (door1_pos_cond = TRUE ∧ door1_closed_cond = TRUE ∧
        door1_opened_cond = TRUE) ∨
        (door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
        door1_closed_cond = TRUE) ∨
        (door1_pos_cond = TRUE ∧
        (door1_opened_cond = FALSE ∨ door1_closed_cond = FALSE))
\( \text{grd}_4.2: \) \( \text{door}_2\/\text{cond} \in \{\text{OK}, \text{DEGRADED}\} \)

\( \text{grd}_4.3: \) \( \text{door}_1\_\text{motor}\_\text{cond} = \text{TRUE} \land \)
\( (\text{door}_1\_\text{pos}\_\text{cond} = \text{FALSE} \land \text{door}_1\_\text{opened}\_\text{cond} = \text{TRUE} \land \)
\( \text{door}_1\_\text{closed}\_\text{cond} = \text{TRUE}) \lor \)
\( (\text{door}_1\_\text{pos}\_\text{cond} = \text{TRUE} \land \)
\( \text{door}_1\_\text{opened}\_\text{cond} = \text{FALSE} \lor \text{door}_1\_\text{closed}\_\text{cond} = \text{FALSE})) \land \)
\( \text{door}_2\_\text{cond} = \text{DEGRADED} \Rightarrow \text{obj}\_\text{presence} = \text{TRUE} \)

\( \text{grd}\_\text{phase}: \) \( \text{phase} = \text{CONT} \)

\( \text{grd}\_\text{pos}: \) \( \text{door}_1\_\text{pos}\_\text{cond} = \text{TRUE} \Rightarrow \text{door}_1\_\text{pos} > \text{min}\_\text{door} \land \)
\( \text{door}_1\_\text{pos} = \text{max}\_\text{door} \Rightarrow \text{door}_1 = \text{OPENED} \land \)
\( \text{door}_1\_\text{pos} < \text{max}\_\text{door} \Rightarrow \text{door}_1 = \text{STOPPED} \)

\( \text{grd}\_\text{closed}: \) \( \text{door}_1\_\text{closed}\_\text{cond} = \text{TRUE} \Rightarrow \text{door}_1\_\text{closed} = \text{FALSE} \)

\( \text{grd}\_\text{opened}: \) \( \text{door}_1\_\text{opened}\_\text{cond} = \text{TRUE} \Rightarrow \)
\( \text{door}_1\_\text{opened} = \text{TRUE} \Rightarrow \text{door}_1 = \text{OPENED} \)

\( \text{then} \)
\( \text{act}_1: \) \( \text{door}_1 := \text{CLOSING} \)
\( \text{act}\_\text{phase}: \) \( \text{phase} := \text{PRED} \)
\( \text{act}\_\text{motor}: \) \( \text{door}_1\_\text{motor} := -1 \)

\( \text{end} \)

\( \text{event closed}_1 \overset{\text{extends}}{=} \text{closed}_1 \)

\( \text{when} \)
\( \text{grd}_1: \) \( \text{door}_1 = \text{CLOSING} \)
\( \text{grd}\_\text{stopped}: \) \( \text{stopped} = \text{FALSE} \)
\( \text{grd}_4.0: \) \( \text{door}_1\_\text{motor}\_\text{cond} = \text{TRUE} \)
\( \text{grd}_4.1: \) \( \text{door}_1\_\text{pos}\_\text{cond} = \text{TRUE} \land \text{door}_1\_\text{closed}\_\text{cond} = \text{TRUE} \land \)
\( \text{door}_1\_\text{opened}\_\text{cond} = \text{TRUE} \land \)
\( \text{door}_1\_\text{closed}\_\text{cond} = \text{TRUE}) \lor \)
\( \text{door}_1\_\text{pos}\_\text{cond} = \text{FALSE} \land \text{door}_1\_\text{opened}\_\text{cond} = \text{TRUE} \land \)
\( \text{door}_1\_\text{closed}\_\text{cond} = \text{TRUE}) \lor \)
\( \text{door}_1\_\text{pos}\_\text{cond} = \text{TRUE} \land \)
\( \text{door}_1\_\text{opened}\_\text{cond} = \text{FALSE} \lor \text{door}_1\_\text{closed}\_\text{cond} = \text{FALSE})) \land \)
\( \text{door}_2\_\text{cond} = \text{DEGRADED} \Rightarrow \text{obj}\_\text{presence} = \text{TRUE} \)

\( \text{grd}\_\text{phase}: \) \( \text{phase} = \text{CONT} \)

\( \text{grd}\_\text{pos}: \) \( \text{door}_1\_\text{pos}\_\text{cond} = \text{TRUE} \Rightarrow \text{door}_1\_\text{pos} > \text{min}\_\text{door} \land \)
\( \text{door}_1\_\text{pos} < \text{max}\_\text{door} \Rightarrow \text{door}_1 = \text{STOPPED} \)

\( \text{grd}\_\text{closed}: \) \( \text{door}_1\_\text{closed}\_\text{cond} = \text{TRUE} \Rightarrow \text{door}_1\_\text{closed} = \text{FALSE} \)

\( \text{grd}\_\text{opened}: \) \( \text{door}_1\_\text{opened}\_\text{cond} = \text{TRUE} \Rightarrow \)
\( \text{door}_1\_\text{opened} = \text{TRUE} \Rightarrow \text{door}_1 = \text{OPENED} \)

\( \text{then} \)
\( \text{act}_1: \) \( \text{door}_1 := \text{CLOSED} \)
act_phase: phase := PRED
act_motor: door1_motor := 0
end

event open2 ≡ extends open2
when
    grd1: door2 = CLOSED ∨ door2 = STOPPED
    grd2: pressure = HIGH_PRESSURE
    grd3: door1 = CLOSED
    grd_stopped: stopped = FALSE
    grd2_0: door2Cond = OK ∨ ((door1_motorCond = TRUE ∧
                              ((door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧
                                door1_closedCond = TRUE) ∨
                               (door1_posCond = TRUE ∧
                                (door1_openedCond = FALSE ∨ door1_closedCond = FALSE)))) ∧
                  door2Cond = DEGRADED ∧ obj_presence = TRUE)
    grd_phase: phase = CONT
then
    act1: door2 := OPENING
    act_phase: phase := PRED
end

event opened2 ≡ extends opened2
when
    grd1: door2 = OPENING
    grd_stopped: stopped = FALSE
    grd4_0: door1_motorCond = TRUE
    grd4_1: (door1_posCond = TRUE ∧ door1_closedCond = TRUE ∧
                       door1_openedCond = TRUE) ∨
           (door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧
                       door1_closedCond = TRUE) ∨
           (door1_posCond = TRUE ∧
            (door1_openedCond = FALSE ∨ door1_closedCond = FALSE))
    grd4_2: door2Cond ∈ {OK, DEGRADED}
    grd4_3: door1_motorCond = TRUE ∧
           ((door1_posCond = FALSE ∧ door1_openedCond = TRUE ∧
             door1_closedCond = TRUE) ∨
            (door1_posCond = TRUE ∧
             (door1_openedCond = FALSE ∨ door1_closedCond = FALSE)))) ∧
           door2Cond = DEGRADED ⇒ obj_presence = TRUE
    grd_phase: phase = CONT
then
    act1: door2 := OPENED
    act_phase: phase := PRED
end

**event close2** \(\cong\) extends close2

**when**

grd1: \(\text{door}2 = \text{OPENED} \lor \text{door}2 = \text{STOPPED}\)

grd_stopped: \(\text{stopped} = \text{FALSE}\)

grd4_0: \(\text{door1\_motor\_cond} = \text{TRUE}\)

grd4_1: 
\[ (\text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE} \land \text{door1\_opened\_cond} = \text{TRUE}) \lor \]
\[ (\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE}) \lor \]
\[ (\text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE}) \]

grd4_2: \(\text{door2\_cond} \in \{\text{OK, DEGRADED}\}\)

grd4_3: \(\text{door1\_motor\_cond} = \text{TRUE} \land \)
\[ ((\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE}) \lor \]
\[ (\text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE}) \) \land \]
\[ \text{door2\_cond} = \text{DEGRADED} \Rightarrow \text{obj\_presence} = \text{TRUE} \]

**then**

act1: \(\text{door2} := \text{CLOSING}\)

act_phase: \(\text{phase} := \text{PRED}\)

end

**event closed2** \(\cong\) extends closed2

**when**

grd1: \(\text{door}2 = \text{CLOSING}\)

grd_stopped: \(\text{stopped} = \text{FALSE}\)

grd4_0: \(\text{door1\_motor\_cond} = \text{TRUE}\)

grd4_1: 
\[ (\text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE} \land \text{door1\_opened\_cond} = \text{TRUE}) \lor \]
\[ (\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE}) \lor \]
\[ (\text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE}) \]

grd4_2: \(\text{door2\_cond} \in \{\text{OK, DEGRADED}\}\)

grd4_3: \(\text{door1\_motor\_cond} = \text{TRUE} \land \)
\[ ((\text{door1\_pos\_cond} = \text{FALSE} \land \text{door1\_opened\_cond} = \text{TRUE} \land \text{door1\_closed\_cond} = \text{TRUE}) \lor \]
\[ (\text{door1\_pos\_cond} = \text{TRUE} \land \text{door1\_opened\_cond} = \text{FALSE} \lor \text{door1\_closed\_cond} = \text{FALSE}) \) \land \]
\[ \text{door2\_cond} = \text{DEGRADED} \Rightarrow \text{obj\_presence} = \text{TRUE} \]
\[ gr\_phase: phase = CONT \]

then

act1: \[door2 := CLOSED\]

act\_phase: \[phase := PRED\]

end

event pump\_up \(\triangleq\) extends pump\_up

when

grd1: \[door1 = CLOSED \land door2 = CLOSED\]

grd2: \[pressure < HIGH\_PRESSURE\]

grd\_stopped: \[stopped = FALSE\]

grd4\_0: \[door1\_motor\_cond = TRUE\]

grd4\_1: \(\{\begin{align*}
& (door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \\
& \hspace{1cm} door1\_opened\_cond = TRUE) \lor \\
& (door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
& \hspace{1cm} door1\_closed\_cond = TRUE) \lor \\
& (door1\_pos\_cond = TRUE \land \\
& \hspace{1cm} (door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))
\end{align*}\]

grd4\_2: \(\{\begin{align*}
& door2\_cond = OK \lor ((door1\_motor\_cond = TRUE \land \\
& \hspace{1cm} (door1\_pos\_cond = FALSE \land door1\_opened\_cond = TRUE \land \\
& \hspace{1cm} door1\_closed\_cond = TRUE) \lor \\
& (door1\_pos\_cond = TRUE \land \\
& \hspace{1cm} (door1\_opened\_cond = FALSE \lor door1\_closed\_cond = FALSE))) \land \\
& \hspace{1cm} door2\_cond = DEGRADED \land obj\_presence = TRUE)
\end{align*}\)

grd\_phase: phase = CONT

then

act1: \[pressure := pressure + 1\]

act\_phase: \[phase := PRED\]

end

event pump\_down \(\triangleq\) extends pump\_down

when

grd1: \[door1 = CLOSED \land door2 = CLOSED\]

grd2: \[pressure > LOW\_PRESSURE\]

grd\_stopped: \[stopped = FALSE\]

grd4\_0: \(\{\begin{align*}
& door1\_pos\_cond = TRUE \land door1\_closed\_cond = TRUE \land \\
& \hspace{1cm} door1\_opened\_cond = TRUE \land \\
& \hspace{1cm} door1\_motor\_cond = TRUE
\end{align*}\)

grd\_phase: phase = CONT

then

act1: \[pressure := pressure - 1\]

act\_phase: \[phase := PRED\]

end

event break1 \(\triangleq\) refines break1

151
when
grd\_phase: phase = CONT
grd\_degradation: (door1\_pos\_cond\_detected = FALSE∧door1\_pos\_cond = TRUE)∨

(door1\_closed\_cond\_detected = FALSE ∧ door1\_closed\_cond = TRUE) ∨
(door1\_opened\_cond\_detected = FALSE ∧ door1\_opened\_cond = TRUE) ∨
(door1\_motor\_cond\_detected = FALSE ∧ door1\_motor\_cond = TRUE)
grd4\_0: door2\_cond ≠ BROKEN
grd4\_1: ¬(door1\_motor\_cond = FALSE ∨ (door1\_pos\_cond = FALSE ∧

(door1\_opened\_cond\_detected = FALSE ∨ door1\_closed\_cond\_detected = FALSE)))
grd4\_2: door1\_motor\_cond\_detected = FALSE∧(door1\_pos\_cond\_detected = FALSE∧

(door1\_opened\_cond\_detected = FALSE∧door1\_closed\_cond\_detected = FALSE))
grd4\_3: (door1\_motor\_cond = TRUE ∧

((door1\_pos\_cond = FALSE ∧ door1\_opened\_cond = TRUE ∧

door1\_closed\_cond = TRUE) ∨
(door1\_pos\_cond = TRUE ∧

(door1\_opened\_cond\_detected = FALSE ∨ door1\_closed\_cond\_detected = FALSE)))) ∧
door2\_cond = DEGRADED ⇒ obj\_presence = TRUE
grd5: stopped = FALSE

with

pos\_cond: pos\_cond = door1\_pos\_cond\_detected
opened\_cond: opened\_cond = door1\_opened\_cond\_detected
closed\_cond: closed\_cond = door1\_closed\_cond\_detected
motor\_cond: motor\_cond = door1\_motor\_cond\_detected

then

act\_phase: phase := PRED
act5\_0: door1\_pos\_cond := door1\_pos\_cond\_detected
act5\_1: door1\_closed\_cond := door1\_closed\_cond\_detected
act5\_2: door1\_opened\_cond := door1\_opened\_cond\_detected
act5\_3: door1\_motor\_cond := door1\_motor\_cond\_detected
act5\_4: stopped := TRUE
end

event stop1 ≜ extends stop1

when

grd1: door1 = OPENING ∨ door1 = CLOSING
grd\_stopped: stopped = FALSE
grd\_phase: phase = CONT
grd\_pos: door1\_pos\_cond = TRUE⇒(min\_door < door1\_pos∧door1\_pos < max\_door)
grd\_closed: door1\_closed\_cond = TRUE ⇒ door1\_closed = FALSE
grd\_opened: door1\_opened\_cond = TRUE ⇒ door1\_opened = FALSE

then
act1: door1 := STOPPED
act_phase: phase := PRED
act_motor: door1_motor := 0
end

event stop2 \(\Rightarrow\) extends stop2

when
  grd1: door2 = OPENING \(\lor\) door2 = CLOSING
  grd_stopped: stopped = FALSE
  grd_phase: phase = CONT
then
  act1: door2 := STOPPED
  act_phase: phase := PRED
end

event degrade_door1 \(\Rightarrow\) refines degrade_door1

when
  grd_phase: phase = CONT
  grd_degradation: (door1_pos_cond_detected = FALSE \(\land\) door1_pos_cond = TRUE) \(\lor\)
                  (door1_closed_cond_detected = FALSE \(\land\) door1_closed_cond = TRUE) \(\lor\)
                  (door1_opened_cond_detected = FALSE \(\land\) door1_opened_cond = TRUE) \(\lor\)
                  (door1_motor_cond_detected = FALSE \(\land\) door1_motor_cond = TRUE)
  grd_stopped: stopped = FALSE
  grd1: door1_pos_cond = TRUE \(\land\) door1_closed_cond = TRUE \(\land\)
       door1_opened_cond = TRUE \(\land\) door1_motorCond = TRUE
  grd7: door2Cond = DEGRADED \(\Rightarrow\) obj_presence = TRUE
  grd_glue: door1_motor_cond_detected = TRUE \(\land\)
            ((door1_pos_cond_detected = FALSE \(\land\) door1_opened_cond_detected = TRUE) \(\land\)
             door1_closed_cond_detected = TRUE) \(\lor\)
            (door1_pos_cond_detected = TRUE \(\land\)
             (door1_opened_cond_detected = FALSE \(\lor\) door1_closed_cond_detected = FALSE)))

with
  pos_cond : pos_cond = door1_pos_cond_detected
  opened_cond : opened_cond = door1_opened_cond_detected
  closed_cond : closed_cond = door1_closed_cond_detected
  motor_cond : motor_cond = door1_motor_cond_detected

then
  act_phase: phase := PRED
  act5_0: door1_pos_cond := door1_pos_cond_detected
  act5_1: door1_closed_cond := door1_closed_cond_detected
act5_2: door1_opened_cond := door1_opened_cond_detected
act5_3: door1_motor_cond := door1_motor_cond_detected
end

event degrade_door2 = extends degrade_door2
when
  grdStopped: stopped = FALSE
  grd4_0: door1_motor_cond = TRUE
  grd4_1: (door1_pos_cond = TRUE ∧ door1_closed_cond = TRUE ∧
            door1_opened_cond = TRUE) ∨
            (door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
            door1_closed_cond = TRUE) ∨
            (door1_pos_cond = TRUE ∧
            (door1_opened_cond = FALSE ∨ door1_closed_cond = FALSE))
  grd4_2: (door1_motor_cond = TRUE ∧
            ((door1_pos_cond = FALSE ∧ door1_opened_cond = TRUE ∧
            door1_closed_cond = TRUE) ∨
            (door1_pos_cond = TRUE ∧
            obj_presence = TRUE)) ⇒
  grd4_3: door2_cond = OK
  grd_phase: phase = OK
then
  act: door2_cond := DEGRADED
  act_phase: phase := PRED
end

event stop_on_degrade_door1 = refines stop_on_degrade_door1
when
  grd_phase: phase = CONT
  grd_degradation: (door1_pos_cond_detected = FALSE ∧
                  door1_pos_cond = TRUE) ∨
                  (door1_closed_cond_detected = FALSE ∧
                  door1_closed_cond = TRUE) ∨
                  (door1_opened_cond_detected = FALSE ∧
                  door1_opened_cond = TRUE) ∨
                  (door1_motor_cond_detected = FALSE ∧
                  door1_motor_cond = TRUE)
  grd_stopped: stopped = FALSE
  grd1: door2_cond = DEGRADED
  grd2: obj_presence = FALSE
  grd_glue: door1_motor_cond_detected = TRUE ∧
  ((door1_pos_cond_detected = FALSE ∧
    door1_opened_cond_detected = TRUE ∧
    door1_closed_cond_detected = TRUE) ∨
  (door1_pos_cond_detected = TRUE ∧
  (door1_opened_cond_detected = FALSE ∨
  door1_closed_cond_detected = FALSE))))

154
with

$\text{pos} \text{ cond} : \text{pos} \text{ cond} = \text{door}1 \text{pos} \text{ cond} \text{ detected}$

$\text{opened} \text{ cond} : \text{opened} \text{ cond} = \text{door}1 \text{opened} \text{ cond} \text{ detected}$

$\text{closed} \text{ cond} : \text{closed} \text{ cond} = \text{door}1 \text{closed} \text{ cond} \text{ detected}$

$\text{motor} \text{ cond} : \text{motor} \text{ cond} = \text{door}1 \text{motor} \text{ cond} \text{ detected}$

then

$\text{act} \text{ phase}: \text{phase} := \text{PRED}$

$\text{act}5_0: \text{door}1 \text{pos} \text{ cond} := \text{door}1 \text{pos} \text{ cond} \text{ detected}$

$\text{act}5_1: \text{door}1 \text{closed} \text{ cond} := \text{door}1 \text{closed} \text{ cond} \text{ detected}$

$\text{act}5_2: \text{door}1 \text{opened} \text{ cond} := \text{door}1 \text{opened} \text{ cond} \text{ detected}$

$\text{act}5_3: \text{door}1 \text{motor} \text{ cond} := \text{door}1 \text{motor} \text{ cond} \text{ detected}$

$\text{act}5_4: \text{stopped} := \text{TRUE}$

end

event stop\_on\_degrade\_door2 $\triangleq$ extends stop\_on\_degrade\_door2

when

$\text{grd} \text{ stopped}: \text{stopped} = \text{FALSE}$

$\text{grd}1: \text{door}2 \text{ cond} = \text{OK}$

$\text{grd}2: \text{obj} \text{ presence} = \text{FALSE}$

$\text{grd}3: \text{door}1 \text{motor} \text{ cond} = \text{TRUE} \land$

\[
\left((\text{door}1 \text{pos} \text{ cond} = \text{FALSE} \land \text{door}1 \text{opened} \text{ cond} = \text{TRUE} \land \text{door}1 \text{closed} \text{ cond} = \text{TRUE}) \lor \right.
\]

\[
\left((\text{door}1 \text{pos} \text{ cond} = \text{TRUE} \land \right.
\]

\[
\left((\text{door}1 \text{opened} \text{ cond} = \text{FALSE} \land \text{door}1 \text{closed} \text{ cond} = \text{FALSE}))
\]

$\text{grd} \text{ phase}: \text{phase} = \text{CONT}$

then

$\text{act}1: \text{door}2 \text{ cond} := \text{DEGRADED}$

$\text{act}2: \text{stopped} := \text{TRUE}$

$\text{act} \text{ phase}: \text{phase} := \text{PRED}$

end

event detect $\triangleq$ extends detect

when

$\text{grd}4_0: \text{door}1 \text{motor} \text{ cond} = \text{TRUE}$

$\text{grd}4_1: (\text{door}1 \text{pos} \text{ cond} = \text{TRUE} \land \text{door}1 \text{closed} \text{ cond} = \text{TRUE} \land$

\[
\left(\text{door}1 \text{opened} \text{ cond} = \text{TRUE} \lor \right.
\]

\[
\left(\text{door}1 \text{pos} \text{ cond} = \text{FALSE} \land \text{door}1 \text{opened} \text{ cond} = \text{TRUE} \land \right.
\]

\[
\left(\text{door}1 \text{closed} \text{ cond} = \text{TRUE} \lor \right.
\]

\[
\left(\text{door}1 \text{opened} \text{ cond} = \text{FALSE} \land \text{door}1 \text{closed} \text{ cond} = \text{FALSE}) \lor \right.
\]

\[
\left(\text{door}1 \text{pos} \text{ cond} = \text{FALSE} \land \text{door}1 \text{opened} \text{ cond} = \text{FALSE} \land \text{door}1 \text{closed} \text{ cond} = \text{FALSE})\right)
\]

$\text{grd}1: \text{door}2 \text{ cond} \in \{\text{OK, DEGRADED}\}$

$\text{grd}2: (\text{door}1 \text{pos} \text{ cond} = \text{TRUE} \land \text{door}1 \text{closed} \text{ cond} = \text{TRUE} \land$

\[
\left(\text{door}1 \text{opened} \text{ cond} = \text{TRUE} \land \text{door}1 \text{motor} \text{ cond} = \text{TRUE} \lor \right.
\]

\[
\left(\text{door}1 \text{opened} \text{ cond} = \text{FALSE} \land \text{door}2 \text{ cond} = \text{OK}\right)\lor
\]

155
grd5: \( \text{stopped} = \text{FALSE} \)
grd_phase: \( \text{phase} = \text{CONT} \)
then
act1: \( \text{obj.presence} :\in \text{BOOL} \)
act_phase: \( \text{phase} := \text{PRED} \)
end

event stopped \( \triangleq \) extends stopped
when
grd: \( \text{stopped} = \text{TRUE} \)
then
skip
end

event object.leave \( \triangleq \) extends object.leave
when
grd_blocked: \( \text{stopped} = \text{FALSE} \)
grd1: \( \text{door1.motor.cond} = \text{TRUE} \land 
\quad ((\text{door1.pos.cond} = \text{FALSE} \land \text{door1.opened.cond} = \text{TRUE} \land 
\text{door1.closed.cond} = \text{TRUE}) \lor 
\quad (\text{door1.pos.cond} = \text{TRUE} \land 
\text{door1.opened.cond} = \text{FALSE} \lor 
\text{door1.closed.cond} = \text{FALSE})) \)
grd2: \( \text{door2.cond} = \text{DEGRADED} \)
grd3: \( \text{obj.presence} = \text{TRUE} \)
grd_phase: \( \text{phase} = \text{CONT} \)
then
act1: \( \text{obj.presence} := \text{FALSE} \)
act_blocked: \( \text{stopped} := \text{TRUE} \)
act_phase: \( \text{phase} := \text{PRED} \)
end

event break2 \( \triangleq \) extends break2
when
grd4.0: \( \text{door2.cond} \neq \text{BROKEN} \)
grd4.1: \( \neg (\text{door1.motor.cond} = \text{FALSE} \lor (\text{door1.pos.cond} = \text{FALSE} \land 
\text{door1.opened.cond} = \text{FALSE} \lor 
\text{door1.closed.cond} = \text{FALSE})) \)
grd4.2: \( \text{door1.motor.cond} = \text{TRUE} \land 
\quad ((\text{door1.pos.cond} = \text{FALSE} \land \text{door1.opened.cond} = \text{TRUE} \land 
\text{door1.closed.cond} = \text{TRUE}) \lor 
\quad (\text{door1.pos.cond} = \text{TRUE} \land 
\text{door1.opened.cond} = \text{FALSE} \lor 
\text{door1.closed.cond} = \text{FALSE})) \land 
\text{door2.cond} = \text{DEGRADED} \Rightarrow \text{obj.presence} = \text{TRUE} \)
grd5: \( \text{stopped} = \text{FALSE} \)
grd_phase: \( \text{phase} = \text{CONT} \)
then
  act\_4\_1: \textit{door\_2\_cond} := \textit{BROKEN}
  act\_4\_4: \textit{stopped} := \textit{TRUE}
  act\_phase: \textit{phase} := \textit{PRED}
end

end
Appendix B: AOCS Case Study Model

This appendix contains full Event-B models for the AOCS case study used in Chapter 5 for method evaluation. The development produced a total of 381 proof obligations, 359 of which were proven automatically, and 22 required interactive proof. A Rodin project containing the proved models can be found in the Examples section at [WIFT].

B.1 Context C0

context c0
constants
UNIT OFF UNIT_ON GPS_COARSE GPS_FINE
axioms
axm1: UNIT_OFF = 0
axm2: UNIT_ON = 1
axm3: GPS_COARSE = 1
axm4: GPS_FINE = 2
end

B.2 Machine M0

machine m0 sees c0
variables
unitES unitGPS unitPLI
invariants
inv1: unitES ∈ {UNIT_ON,UNIT_OFF}
inv2: unitGPS ∈ {UNIT_OFF,GPS_COARSE,GPS_FINE}
inv3: unitPLI ∈ {UNIT_OFF,UNIT_ON}
events
event INITIALISATION
then
  act0: unitES := UNIT_OFF
  act1: unitGPS := UNIT_OFF
  act2: unitPLI := UNIT_OFF
end

event ES_work ≡
  when
    grd1: unitES = UNIT_ON
  then
    skip
  end

event ES_switch ≡
  any
    newState
  where
    grd1: newState ∈ \{UNIT_ON, UNIT_OFF\}
    grd2: newState ≠ unitES
  then
    act1: unitES := newState
  end

event GPS_work ≡
  when
    grd1: unitGPS ∈ \{GPS_COARSE, GPS_FINE\}
  then
    skip
  end

event GPS_switch ≡
  any
    newState
  where
    grd1: newState ∈ \{UNIT_OFF, GPS_COARSE, GPS_FINE\}
    grd2: newState ≠ unitGPS
    grd3: unitGPS = UNIT_OFF ⇒ newState = GPS_COARSE
  then
    act1: unitGPS := newState
  end

event PLI_work ≡
  when
    grd1: unitPLI = UNIT_ON
  then
event PLI_switch ≡
  any
  newState
where
  grd1: newState ∈ \{UNIT_ON, UNIT_OFF\}
  grd2: newState ≠ unitPLI
then
  act1: unitPLI := newState
end
end

B.3 Machine M1

machine m1 refines m0 sees c0
variables
  unitES unitGPS unitPLI stopped
invariants
  inv_stopped: stopped ∈ BOOL
events
event INITIALISATION
extended
then
  act0: unitES := UNIT_OFF
  act1: unitGPS := UNIT_OFF
  act2: unitPLI := UNIT_OFF
  act_stopped: stopped := FALSE
end

event ES_work ≡ extends ES_work
when
  grd1: unitES = UNIT_ON
  grd_stopped: stopped = FALSE
then
  skip
end

event ES_switch ≡ extends ES_switch
any

\textit{newState}

\textbf{where}

\textit{grd}1: \textit{newState} \in \{UNIT\_ON, UNIT\_OFF\}
\textit{grd}2: \textit{newState} \neq \textit{unitES}
\textit{grd\_stopped}: \textit{stopped} = FALSE

\textbf{then}

\textit{act}1: \textit{unitES} := \textit{newState}

\textbf{end}

\textbf{event} GPS\_work \triangleq \textit{extends} GPS\_work

\textbf{when}

\textit{grd}1: \textit{unitGPS} \in \{GPS\_COARSE, GPS\_FINE\}
\textit{grd\_stopped}: \textit{stopped} = FALSE

\textbf{then}

\textit{skip}

\textbf{end}

\textbf{event} GPS\_switch \triangleq \textit{extends} GPS\_switch

\textbf{any}

\textit{newState}

\textbf{where}

\textit{grd}1: \textit{newState} \in \{UNIT\_OFF, GPS\_COARSE, GPS\_FINE\}
\textit{grd}2: \textit{newState} \neq \textit{unitGPS}
\textit{grd}3: \textit{unitGPS} = UNIT\_OFF \Rightarrow \textit{newState} = GPS\_COARSE
\textit{grd\_stopped}: \textit{stopped} = FALSE

\textbf{then}

\textit{act}1: \textit{unitGPS} := \textit{newState}

\textbf{end}

\textbf{event} PLI\_work \triangleq \textit{extends} PLI\_work

\textbf{when}

\textit{grd}1: \textit{unitPLI} = UNIT\_ON
\textit{grd\_stopped}: \textit{stopped} = FALSE

\textbf{then}

\textit{skip}

\textbf{end}

\textbf{event} PLI\_switch \triangleq \textit{extends} PLI\_switch

\textbf{any}

\textit{newState}

\textbf{where}

\textit{grd}1: \textit{newState} \in \{UNIT\_ON, UNIT\_OFF\}
\textit{grd}2: \textit{newState} \neq \textit{unitPLI}
grd\_stopped: \( stopped = FALSE \)
then
act1: unitPLI := newState
end

event stop \( \approx \)
when
grd\_stopped: \( stopped = FALSE \)
then
act\_stopped: \( stopped := TRUE \)
end

event stopped \( \approx \)
when
grd\_stopped: \( stopped = TRUE \)
then
skip
end

end

**B.4 Context C2**

context c2 extends c0
constants
\( OFF \quad NOMINAL \quad SCIENCE \quad MODE \)
axioms
axm1: \( OFF = 0 \)
axm2: \( NOMINAL = 1 \)
axm3: \( SCIENCE = 2 \)
axm4: \( MODE = \{0, 1, 2\} \)
end

**B.5 Machine M2**
machine m2 refines m1 sees c1
variables
\( unitES \quad unitGPS \quad unitPLI \quad stable \quad mode \quad stopped \)
invariants
inv1: stable ∈ BOOL
inv2: mode ∈ MODE

events

event INITIALISATION
extended
then
act0: unitES := UNIT_OFF
act1: unitGPS := UNIT_OFF
act2: unitPLI := UNIT_OFF
act_stopped: stopped := FALSE
act1_0: stable := TRUE
act1_1: mode := OFF
end

event goAdvance ≜
when
grd_stopped: stopped = FALSE
grd1: stable = TRUE
grd2: mode < SCIENCE
then
act1: mode := mode + 1
act2: stable := FALSE
end

event downgrade ≜
any
newMode

where
grd_stopped: stopped = FALSE
grd_par: newMode ∈ MODE
grd1: mode > OFF
grd2: newMode < mode
then
act1: mode := newMode
act2: stable := FALSE
end

event ES_work ≜ extends ES_work
when
grd1: unitES = UNIT_ON
grd_stopped: stopped = FALSE
grd_mode: mode = NOMINAL
then
  skip
end

event ES_switch_on ≜ refines ES_switch
  when
    grd_stopped: stopped = FALSE
    grd1: unitES = UNIT_OFF
    grd2: mode = NOMINAL
  with
    newState : newState = UNIT_ON
  then
    act1: unitES := UNIT_ON
  end

event ES_switch_off ≜ refines ES_switch
  when
    grd_stopped: stopped = FALSE
    grd1: unitES = UNIT_ON
    grd2: mode ∈ \{OFF, SCIENCE\}
  with
    newState : newState = UNIT_OFF
  then
    act1: unitES := UNIT_OFF
  end

event GPS_work ≜ extends GPS_work
  when
    grd1: unitGPS ∈ \{GPS_COARSE, GPS_FINE\}
    grd_stopped: stopped = FALSE
    grd1_2: mode ∈ \{NOMINAL, SCIENCE\}
  then
    skip
  end

event GPS_switch_on ≜ refines GPS_switch
  when
    grd_stopped: stopped = FALSE
    grd1: unitGPS < GPS_FINE
    grd2: mode ∈ \{NOMINAL, SCIENCE\}
  with
    newState : newState = unitGPS + 1
then
  act1: unitGPS := unitGPS + 1
end

event GPS_switch_off ≜ refines GPS_switch
  any
  newState
where
  grd_stopped: stopped = FALSE
  grd1: unitGPS > UNIT_OFF
  grd2: newState ∈ {UNIT_OFF, GPS_COARSE}
  grd3: newState < unitGPS
  grd4: mode ∈ {NOMINAL, OFF}
then
  act1: unitGPS := newState
end

event PLI_work ≜ extends PLI_work
  when
    grd1: unitPLI = UNIT_ON
    grd_stopped: stopped = FALSE
    grd_mode: mode = SCIENCE
then
  skip
end

event PLI_switch_on ≜ refines PLI_switch
  when
    grd_stopped: stopped = FALSE
    grd1: unitPLI = UNIT_OFF
    grd2: mode = SCIENCE
with
  newState : newState = UNIT_ON
then
  act1: unitPLI := UNIT_ON
end

event PLI_switch_off ≜ refines PLI_switch
  when
    grd_stopped: stopped = FALSE
    grd1: unitPLI = UNIT_ON
    grd2: mode ∈ {NOMINAL, OFF}
with

newState :\; newState = \text{UNIT\_OFF}

then
act1: unitPLI := UNIT\_OFF
end

event standby ≜
when
grd\_mode: mode = OFF
grd\_stopped: stopped = FALSE
then
skip
end

event reconf\_finish ≜
when
grd\_stopped: stopped = FALSE
grdi: stable = FALSE
then
act1: stable := TRUE
end

event stop ≜ \text{extends} \; \text{stop}
when
grd\_stopped: stopped = FALSE
then
act\_stopped: stopped := TRUE
end

event stopped ≜ \text{extends} \; \text{stopped}
when
grd\_stopped: stopped = TRUE
then
skip
end
end

B.6 Machine M3

machine m3 refines m2 sees c2
variables
unitES unitGPS unitPLI mode stable stopped

invariants

inv\_modes1: mode = OFF ∧ stable = TRUE ⇒
\hspace{1cm} unitES = UNIT\_OFF ∧ unitGPS = UNIT\_OFF ∧ unitPLI = UNIT\_OFF

inv\_modes2: mode = NOMINAL ∧ stable = TRUE ⇒
\hspace{1cm} unitES = UNIT\_ON ∧ unitGPS = GPS\_COARSE ∧ unitPLI = UNIT\_OFF

inv\_modes3: mode = SCIENCE ∧ stable = TRUE ⇒
\hspace{1cm} unitES = UNIT\_OFF ∧ unitGPS = GPS\_FINE ∧ unitPLI = UNIT\_ON

inv\_modes4: unitES = UNIT\_ON ∨ unitGPS ∈ \{GPS\_FINE, GPS\_COARSE\} ∨
\hspace{1cm} unitPLI = UNIT\_ON ⇒ ¬(stable = TRUE ∧ mode = OFF)

events

event INITIALISATION

extended

then

act0: unitES := UNIT\_OFF
act1: unitGPS := UNIT\_OFF
act2: unitPLI := UNIT\_OFF
act\_stopped: stopped := FALSE
act1\_0: stable := TRUE
act1\_1: mode := OFF

end

event goAdvance ≡ extends goAdvance

when

grd\_stopped: stopped = FALSE
grd1: stable = TRUE
grd2: mode < SCIENCE

then

act1: mode := mode + 1
act2: stable := FALSE

end

event downgrade ≡ extends downgrade

any

newMode

where

grd\_stopped: stopped = FALSE
grd\_par: newMode ∈ MODE
grd1: mode > OFF
grd2: newMode < mode

then

act1: mode := newMode
act2: stable := FALSE
end

event ES_work \hat{=} extends ES_work
  when
    grd1: unitES = UNIT_ON
    grd_stopped: stopped = FALSE
    grd_mode: mode = NOMINAL
  then
    skip
end

event standby \hat{=} extends standby
  when
    grd_mode: mode = OFF
    grd_stopped: stopped = FALSE
  then
    skip
end

event ES_switch_on \hat{=} extends ES_switch_on
  when
    grd_stopped: stopped = FALSE
    grd1: unitES = UNIT_OFF
    grd2: mode = NOMINAL
    grd2_0: stable = FALSE
  then
    act1: unitES := UNIT_ON
end

event ES_switch_off \hat{=} extends ES_switch_off
  when
    grd_stopped: stopped = FALSE
    grd1: unitES = UNIT_ON
    grd2: mode \in \{OFF, SCIENCE\}
    grd2_0: stable = FALSE
  then
    act1: unitES := UNIT_OFF
end

event GPS_work \hat{=} extends GPS_work
  when
    grd1: unitGPS \in \{GPS_COARSE, GPS_FINE\}
    grd_stopped: stopped = FALSE
    grd1_2: mode \in \{NOMINAL, SCIENCE\}
  then
skip
end

event GPS\_switch\_on \overset{\text{extends GPS\_switch\_on}}{=} 
\text{when}
\begin{align*}
grd\_\text{stopped}: & \hspace{1em} \text{stopped} = \text{FALSE} \\
grd1: & \hspace{1em} \text{unitGPS} < \text{GPS\_FINE} \\
grd2: & \hspace{1em} \text{mode} \in \{\text{NOMINAL, SCIENCE}\} \\
grd2\_0: & \hspace{1em} \text{stable} = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
act1: & \hspace{1em} \text{unitGPS} := \text{unitGPS} + 1
\end{align*}
end

event GPS\_switch\_off \overset{\text{extends GPS\_switch\_off}}{=} 
\text{any}
\begin{align*}
\text{newState}
\text{where}
\begin{align*}
grd\_\text{stopped}: & \hspace{1em} \text{stopped} = \text{FALSE} \\
grd1: & \hspace{1em} \text{unitGPS} > \text{UNIT\_OFF} \\
grd2: & \hspace{1em} \text{newState} \in \{\text{UNIT\_OFF, GPS\_COARSE}\} \\
grd3: & \hspace{1em} \text{newState} < \text{unitGPS} \\
grd4: & \hspace{1em} \text{mode} \in \{\text{NOMINAL, OFF}\} \\
grd2\_0: & \hspace{1em} \text{stable} = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
act1: & \hspace{1em} \text{unitGPS} := \text{newState}
\end{align*}
end

event PLI\_work \overset{\text{extends PLI\_work}}{=} 
\text{when}
\begin{align*}
grd1: & \hspace{1em} \text{unitPLI} = \text{UNIT\_ON} \\
grd\_\text{stopped}: & \hspace{1em} \text{stopped} = \text{FALSE} \\
grd\_\text{mode}: & \hspace{1em} \text{mode} = \text{SCIENCE}
\end{align*}
\text{then}
\begin{align*}
\text{skip}
\end{align*}
end

event PLI\_switch\_on \overset{\text{extends PLI\_switch\_on}}{=} 
\text{when}
\begin{align*}
grd\_\text{stopped}: & \hspace{1em} \text{stopped} = \text{FALSE} \\
grd1: & \hspace{1em} \text{unitPLI} = \text{UNIT\_OFF} \\
grd2: & \hspace{1em} \text{mode} = \text{SCIENCE} \\
grd2\_0: & \hspace{1em} \text{stable} = \text{FALSE}
\end{align*}
\text{then}
\begin{align*}
act1: & \hspace{1em} \text{unitPLI} := \text{UNIT\_ON}
\end{align*}
end

event PLI_switch_off := extends PLI_switch_off
when
    grd_stopped: stopped = FALSE
    grd1: unitPLI = UNIT_ON
    grd2: mode ∈ {NOMINAL, OFF}
    grd2.0: stable = FALSE
then
    act1: unitPLI := UNIT_OFF
end

event reconf_finish := extends reconf_finish
when
    grd_stopped: stopped = FALSE
    grd1: stable = FALSE
    grd_mode_off: mode = OFF ⇒
        unitES = UNIT_OFF ∧ unitGPS = UNIT_OFF ∧ unitPLI = UNIT_OFF
    grd_mode_nominal: mode = NOMINAL ⇒
        unitES = UNIT_ON ∧ unitGPS = GPS_COARSE ∧ unitPLI = UNIT_OFF
    grd_mode_science: mode = SCIENCE ⇒
        unitES = UNIT_OFF ∧ unitGPS = GPS_FINE ∧ unitPLI = UNIT_ON
then
    act1: stable := TRUE
end

event stop := extends stop
when
    grd_stopped: stopped = FALSE
    grd_units: unitES = UNIT_ON ∨ unitGPS ∈ {GPS_FINE, GPS_COARSE} ∨
        unitPLI = UNIT_ON
then
    act_stopped: stopped := TRUE
end

event stopped := extends stopped
when
    grd_stopped: stopped = TRUE
then
    skip
end
end
B.7 Machine M4

machine m4 refines m3 sees c2

variables

unitES unitGPS unitPLI mode stable unitES_cond unitGPS_cond unitPLI_cond stopped

invariants

inv_ES_cond: unitES_cond ∈ {0, 1, 2}
inv_GPS_cond: unitGPS_cond ∈ {0, 1, 2}
inv_PLI_cond: unitPLI_cond ∈ {0, 1, 2}
inv_unitPLI_cond_mode: unitPLI_cond = 0 ∧ unitES_cond > 0 ∧ unitGPS_cond > 0 ⇒ mode ∈ {OFF, NOMINAL}
inv_units2: mode = NOMINAL ∧ stopped = FALSE ⇒ unitES_cond > 0 ∧ unitGPS_cond > 0
inv_units3: mode = SCIENCE ∧ stopped = FALSE ⇒ unitES_cond > 0 ∧ unitGPS_cond > 0 ∧ unitPLI_cond > 0
inv_glue: stopped = TRUE ⇔ unitES_cond = 0 ∨ unitGPS_cond = 0
inv_glue1: stopped = FALSE ⇔ unitES_cond > 0 ∧ unitGPS_cond > 0

events

event INITIALISATION

extended

then

act0: unitES := UNIT_OFF
act1: unitGPS := UNIT_OFF
act2: unitPLI := UNIT_OFF
act_stopped: stopped := FALSE
act1_0: stable := TRUE
act1_1: mode := OFF
act3_0: unitES_cond := 2
act3_1: unitGPS_cond := 2
act3_2: unitPLI_cond := 2

end

event goAdvance ≜ extends goAdvance

when

grd_stopped: stopped = FALSE
grd1: stable = TRUE
grd2: mode < SCIENCE
inv_nominal: mode + 1 = NOMINAL ⇒ unitES_cond > 0 ∧ unitGPS_cond > 0
inv_science: mode + 1 = SCIENCE ⇒ unitES_cond > 0 ∧ unitGPS_cond > 0 ∧ unitPLI_cond > 0

then
act1: \( \text{mode} := \text{mode} + 1 \)
act2: \( \text{stable} := \text{FALSE} \)

end

event ES\_break \models \text{refines} \text{stop}
when
  grd1: \( \text{unitES\_cond} = 1 \)
  grd2: \( \text{unitES} = \text{UNIT\_ON} \)
  grd3: \( \text{unitGPS\_cond} > 0 \)
then
  act1: \( \text{unitES\_cond} := 0 \)
  act2: \( \text{stopped} := \text{TRUE} \)
end

event GPS\_break \models \text{refines} \text{stop}
when
  grd1: \( \text{unitGPS\_cond} = 1 \)
  grd2: \( \text{unitGPS} \in \{\text{GPS\_COARSE}, \text{GPS\_FINE}\} \)
  grd3: \( \text{unitES\_cond} > 0 \)
then
  act1: \( \text{unitGPS\_cond} := 0 \)
  act2: \( \text{stopped} := \text{TRUE} \)
end

event ES\_work \models \text{extends} ES\_work
when
  grd1: \( \text{unitES} = \text{UNIT\_ON} \)
  grd\_stopped: \( \text{stopped} = \text{FALSE} \)
  grd\_mode: \( \text{mode} = \text{NOMINAL} \)
  grd3\_0: \( \text{unitES\_cond} > 0 \)
then
  skip
end

event standby \models \text{extends} standby
when
  grd\_mode: \( \text{mode} = \text{OFF} \)
  grd\_stopped: \( \text{stopped} = \text{FALSE} \)
  grd1: \( \text{unitES\_cond} > 0 \land \text{unitGPS\_cond} > 0 \)
then
  skip
end

event ES\_switch\_on \models \text{extends} ES\_switch\_on
when
grd\_stopped: stopped = FALSE
grd1: unitES = UNIT\_OFF
grd2: mode = NOMINAL
grd2\_0: stable = FALSE
grd3\_0: unitES\_cond > 0
then
act1: unitES := UNIT\_ON
end

event ES\_switch\_off \equiv extends ES\_switch\_off when
grd\_stopped: stopped = FALSE
grd1: unitES = UNIT\_ON
grd2: mode \in \{OFF, SCIENCE\}
grd2\_0: stable = FALSE
then
act1: unitES := UNIT\_OFF
end

event GPS\_work \equiv extends GPS\_work when
grd1: unitGPS \in \{GPS\_COARSE, GPS\_FINE\}
grd\_stopped: stopped = FALSE
grd1\_2: mode \in \{NOMINAL, SCIENCE\}
grd3\_0: unitGPS\_cond > 0
then
skip
end

event GPS\_switch\_on \equiv extends GPS\_switch\_on when
grd\_stopped: stopped = FALSE
grd1: unitGPS < GPS\_FINE
grd2: mode \in \{NOMINAL, SCIENCE\}
grd2\_0: stable = FALSE
grd3\_0: unitGPS\_cond > 0
then
act1: unitGPS := unitGPS + 1
end

event GPS\_switch\_off \equiv extends GPS\_switch\_off any
newState
where
grd\_stopped: \( stopped = FALSE \)
grd1: \( unitGPS > UNIT\_OFF \)
grd2: \( newState \in \{UNIT\_OFF, GPS\_COARSE\} \)
grd3: \( newState < unitGPS \)
grd4: \( mode \in \{NOMINAL, OFF\} \)
grd2\_0: \( stable = FALSE \)

then

act1: \( unitGPS := newState \)

end

event PLI\_work \xrightleftharpoons{} extends PLI\_work

when

grd1: \( unitPLI = UNIT\_ON \)
grd\_stopped: \( stopped = FALSE \)
grd\_mode: \( mode = SCIENCE \)
grd3\_0: \( unitPLI\_cond > 0 \)

then

skip

end

event PLI\_switch\_on \xrightleftharpoons{} extends PLI\_switch\_on

when

grd\_stopped: \( stopped = FALSE \)
grd1: \( unitPLI = UNIT\_OFF \)
grd\_mode: \( mode = SCIENCE \)
grd2\_0: \( stable = FALSE \)
grd3\_0: \( unitPLI\_cond > 0 \)

then

act1: \( unitPLI := UNIT\_ON \)

end

event PLI\_switch\_off \xrightleftharpoons{} extends PLI\_switch\_off

when

grd\_stopped: \( stopped = FALSE \)
grd1: \( unitPLI = UNIT\_ON \)
grd2: \( mode \in \{NOMINAL, OFF\} \)
grd2\_0: \( stable = FALSE \)

then

act1: \( unitPLI := UNIT\_OFF \)

end

event reconf\_finish \xrightleftharpoons{} extends reconf\_finish

when

grd\_stopped: \( stopped = FALSE \)
grd1: \( stable = FALSE \)
grd\_mode\_off: \ mode = OFF \Rightarrow
\begin{align*}
\text{unitES} &= \text{UNIT\_OFF} \land \text{unitGPS} = \text{UNIT\_OFF} \land \text{unitPLI} = \text{UNIT\_OFF}
\end{align*}

grd\_mode\_nominal: \ mode = NOMINAL \Rightarrow
\begin{align*}
\text{unitES} &= \text{UNIT\_ON} \land \text{unitGPS} = \text{GPS\_COARSE} \land \text{unitPLI} = \text{UNIT\_OFF}
\end{align*}

grd\_mode\_science: \ mode = SCIENCE \Rightarrow
\begin{align*}
\text{unitES} &= \text{UNIT\_OFF} \land \text{unitGPS} = \text{GPS\_FINE} \land \text{unitPLI} = \text{UNIT\_ON}
\end{align*}

\text{then}
\begin{align*}
\text{act1: stable} &:= \text{TRUE}
\end{align*}
\text{end}

\text{event stopped} \Rightarrow \text{extends stopped}
\begin{align*}
\text{when}
\begin{align*}
grd\_stopped: \ stopped &= \text{TRUE}
\end{align*}
\text{then}
\begin{align*}
\text{skip}
\end{align*}
\text{end}
\end{align*}

\text{event ES\_downgrade} \Rightarrow \text{refines downgrade}
\begin{align*}
\text{when}
\begin{align*}
grd1: \ mode > OFF
\end{align*}
\begin{align*}
grd2: \ unitES = \text{UNIT\_ON}
\end{align*}
\begin{align*}
grd3: \ unitES\_cond = 2
\end{align*}
\begin{align*}
grd4: \ unitGPS\_cond > 0
\end{align*}
\text{with}
\begin{align*}
\text{newMode} : \ newMode = \text{OFF}
\end{align*}
\text{then}
\begin{align*}
\text{act1: mode} &:= \text{OFF}
\end{align*}
\begin{align*}
\text{act2: stable} &:= \text{FALSE}
\end{align*}
\begin{align*}
\text{act3: unitES\_cond} &:= 1
\end{align*}
\text{end}
\end{align*}

\text{event GPS\_downgrade} \Rightarrow \text{refines downgrade}
\begin{align*}
\text{when}
\begin{align*}
grd1: \ mode > OFF
\end{align*}
\begin{align*}
grd2: \ unitGPS > \text{UNIT\_OFF}
\end{align*}
\begin{align*}
grd3: \ unitGPS\_cond = 2
\end{align*}
\begin{align*}
grd4: \ unitES\_cond > 0
\end{align*}
\text{with}
\begin{align*}
\text{newMode} : \ newMode = \text{OFF}
\end{align*}
\text{then}
\begin{align*}
\text{act1: mode} &:= \text{OFF}
\end{align*}
\begin{align*}
\text{act2: stable} &:= \text{FALSE}
\end{align*}
act3: $unitGPS\_cond := 1$
end

event PLI\_downgrade \supseteq \text{refines} \ downgrade
  when
  grd1: mode = SCIENCE
  grd2: $unitPLI = UNIT\_ON$
  grd3: $unitPLI\_cond = 2$
  grd4: $unitES\_cond > 0 \land unitGPS\_cond > 0$
  with
  newMode : newMode = NOMINAL
then
  act1: mode := NOMINAL
  act2: stable := FALSE
  act3: $unitPLI\_cond := 1$
end

event PLI\_break \supseteq \text{refines} \ downgrade
  when
  grd1: mode = SCIENCE
  grd2: $unitPLI = UNIT\_ON$
  grd3: $unitPLI\_cond = 1$
  grd4: $unitES\_cond > 0 \land unitGPS\_cond > 0$
  with
  newMode : newMode = NOMINAL
then
  act1: mode := NOMINAL
  act2: stable := FALSE
  act3: $unitPLI\_cond := 0$
end
end