Farm Efficiency in Bangladesh

by

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(B.S.S. Honours in Economics, M.S.S. in Economics)

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Declaration: No portion of this work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

Abstract

This thesis examines farm-level efficiency of rice farmers in the High Barind region of Bangladesh by estimating technical, allocative and economic efficiency using farm level cross section survey data. Two contrasting methods for measuring efficiency are applied: the stochastic econometric frontier and Data Envelopment Analysis (DEA). These measures are used to investigate the factors associated with technical, allocative and economic inefficiency. First, technical efficiency is computed by estimating the translog stochastic frontier in which technical inefficiency effects are modelled as a function of socioeconomic, infrastructure and environmental degradation factors in a single stage estimation technique using maximum likelihood method. Technical and scale efficiency are calculated by solving output- and input-oriented constant returns to scale (CRS) and variable returns to scale (VRS) DEA frontiers. A Tobit model is used to evaluate factors associated with technical and scale inefficiency from both input-oriented and output-oriented CRS and VRS frontiers. Same factors are analyzed as in the translog stochastic frontier.

The translog stochastic frontier results show that farm households are, on average, 79 per cent technically efficient. The output-oriented DEA frontier results show that the average technical efficiency estimates are 79 and 86 per cent under CRS and VRS assumptions and the average scale efficiency is 92 per cent. The average values for technical efficiency measures and scale efficiency from the input-oriented CRS and VRS frontiers are 79, 85 and 93 per cent respectively. The translog stochastic frontier exhibits decreasing returns to scale, whereas the DEA frontier exhibits decreasing, constant and increasing returns to scale. The technical inefficiency effects model in the translog stochastic frontier and Tobit analysis for DEA frontier show that irrigation infrastructure and environmental degradation are significant factors in determining technical inefficiency.

We then measure technical, allocative and economic efficiency by estimating the Cobb-Douglas stochastic frontier following the Kopp and Diewert cost decomposition technique

and by running input-oriented CRS and VRS DEA frontier models. We estimate the Tobit model to analyze the factors associated with technical, allocative and economic inefficiency from the DEA frontiers. In addition, we compare the results obtained from both the Cobb-Douglas stochastic frontier and DEA frontiers.

The results from the Cobb-Douglas stochastic frontier shows that the average technical, allocative and economic efficiency of farm households are 80, 77, and 61 per cent respectively. The input-oriented CRS frontier results show that farm households have, on average, 86, 91 and 78 per cent technical, allocative and economic efficiency and the corresponding VRS frontier shows that farm households are, on average, 91, 87 and 79 per cent technically, allocatively and economically efficient. An evaluation of factors associated with technical, allocative and economic inefficiency from both the Cobb-Douglas stochastic frontier and DEA frontier reveals that irrigation infrastructure and environmental degradation are the most statistically significant factors affecting technical, allocative and economic inefficiency. This implies that diesel-operated pumps and environmental degradation are not only reducing output from given inputs but are also causing sub-optimal cost-minimizing input decisions.

Assessing efficiency suggests that there is a considerable amount of inefficiency among farm households and there is room for enhancing rice production through the improvement of technical, allocative and economic efficiency without resort to technical improvements. Farm households could reduce their variable production costs, on average, between 21 - 31 per cent if they could utilize their inputs in a technically and allocatively efficient manner. An evaluation of factors associated with inefficiency concludes that government electrification programmes which convert diesel pumps into electricity-operated pumps for irrigation in rural areas and policies which lead to reduced environmental degradation would reduce inefficiency, thereby increasing rice production and the welfare of farm households.

This PhD research is dedicated

To My Parents

who permitted me to come to Newcastle

and

complete the PhD.

Biographical Note

Md Abdul Wadud graduated in 1992 with a degree in Economics (Honours) from Rajshahi University, Bangladesh. He received his Master of Social Science (M.S.S.) result in Economics from the same University on 23 September 1994. On 5 October 1994, he was appointed as a lecturer in the Department of Economics, Rajshahi University, Bangladesh. In 1997, he received Technical Cooperation Training (TCT) award of The British Council to pursue the PhD studies at Newcastle upon Tyne. The author also received Research Studentship of the University of Newcastle upn Tyne to substitute the TCT award in 1999.

In 1992 the author was honoured with University Grant Commission award, Hamida Haque Award, Rajshahi University award and Agrani Bank Gold Medal award for the best B.S.S student in the Social Science Faculty. The author was also honoured with the Rajshahi University award for the best M.S.S degree in Economics.

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Farm Efficiency in Bangladesh

Contents

Chapter 1

Introduction to Farm Efficiency in Bangladesh

1.1. Introduction

Agriculture in Bangladesh accounts for about 59.56 per cent of the total land area, employs about 66 per cent of the labour force and provides the main sources of income for 80 per cent of the population. The average growth rate of agricultural sector from 1990 to 1996 is 1.34 per cent and that of crop sector is only about 0.18 per cent (Bangladesh Economic Review, 1997). The share of agriculture in GDP has fallen from 57 per cent in the 1970s to 35 per cent in the 1990s. The rice crop accounts for 74 per cent of the cultivated area, 83 per cent of irrigated area, 88 per cent of fertilizer consumption and 68 per cent of caloric intake. This is set against a population of 114.4 million with a growth rate of 2.17 per cent in 1991. The average overall food deficit over recent years is about 1.5 million metric tonnes of rice per annum.

Farm households in Bangladesh are generally large with a low level of literacy. Production is hampered by land fragmentation, environmental degradation, in particular land degradation, weak irrigation infrastructure, ineffective and bureaucratic extension services, restricted access to credit, poor transport systems, and storage facilities. This weak resource base in combination with a growing population are aggravating the problems of the agricultural sector in Bangladesh.

In the 1960s Bangladesh agriculture started to adopt the prescriptions of the Green Revolution. There has been a widespread adoption of new varieties and modern inputs. However, the late 1990s sees Bangladesh facing low and declining levels of rural income and a set of environmental problems which are, in part, due to modern agricultural inputs and practices. The long turn aim in Bangladesh should be to make the system more resilient to environmental shocks. There is therefore an urgent need to reduce soil erosion and land degradation and increase the efficiency of resource use.

The policy makers might consider two issues: first how to enhance agricultural productivity and second how to encourage farms to adopt new technology. Many studies have been conducted on the slow rate of technical change, but most ignore efficiency aspects of farm households. This thesis is concerned with the efficient utilization of the resources allocated to agricultural production.

Output gains stemming from productivity improvement through improvements in efficiency are important to Bangladesh considering that the scope to enhance farm production by bringing land into cultivation has reduced to an insignificant level. Measuring efficiency and productivity is important in Bangladesh for several reasons. First of all, the performance of farm households is evaluated by efficiency and productivity which are performance measures and success indicators. Second, the determinants of inefficiency or productivity differentials can be hypothesized by estimating efficiency and productivity, and isolating their effects from the effects of the environment in which production occurs. Identifying sources of inefficiency plays an important role in designing policies to improve the performance of farm households.

1.2. Aim and Objectives of this Study

The aim of this thesis is to investigate the determinants of efficiency of individual farm households in the High Barind Bangladesh. The objectives include, first, to assess different methods of efficiency measurement, second, to determine if inefficiency is related to aspects of the production environment and third, to assess the implications of variability in efficiency to policy makers.

1.3. Plan of the Thesis

This thesis proceeds from an overview of the study area to identify and measure inefficiencies for farm households. The chapters are organized as follows: Chapter 2 provides an overview of the socioeconomic and environmental condition of the High Barind Bangladesh. It starts with describing location, physiography, geology and topography. The next section discusses the utilization of inputs, irrigation and water resources, cropping pattern, farming and livestock resources. The final section discusses environmental degradation. Chapter 3 gives survey methodology and results. It begins with explaining the methodology of the survey and it then describes the results.

Chapter 4 sets out some basic theoretical concepts from production theory. Section 2 describes the production function with some related concepts. We then derive the conditions for the least-cost input combination which is associated with allocative efficiency. We then turn to explain the concepts of technical, allocative and economic efficiency and describe cost decomposition technique to calculate technical, allocative and economic efficiency.

Chapter 5 presents the stochastic frontier approach to measuring efficiency. We first describe the stochastic frontier model of efficiency measurement. We then discuss alternative forms of production function and hypothesis testing strategy. We finally discuss the parametric approach to decompose efficiency into technical, allocative and economic efficiency using the self-dual Cobb-Douglas stochastic frontier model.

Chapter 6 gives results from the stochastic frontier production model. After discussing summary statistics of variables and factors affecting efficiency, the results from the translog stochastic frontier model, technical efficiency estimates and technical inefficiency effects are considered. We then discuss the estimates of technical, allocative and economic efficiency computed from the self-dual Cobb-Douglas stochastic frontier model and quantify the effects of factors associated with inefficiency. The final section compares the results from the translog and Cobb-Douglas models.

Chapter 7 introduces Data Envelopment Analysis. First, the construction of input-oriented and output-oriented constant returns to scale (CRS) and variable returns to scale (VRS) DEA model is described from its simple to multi-stage framework. We then discuss an input-oriented CRS and VRS DEA model to calculate technical, allocative and economic efficiency simultaneously. The final section explains a Tobit model to quantify the sources of farm-specific factors affecting inefficiency.

Chapter 8 reports the DEA frontier results. First, the results of technical efficiency estimates and estimates of the effects of farm-specific characteristics on technical inefficiency derived from input-oriented and output-oriented CRS and VRS models are described. We then discuss the results of estimates of technical, allocative and economic efficiency and discuss the effects of factors associated with inefficiency.

Chapter 9 presents the summary and main results, draws some conclusions and policy implications. It also identifies some issues where further research is needed.

Chapter 2

Socioeconomic and Environmental Conditions in High Barind

2.1. Introduction

The High Barind consists of alluvial deposits from the flood waters of the tributaries feeding the Ganges (Padma) to the south and the Brahamaputra (Jamuna) to the east. It rises gradually in a sequence of terraced steps to the Indian border extending into the West Bengal.

The crop year in the High Barind includes three distinct seasons: Season I from March to May, Season II from May to September and Season III from October to February; transplanted aman (T.Aman) dominates the cropping pattern in Season II; aman rice accounts for the largest area followed by Boro rice. Boro rice grows in Season III if irrigation is available. Aus and broadcast aus (B.Aus) grow in Season I. Other crops like, potato, wheat, sugarcane, cabbage are also grown. Land utilization patterns indicate that 90 per cent of the total land area is already under cultivation with a small proportion of forest and fallow area. Increasing the frequency of cultivation and increasing yields are the only ways to increase productivity. Land is highly fragmented with an average plot size of one-third of an acre which restricts the use of modern equipments, especially tractors and irrigation equipments.

The demand for irrigation from subsistence farmers is increasing partly in response to rapid population growth. The population of Bangladesh is 114.4 million with a growth rate of 2.17 per cent. Moreover, surface and ground water irrigation is restricted to those farmers who have access to equipment and finance to pay for operating costs. Irrigation management and infrastructure are weak; the availability of spare parts and equipment, maintenance services are limited; and this affects tubewell owners profitability, farm efficiency and land productivity. Groundwater resources are overexploited.

Agricultural production is also impeded by environmental degradation. Due to high population pressure, most land is under cultivation, farmers do not produce fodder crops to feed draft animals; instead draft animals graze on field boundaries and communal grazing areas creating pressure on the land; this causes land degradation. Only a small amount of organic matter is recycled back to the soil in the form of animal dung, wood, leaves and crop residues. The region faces environmental problems regarding water management and land degradation.

This Chapter describes the resources available for agriculture and the factors affecting farm efficiency, productivity and welfare with special emphasis on irrigation infrastructure and environmental degradation in the High Barind. This Chapter is outlined as follows: Section 2 provides location, physiography, geology and topography; Section 3 points out the socioeconomic indicators of development; Section 4 describes agriculture, input utilization, irrigation, farming system and livestock in the High Barind; Section 5 discusses the environmental factors which affect farm efficiency and production; and finally Section 6 presents a summary and conclusion.

2.2. Location, Physiography, Geology and Topography

The High Barind is situated in the north-western part of Bangladesh approximately between $24^{o}35'$ and $24^{o}50'$ North latitude and between $88^{o}16'$ and $88^{o}30'$ East longitude. The High Barind consists of 14 thanas of the three north-west districts, Rajshahi, Naogoan and Nawabganj in Bangladesh. The map shows the locations of the High Barind thanas. It is an area of about 870,492 acres and lies in the driest north-western part of the country (Table 2.1).

Geologically, it is old alluvium, characterized by abundant tainted calcareous material in the shape of random concretions. Topographically, it is dome-shaped with vast level land passing into relatively low lying area with gradual slopes, lying west of the Atrai river and it ranges from 40 to 150 feet above mean sea level. It is mainly undulating having level summits and terraced slopes. It is semi-arid.

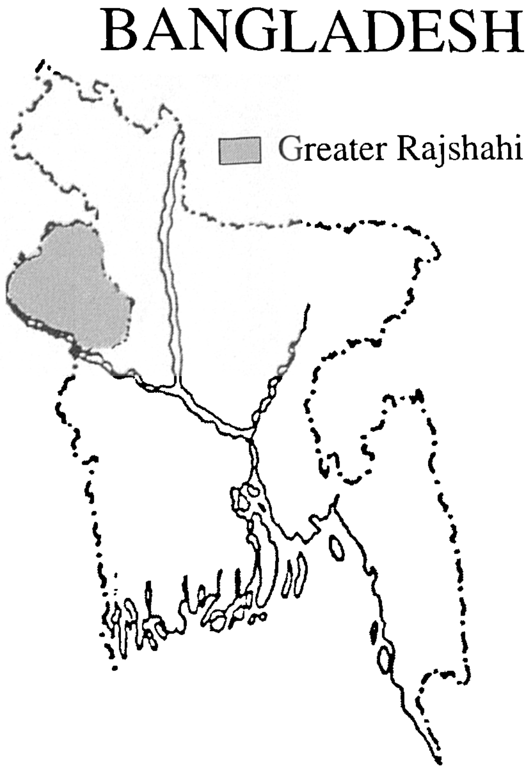
Table 2.1: High Barind Area (Acres)

District: Rajshahi		District: Naogoan		District: Nawabganj	
Thana	Area	Thana	Area	Thana	Area
Godagari	91343	Badalgachi	17510	Bholahat	3592
Tanore	65982	Dhamorhat	55925	Gomostapur	37037
		Manda	16207	Nachole	61953
		Mohadevpur	71674	Nawabganj	20137
		Niamotpur	107961	Shibganj	129629
		Patnitola	87236		
		Porsa	104306		
Total	157325		460819		252348

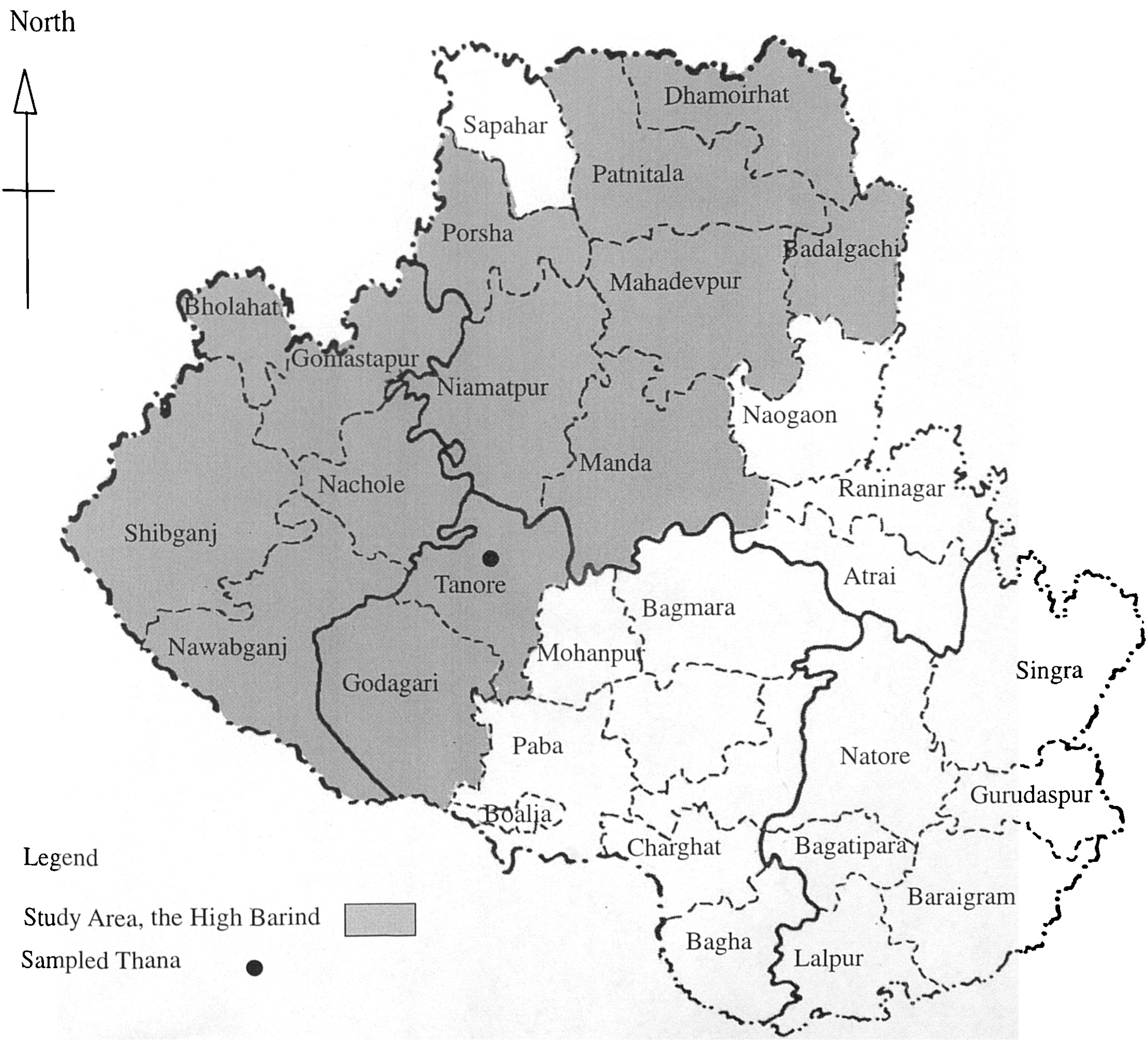
Source: Hunt, 1984, p.18.

The High Barind receives sufficient rainfall for crop growth during the Season II (the hot, rainy season) from May to September, rain water is contained within fieldbounds on terraced fields with some valley bottoms subject to shallow seasonal flooding and flash floods. The land then becomes parched during the Season III (dry, rabi season) from October to March. Relatively high temperature variations are experienced: maximum temperature is 30^{o} - 40^{o} C and minimum temperature is 5^{o} - 15^{o} C. The Barind is dominated by poorly drained soils which vary from silty-clay to clay-loam. In general the structural stability of the soils is poor resulting in soft and sticky soil when wet and hard when dry (Zuberi and Rahman, 1994).

Map of Bangladesh showing the High Barind



Greater Rajshahi District



2.3. Socioeconomic Indicator in the High Barind

Key development indicators in Bangladesh are shown in Table 2.2. The size of Gross Domestic Product (GDP) of Bangladesh for the year 1993-'94 at constant market price (base: 1984-'85=100 and Tk.70 = £1)) is Tk.638,173 million; and the growth rate of real GDP is 4.2 per cent. The contribution of agricultural sector to GDP is 32.77 per cent and that of the crop sector is 24.28 per cent. The growth rate of agriculture is 3.7 per cent and that of the crop sector is 2.8 per cent for the year 1995-'96 (Bangladesh Economic Review, 1997). Per capita GDP is Tk.5181 at market price, and growth rate of per capita GDP is 2.57 per cent. Life expectancy at birth is 58.7 years. The growth rate of population was 2.17 per cent in 1991.

Table 2.2: Socioeconomic Indicators in Bangladesh (1993-'94; base: 1984-'85=100)

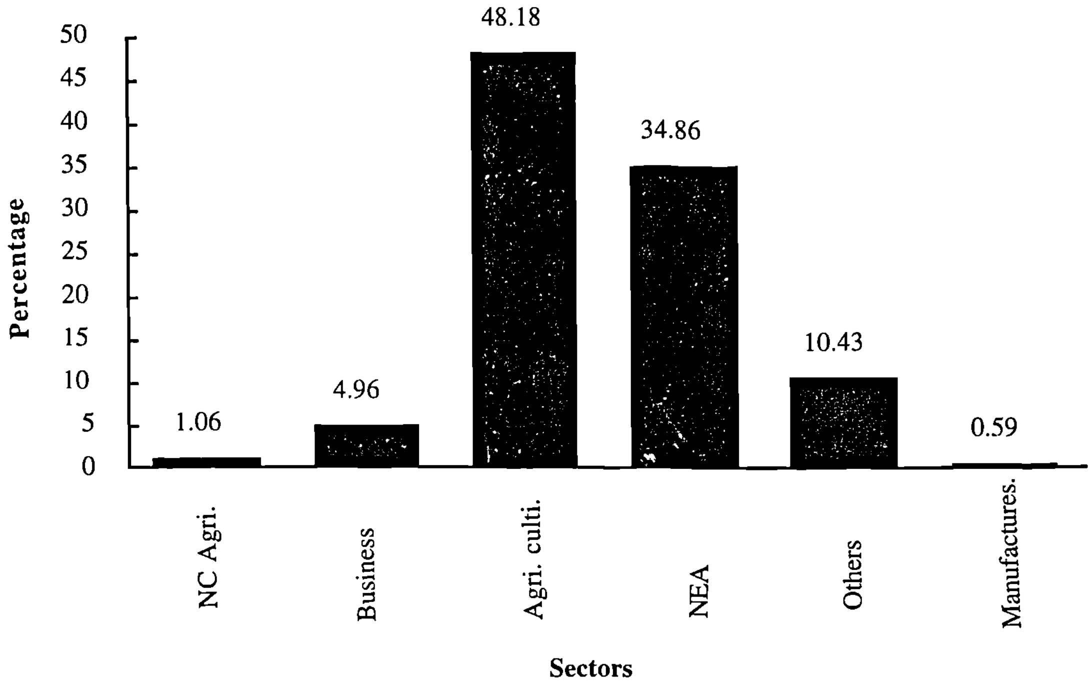
GDP at constant market price	Tk.638173 Million
GDP growth rate at constant price	4.2 per cent
Agriculture as a percentage of GDP	32.77 per cent
Crops as percentage of Agriculture	24.28 per cent
Per Capita GDP at market Price	Tk.5181
Per Capita GDP Growth at market price	2.57 per cent
Population in 1991 (Census, 1991)	114.4 millions
Life Expectancy	58.7 years
Agriculture as percentage of total employment	68.5 per cent

Source: Statistical Yearbook of Bangladesh, 1995; Statistical Pocketbook, Bangladesh, 1997.

The population of the Barind was 3.34 million in 1996 with a density of 1035 per square mile. The literacy rate is 40 per cent compared to a national average of 45 per cent in 1996 (Statistical Pocketbook, Bangladesh, 1997). The female literacy rate is 19 per cent (Census, 1991). This low level of literacy may affect farmers' efficiency as literacy enables them to process information relevant to the use of modern inputs.

The employment pattern of the work force in Figure 2.1 implies that part of the agricultural labour force remains underemployed for a major part of the year when land remains uncropped. Figure 2.1 shows that almost 35 per cent of population are not economically active.

Figure 2.1: Male Population engaged in Employment



Note: (1) NC Agri. stands for non-crop agricultural activities, (2) Agri.culti for agricultural cultivation, (3) NEA for not economically active.

Source: Government of Bangladesh, 1984

The Figure 2.1 also shows that 48 per cent of people are engaged in agricultural cultivation; 5 per cent of the population are employed in business; 1 per cent of the population is engaged in non-crop agricultural activities; 1 per cent population is employed in manufacturing industries and 10 per cent of population are engaged in other activities. The average age of farmers is about 59 years (Statistical Pocketbook, 1997).

Agriculture is the single most important sector of Bangladesh's economy and plays a significant role in the economic development of Bangladesh. The majority (80 per cent) of the population earn their livelihood from agriculture and agriculture is the major income source employing about 66 per cent of the labour force; within this sector the crop sector employs almost 55 per cent of the labour force and provides about 78 per cent of the value added in the agricultural sector. Within the crop sector, the main crops are rice, wheat, pulses and jute. Of these, rice is the dominant crop and accounts for more than 74 per cent of the cultivated area, 83 per cent of all irrigated areas, 88 per cent of the total fertilizer consumption and provides about 71 per cent of the gross output value of crops in the country. The vast majority of farm households in Bangladesh maintain their livelihood from the cultivation of rice. Rice, the major staple food of the Bangladeshi people, constitutes 95 per cent of the cereals consumed. About 68 per cent of the caloric intake and 54 per cent of the protein intake attain from rice in the Bangladeshi diet. The consumer price index accounts about 62 per cent as the weight of rice. With the introduction of high-yielding varieties (HYVs) and the adoption of irrigation and fertilizer technologies which spearheaded the so-called Green Revolution, rice production contributed significantly to Bangladesh's increase in self-sufficiency (FAO, 1997).

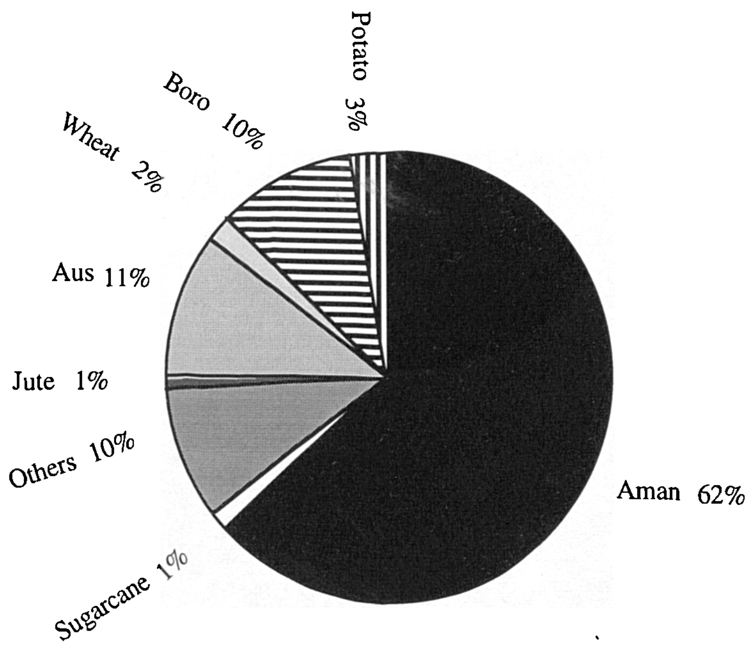
In the 1980s rice production increased at a rate of 2.7 per cent (Goletti, 1994). Through agricultural reform policy the government of Bangladesh has liberalized the markets of agricultural inputs and outputs. In spite of this, Bangladesh imports significant quantities of foodgrain including 1.5 million tonnes of rice per annum. This food import requirement limits development by reallocating funds from other vital sectors. The main problem of Bangladesh agriculture is the fragmentation of land plot size which constraints the use of modern agricultural tools and equipments. The restricted use of modern agricultural tools and equipment affects the capability of farm households to produce output at optimal levels and thereby reduces farm efficiency and productivity. We discuss agriculture in the Barind in the following subsections.

2.4. Agriculture in the High Barind

2.4.1. Cropping Systems, Soil and Vegetation

The High Barind has a semi-arid monsoon climate. The climatic factors - rainfall, evaporation, temperature and hours of light - determine the cropping pattern. The potential productivity of crops and the cropping intensity is influenced by the time of the onset of the monsoon rains and the quantity and distribution of rainfall (Hossain, 1991, p.11). The crops in this region are grown in three distinct seasons throughout the year: Season I, Season II and Season III. Season I lasts from the end of March to May; it is hot spring or pre-monsoon season with moderate humidity; rainfall occurs in this season with occasional heavy thunderstorms. Season II lasts from May to September; it is the hot monsoon season and characterized by high humidity; more than 80 per cent of the total rainfall occurs in this season. Season III lasts from October to February; it is a dry, cool winter season with negligible rainfall.

Figure 2.2: Cropping System in the Barind Region

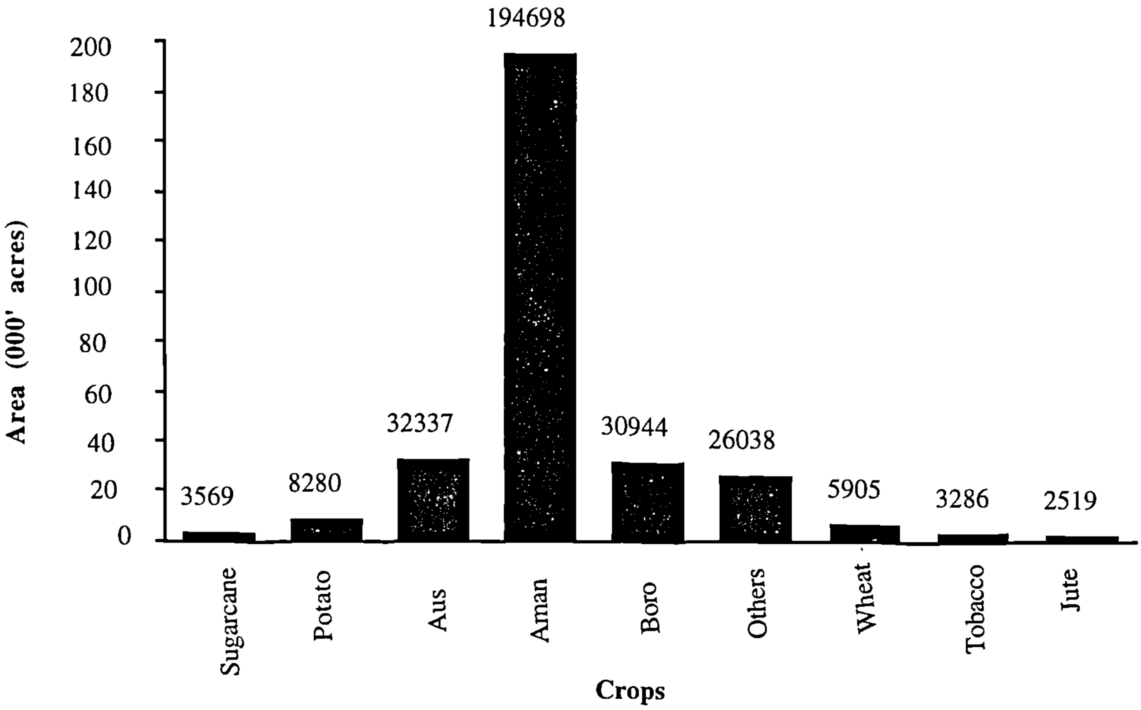


Source: Government of Bangladesh, 1994.

The overall intensity of cropping is 120 - 130 per cent. About 80 per cent of the land is high in altitude and floods do not affect the cropping pattern but T.Aman often suffers drought stress. The average yield of land varies from about 0.45 tonnes per acre to 0.55 tonnes per acre (Zuberi and Rahman, 1994).

The cropping pattern shows that a single rainfed T.Aman is harvested predominantly in Season II followed by Boro rice in Season III (Huke and Huke, 1983; Brammer 1985). Traditionally the cropping pattern in the Barind has been a single crop of T.Aman paddy and in localised areas, where there is double cropping, B.Aus rice is grown in Season I, followed by T.Aman in Season II. Figure 2.2 shows 62 per cent of the total cropped area is covered with Aman paddy crop in 1992. Season III (the dry, rabi season) has traditionally been a fallow period as most land is left fallow. During this season about 90 per cent of the land remained uncropped in the rest of the period. The provision of tubewells has resulted in the cultivation of Boro paddy, but the cost of tubewell water prohibits cultivation by some small farmers. Given the predominance of rice in Bangladesh diet, Boro rice has been grown in Season III, if irrigation is available, it is transplanted in January/February, and harvested in April/May.

Figure 2.3: Cropping Methods in the Barind Region



Source: Government of Bangladesh, 1992.

Potato, wheat, sugarcane, cabbage, radish, cauliflower, garlic, onion, ginger, coriander, turmeric, cucumber and vegetable-spices are also grown in the Season III. The share of land under these crops is shown in Figure 2.3 which indicates that Aman rice occupies most of the land followed by Aus and Boro rice. Among other crops, potatoes occupy most land followed by wheat, then sugarcane, tobacco and jute. Land utilization for crops is shown in Table 2.3 and Figure 2.4 which indicate that except rice and wheat, the shares of rabi crops and others are small and decreasing. Homestead production covers livestock and poultry, fruit and fibre, and vegetables but most homestead areas remain uncultivated or underutilized due to poor management.

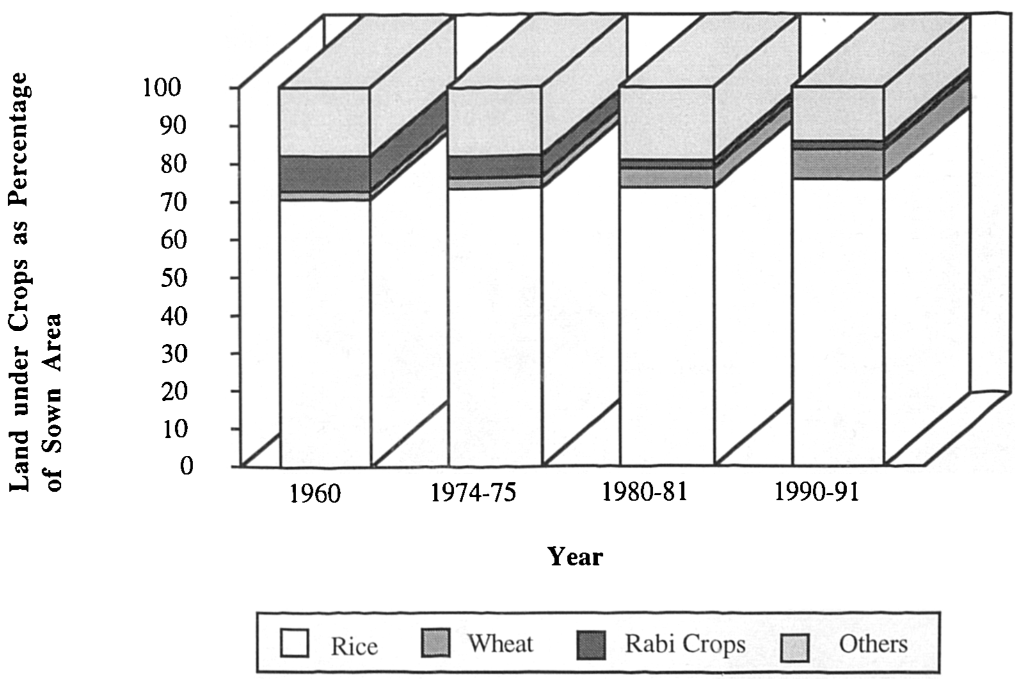
Table 2.3: Utilization of Land and Cropping Pattern Changes in the Barind

Year	Cultivable land as percentage of total land	Lands under crops (as percentage of cropped area)			
		Rice	Wheat	Rabi crops	Others
1960	69.00	71.00	2.20	8.80	18.00
1974-'75	71.30	74.00	2.40	5.60	18.00
1980-'81	70.40	74 .00	5.00	2.20	19.00
1990-'91	90.40	76 .00	8.00	2.00	14.00

Source: Zuberi and Rahman, 1994.

There is a lack of fuelwood and fodder which are related to the depletion of forest areas and vegetative cover. Animals mostly depend on aman stubble and overgraze the available vegetation on plot boundaries because farmers do not produce fodder crops. The dung of animal grazing on stubble along with leaves and twigs collected for fuel minimizes the recycling of organic matter causing deficiency in humus, zinc and boron in the soil. The low availability of forage has led to a decline in livestock farming to insignificant levels.

Figure 2.4: Changes of Cropping in the Barind Area



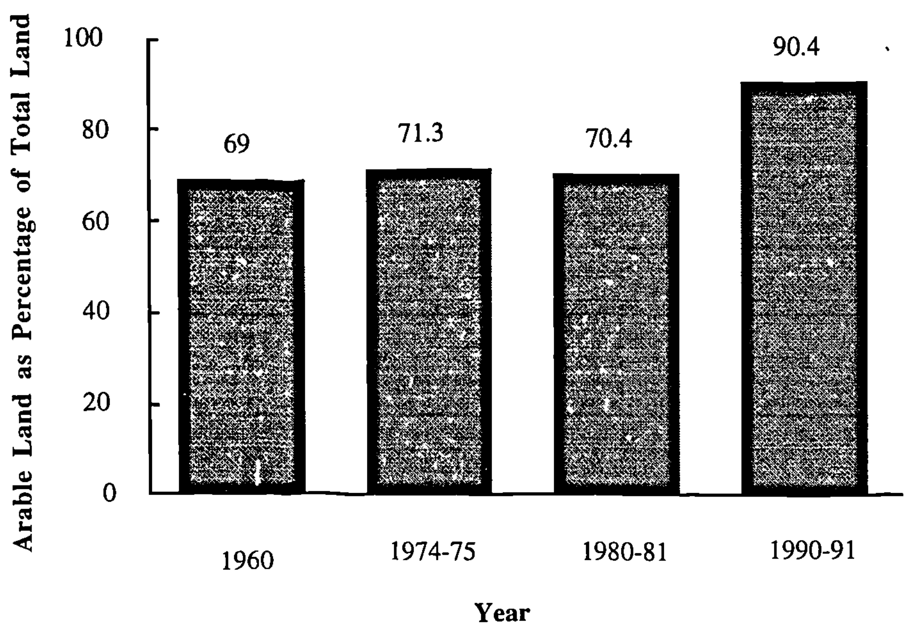
Source: Government of Bangladesh, 1992.

Mango, trees, jackfruit trees, palm trees and neem trees are found in most of the homestead areas and some housewives produce vegetables in the homestead area for household use.

2.4.2. Land Tenure System and Land Utilization

The Agricultural Census of 1983-'84 defines that a farm household is one that has at least 0.05 acres of cultivated land. Farms are classified into categories: landless farm, small farm, medium farm and large farm. A farm household is landless if he operates an area between 0.00 - 0.5 acres, is small if his operated area is between 0.51 - 2.5 acres, is medium if his operated area is between 2.51 - 5.00 acres and is large if his operated area is between 5.01 acres and above (Hossain, 1981, p.53). More lands are rented by medium farmers than others. Figure 2.5 shows that, in the Barind, the increase of arable land during 1960 to 1990 was 69 - 90 per cent; 90 per cent of the available land is cultivated. Further, with no significant forest area or waste land area, there is little scope to expand the arable area. Thus productivity can only be increased through 'vertical expansion', that is, increasing the frequency of cultivation and increasing yields.

Figure 2.5: Changes of Land Use Pattern



Source: Government of Bangladesh, 1992.

The farm area ranges from one-third to about twenty seven acres with a positively skewed distribution (Gill, 1981). [2.1] Farms in Bangladesh are typically small and highly fragmented and farmers in the Barind are no exception. The average plot size is one-third of an acre and it is skewed (Gill, 1981). The average plot size in a farm is less than 0.20 acres (Hossain, 1991, p.378). The larger the number of plots the greater the quantity of land left uncultivated. This high degree of land fragmentation does not allow the application of modern equipments, especially tractors and irrigation equipments, and reduces labour efficiency in farming activities causing low efficiency and productivity.

2.4.3. Water Resources, Market and Infrastructures

Water management policies are important because they extend the cropping season in the High Barind and greatly increase rice production. The issue of water markets for irrigation is essentially questions of property rights and, in the High Barind, such rights are complex. Surface and ground water raise different problems.

Surface water includes ponds, khals and bheels. There are 4,573 khas ponds and 139 miles of khas khals (BSO, 1987). Ponds are, on average, about one acre and average volume is close to about 200,000 cubic feet. Khals vary significantly in size and range between 1-30 miles. The crest width varies from 10 feet to over 200 feet while the depth varies from 3-27 feet; and the bed slope of khals also varies considerably with an average slope of 0.66 ft/mile (BSO, 1987). Ponds may be privately owned where access is restricted, in contrast to khals and bheels, which can be regarded as common property although government leases sometimes favour the local rich. With surface water sources, however, other variables determine access and use, such as the proximity of land and capital or the ability to pay labour costs. Sometimes capital is required in the form of low-lift pumps, where

[2.1] The 1983-'84 agricultural census (p.32) shows that 70.3 per cent of 10.05 million farm households are small farm households and share 29 per cent of the total land operated; the medium farm households are 24.7 per cent of total farm households and share about 45 per cent of total land area; and 5 per cent of total farm households share 25.9 per cent of total farm area.

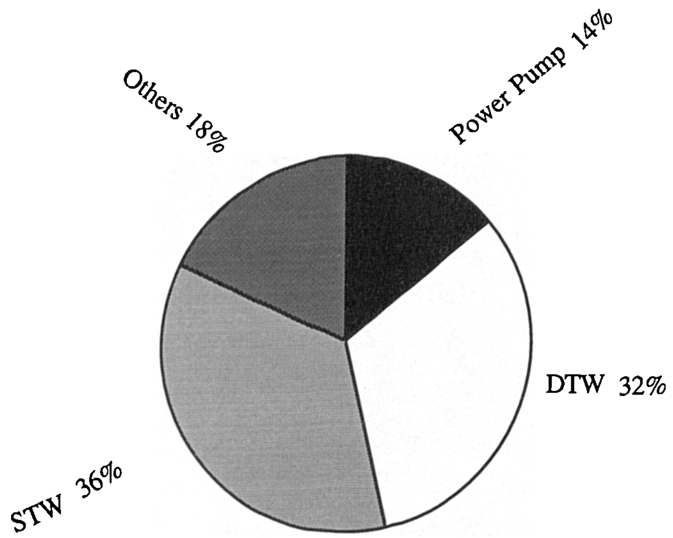
surface water rights become a function of access to capital which could affect efficiency and productivity.

Groundwater presents different issues. Groundwater rights are concentrated in the hands of those people who have access to capital equipment. The policy to privatize shallow tubewells (STWs) encouraged investment in STWs by private operators who offer commercial irrigation service as well as irrigating their own land. Here the state is in effect creating property rights over the acquifer beneath an owner's land and endorsing the transfer of rents to STW operators.

Population growth and increased food demand have brought an increase in irrigation water use. The market for irrigation water due to large-scale privatization is expanding. Some of the features of this market are: the demand for water is for the production of mainly HYV Boro rice and less for the production of non-rice crops; competition between tubewell owners for the 'command area', that is, area over which they control the irrigation, plots exists; the supply of water is variable and income-elastic; payment for irrigation water is mainly by cash and an instalment system of payment is typically practized. Water sellers and water buyers choose to include or exclude plots for irrigation depending on soil quality and elevation. Figure 2.6 shows the area irrigated by different means; 36 per cent of total irrigated land is irrigated by STWs; 32 per cent of total irrigated land is irrigated by deep tubewells (DTWs); 14 per cent of total irrigated land is irrigated by power pump and 18 per cent of total irrigated land is irrigated by other methods.

The cropping pattern in Bangladesh has shifted from the traditional variety of Aman to the HYV Boro variety which requires intensive irrigation; hence the demand for irrigation water is increasing. There has been a rising dependence on groundwater because surface water sources been silted up. The Barind Multipurpose Development Authority (BMDA) has installed a number of DTWs to create irrigation facilities and perform some infra-structural development like re-excavation of canals, afforestation etc. to improve the quality of life and to sustain agricultural growth. However, the area has low potential for groundwater exploitation and the over-use of DTWs results in STWs drying up.

Figure 2.6: Percentage of Irrigated Area by different Means of Irrigation



Source: Centre for Environmental Research, University of Rajshahi, Bangladesh, 1994.

Despite attempts to improve cropping potential through DTWs and to solve associated problems of over-extraction, the region is still less developed for irrigation than the rest of Bangladesh: in the High Barind, irrigation intensity is 19 per cent, while nationally it is 32 per cent. This is reflected in a lower cropping intensity in Season III compared with other regions. The national average of cropping intensity is 173 per cent, but in the High Barind, it is 130 per cent (Zuberi, 1996).

The tubewell owners/operating committee pay the costs of fuel, lubricants, repairs, canal repair and salary to the distributors (lineman) and charge the farmers a fixed fee per acre of land for supplying water within the command areas. Repair and spare parts costs sometimes appear to be critical factors affecting tubewell owners' returns from selling water which also influence productivity. Expansion of the spare parts markets and development of private workshops and mechanical services have the potential for improving irrigation markets in the region. The water market has a profound implication for tubewell capacity utilization as well as for returns to tubewell owners and farm households as the amount paid for irrigation determines both the returns of the tubewell owners and the profit of the farm households from irrigated crops.

2.4.4. Farming Systems

Many farms in the Barind have limited resources and can not afford modern inputs. They are left behind in the development process. There are wide variations in physical, social and economic conditions. Generally, farm households supply inputs for crop production such as draft power and human labour. Farm size averages 0.20 acre which does not permit mechanization, even to the extent of a power tiller. Therefore farm families have to cultivate using traditional methods, for example, with draught animals, wooden ploughs, wooden comb harrows and bamboo ladders for crushing and levelling. Typically they reduce risk by inter-cropping instead of using a monocrop or diversifying crops according to local market demands. Often the whole family is involved in agricultural production and they also involve other farm families at critical moments of the production cycle for joint efforts such as labour-sharing and plough-sharing. Draft animals and human labour are the main sources of farm power for cultivation in the Barind although it is difficult to cultivate with animal drawn ploughs.

2.4.5 Agricultural Marketing System

Modern inputs are scarcely used in the High Barind region. Cold storage, warehousing, processing facilities are not available so that the output is of an inferior quality and this reduces output price and hence household incomes. Further, transportation infrastructure is poorly developed making exchange and procurement of agricultural inputs and outputs difficult. An efficient market supplying inputs of seeds, fertilizer, irrigation water, etc. helps exploit benefits of new technologies (Islam, 1978). In addition, a chain of intermediaries operates in most markets and take a significant share of the consumer price and reduce farm-gate prices (Hossain, 1991). Allocative inefficiency may be partly due to unavailability of some inputs, that is, where farmers are unable to obtain the inputs required for a cost minimizing input mix.

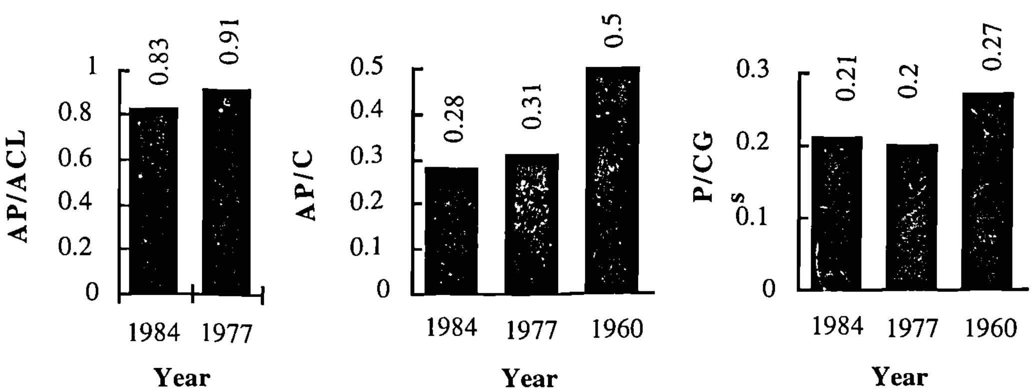
2.4.6. Agricultural Services

The adoption of modern technology in agriculture requires proper extension services, adequate credit supply and cooperation. In the High Barind region, agricultural cooperatives, loan providing banks and Non-Government Organisations (NGOs) and agricultural extension services are poorly developed. Few farm households are able to obtain money to purchase modern equipment; moreover, of those who can afford the equipment, many do not purchase it because they lack the training to operate and repair it (Hossain, 1991).

2.4.7. Livestock

In the Barind region, the principal source of draught power for agriculture is livestock. Draught animals are regarded as a versatile power source and are used for cultivation, weeding, transportation and crop processing. Apart from affording food for protein in terms of meat, milk and eggs, livestock also supply hides, bones, horns as raw material for industry as well as manure and fuel in the form of dung. The greater Rajshahi district including the Barind experiences a shortage of draft power (Hossain, 1991) in Season II. Scarcity of pasture means that livestock sometimes live on dry straw in the T.Aman season and graze in the open fields in the dry season increasing the rate of land degradation (Zuberi and Rahman, 1994) and results in animal in a very poor condition.

Figure 2.7: Livestock Availability (per capita)



Source: Government of Bangladesh, 1990. Note: AP/ACL implies animals per acre of cultivated land, AP/C per capita animals and P/CGS per capita goats and sheep.

Figure 2.7 shows the availability of the domestic animals of the greater Rajshahi district which encompasses the High Barind.

2.4.8. Agricultural Project in the High Barind

The Government of Bangladesh has launched a number of development projects in the Barind to increase agricultural production. Two of the most important are: the Barind Integrated Area Development Project and the Bangladesh Agricultural Research Project Phase-II: Area Development of the Barind.

Barind Integrated Area Development Project: Barind Integrated Area Development Project, initiated in 1985, aims to increase and sustain agricultural growth. The executive authority of this project is Barind Multipurpose Development Authority (BMDA) and the Ministry of Agriculture is the sponsoring Ministry. The instalment of DTWs is not the only objective but infra-structural development, like afforestation, canal digging are others. BMDA reexcavates derelict ponds to provide supplementary irrigation. Such supplementary irrigation stimulates increasing rabi crops which are beneficial for soil fertility retention (Idris, 1990).

Bangladesh Agricultural Research Project Phase-II: Area Development of the Barind: Bangladesh Agricultural Research Council (BARC) manages a project called Bangladesh Agricultural Research Project Phase-II: Area Development of the High Barind. BARC aims to assess research and planning needs for agricultural growth. BARC find that the High Barind suffers from environmental degradation. It suggests linkages between a progressive environmental policy and progressive social policy for sustainable agriculture in the Barind.

2.5. Environmental issues

Development in the High Barind has been impeded due to overexploitation of natural resources and environmental degradation. Table 2.4 shows some parameters of environmental problems.

Table 2.4: Some Parameters of Resources and Environmental Problems

1. Human Resources	**2. Land Resources**
Population growth	Soil erosion from cultivated land
Urban congestion	Fragmentation and farm land deterioration
Rural involution	Forest destruction
3. Natural Hazards	**4. Environmental Pollution**
Drought	Fresh water pollution
Flood	Saline water pollution
Endemic disease and fire	Domestic garbage
5. Water Resources	**6. Atmospheric Resources**
Water related diseases	Nitrogen and Oxygen
Ecology of fresh water fishes	Carbon dioxide

Source: Islam, 1991.

There are rising environmental concerns about water projects and the exploitation of groundwater. In addition to supplying water to domestic, industrial and agricultural users, the region is increasingly faced with severe environmental problems related to the management of water resources. For example, fisheries depend on continuous flows of high quality water and are threatened by increasing water withdrawals. In turn, a reduction in fish availability reduces protein intake.

The replacement of natural ecosystem, especially tropical forest, for agricultural purposes due to population pressure results in a loss of biodiversity. A large diversity of species is vital to agriculture and forestry, and plays an important role in recycling the essential elements for the living system, such as, carbon, nitrogen and phosphorus as well as in maintaining a quality environment. The Barind does not have a stable ecosystem and the farming pattern is vulnerable to disruption due to droughts and flooding. The challenge is to maintain a sustainable agricultural environment (Hunt, 1984).

2.6. Summary and Conclusion

The Chapter presents an overview of agriculture in the High Barind with an emphasis on socioeconomic, irrigation infrastructure and environmental factors. Farm efficiency is likely to be influenced by these factors. The crop year is characterized by three seasons: Season I, Season II and Season III; a single rainfed T.Aman is harvested predominantly in season II; Boro rice dominates in Season III if irrigation is available; other major crops are potato, wheat, sugarcane and vegetables. The farming system is one of small family farms which are too fragmented to operate with modern equipments. Surface and groundwater access are limited to those who have better access to finance to pay operating costs. Generally, the irrigation infrastructure and management are poorly developed. Grazing domestic animals in the open fields in the dry season, because of a lack of pasture, accelerates soil erosion and land degradation. A partially restored ecosystem and integrated land use plan are essential to the establishment of a sustainable agricultural environment. Some agricultural inputs are not available to all farmers and agricultural service systems are underdeveloped. Agricultural projects, if implemented, can help improve the efficiency and productivity of farms and income and welfare of farm households.

Chapter 3

Survey Methodology and Results

3.1. Introduction

Farm level data is a prerequisite for an empirical study of farm efficiency. The lack of farm level data for the study region necessitates a farm survey. Two villages were selected in the High Barind Bangladesh through purposive sampling technique. Farm households were selected according to farm size as the distribution of farm area is skewed and a simple random sampling technique is applied to each strata. A pilot survey was used to help design the main survey. The survey recorded two types of information, farm production data and farm characteristics.

Broadly the farm households can be characterized as follows. The average age of farmers is 39 years with farmers generally having low levels of education. Land holding distribution is skewed and land holdings are unequally distributed. Most land has already been exploited for cultivation leaving a small amount of forest and fallow area implying a threat to the environment. The farming system is semi-subsistence and is dominated by rice which accounts for 95 per cent of cultivated land. Farm households use mostly traditional agricultural equipments. Medium and large farmers hire labour; labour utilization pattern shows seasonality in use and labour demand in Season II is higher which pushes the agricultural wage rate up. The success of HYVs depends upon the use of fertilizer and irrigation. Rice crop accounts for 87 per cent of total fertilizer cost. Irrigation using DTWs and STWs covers 80 per cent of the cultivated land. Most of the income of farm households come from on-farm crop production. Poorer farm households are also involved in fishing, petty trading and wage labour. This Chapter aims to describe the survey methodology and results.

The plan of this Chapter is as follows: Section 2 explains the methodology of the survey; Section 3 describes the results of the survey and Section 4 concludes.

3.2. Methodology of the Survey

3.2.1. Framework of the Fieldwork Survey

Two villages in the Tanore thana located in the High Barind region are selected on the basis of purposive sampling subject to the availability of irrigation water. The farmers of both villages take irrigation water from irrigation scheme which includes DTWs and STWs. The irrigation scheme in one of the villages called Kamargonj is operated with electricity and in the other village called Manikkanna with diesel. The data are obtained using a structured questionnaire administered personally with the help of one assistant either in the respondent's house or in one of the local tea shops where farmers meet regularly. The questionnaire was structured in English and translated to Bengali verbally when administered. In total 150 farm households were interviewed. The fieldwork research took place from August 7 to September 30, 1997.

3.2.2. Questionnaire Design

The questionnaire (see Appendix 3) aimed to achieve two goals. First, to gather data relevant to the objectives of the survey and second, to gather data which are reliable and valid. These goals can be called relevance and accuracy (Warwick and Lininger, 1975). A pilot survey was carried out on 20 farmers to check whether the questionnaire was capable of generating the required data, the respondents grasp of the survey and the time taken to complete the questionnaire. The pilot survey examines not only the questionnaire aspects but also the effectiveness of the framework of fieldwork, the quality of the interviews, the justification and adequacy of the sample instruction, the frequency of different reasons for refusals and the overall correctness of the survey methods. After this pilot survey an integrated questionnaire was prepared.

The questionnaire consists of five major sections. The first section contains a number of personal questions discussing name, age, marital status, educational status, demographic characteristics and social status of the farm household. The second section covers production and includes questions on total land owned, total cultivable land, homestead area, forest area, total cultivated area, net cultivated area, total irrigated area, number of plots, average plot size, average plot distance, sharecropping area, homestead utilization, land and labour utilization, irrigation information, fertilizer utilization, pesticides utilization, water seller's information and yield and output and input prices. The third section concerns non-farm income. The fourth section covers the consumption side of farm households and the fifth section includes livestock information. Apart from this detailed household questionnaire, Participatory Rural Appraisal (PRA) is undertaken in each village which involves a range of socioeconomic and environmental issues.

3.2.3. Definitions Used

Some definitions used are critical to questionnaire design. The definitions aim to avoid ambiguity and double-counting of farm household resources when data are recorded.

Agricultural farm household: The agricultural farm household consists of persons or individuals who work together or subscribe to a general wealth fund, that is, share in household funds through wages and salaries, other cash and in-kind income as well as share food from a common source, that is, cooked and eaten together. This definition excludes visitors (SALDRU, 1994).

Farmer: A farmer is the head of a farm household who takes production decisions.

Agricultural farm: A farm consists of the total area tilled by the members of the household during the survey period.

Plot: A piece of cultivated land containing a single crop or single homogeneous mixture of crops (Casley and Kumar, 1988).

3.2.4. Sampling Strategy and Sample Size

A list of the number of Agricultural farm households consisting of large farmers, medium farmers, marginal farmers and landless farmers and the total area of arable lands of the study area are collected from the Local Government Offices (LGOs), namely, the Thana Agricultural Development Offices. Two villages in Tanore Thana are chosen using the technique of purposive sampling. Purposive samples are selections from certain subgroups in the population chosen to allow hypotheses to be tested (Warwick and Lininger, 1975). In the High Barind, the land distribution of the farm households is skewed. Therefore if the simple random sampling procedure is applied to such a distribution of farm households in each village, there is a chance that either none or too many very large farms may be included in the sample. As a result, the sample may not adequately represent this group in the population. Simple random sampling by stratification may improve the representativeness of the sample drawn from a population when we know something about the make up of the population relevant to our research. Stratification can reduce the sampling error, a measure of the variability of the population estimates from repeated samples around the population value (Warwick and Lininger, 1975), which depends not only on the sample size but also on the sample design (Casley and Kumar, 1988).

Therefore, it is advantageous to specify strata according to land holdings, such as landless farm, marginal farm, middle farm and large farm. Simple random sampling selection procedures could then be applied separately to each of the strata to give each farm household in the population an equal chance of being selected in such a way that there is some relationship between being in a particular stratum and the answer sought in the survey research and that within the separate strata there is as much homogeneity as possible. Then information concerning individual stratum may be desirable which may increase precision. Population characteristics can be estimated more efficiently from a stratified sample than from an overall random sample if strata means vary significantly.

The stratified sample mean ($\bar{x}_{st}$) is more efficient than the simple random sample mean ($\bar{x}_{sr}$) if the strata means differ widely compared with within-strata variation: the greater this effect of stratification the greater the efficiency of $\bar{x}_{st}$ compared to $\bar{x}_{sr}$ (Barnett, 1974). Following (Barnett, 1974), if a simple random sample of size n from a population of size N is drawn in such a way that values are $x_1, x_2, ..., x_n$, then the mean and variance of this sample are as follows:

The sample mean: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

and the variance of the mean: $\operatorname{var}(\bar{x}) = \frac{(1-f)S^2}{n}$

where $f = \frac{n}{N}$ and S^2 is the population variance. If the set of values $x_1, x_2, ..., x_n$ in a finite population of size N has been classified into k strata of sizes

$$\left(\sum_{i=1}^{k} n_i = N\right)$$

with members x_{ij} $(i = 1,2,...,k;\ j = 1,2,...,N_i)$ then the stratified sample mean:

$$\bar{x}_{st} = \sum_{i=1}^{k} w_i \bar{x}_i$$

where $\bar{x}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}$ is sample mean of ith stratum and $w_i = \frac{n_i}{N}$

and the variance of the mean:

$$\operatorname{var}(\bar{x}_{st}) = \frac{1-f}{n}\sum_{i-1}^{k}\frac{n_i}{N}S_i^2$$

where $f = \frac{n}{N}$ by proportional allocation, the sampling fraction, is identical for all strata.

Now we can have:

$$Var(\bar{x}_{sr}) - Var(\bar{x}_{st}) = \frac{(1-f)}{n}\left\{S^2 - \frac{1}{N}\sum_{i=1}^{k} n_i S^2{}_i\right\}$$

Simplification of this expression yields:

$$Var(\bar{x}_{sr}) - Var(\bar{x}_{st}) = \frac{(1-f)}{nN}\sum_{i=1}^{k} n_i\left(\bar{x}_i - \bar{X}\right)^2$$

$$= \frac{(1-f)}{n}\sum_{i=1}^{k} w_i\left(\bar{x}_i - \bar{X}\right)^2 > 0$$

unless the $\bar{x}_i$ are all the same; where $\bar{X}$ is population mean. This implies that the stratified sample mean will always be more efficient than the simple random sample mean and the efficiency increases with the variations in the strata means.

A sample size of 150 farmers from two villages is selected as a stratified random sample which includes landed farmers, middle farmers, marginal farmers and landless farmers. In order to have a representative sample a number of villages were visited in Tanore thana to assess irrigation characteristics.

3.2.5. Policy to Ensure Correctness in the Accumulation of Data

The questionnaire was piloted on 20 farmers with a view to check and pre-test the appropriateness and relevance of the questions being asked and to ensure data accuracy. This pilot survey identified the possible problems which might be encountered during the main fieldwork survey. In the accumulation of farm level primary data from the largely illiterate farming villages the degree of precision is dependent on the following aspects: adequate knowledge about the questionnaire and survey of the staff employed in the collection of the primary data; the collaboration of farm households in responding to questions. The researcher himself along with a well trained assistant engaged in collecting the data. In order to achieve highest cooperation and lowest distortion of farm households the questionnaire was prepared in such a way so that it was interesting and not hard to answer, embarrassing or time-consuming. The researcher attempted to gain the household's confidence. Stephen (1964) notes, "The greater part of the research rests on kindness and confidence: kindness in the willingness of respondents to give time to the interview and to do what is requested, confidence in accepting the implicit assurance of the interviewer that he will not take advantage of the respondent and that the survey will in no way harm his interests". Farmers were reassured by explaining the importance of the survey and the survey data. Some farmers say that a lot of surveys have already been undertaken in this region but no development steps have been taken. They blame the government authorities and some NGOs. Some farmers also hide information in fear of the tax authorities. Farmers are assured that the data collected will be utilized for personal research for a higher degree and not for tax motives and that their identity will remain anonymous.

Most farmers in this largely illiterate agricultural farming region do not keep any written records of their farming resources, activities and utilities. Most of the primary data was obtained through memory recall of the farmers. Since most of the farmers in this region are middle-aged, experienced and full-time it was easy to collect important information on various farming activities for the cropping operations in stages and by reminding them of previous answers. The quality of the field staff employed and the cooperation of farmers alone do not ensure data accurately. The best approach is to resort to a work study approach but this would involve more resources than were available (Ekine, 1996). Subject to the budgetary restrictions and time available for research, the collection of data in stages and cross-checking are the best approaches available. A number of group meetings or Participatory Rural Appraisal (PRA) with the village farm household are held in order to ensure consistency or correctness of information and to help the researcher communicate with farm households until the survey finishes.

3.2.6. Primary Data Collected

The primary data collected from the survey for the year 1997 can be categorized in Table 3.1 and in the sub-headings set forth below:

Table 3.1: Primary Data Collected

	Variables recorded	Units
Output	Output per acre	Maund (1 maund = 37.32 kg)
	Output price per maund	Tk.
	Revenue from output	Tk.
Land	Total cultivated land	acres
	Price of land per acre	Tk.
	Land value	Tk.
	total area owned	acres
	homestead area	acres
	forest area	acres
	fallow area	acres
	net cultivated land	acres
	total irrigated land	acres
	net irrigated land	acres
	number of plot	
	average plot size	acres
	plot distance	mile
Labour	Labourer per acre	per manday
	Wage	Tk.
	Total labour costs	Tk.
Irrigation	Irrigated land	per acre
	Irrigation price per acre	Tk.
	Irrigation price per day	Tk.
	Total cost of irrigation	Tk.
Fertilizer	Fertilizer applied per acre	kg
	Fertilizer price per acre	Tk.
	Total fertilizer costs	Tk.
Pesticides	Pesticides used per acre	millilitre
	Pesticides price per acre	Tk.
	Total pesticides cost	Tk.
Factors associated with inefficiency	Age of farmers	years
	Schooling	years
	Land fragmentation, i.e., plot size	acres
	Irrigation infrastructure	dummy
	Environmental degradation	dummy

As well as the data collected shown in Table 3.1, the data on the following issues are also collected:

(i) **Non-farm income data:** include non-farm work hours, days, costs and income in each season for various non-farm activities.

(ii) **Livestock data**: include livestock numbers, hours spent on livestock husbandry, livestock costs and income from livestock.

(iii) **Miscellaneous data**: These data include the household's name, social status, household sharecropping information and some information about peak period of farming.

3.2.7. Participatory Rural Appraisal (PRA)

To complement the individual farm data, PRA was undertaken to broaden the information. PRA is a technique which involves group discussion and informal discussion meetings; groups include people who have an appreciation of socioeconomic, irrigation and environmental factors which affect efficiency. PRA in the survey was used to discuss issues relating to social, economic and environmental aspects of farming in the High Barind and it identifies the factors associated with inefficiency and the main survey collected data on those factors. PRA also discussed the role of the agricultural extension department officials, reasonability of farm product prices, the role of NGOs in agriculture, the availability of the non-farm activities and irrigation payments.

3.2.8. Shortcomings of the Cross-Section Primary Data

The memory recall process is used to collect the cross-section primary data during the survey because of the non-availability of written farm records. Time, budgetary constraints and illiteracy of farm households are the other problems which make it difficult to collect the data. Farmers were unable to grasp the idea of research and sometimes they refuse to give information due to illiteracy and fear of paying taxes. Since the survey is administered during August-September, a time when farmers are not busy, all the farming operations in this season are carried out before the field survey period. However, methods were adopted to assist the farmers in thinking back to the farming operations; farmers are reminded about the costs of farming activities and revenues from crops in previous answers to help in their memory recall process. Given the limitation of time and financial resources for this work, the memory recall process is the most practicable and reasonable way to obtain the primary data (Norman, 1972; Okuneye, 1985; Orul, 1992).

3.3. Results of the Survey

3.3.1. Farm and Agricultural Farm Household Characteristics

One way analysis of variance is carried out to test the hypotheses that there are no differences in means for all variables across the villages. An F statistic instead of t statistic is preferable. The more t tests are made, the greater is the possibility that a spuriously significant t is obtained (Comrey, 1975). The structure of the one way analysis of variance is given in Table 3.2.

Table 3.2: One Way Analysis of Variance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares (Variance estimate)	F ratio
Between-Groups	$\sum X_i^2$	k-1	$S_B^{\ 2}$	
Within-Groups	$\sum X_j^2$	N-k	$S_W^{\ 2}$	$F = \frac{S_B^{\ 2}}{S_W^{\ 2}}$
Total	$\sum X_i^{\ 2} + \sum X_j^{\ 2}$	N-k+k-1		

The null hypothesis: H_0 = there are no differences between the villages and

The alternative hypothesis: H_A = there are differences between the villages.

The null hypothesis H_0 is not rejected if the F value is less than the critical value at the 5 per cent significance level.

Farm resource inequality is predicted using a Gini coefficient which is a numerical representation of inequality and the corresponding graphical illustration is the Lorenz curve. The Gini coefficient is estimated using the formula derived by Pyatt et al. (1980) and applied by Lerman and Yitzhaki (1984), Garner (1993) and Yitzhaki (1994) as:

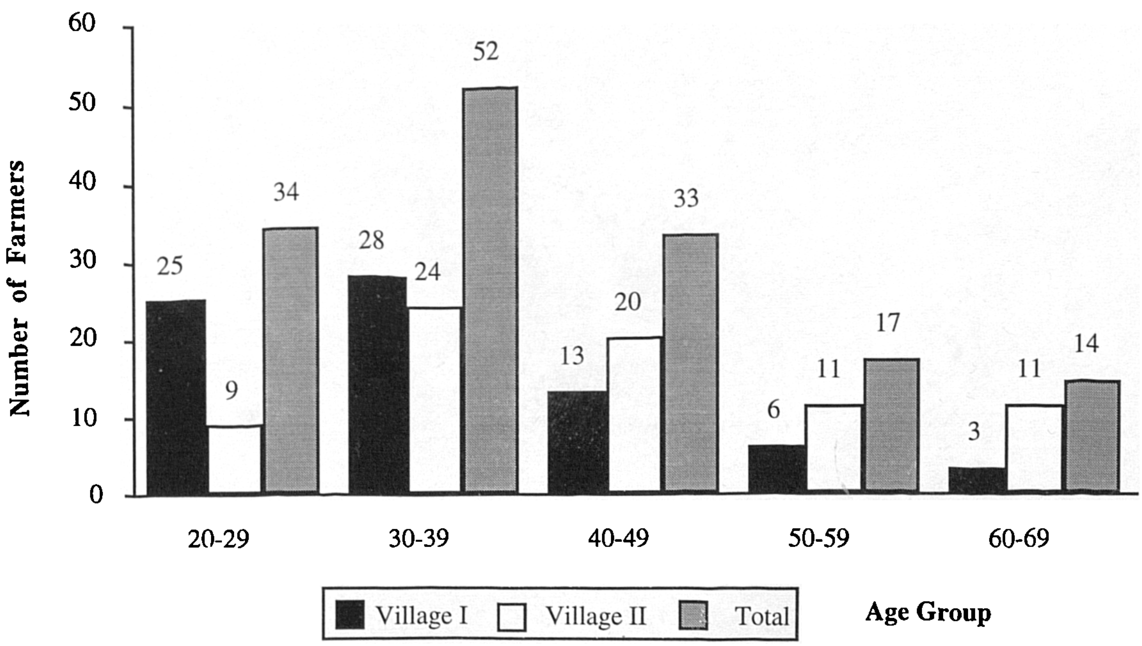
$$\text{Gini coefficient } G = \frac{2Cov(y, r_y)}{n\bar{y}}$$

where $Cov(y, r_y)$ is covariance between y and r_y and here y is any variable and r_y are ranks of all individuals according to y, n is total number of individuals and $\bar{y}$ is the mean value of y.

3.3.2. Age Distribution and Education Level

The age range of farm households in the sample area varies from 20 - 69 years with an average of 39 years (Figure 3.1). It is notable from the Figure 3.1 that 35 per cent of farmers fall in the age group of 30 - 39 years; 23 per cent of farmers fall in the age group of 20-29 years; 22 per cent of farmers fall in the age group of 40 - 49 years; 11 per cent of farmers fall in the age group of 50 - 59 years and the least number of farmers, 9 per cent, fall in the age group of 60 - 69 per cent. Figure 3.1 shows that Village I has more young farmers. Table 3.3 shows the years of farming experience of farmers. The minimum farming experience is 3 years and the maximum is 50 years with an average farming experience of 20 years.

Figure 3.1: Age Distribution of the Farm Households



Source: Survey data, 1997.

The F statistic shows there is no difference of age of farm households between Village I and Village II.

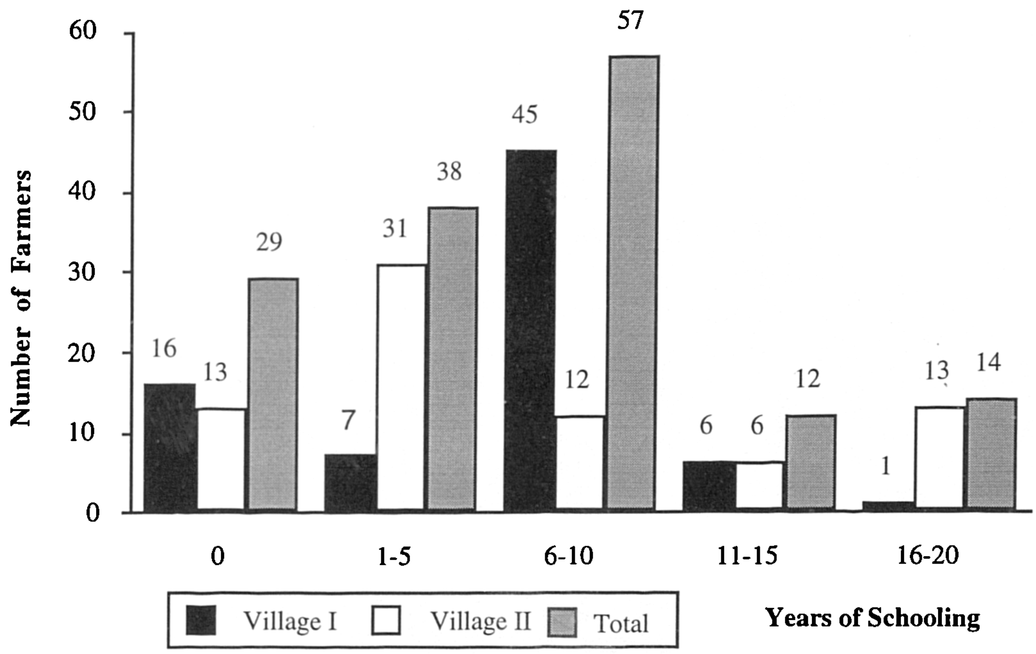
Table 3.3: Years of Farming Experience

	Minimum	Maximum	Mean	C.V. [3.1]
Village I	3	50	17	55.95
Village II	6	50	23	46.70
Total	3	50	20	52.46

Source: Survey data, 1997.

One of the important and crucial aspects concerning planning decisions about production is the education level of the farmers. It is expected that the educational level of farmers has bearing on their ability to adopt new productivity increasing technologies.

Figure 3.2: Year of Schooling of the Farm Households



Source: Survey data, 1997.

Education plays an important role in making decisions concerning the adoption of seeds of an improved variety and contact agricultural extension officials for suggestions and advice regarding agricultural systems (Tetlay et al., 1991). None of the farmers in the study area has any university degree or professional training in agriculture; 19 per cent of farmers have no formal education; 25 per cent of farmers are between 1 - 5 years of schooling; 38

[3.1] C.V. is the coefficient of variation defined as the ratio of standard deviation to mean multiplied by 100.

per cent of farmers are between 6 - 10 years of schooling; 12 per cent of farmers are between 11 - 15 years of schooling; and 9 per cent of farmers are between 16 - 20 years of schooling. The rate of illiteracy is higher in village I than in Village II (Figure 3.2). The F statistic rejects any difference of educational levels of farm households between the two Villages.

3.3.3. Farm Resources

The distribution of land holding in the survey of the High Barind Bangladesh is moderately skewed as in Table 3.4 and Figures 3.3 which show that most of farms, 34 per cent, possess land within 2 - 3.99 acres; 31 per cent of farms possess land within 0 - 1.99 acres; 25 per cent of farms possess land within 4 - 5.99 acres and 10 per cent of farms possess land 6 acres and above. Most of farms, 36 percent, in Village II have their land holding within 2 - 3.99 acres while 34.67 percent of the farmers possess land holding within 4 - 5.99 acres in Village I in the survey area.

Table 3.4: Distribution of Farm Sizes

Farm Size	Farmers in Village I		Farmers in Village II		Total Farmers	
(acres)	Number	Percentage	Number	Percentage	Number	Percentage
0-1.99	21	28.00	25	33.33	46	30.67
2-3.99	24	32.00	27	36.00	51	34.00
4-5.99	26	34.67	12	16.00	38	25.33
6 + Above	4	5.33	11	14.67	15	10.00
Total	75	100.00	75	100.00	150	100.00

Source: Survey data, 1997.

Inequality in land holding predicted by Gini Coefficient is presented in Table 3.5. The overall Gini Coefficient is 0.4. Gini Coefficient of Village I is 0.33 and that of Village II is 0.45 which shows that land holding inequality in Village I is higher than in Village II.

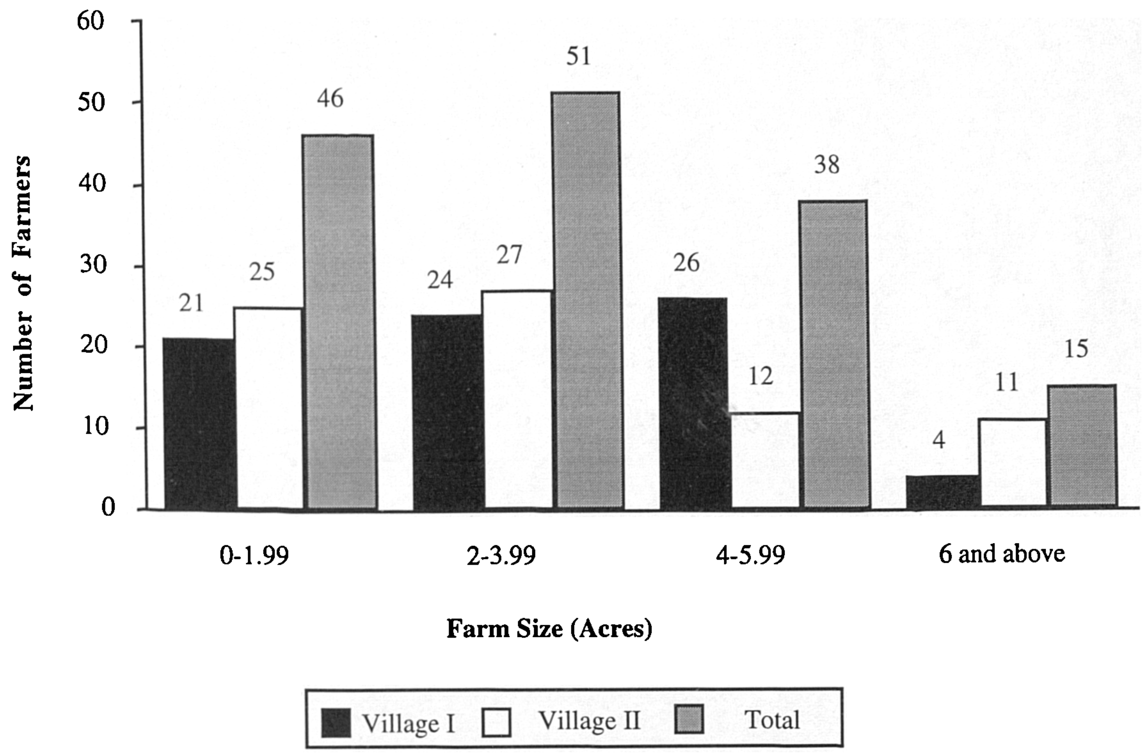
Table 3.5: Land Inequality

	Gini Coefficients		
	Village I	Village II	Both Villages
Land	0.329	0.449	0.397

Source: Survey data, 1997.

The F statistic with probability 0.91 rejects the hypothesis of any significant differences between farm areas between the two villages.

Figure 3.3: Frequency Distribution of Farm Size



Source: Survey data, 1997.

3.3.4. Land Utilization Pattern

The survey results in Table 3.6 show that 88 percent of the total land area has already been exploited for cultivation. It also shows that the forest area is only 3 per cent implying a potential threat to the environment. The fallow area is only 2 per cent indicating no excess lands left to bring under cultivation and implying a reduction of the water bodies creating a threat to fisheries resources since fallow also includes water bodies and homestead area is only 7 per cent. The main way in this region for a rise in the production of output is to increase cropping intensity and increase yields per acre rather than expand the cultivated area.

Table 3.6: Land Use Pattern

Area	Sum	Mean	Std Error	C.V.	Minimum	Maximum
Total Area Owned(TAO)	530.25	3.50	0.25	88.19	0.05	28.00
Homestead Area(HSA)	37.05	0.25	0.02	90.45	0.00	1.00
Forest Area(FOA)	14.53	0.10	0.02	195.77	0.00	2.00
Fallow Area(FA)	9.88	0.06	0.02	362.26	0.00	2.00
Total Cultivable Land(TCL)	469.06	3.12	0.22	85.86	0.00	23.20
Total Cultivated Area(TCA)	469.06	3.13	0.22	85.11	0.00	23.00
Net Cultivated Area(NCA)	820.62	5.47	0.34	77.36	0.00	24.33
Total Irrigated Area(TIA)	376.84	2.41	0.17	87.61	0.00	16.66
Net Irrigated Area (NIA)	401.73	2.81	0.27	88.44	0.00	17.66

Source: Survey data, 1997. Note: Std Error = Standard Error.

The coefficient of variation, 362.26, for fallow area is the highest which indicates extreme inequality of fallow land holdings; the second highest coefficient of variation is for forest area. The F statistic rejects any significant differences between land use of farmers between the two villages.

3.3.5. Farming Patterns

The farming system is dominated by rice production and is semi-subsistence; rice accounts for 95 per cent of the cultivated area as indicated in Table 3.7. The traditional system of rainfed paddy production has been transformed through the unplanned adoption of new technologies; HYV aman, irrigated Boro and IRRI paddy and rabi crops supplement rainfed crops. With inorganic fertilizer, HYV seeds and ground water irrigation, an intensive multiple paddy cropping pattern has replaced the indigenous less-intensive semi-irrigated rice production system.

Table 3.7: Cropping Area of Rice and Other Crops

	Sum	Average	Std Error	C.V.
Total Cultivated Area	469.06	3.12	0.23	85.11
Net Cultivated Area	820.62	5.47	0.34	77.36
Rice Cultivated Area (S1)	170.88	1.14	0.11	114.84
Rice Cultivated Area (S2)	415.58	2.77	1.68	503.95
Rice Cultivated Area (S3)	269.72	1.80	0.14	96.22
Other Cultivated Area (S1)	6.19	0.04	0.01	325.06
Other Cultivated Area (S2)	1.26	0.01	0.01	988.07
Other Cultivated Area (S3)	43.04	0.29	0.06	212.87

Source: Survey data, 1997.

Table 3.8 shows that the overall cropping intensity of the Villages is 174.95 per cent indicating a large proportion of the land remain uncropped after a single crop of T.Aman rice. Comparatively little area remains uncropped after T.Aman rice harvesting in Village I indicated by the cropping intensity of 204 per cent. Boro and IRRI rice paddy, irrigated with DTWs and STWs, are grown in Season III but the expansion of the land under these crops depends on the availability of irrigation water, fertilizer and inputs at reasonable prices. Farm households use mostly simple traditional tools such as cutlasses, hoes, ploughs, ladders and spades in their farm activities.

Table 3.8: Cropping Pattern

	Village I	Village II	Both
Cropping Intensity [3.2]	204.67	146.72	174.95
Net rice cropping area	464.73	591.45	1056.18
Net other cropping area	24.91	25.58	50.49

Source: Survey data, 1997.

Some secondary crops are observed throughout the survey area with some variation across villages; these include potato, dal and other vegetables. Perennial cash crops are not found in the survey area. Land degradation affects rice crop. Plate 1 shows a rice crop affected by land degradation.

[3.2] Cropping Intensity is defined as the ratio of net cultivated area divided by total cultivated area multiplied by 100. The maximum possible cropping intensity in this region is 300 per cent which indicates three rice crops.

Plate 1: View of an environmentally degraded rice crop land in the Barind a boy called 'Md Faridul Islam' was weeding.

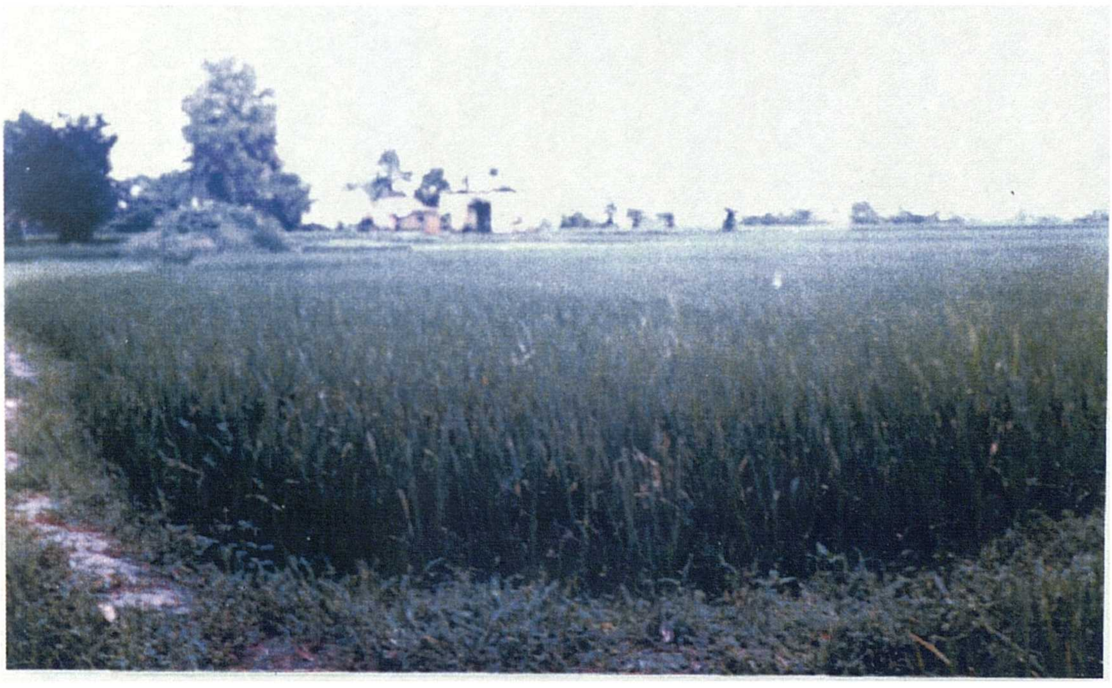


Plate 2: View of an Irrigation Scheme in the Barind during Season II

3.3.6. Agricultural Inputs

3.3.6.1 Labour Use

Large and medium farmers hire labour, but small farmers employ mainly family labour. The local availability of labour for agricultural activities remains constant except during the peak period in season II. This shortfall is met by migrant labourers from nearby districts and this requires more wages. Wage rate varies between Tk.45.00 per manday in the slack season up to Tk.70.00 per manday in the peak season with an average wage rate of Tk.55.00 in the peak period and Tk.50.00 in the slack period. Farm households with small land areas devote more labour to their land on a per acre basis.

Table 3.9: Labour Utilization, Seasonality and Labour Market

Labour Input	Sum	Mean	Standard Error	Coefficient of variation
Family Labour				
Season I	2067.00	13.78	1.12	105.60
Season II	3149.00	20.99	1.52	88.79
Season III	2580.00	17.20	1.04	73.95
Total	7796.00	51.97	3.51	180.44
Hired Labour				
Season I	7796.00	51.97	3.51	82.57
Season II	3140.18	20.93	3.08	180.44
Season III	9789.20	65.26	19.69	369.46
Total	5313.86	35.43	5.17	178.66
Both Total	18243.24	121.62	23.62	237.89

Source: Survey data, 1997.

Labour use and its seasonal distribution in Table 3.9 reveal the coefficients of variation exhibit considerable seasonality for hired labour input. The coefficient of variation of family labour use in Season III is lower than in other seasons; this implies that family labour use is less seasonal in Season III. The coefficient of variation of hired labour utilization is Season I is lower than in other seasons; this indicates hired labour use is less seasonal in Season I.

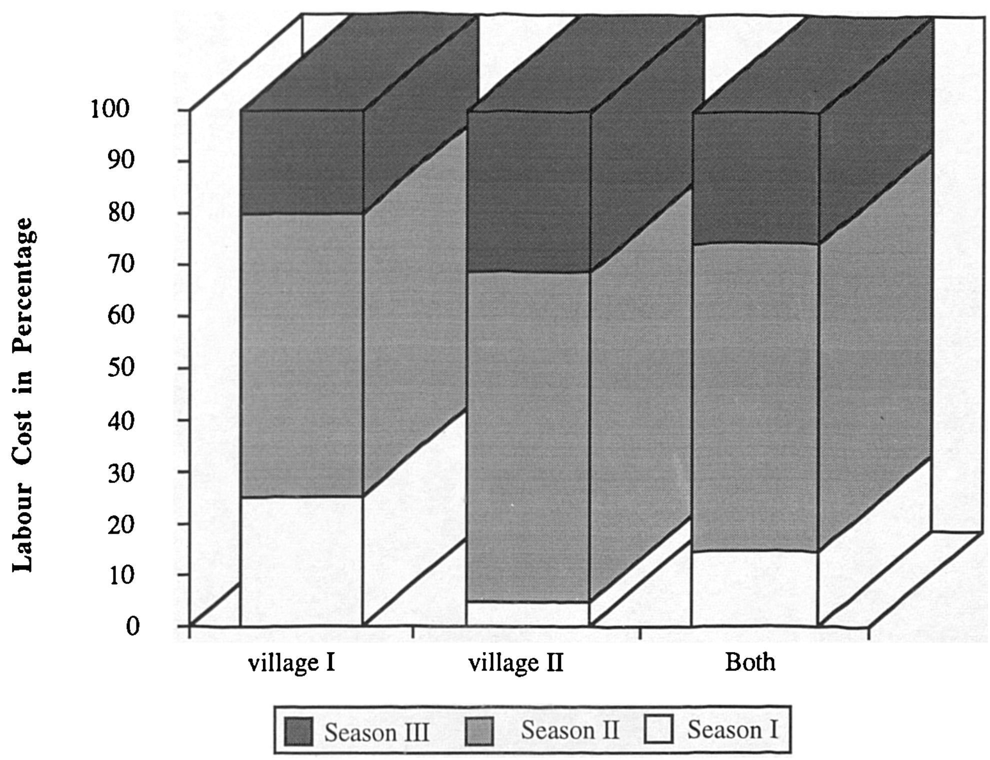
Table 3.10: Labour Costs

Labour Cost	Wage rate	Averages	Standard Error	Coefficient of variation
Season I	50.00	897.86	103.42	141.07
Season II	55.00	3574.07	1228.37	420.93
Season III	50.00	1540.70	164.72	130.94
Total	52.00	6012.63	1292.78	263.33

Source: Survey data, 1997.

Labour cost presented in Table 3.10 implies that labour cost is highest in Season II and lowest in Season I. The coefficient of variation in Season II among the three seasons' coefficients indicates considerable fluctuations in labour costs.

Figure 3.4: Labour Costs in three Seasons by Village



Source: Survey data, 1997.

The coefficient of variation in Season II, 420.93, is higher than the coefficient of variation of total labour cost, 263.33. The average labour cost of the farm households is Tk.6012.63. The F statistics can not explain any difference in the hired labour utilization pattern between the two villages although there is some significant difference in the pattern of family labour use. Labour costs in three seasons by Village are presented in Figure 3.4. The labour costs in Season II are greater than the labour costs in Season I and Season II together. This is the case for both Villages and in total. The peak labour demand in Season II explains the higher wage rate paid during this season. The analysis of variance shows there are no significant differences in labour costs between Village I and Village II.

3.3.6.2. Fertilizer Applications

Increased fertilizer use tends to accompany the adoption of HYVs. The scarcity of fertilizer, specially triple super phosphate (TSP), causes the reduction of productivity of rice crop. Fertilizers include cowdung, urea, TSP, single super phosphate (SSP), murate of potash (MP). Reduced land fertility due to crop intensification is remedied by higher fertilizer use. Fertilizer cost per acre of land is about Tk.1000 on average.

Table 3.11: Fertilizer Costs Statistics of Rice and Other Crops for Various Seasons by Village

	Rice					
	Village I		Village II		Both Villages	
	Average	C.V.	Average	C.V.	Average	C.V.
Season I	2526.93	103.60	3912.96	248.43	1456.36	153.84
Season II	4293.21	84.28	6604.54	559.94	5405.10	481.74
Season III	1922.65	90.768	3298.59	95.56	2590.48	101.30
Total	8742.79	83.66	13826.10	365.576	9451.94	284.19
	Others					
	Village I		Village II		Both Villages	
	Average	C.V.	Average	C.V.	Average	C.V.
Season I	595.54	419.60	0.00	0.00	297.77	599.86
Season II	88.21	756.98	0.00	0.00	44.10	1071.64
Season III	1377.30	387.78	812.37	367.75	1089.42	396.17
Total	2061.05	283.06	812.37	367.75	1431.30	325.31
Both Total	10803.84	84.18	11107.47	362.80	10883.24	280.31

Source: Survey data, 1997. Note: All statistics are expressed in Bangladesh Taka.

Most of farmers use less than the recommended dosage of fertilizers per acre; moreover they often have to use suboptimal combinations of fertilizer types because of the unavailability of some fertilizers. Table 3.11 reports the summary statistics of fertilizer costs. Rice, as the principal and staple crop in this region, accounts for 87 per cent of total fertilizer costs. IRRI and Boro paddy require more fertilizers per acre than Aman and Aus paddy. The average fertilizer cost for rice and other crops are Tk.9451.94 and Tk.1431.3 respectively (Table 3.11). The average cost of fertilizer in Season III (for the IRRI and Boro season) is Tk.2590.48, this amount is Tk.1456.36 in Season I. The coefficients of variation for fertilizer cost, and hence fertilizer use, of the farm households in Village II are higher than those in Village I. Farmers in Village II expend more money on fertilizer in Season II and Season III than farmers in Village I. Most of the farmers are not satisfied with this amount of fertilizers utilized perhaps because of high prices of quality fertilizers.

The F-ratio both at 5 percent and 1 percent level of significance rejects the hypothesis that fertilizer costs are different between the Villages. The survey results show that the mixing of quality and low quality inferior fertilizer and black marketing of fertilizer along with high prices are major problems of the fertilizer market. Only the well-off farmers and those who have good relations with dealers obtain quality fertilizers.

3.3.6.3. Irrigation Application

The irrigation schemes are operated by a committee nominated by farm households. The committee pays a fee to the Barind Multipurpose Development Authority (BMDA), the overall regulator of the irrigation schemes for spare parts and maintenance. Before the introduction of DTWs, Dhone, Swing Basket, Low Lift Pump (LLPs) and large-scale canal systems were used to irrigate wheat, Boro and IRRI Paddy fields. The expansion of irrigated production area is viewed as necessary for the adoption of HYV technology to spread (Ahmed and Sampath, 1992). A diesel pump is shown in Plate 2 in a rice field in Season II. A homestead area is shown in Plate 3 with some uncultivated land due to lack of an irrigation. 'Shubi' river is shown in Plate 4. This river was used for irrigation before the introduction of DTWs and STWs.

Plate 3: View of a homestead area with some uncultivated land.

Plate 4: View of a river during the rainy season.

All the STWs are installed privately and are not subject to restrictions. Irrigation management and infrastructure are poor and are differentiated by irrigation fuel, that is, by diesel and electricity. Some irrigation schemes are powered by diesel and some are by electricity. The operating costs per hour of diesel tubewells are higher than electric tubewells. Accordingly the irrigation price is higher in areas where diesel tubewells are predominant. The impact of agricultural technology on farm income and income distribution is a function of both the character of the technology itself and its interaction with the economic and social environment into which it is introduced (Goldman and Squire, 1982). With the expansion of irrigation a conflict of interest between irrigation users and water sellers is growing. Most farm households allege the prices of irrigation water are increasing at a greater rate than their product prices.

Table 3.12: Irrigation Application

	Total Area Irrigated with (acres)				
	DTWs	STWs	Others	Total	Percentage of cultivated area
Village I					
Season I	62.34	0.00	0.00	62.34	27.28
Season II	0.00	0.00	0.00	0.00	0.00
Season III	119.78	0.00	0.00	119.7	52.38
Total	182.12	0.00	0.00	182.12	79.7
Village I					
Season I	19.96	2.83	0.00	22.79	9.47
Season II	0.00	0.00	0.00	0.00	0.00
Season III	132.53	34.54	3.7	170.77	70.99
Total	152.49	37.37	3.7	193.56	80.47
Both					
Season I	82.30	2.83	0.00	85.13	18.15
Season II	0.00	0.00	0.00	0.00	0.00
Season III	252.31	35.70	3.70	291.71	62.19
Total	334.61	38.53	3.70	376.84	80.00

Source: Survey data, 1997.

Total area irrigated by different means of irrigation is presented in Table 3.12 which shows that irrigation covers 80 per cent of the cultivated area in the survey year 1997. The

High Barind has more potential for expanding the irrigated area. The divergence of farm production and subsistence demand will be greater unless irrigation area expands rapidly. The F statistics show there is no difference between Village I and Village II with respect to total irrigated area, total irrigation costs and area irrigated by DTWs, the dominant means of irrigation.

Table 3.13: Basic Statistic for Total Irrigated Area

	Mean	C.V.
Village I		
Season I	0.41	75.52
Season II	0.00	0.00
Season III	0.8	95.84
Total	1.21	84.34
Village II		
Season I	0.15	230.21
Season II	0.00	0.00
Season III	1.34	94.75
Total	1.29	94.93
Both		
Season I	0.57	125.76
Season II	0.00	0.00
Season III	1.94	97.95
Total	2.52	89.90

Source: Survey data, 1997.

The seasonal distribution of irrigation utilization and each household average irrigated area for the survey year are presented in Table 3.13. The coefficients of variation imply significant variation of total irrigation application in Village II implying that the seasonal character of farming may impact more on farm household income than in Village I.

3.3.7. Income of the Agricultural Farm Households

In the High Barind many farm households earn a significant part of their income from non-farm activities; these include on-farm non-agricultural products and off-farm income; on-farm non-agricultural products include fishing and water selling and off-farm income involves rickshaw pulling, waged labour and business. The major sources of farm income are HYV T.Aman paddy, Boro paddy, IRRI paddy crops and potatoes. Farmers have some other minor sources include mustard, dal and vegetables. The introduction of HYVs and irrigation has enhanced the physical productivity of family-owned assets. Considering land as a fixed factor and ignoring rent payment the net farm income is expressed as:

$$Y = \left(pQ - p_i I - p_f f\right) - w(L - F) + Y_0$$

where p and Q are price of paddy and amount of paddy, w is the wage rate, L is total hired labour input and F is family labour, p_i is irrigation price per acre and I is per acre area irrigated, p_f is fertilizer price and f is amount of fertilizer, Y_0 is other income. Variations in farm income embody changes in input use and productivity as well as in relative prices. Farm income derived from irrigation facilities varies among farmers with respect to their operational holding size and the number of working family members. Table 3.14 shows that farmers in Village I and Village II earn 84 per cent and 88 per cent of their total income from farm activities. Treating both villages together 87 per cent of total income comes from farm production.

Most of farm households can not subsist on their farm earnings alone. They depend on a range of non-farm income for their livelihood. Wage labourer, petty trading, fishing, rickshaw pulling are common forms of non-farm work; 16 per cent and 12 per cent of total income in Village I and Village II respectively derive from non-farm work activities. Poor farm households tend to catch fish from beels and rivers. Rivers were traditionally regarded as a common property resources. With the intensification of agriculture due to population pressure these have either been taken into private ownership or no longer exist; beels and river sides are being converted into paddy fields; this not only reduces the income that poor households derived from fishing, it also reduces their protein intake.

Table 3.14: Farm Income Pattern

Income	Mean	Per Capita	C. V.	Gini coefficient
Village I				
Total farm income	40787.40 (84.33)	7479.35 (84.33)	64.13	0.345
Total non-farm income	7581.00 (15.67)	1390.16 (15.67)	266.38	0.688
Total Income	48368.40	8869.51	62.29	0.343
Village II				
Total farm income	95881.67 (87.50)	20030.99 (90.48)	381.18	0.687
Total non-farm income	13691.22 (12.50)	2860.28 (9.52)	114.32	0.823
Total income	109572.89	22139.18	335.10	0.682
Both				
Total farm income	67902.81 (86.56)	13262.27 (86.56)	379.92	0.593
Total non-farm income	10544.83 (13.44)	2059.54 (13.44)	172.97	0.775
Total income	78447.64	15321.80	331.08	0.587

Source: Survey data, 1997. Figures in the parentheses represent respective percentages.

On the basis of different sources of income, total farm income, total non-farm income, total income and per capita income in Village I are lower than those of Village II as shown in Table 3.14. The coefficients of variation of Village I exhibit farm income in Village I is less variable and non-farm income is more variable. Coefficients of variation for the Village II show the reverse. The analysis of variance reveals F-tests reject the hypothesis that there are any significance differences concerning total farm income, total non-farm income, total non-farm cost and total overall income between Village I and Village II.

3.3.8 Livestock

The livestock resources are bullock, cow, goat, poultry, duck and pigeon. The breeds of animals are mostly local with a very few improved poultry and duck breeds. Livestock feeding practices are mostly traditional such as feed supply from internal sources with only large and medium farmers using purchased supplementary feeds from markets.

Table 3.15: Livestock Statistics

	Sum	Averages		Sum	Averages
Bullock	160	1.07	Goat	240	1.60
Cow	200	1.33	Chicken	1420	9.47

Source: Survey data, 1997.

Table 3.15 shows that average chicken holding is 9 but most of these are sold to provide monetary income.

Bullock is the main among draught animals. Bullocks have been traditionally used for land preparation, sowing, making drainage ditches, carrying agricultural products to home, threshing agricultural products and chrushing sugarcane for molasses. Cows are used mainly for breeding and for milking. Poor farm households use cows for both milking and cultivation.

3.3.9. Subjective Contents from Households obtained from PRA

Despite the government price-support policy a substantial proportion of farmers sell their products at lower prices than the minimum fixed by the government. This is due to the chains of intermediaries that exist between farmers and regional markets.

A farmer gives the following estimate of gross margin from one acre of Boro rice land. Table 3.16 shows that total cost of producing Boro rice on one acre of land is Tk.5600 and output value from rice produced is Tk.9450 and the gross margin is Tk.3850.

Table 3.16: Gross Margin from Boro Rice (per acre)

Output Value (per acre)	
Output per acre (maund)	45
Output price (per maund)	210
Output value per acre (Tk.)	9450
Variable Costs	
Plough Costs (Tk.)	600
Seed Costs (Tk.)	500.
Fertilizer Costs (Tk.)	1000
Irrigation Costs (Tk.)	1200
Labour Costs (Tk.)	2000
Pesticides Costs (Tk.)	300
Total Costs (Tk.)	5600
Gross Margin (Tk.)	3850

Source: Survey data, 1997.

PRA indicated that the increase in crop yield almost entirely depends on use of improved seeds, the foundation of crop production. Due to ignorance and superstitions the new rice varieties were not considered important by farmers until the introduction of HYV paddy. The acceptability of new varieties, fertilizer and irrigation utilization along with pesticides has been enhanced gradually with the introduction of HYV paddy. Farm households themselves preserve seeds for some cereals because farmers claim seeds supplied by Bangladesh Agricultural Development Corporation (BADC), entrusted with providing improved varieties through some dealers are impure. A number of farmers were of the view that home-preserved seeds are more of a better quality.

Many small farmers feel the necessity of credits to accommodate input costs of production. Apart from the time-consuming procedure and bribes involved in obtaining credit, higher interest rates and non-availability of credit, farm households need lands to secure credit. This effectively excludes a significant rent of the rural population from obtaining credit. PRA results exhibit that most of farms get credit from non-institutional sources which they would not like to mention probably because of high interest rates. Usual sources of credit include bank, landlords, shopkeepers, relatives and friends. Consequently faced with restricted access to government credit and restricted capital for farm investment, small farming households realise low yields, which in turn provide low incomes and the downward spiral continues until, pauperized, they are forced to give up their land. Because of low income most of farmers can not provide fees for their children in school leaving them illiterate and keeping them in a low standard of living.

Farm households also discussed some environmental problems. Declining groundwater levels occur in the Season III due to overexploitation of groundwater for IRRI and Boro paddy cultivation. During November-February most of drinking-water-supplied wells and tubewells dry up makes drinking water scarcity. During the monsoon bad water stocks in ponds and tanks. Farm households use animal dungs, leaves and twigs, crop residues for fuel due to lack of natural gas and electricity supply for domestic fuel. This contributes to the environmental degradation because these, if recycled back to the soils, would contribute to reduction of soil erosion, soil structure degradation and loss of soil nutrients.

Moreover, the lack of pasture has led to livestock grazing in the communal fields and land boundaries which increase the rate of land degradation.

3.4. Summary and Conclusion

In the study area a sample of 150 agricultural farm households from two villages is drawn using stratified sampling technique after selecting these villages applying purposive sampling methods keeping in view irrigation facilities. The cross-section primary data for the study are collected by interviewing farm households through personal interviewing technique. Data on farm output and output prices, input and input prices, socioeconomic characteristics and other information are mainly collected. Participatory Rural Appraisal technique complements the survey by helping to identify factors associated with inefficiency. The memory recall process by which farmers remember series of production-related data and information is, among other limitations, the most serious shortcoming of the survey method.

The comprehensive fieldwork survey, held in the period 1997 and collected data on farm households' socio-economic characteristics, land use pattern, resource availability and requirements, farming systems, input applications including labour utilization pattern, irrigation use and fertilizer application from two villages in Tanore thana in the High Barind Bangladesh, draws inference through analysis of variance that there are no considerable differences in two villages regarding their farming systems, input applications. The survey results show a low literacy rate of households with none of them having any formal training in agriculture, but most of the farmers are experienced with an average of 20 years farming experience. The majority of farmers, 34 per cent, have land holding between 2-3.99 acres and 30.67 per cent have land holding between 0-1.99 acres. Land resources have been over-exploited leaving no extra land to convert into cultivable land. The farming system is semi-subsistence in nature with rice, the principal food crop with a cropping intensity of 174.95 per cent. The farm households in both villages utilize most of the hired labour in Season II allocating about 55 per cent of total labour costs.

Irrigation technology has enhanced productivity making the production of IRRI and Boro paddy successful. The major problem identified by the farmers are: high prices of agricultural inputs and manufactured goods and low prices of agricultural products. Most of the farmers' incomes are too low to meet consumption requirements. Participatory Rural Appraisal technique identifies some socioeconomic problems of the study area, the High Barind Bangladesh.

Appendix: 3: Questionnaire

Personal Information

(1) Name of the Farmer : ______________________

(2) Village : ______________________

(3) Thana and district : ______________________

(4) Sex : (i) Male ☐ (ii) Female ☐

(5) Age : ______________________

(6) Marital status : ______________________

(i) Single ☐

(ii) Married ☐ No of wives: ______________

(iii) Divorced ☐

(iv) Separated ☐

(v) Widowed ☐

(7) Local Government Area: ______________________

Formal Education

		Years of schooling
(8) (i) Never attended school	☐	________
(ii) Below class Five	☐	________
(iii) Above class five but below SSC certificate	☐	________
(iv) SSC certificate	☐	________
(v) HSC certificate	☐	________
(vi) Commercial schooling	☐	________
(vii) First University Degree and above	☐	________
(viii) Technical education and other training	☐	________
(ix) Total no. of years of education	☐	________

Household Characteristics

(9) Number of children of the household:

(10) Number of people living in the household:

(11) Names, age, sex, education, income and relationship of the members of the household (children and others):

Name	Age	Sex	Education	income	relationship
(i)					
(ii)					
(iii)					
(iv)					
(v)					

Occupation

(12) Main occupation of the household: ______________________

(13) Other sources of income (enlist names): Time spent per year:

(i) ______________________ ______________________

(ii) ______________________ ______________________

(iii) ______________________ ______________________

(iv) ______________________ ______________________

(v) ______________________ ______________________

(14) Duration of the occupation: ______________________

Special Status

(15) The household:

(i) School teacher ☐

(ii) Official ☐

(iii) U.P. / Ward leader ☐

(iv) Ordinary member of the society ☐

(16) **Farm Size of the households**

(i) Total area owned : ________________

a) Homestead area : ________________

b) Forest area : ________________

c) Fallow area : ________________

(ii) Total cultivable land : ________________

(iii) Total land cultivated (1997): __________

(iv) No. of plot : ____________________

(v) Plot size (average) : ________________

(vi) Distance from homestead (average): _____

(17) Have you got land for sharecropping? Yes ❑ No ❑

(18) For sharecropping, what's the mode of payment?

(i) In cash ❑ Please specify: ____________________________

(ii) In kind ❑ Please specify: ____________________________

(19) Do you think of any major problem of sharecropping systems?

____

(20) Do you have any rented land? Yes ❑ No ❑

(21) For rented land, what's the mode of payment?

(i) In cash ❑ Please specify: ____________________________

(ii) In kind ❑ Please specify: ____________________________

22. **Land and Irrigation Information**

Season	CL	FL	AC	TCA	NCA	TIA	NIA	Problems
Season 1								
Season 2								
Season 3								

23. Production Side of the Household

(a) Homestead utilization

Homestead use	No of family labour	Hired labour	Hrs of work each season	Costs each season	Revenue each season
Crops	M F C	Male Female			
Season 1 a. b. c.					
Season 2 a. b. c.					
Season 3 a. b. c.					

Farm activities

(b) Land Utilization

Crops	Season 1	Season 2	Season 3	M. price P/A	Total Value
Tilled land a. b. c. d. e.					
Sharecrop-ping					
Total					

(i) Do you think your land is degraded? yes ❑ no ❑

(ii) If yes, could you tell the reasons?

(c) **Labour Utilization** (per acre)

	Family Labour	Hired labour	Hrs of work	Days of work	Wage rate	Cost of labours
	M - F - C	M F	Per day			
Season 1						
Season 2						
Season 3						
Total						
Pre. Exp.						

(i) Mention the peak period of farming : ______________________

(d) **Peak period**

Months	Operation	Work Hrs	Wage rate	Variation	L.Demand	Problems
Season 1						
Season 2						
Season 3						
Pre. exp.						

(e) **Irrigation**

	C.A.	Irrigation Area			Price per acre (Tk.)			Total	Crops
		DTW	STW	Others	DTW	STW	Others	Costs	irrigated
Seas 1									
Seas 2									
Seas 3									
Prev.									

(i) Who owns the means of Irrigation? ____________________________

(ii) Modes of Payments for Irrigation: ____________________________

(iii) Any fluctuations of Irrigation price: ____________________________

(iv) Your idea about productivity due to Irrigation: ____________________

(f) Water Sellers

Water Source	No of customers	Price	Quantity	Length of time	
Season 1					
Season 2					
Season 3					

(i) Competition among other water sellers: ____________________________

(ii) Basis of contract: ____

(iii) Controls over the irrigation project: ______________________________

(iv) Shortfalls in water supply: _____________________________________

(v) Limitations in supplying water: __________________________________

(g) **Fertilizer Utilization** (per acre)

	cowdung	T.S.P.	M.P.	Urea	Others	Pesticides
Season 1 a. b. c. d.						
Season 2 a. b. c. d. f. g.						
Season 3 a. b. c. d. e. f. h. i. i.						

(h) **Market Prices of Fertilizers**

	Cowdung per maund	T.S.P. per kg	M.P. per kg	Urea per kg	Others per kg	Pesticides per 100 ml
Season 1						
Season 2						
Season 3						

(i) Is there any price fluctuation? No ❑ yes ❑

(ii) if yes, please specify the reasons: ______________________

(j) **Yield and Revenue**

	Yield per acre	Market price	Revenue	Previous Rev.
Season 1 a. b. c. d. e. f.				
Season 2 a. b. c. d. e.				
Season 3 a. b. c. d. e. f. g. h.				
Total				

24. Non-farm activities

Activities	Hrs. of work/week	Days of work/month	Costs in this season	Income in the season	Is it easily available
Season 1 a.Labourer b.Fishing c.Business d.Rickshaw pulling e.Others					
Season 2 a.Labourer b.Fishing c.Business d.Rickshaw pulling e.Others					
Season 3 a.Labourer b.Fishing c.Business d.Rickshaw pulling e.Others					
Total					

25. **Consumption Side of the Household**

(a) **Consumption of own-produced goods and costs**

Seasons & Goods	Amount per week	Amount in this season	Marker price per kg	Total costs	Satisfiable consumption
Season 1 Rice Wheat Potatoes Dal Vegetables Chicken Fish Vegetables Others					
Season 2 Rice Wheat Potatoes Dal Beans Vegetables Chicken Others					
Season 3 Rice Wheat Potatoes Chicken Vegetables Others					
Total					

(i) Any other comments and suggestions about consumption:

(b) **Market-Purchased goods for consumption**

Seasons & Goods	Amount per week	Amount in this season	Market price per kg	Total costs	Problems & comments
<u>Season 1</u> Rice Wheat Vegetables Meat Fish Others					
<u>Season 2</u> Rice Wheat Vegetables Meat Fish Others					
<u>Season 3</u> Rice Wheat Vegetables Meat Fish Others					
Total					

(i) Do you think the market prices of the goods purchased are reasonable?

No ❑ Yes ❑

(ii) Why? ________________________________

26. **Livestock**

Types of Livestock	No. of livestock	Hrs p/w M. F. C	Diseases	Costs = L+M+F	Bought this year	Income
a.Bullock b.Cow c.Goat d.Chicken e.						
Total						

Participatory Rural Appraisal (PRA) Questions

(1) Do you think that the water extraction capacity of the diesel-operated irrigation pumps is lower? No ❑ Yes ❑

(2) Do you think this affects production? No ❑ Yes ❑

(3) What is your suggestion regarding this aspect?

____

(4) Any environmental problems due to irrigation?

No ❑ Yes ❑ specify: ______________________

(5) Your suggestions to reduce the problems: ______________________

(6) Any extension officials come to help you giving ideas about the different aspects of production systems? No ❑ Yes ❑

(7) If yes , how many times in a season? : ______________________

(8) Do you read any newspapers / magazines or see Television programme about farming? No ❑ Yes ❑ which sorts ______________________

(9) Do you think land is degrading in this region? No ❑ Yes ❑

(10) What are the main reasons for land degradation? ____________________________

____

(11) Do you think the following causes land degradation:

(i) Use cow dung for domestic fuel No ❑ Yes ❑

(ii) Use crop residues for domestic fuel No ❑ Yes ❑

(iii) Use leaves and twigs for domestic fuel No ❑ Yes ❑

(iv) Grazing domestic animal in the open field No ❑ Yes ❑

(v) Please tell any others ____

(12) Is there any social conflict in this area about any aspects? why ? ________________

____

(13) (i) Do you have any comment about the labour market?

(ii) Any suggestions for improvements of the market?

(iii) Any socioeconomic problems relating to labour market?

(14) (i) Do you think the market prices of your products are reasonable?

No ❑ Yes ❑

(ii) If no , what's the problem with it? ______________________________

(15) (i) Do you think non-farm income is reasonable?

No ❑ Yes ❑

(ii) Why? ______________________________________

(16) Does lack of credit affect your use of produced inputs e.g. seed, fertilizer, water use?

(17) Source of inputs? ____

Chapter 4

Production Functions and Efficiency: Some Theoretical Issues

4.1 Introduction

This Chapter discusses production functions and some related concepts which form the basis of measuring the efficiency of farms. We explain the basic concepts of technical, allocative and economic efficiency. The measurement of efficiency begins with Farrell (1957). The failure to produce the maximum output from a given input mix at minimum cost results in inefficiency. Inefficiency is explained by, *inter alia*, restricted access to technology, a lack of knowledge, restricted access to extension services, an inappropriate scale of production and sub-optimal allocation of resources. The efficiency of a farm consists of two components: technical and allocative efficiency. Technical efficiency concerns the ability of a farm to produce maximum output from a given set of inputs using existing technology; allocative efficiency reflects the ability of a farm to choose the inputs in optimal proportions, given their input prices; and a combination of these two measures provides a measure of economic efficiency. Thus economic efficiency concerns the ability of a farm to produce output at minimum cost; to obtain this minimum cost, the farm uses inputs in an efficient manner (technical efficiency) and chooses a cost-minimizing combination of inputs, given input prices and marginal productivities.

The plan of this Chapter is as follows: Second 2 considers production functions and related concepts; Section 3 presents the condition for the cost-minimizing input vectors; Section 4 discusses the measures of efficiency; and Section 5 summarizes.

4.2. Production Functions

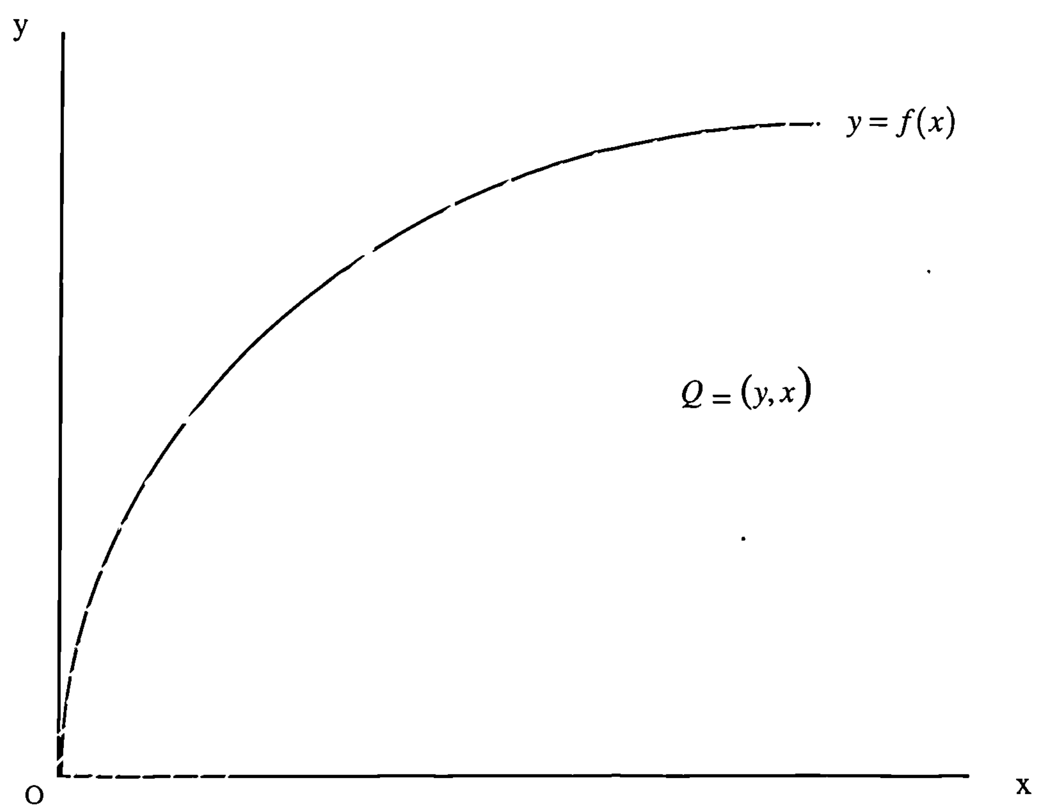
In microeconomic theory, the production function explains the technical or physical relationship between output and inputs. Specifically it shows the maximum output obtainable from a given set of inputs. Inputs are rates of resource use and output is the rate of production over a specific time period. Let $(x_1, x_2, ..., x_q)$ denote the inputs used in the production of output y; the production function can be written as:

$$y_i = f(x_1, x_2, ..., x_q) \tag{4.1}$$

This formulation excludes the possibility of technical inefficiency because output is a maximum for any level of inputs.

The production function is the boundary of a production set. Consider the Figure 4.1 where, for simplicity, one input x is used to produce a single output y. The production set, Q, denotes the technically feasible production set (y, x), i.e., $Q = (y, x)$. The shaded region in the Figure 4.1 represents the production set.

Figure 4.1: The production function



The production combinations which maximize y for given x or minimize x for given y are technically efficient combinations constitute the boundary to the production set $Q = (y, x)$. Thus the production function $y = f(x)$ is the set of technically efficient combinations, and all technically inefficient combinations belong to the interior of the production set.

Production functions involve concepts some of which are used in our analysis: the marginal productivities of the factors of production, output elasticities, the marginal rates of technical substitution, the elasticity of substitution, and returns to scale.

The marginal productivity of a factor is defined as the change in output for an infinitesimal change in a factor, holding all other factors constant. Mathematically, the marginal productivity of each input is obtained by the partial derivative of the production function with respect to this input. Consider the production function in (4.1), the marginal productivity of x_i is:

$$f_i = \frac{\partial f}{\partial x_i} \qquad (i = 1,2,3,\ldots,q)$$

The basic production theory concentrates on the range of output over which the marginal productivity is positive and diminishing, that is:

$$f_i > 0 \qquad \text{and} \qquad f_{ii} = \frac{\partial^2 f}{\partial x_i^2} < 0 \qquad (4.2)$$

where f_{ii} is the second order derivative.

Output elasticity measures the percentage change in output resulting from a percentage change in an input, holding all other inputs constant. Considering the production function in (4.1), it is defined as:

$$E_i = \frac{\partial f_i}{\partial x_i}\frac{x_i}{y_i}$$

It is a unit-free measure of marginal productivity (Chambers, 1988, p.18). If $E_i = 1$, a proportional increase in input i results in the same proportional increase in output; if $E_i >$ 1, the proportional increase in output is greater than the proportional increase in the input

i; and if $E_i < 1$, the proportional increase in output is less than the proportional increase in the input i.

An isoquant or production indifference curve is defined as the locus of all the technical efficient combinations of inputs which produce the same output. It shows the rate at which inputs are substituted in production holding output constant. For simplicity consider the two variable production function:

$$y = f(x_1, x_2) \tag{4.3}$$

The equation of an isoquant is obtained by the production function (4.3) when output is held constant at say y_0:

$$y_0 = f(x_1, x_2) \tag{4.4}$$

This represents the isoquant which displays all combinations of inputs that can be used to produce output y_0. It is illustrated in Figure 4.2. The slope of the isoquant at any point is derived by differentiating (4.4) implicitly with respect to one of the inputs, say x_1. This yields:

$$f_1 + f_2 \frac{dx_2}{dx_1} = 0$$

or

$$\frac{dx_2}{dx_1} = -\frac{f_1}{f_2}$$

The negative of the slope of an isoquant is the marginal rate of technical substitution (MRTS) which measures the rate at which inputs can be substituted, keeping output constant. The MRTS is not independent of units of measurement.

The elasticity of factor substitution is a better measure of factor substitution as it does not depend on the units of measurement. It is defined as the proportionate rate of change of the input ratio divided by the proportionate rate of change in MRTS:

$$\sigma = \frac{d(x_2/x_1)/(x_2/x_1)}{d(MRTS)/(MRTS)}$$

The larger the value of σ, the greater the degree of substitutability between the two factors. In general, we expect variable elasticity of substitution production function, however, some production functions have a constant elasticity of substitution. For example, a Cobb-Douglas function has a constant and unitary elasticity of substitution.

Returns to scale measures the proportional change in output as all inputs change by the same proportion. It is mathematically defined as:

$$\epsilon = \sum_{i=1}^{k} \frac{\partial y}{\partial x_i} \frac{x_i}{y} \tag{4.5}$$

Returns to scale delineates three important characterizations of productions. If $\epsilon = 1$, the production function shows constant returns to scale, that is, output increases by the same proportion as the inputs; if $\epsilon < 1$, the production function exhibits decreasing returns to scale, which implies that output increases less than proportionally with the increase in the inputs; and if $\epsilon > 1$, the production function reveals increasing returns to scale, which implies that output increases in greater proportion than the increase in the inputs. Returns to scale can be shown as the sum of the output elasticities.

The isocost line shows the rate at which inputs are exchanged in the market (their relative prices). It is the locus of all combinations of inputs that can be purchased with a given cost outlay, that is, the isocost line is the locus of input combinations that entails the same total cost C_0:

$$C_0 = p_1x_1 + p_2x_2 \tag{4.6}$$

where p_1 and p_2 are the input prices of x_1 and x_2. The isocost line is shown in Figure 4.2. Its slope is found by differentiating the isocost line:

$$\frac{dx_2}{dx_1} = -\frac{p_1}{p_2}$$

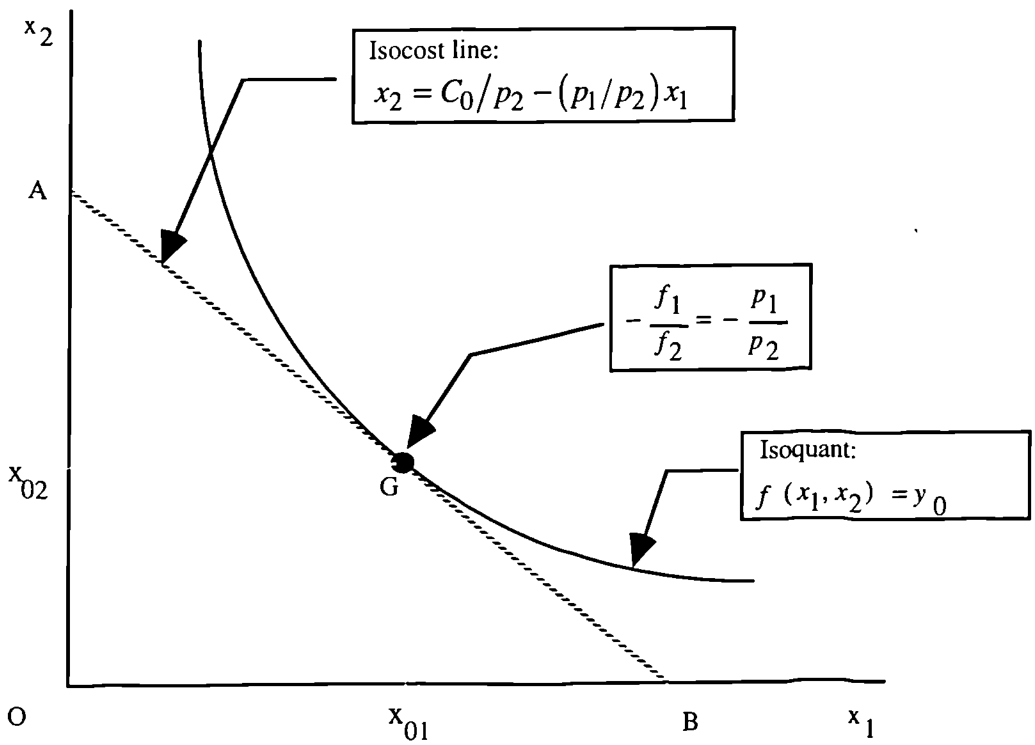
which is the negative of the ratio of the input prices.

4.3 Choice of Cost Minimizing Input Mix

We explain the problem of finding the least-cost input combination the farm chooses for the production of a specified level of output y_0. The choice of a cost-minimizing input combination deals with the issue of how cost can be minimized? We assume that output and the price of the inputs are given.

Cost minimization requires the tangency of the given isoquant with the lowest possible isocost line.

Figure 4.2: Isoquant, Isocost line and Cost Minimization



The farm minimizes its costs by using input combination (x_{01}, x_{02}) determined by the tangency point of the given isoquant y_0 with the isocost line AB.

We derive the conditions obtaining the least-cost combination of inputs by formulating a minimization problem. Minimize the cost in (4.6) subject to the output constraint in (4.4). Hence the Lagrangian function is:

$$Z = p_1 x_1 + p_2 x_2 + \lambda\left[y_0 - f(x_1, x_2)\right]$$

where λ is the Lagrangian multiplier. The input levels must satisfy the following simultaneous first-order conditions for a minimum cost:

$$p_1 - \lambda f_1 = 0 \tag{4.7}$$

$$p_2 - \lambda f_2 = 0 \tag{4.8}$$

$$y_0 - f(x_1, x_2) = 0 \tag{4.9}$$

Equations (4.7) and (4.8) give the conditions which ensure the least-cost input combination:

$$\frac{p_1}{f_1} = \frac{p_2}{f_2} = \lambda$$

that is, the input-price to marginal productivity ratio must be the same for each input. Alternatively we can write:

$$\frac{p_1}{p_2} = \frac{f_1}{f_2} \tag{4.10}$$

which indicates that the cost-minimizing input combination is obtained at a point where the slopes of the isoquant and the isocost line are equal. Obtaining the cost-minimizing input vector ensures allocative efficiency.

Equation (4.10) provides the first-order conditions for cost minimization. To ensure this minimum cost, the following second-order conditions must hold for the negative bordered Hessian:

$$\begin{vmatrix} 0 & f_1 & f_2 \\ f_1 & \lambda f_{11} & \lambda f_{12} \\ f_2 & \lambda f_{21} & \lambda f_{22} \end{vmatrix} = \lambda\left(f_{11}f_2^2 - 2f_{12}f_1f_2 + f_{22}f_1^2\right) < 0$$

Since the optimal value of λ is positive, the expression in parenthesis is negative if the production function is strictly quasi concave.

4.4 Measures of Efficiency

4.4.1. Defining Efficiency

The term 'efficiency' implies the success with which a farm best utilizes its available resources to produce maximum levels of potential outputs (Dinc et al., 1998). A farm is efficient if and only if it is not possible to increase output (decrease inputs) without more inputs (without decreasing output) (Cooper et al., 1995). Failure to obtain this potential maximum output results in inefficiency.

The neoclassical theory of production defines the production function based on the notion of efficiency that gives the maximum possible output for given amounts of inputs. It is not realistic to recognize this 'maximum' output simply by observing the actual amount of output unless the observed output is assumed to be a maximum: different farms produce different output levels even if they utilize the same input vector (Kumbhakar, 1994). Variations in output among farmers can be explained through differences in efficiency.

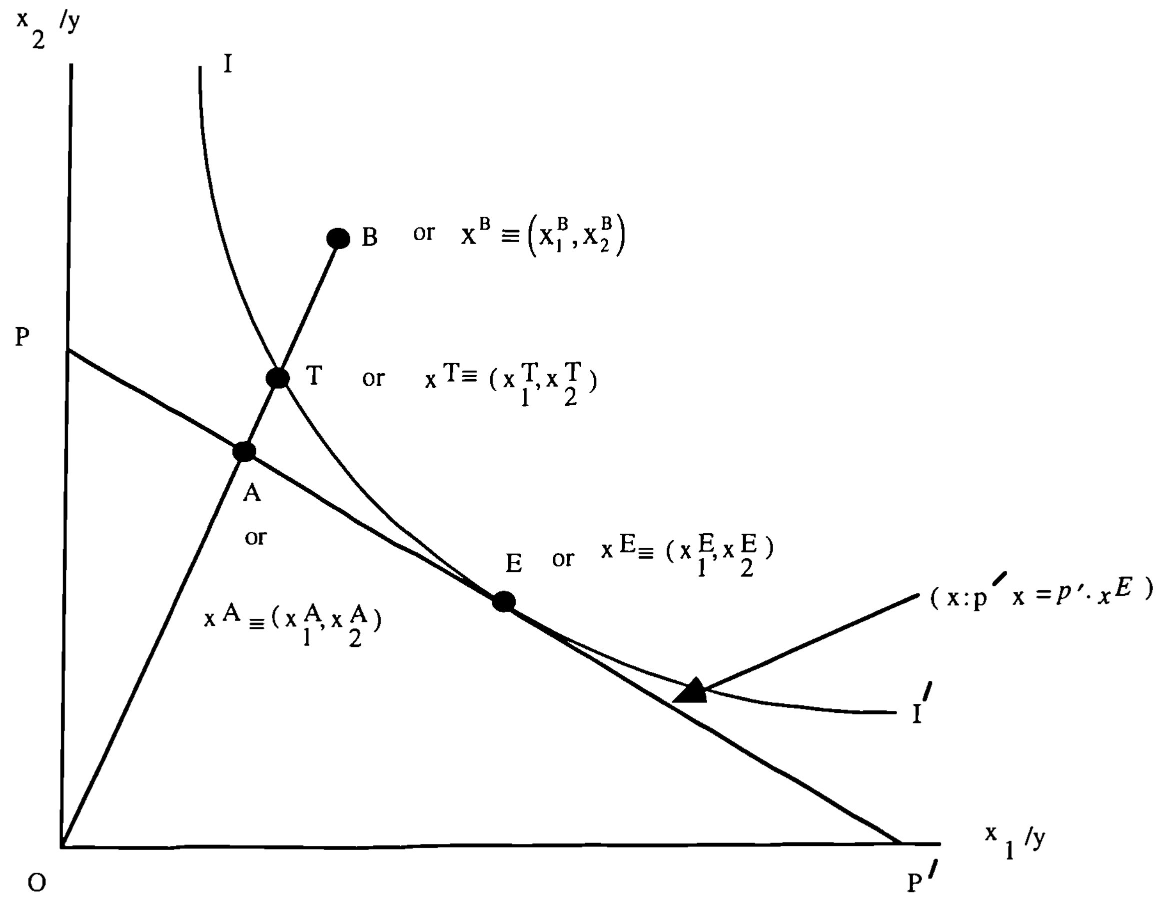
The production process of a farm may reflect technical inefficiency, allocative inefficiency or both. The concept of technical inefficiency is due to Farrell (1957). A farm is technically efficient if it produces a maximum output, given the amount of inputs and technology. Thus the production frontier is associated with the maximum obtainable level of output, given a level of inputs, or the minimum level of inputs required to produce a given output. In other words, it is the locus of maximum attainable output for each input mix. Technical inefficiency is attributed to a failure of the farm to produce the frontier level of output, given the quantities of inputs (Kumbhakar, 1994).

Allocative inefficiency arises if farms fail in allocating inputs which minimize the cost of producing a given output, given relative input prices. This results from not allocating inputs in the most efficient manner, i.e., there exists resource misallocation or allocative inefficiency. Failure in allocating resources optimally results in increased cost and decreased profit. In particular, a farm is said to be allocatively inefficient if the marginal rate of technical substitution between any two inputs is not equal to the corresponding ratio of input prices, that is, allocative inefficiency is when the farm fails to use cost-minimizing input mixes. This can be attributed to sluggish adjustment to price changes and regulatory constraints (Atkinson and Cornwell, 1994). Thus allocative efficiency is defined as the ability of farmers to adjust inputs and output to reflect relative prices, given the production technology. The distinction between technical and allocative efficiency provides four ways for explaining the relative performance of farms. First, a farm might show both technical and allocative inefficiency; second, it may be technically efficient but allocatively inefficient; third, it may display allocative efficiency but technical inefficiency; and fourth it may be both technically and allocatively efficient.

Economic efficiency combines technical and allocative efficiency that reflects the ability of a farm to produce output at minimum cost. Thus either one of the efficiencies may be necessary but not sufficient conditions to ensure economic efficiency for a farm. The simultaneous attainment of both efficiencies gives the sufficient condition to ensure economic efficiency (Ellis, 1988, p.66).

To explain diagrammatically the three concepts, consider the production activity of a farm, following Kopp and Diewert (1982). In Figure 4.3, assume that the farm uses two inputs x_1 and x_2 to produce a single output y, and that the production technology is summarized by a linearly homogeneous production function following Farrell. The frontier unit isoquant for this technology and an inefficient production activity are depicted by II' and B respectively. Along the ray OB, the production activity, denoted by T and defined by the intersection of line segment OB with the isoquant II', represents a technically efficient input combination as it lies on the frontier isoquant. The technical inefficiency of the farm producing at point B is represented by the distance TB because this is the amount by which both inputs could be proportionally reduced producing the same level of output. In percentage terms, this is usually written as the ratio TB/OB.

Figure 4.3: Measures of Technical, Allocative and Economic Efficiency



The technical efficiency of the farm operating at point B is expressed as:

$$TE = \frac{OT}{OB} = 1 - \frac{TB}{OB} = 1 \text{ - Technical inefficiency} \qquad (0 \leq TE \leq 1).$$

The farm operating at point T is fully technically efficient farm because it is located on the efficient and frontier isoquant and $TE = 1$. Given competitive factor markets and the relative factor prices $p = (p_1, p_2)$, the isocost line is PP' and the point E is the corresponding cost-minimizing point of input combination. The allocative efficiency of the farm operating at point B is defined as:

$$AE = \frac{OA}{OT} \qquad (0 \leq AE \leq 1)$$

The distance AT indicates the production cost which could be reduced if the farm produces at the technically and allocatively efficient point E instead of at the technically but not allocatively efficient point T.

Overall economic efficiency (EE) or productive efficiency is then defined as the product of technical and allocative efficiency measures:

$$EE = TE \times AE = \frac{OT}{OB} \times \frac{OA}{OT} = \frac{OA}{OB} \qquad (0 \leq EE \leq 1)$$

Hence the economic inefficiency is:

$$\text{Economic Inefficiency } = 1 - EE = 1 - \frac{OA}{OB} = \frac{AB}{OB}$$

where the distance AB represents the cost which can be reduced producing on economically efficient point of the isoquant.

Farrell's radial measures of efficiency are originally characterized by constant returns to scale and these measures have been generalized to less restrictive technologies by Fare and Lovell (1978) and Forsund and Hjalmarsson (1979).

4.4.2. Decomposition of Cost Function and Efficiency

Kopp and Diewert (1982) derive the cost function using neoclassical duality theory and calculate the technical, allocative and economic efficiencies of Farrell in terms of both vectors and cost. Adding a technical inefficiency effects term ζ_i in (4.1) provides the frontier function as (with n inputs):

$$\tilde{y}_i = f(x_1, x_2, \ldots, x_n)e^{-\zeta_i} \quad \text{and } \zeta_i \geq 0 \tag{4.11}$$

where $\tilde{y}_i$ is the observed output of the ith farm. Assume that the frontier cost function dual to (4.11) can be obtained analytically and expressed in general form as follows:

$$C_i = C(p_i, \tilde{y}_i)$$

where C_i represents the minimum cost of the ith farm associated with the production of output vector $\tilde{y}_i$, p_i the vector of input prices for the ith farm. Assume that $p > 0$ is the price vector which corresponds to the isocost line PP' in Figure 4.3. Applying Shephard's (1953) Lemma [4.1] results in the coordinates of the point E, x^E, as:

$$x^E = \frac{\partial C_i}{\partial p_i} = \left[\frac{\partial C(p_i, \tilde{y}_i)}{\partial p_1}, \ldots, \frac{\partial C(p_i, \tilde{y}_i)}{\partial p_n}\right] = x_i(p_i, \tilde{y}_i)$$

which is a system of cost-minimizing input demand functions, $\tilde{y}_i > 0$ is the output produced by the inefficient farm operating at point x^B. In Figure 4.3, point x^A can be obtained as the intersection of the line segment joining the origin to x^B with the cost plane PP', that is, we can equate:

$$\{x: p.x = px^A\} = \{x: p.x = C(p_i, \tilde{y}_i)\} \tag{4.12}$$

Solving the equality (4.12) provides $x^A = \eta^A x^B = \frac{C(p_i, \tilde{y}_i)}{p.x^B} x^B$ where $\eta^A = \frac{C(p_i, \tilde{y}_i)}{p.x^B}$.

Point x^T lies on the efficient production surface and hence we obtain:

$$x^T = \frac{\partial C(p_i^T, \tilde{y}_i)}{\partial p_i^T} \tag{4.13}$$

because this point lies on the efficient production surface for some set of input prices $p^T > 0$ and it also lies on the line segment joining the origin to x^B so that

$$x^T = \eta^T x^B \tag{4.14}$$

4.1 If the cost function $C(p, y)$ is differentiable in p, then there exists a unique vector of cost-minimizing input demands that is equal to the gradiant of $C(p, y)$. That is, if $x_i(p, y)$ is the ith, unique, cost-minimizing demand, then $x_i(p, y) = \partial C(p, y)/\partial p_i$.

where $\eta^T > 0$ is an unknown scalar. Equations (4.13) and (4.14) involve 2n equations in 2n+1 unknowns, x^T, p^T and η^T. Therefore an additional equation is required which is obtained by placing a normalization restriction on the input price vector p^T so that only relative prices are determined. By setting $p_n^T = 1$, the n equations $x^T / x_1^T = x^B / x_1^B$ can be obtained by dividing the left-hand side vector of (4.14) by x_1^T, the first component of x^T and by dividing the right-hand side vector by its first component $\eta^B x_1^B$ which eliminates η^T. The first of the n equations is the identity 1=1 and the remaining n-1 equations can be expressed as:

$$x_2 / x_1 = x_2^B / x_1^B$$

$$x_3 / x_1 = x_3^B / x_1^B$$

....

$$x_n / x_1 = x_n^B / x_1^B$$

where the superscript T has been dropped on the left-hand side of equations for notational brevity. The following n equations can be written by substituting equation $p_n^T = 1$ into (4.13) and omitting the superscripts as:

$$x_1 = \frac{\partial C(p_1, p_2, \ldots, p_{n\;1}, 1, \tilde{y}_i)}{\partial p_1}$$

$$x_2 = \frac{\partial C(p_1, p_2, \ldots, p_{n-1}, 1, \tilde{y}_i)}{\partial p_2}$$

....

$$x_n = \frac{\partial C(p_1, p_2, \ldots, p_{n-1}, 1, \tilde{y})}{\partial p_n}$$

Hence we obtain the 2n-1 equations in the 2n-1 unknowns $x_1, x_2, \ldots x_n$, and $p_1, p_2, \ldots, p_{n-1}$ that can be used to solve the vector for x^T, which represents the technically efficient input vector. Therefore, the technically efficient input vector for the ith farm, x_i^T for a given level of output, $\tilde{y}_i$, is obtained by solving simultaneously (4.11) and the input ratios $x_1 / x_i = k_i$ $(i > 1)$, where k_i is the ratio of observed inputs x_1 and x_i at output level $\tilde{y}_i$.

Substituting the input prices and output level of the farms into the system of minimum cost input demand functions provides the economically efficient (technically and allocatively efficient) input vector x_i^E in Figure 4.3 where the efficient isoquant is II' associated with the farm at point B and the observed input vector of the farm operating at point B is x_i^B. The technical efficiency of farm B is defined as the ratio of the two vector norms as:

$$TE = \frac{\left\|x_i^T\right\|}{\left\|x_i^B\right\|} \tag{4.15}$$

where x_i^T and x_i^B also represent the coordinates corresponding to the points T and B in Figure 4.3. Overall economic efficiency is:

$$EE = \frac{\left\|x_i^E\right\|}{\left\|x_i^B\right\|} \tag{4.16}$$

where x_i^E represents x_i^A and x_i^A also represents the coordinates corresponding to point A. Note that although point x_i^A is not technically feasible to produce output y, it still provides a cost equal to the technically feasible point x^E, given factor prices embodied in the isocost line. The allocative efficiency of farm at point B is defined as:

$$AE = \frac{EE}{TE} = \frac{\left\|x_i^E\right\|}{\left\|x_i^T\right\|} \tag{4.17}$$

The measures of efficiency defined in (4.15), (4.16) and (4.17) correspond to the original efficiency measures given by Farrell.

The efficiency measures are interpreted in terms of costs. The cost of the observed operating input mix is $x_i p_i$ while the technically efficient and economically efficient cost of production are estimated as $x_i^T p_i$ and $x_i^E p_i$ respectively, given the actual level of output. These three measures of production costs are the basis for calculating the technical efficiency (TE), economic efficiency (EE) and allocative efficiency (AE).

The technical efficiency of the farm operating at point B is defined as the ratio of total cost at point B to total cost at point T, that is, the ratio of technically efficient cost to observed cost:

$$TE = \frac{x_i^T p_i}{x_i p_i}$$

Economic efficiency is defined as the ratio of total cost at point E to the total cost at point B, that is, the ratio of frontier cost to observed cost:

$$EE = \frac{x_i^E p_i}{x_i p_i}$$

where $p.x^A$, $p.x^T$ and $p.x^B$ denote the total cost at point A, T and B respectively. The technically infeasible point x^A yields a cost outlay equal to that of the technically feasible point x^E.

Allocative efficiency (AE) is derived by combining technical efficiency and economic efficiency following Farrell (1957) and is defined as the ratio of total cost of A to total cost at point T, that is, as the ratio of frontier cost to technically efficient cost as follows:

$$AE = \frac{EE}{TE} = \frac{x_i^E p_i}{x_i^T p_i}$$

Observed cost is known and frontier cost is determined from the estimated cost function. The economic inefficiency of the ith farm $x_i p_i - x_i^E p_i$ can be decomposed into the technical inefficiency, $x_i p_i - x_i^T p_i$, and allocative inefficiency, $x_i^T p_i - x_i^E p_i$.

Forsund et al. (1980) discuss different efficiency measures relative to production and cost functions. Consider a farm which utilizes inputs $x_1, x_2, \ldots, x_n$ where input prices are $p_1, p_2, \ldots, p_n$, $(p_i > 0)$ to produce y. The production function $f(x)$ transforms the inputs into outputs to obtain the maximum level of output from various input vectors. Alternatively, it explains the minimum quantities of inputs to produce a given output level.

The efficient technology can also be represented by a cost function under some regularity conditions as:

$$C(y,p) = \min\left\{\sum_{i=1}^{n} p_i x_i \middle| f(x) \geq 0, x \geq 0\right\}$$

This cost function denotes the minimum cost to produce output y at input prices p_i. The functions $f(x)$ and $C(y,p)$ are known as frontier functions as they optimize farm objectives and restrict the possible values of the dependent variables.

As technical inefficiency refers to the inability to produce the maximum possible output from given input mix and technology, the production plan $\left(y^0, x^0\right)$ is technically inefficient if $y^0 < f\left(x^0\right)$; so cost is not minimized as excessive input use causes technical inefficiency, that is, $\sum_{i=1}^{n} p_i x_i \geq C\left(y^0, p\right)$. The farm is technically efficient if $y^0 = f\left(x^0\right)$ and hence $\sum_{i=1}^{n} p_i x_i = C\left(y^0, p\right)$. The production plan is said to be allocatively efficient if $f_i\left(x^0\right)/f_j\left(x^0\right) = p_i/p_j$ and allocatively inefficient if $f_i\left(x^0\right)/f_j\left(x^0\right) \neq p_i/p_j$, where $f_i(x)$ is the marginal product of the ith input, as allocative inefficiency arises from utilizing inputs in sub-optimal cost minimizing proportions. Thus if the farm is both technically and allocatively efficient, observed cost $\sum_{i=1}^{n} p_i x_i$ coincides with minimum cost $C\left(y^0, p\right)$ and $\sum_{i=1}^{n} p_i x_i > C\left(y^0, p\right)$ occurs due to technical or allocative inefficiency or both. This also implies that if the farm is technically and allocatively efficient, observed input utilization coincides with cost minimizing input demands.

4.5 Summary

This Chapter examines the concept of a production function which is the technical relationship between output and inputs which describes the maximum output obtainable from a given set of inputs. We also discuss some concepts which are used in our empirical analysis: marginal productivities, output elasticities, marginal rates of technical substitution and returns to scale; the marginal productivity of an input explains the change of output for a very small change in that input, holding all other input fixed; the output elasticity is a unit-free measure of marginal productivity and it describes the percentage change in output resulting from a percentage change in an input, keeping all other inputs constant; the marginal rate of technical substitution measures the rate at which inputs are substituted, holding output constant; the elasticity of substitution is unit free and measures the degree of substitution between inputs; returns to scale is the proportional change in output resulting from the proportional changes in all inputs and is shown as the sum of the output elasticities. We also explain that the farm obtains the least-cost combination of inputs at the point of ratio of input prices and the marginal rate of technical substitution are equal.

We discuss the concepts of efficiency. The efficiency implies the success with which a farm produces maximum output utilizing its available resources with minimum cost. In other words, a production function describes the maximum potential output from a given input mix and failure to achieve this output with minimum cost results in inefficiency. Efficiency consists of technical and allocative components: technical efficiency reflects the capability to produce maximum output with a given input mix utilizing the existing technologies; allocative efficiency reflects the capability to use cost-minimizing input proportions, given input prices; in other words, failure to produce with the least-cost input combination results in allocative inefficiency. The economic efficiency measure combines the two. Moreover, we discuss Kopp and Diewert (1982) cost decomposition procedure which obtains the dual cost function and input ratios; this dual cost function and input ratios in turn provide technically efficient and economically efficient cost; these costs along with observed cost produce measures of technical, economic and hence allocative efficiency: technical efficiency is the ratio of technically efficient cost to observed cost, economic efficiency is the ratio of frontier cost to observed cost, and allocative efficiency is the ratio of frontier cost to technically efficient cost.

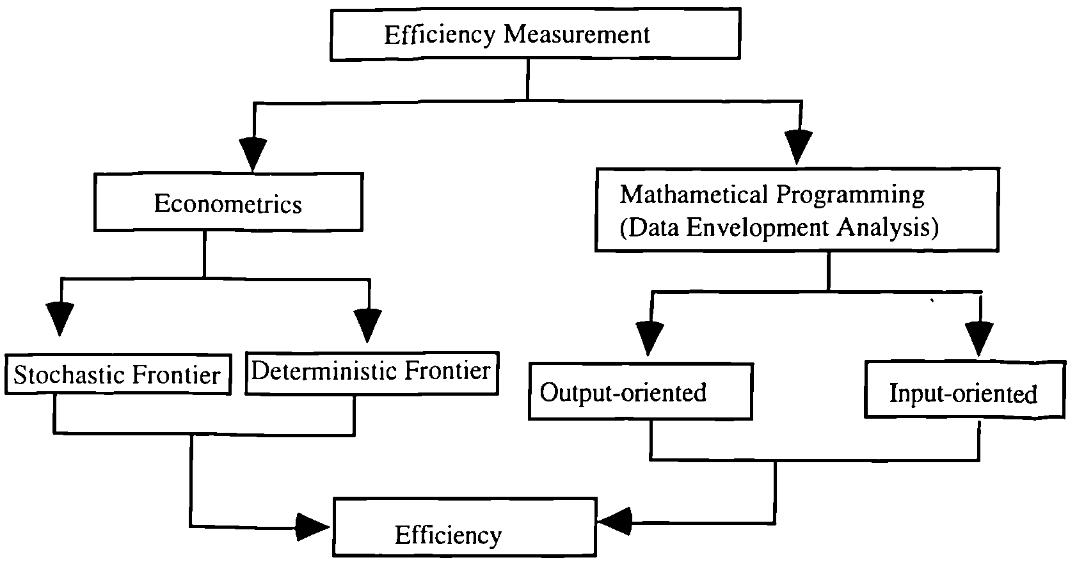
Chapter 5

The Stochastic Econometric Frontier Approach to Measuring Efficiency: Empirical Methodology

5.1. Introduction

The seminal work of Farrell (1957) on efficiency pioneered the development of different approaches to efficiency measurement. These approaches are summarized in Figure 5.1. The stochastic econometric frontiers and mathematical programming frontiers are the two main methods.

Figure 5.1: Approaches to Efficiency Measurement



The econometric approach includes both the stochastic econometric frontier (SF) and deterministic frontier (DF) models. The deterministic frontier approach does not allow for a stochastic random error component in the error term and hence is subject to the criticism that all deviations from the frontier are attributed to inefficiency. Accordingly, this Chapter focuses on the stochastic econometric frontier approach to measuring efficiency. Data Envelopment Analysis is discussed in Chapter 7.

Production function models estimated by OLS assume that farms maximize expected profit so that a stochastic error term, with zero mean, accounts for the difference between observed and expected output and are ascribed to factors outside the control of the farmers (Zellner et al., 1966). Thus, all farms are equally efficient. However, it is unlikely that all farms are equally efficient. Productivity differs because of differences in technology, the efficiency of the production process, and the environment in which production process happens (Lovell, 1993), and managerial ability (Dawson and Lingard, 1982). A frontier production function relaxes the assumption of equal efficiency and hence relaxes the assumption of stochastic error terms with zero means.

The approaches to the measurement of efficiency and the analysis of productivity stem from Farrell (1957) who measured technical efficiency by estimating a fully-efficient frontier production model using linear programming. The general stochastic frontier production function model, independently proposed by Aigner et al. (1977) and Meeusen and van den Broeck (1977), decomposes the composed error term into two components: a stochastic random error component and a technical inefficiency component. This approach is closer to the theoretical production function, which gives the maximum output from a given input mix, than the average production function and is more realistic than the deterministic frontiers of Farrell (1957) and Aigner and Chu (1968).

The stochastic approach attempts to distinguish the effects of stochastic noise from the effects of inefficiency. Addressing the stochastic noise problem associated with the deterministic frontier, and statistical hypothesis testing are the main strengths of the stochastic frontier approach; assumptions regarding the parametric functional form for the frontier technology and the distributional assumptions for the technical inefficiency term are its major drawbacks. Coelli (1995) provides a review and critique of the recent developments and applications of frontier techniques of efficiency measurement. Comprehensive reviews of the various stochastic frontier functions and econometric estimation of frontiers are provided also by Førsund et al. (1980), Schmidt (1985), Bauer

(1990), Battese (1992), Brevo-Ureta and Pinheiro (1993), Fried et al. (1993), and Greene (1993).

Most empirical applications of stochastic frontiers in agriculture have investigated the sources of farmer technical inefficiency using a two-stage approach (for example, Tadesse and Krishnamoorthy, 1997; Hallam and Machado, 1996; Parikh and Shah, 1994). The first stage estimates a stochastic frontier by maximum likelihood techniques and calculates the technical efficiency for each farm under the assumption that these inefficiency effects are identically distributed. It ignores the fact that the technical inefficiency is a function of farm-specific variables. Once technical inefficiency is estimated, it is further regressed in the second stage on a set of farm-specific factors that may explain differences in technical inefficiency among farms using OLS. The OLS results in the second step contradict the assumption of identically distributed inefficiency effects in the stochastic frontier model since the technical inefficiency - the dependent variable - is one sided (Kumbhakar et al., 1991). Thus, in the second stage, the estimated technical inefficiency effects are modelled as a function of some farm-specific characteristics which implies that inefficiency effects are not identically distributed unless the coefficients of the farm-specific factors are simultaneously equal to zero (Coelli et al. 1998). This two-stage approach, using a stochastic frontier, has been applied by Kalirajan (1981) and Pitt and Lee (1981) and by Heshmati and Kumbhakar (1997) for pseudo panel data, and Sharma et al. (1999) for cross sectional data. Timmer (1970) was one of the first to apply this approach albeit using covariance analysis in stage one.

The problems of this two-stage method can be addressed using a one-stage formulation. This specifies the technical inefficiency effects (Kumbhakar et al., 1991) and estimates the stochastic frontier and the inefficiency effects simultaneously, given appropriate distributional assumptions (Battese and Coelli, 1995). The simultaneous estimation of the stochastic production frontiers and models of technical inefficiency using maximum likelihood techniques has been proposed by Kumbhakar et al. (1991), Reifschneider and Stevenson (1991), Huang and Lui (1994), Battese and Coelli (1995). This one-stage approach is statistically consistent and leads to more efficient inference with respect to the parameters (Coelli and Battese, 1996). The approach has been applied empirically by, among others, Coelli and Battese (1996), Coelli (1996), Battese and Broca (1997), Ajibefun et al. (1996) and Seyoum et al. (1998). We now discuss the single-stage approach in more detail as it forms the basis of the empirical analysis reported in Chapter 6.

Suppose that the farm seeks to produce output at minimum cost (economic efficiency). To achieve this goal, it must utilize its inputs in the most efficient manner (technical efficiency) as well as choosing a combination of inputs which recognizes the relative input prices and marginal products (allocative efficiency) (Kopp and Diewert, 1982). As discussed in Chapter 4, the efficiency of a farm can be measured and divided into its technical and allocative components (Farrell, 1957). Farrell's frontier unit isoquant approach of a linearly homogeneous technology associates the deviation of output from the frontier isoquant with technical inefficiency, and deviation from the cost minimizing input ratios with allocative inefficiency.

Kopp (1981) generalizes Farrell's method to allow efficiency measurement with increasing returns-to-scale by using a frontier production function. Kopp and Diewert's (1982) approach, which requires direct estimation of the production frontier and then analytically derives the dual cost function equivalently, computes Farrell's generalized efficiency measures of Kopp (1981) (Taylor et al., 1986). This approach produces measures of technical efficiency and allocative efficiency. Economic efficiency is calculated as their product. Bravo-Ureta and Rieger (1991), Bravo-Ureta and Evenson (1994), Sharma et al. (1999) have applied this method.

This chapter is structured as follows: Section 2 discusses the stochastic frontier approaches to efficiency measurement; Section 3 discusses functional forms of production function and hypothesis tests which relate to the model in Section 2; Section 4 describes the dual cost decomposition techniques to measure technical efficiency (TE), allocative efficiency (AE) and economic efficiency (EE); and Section 5 summarizes.

5.2. Stochastic Frontier Production Models and Measurement of Efficiency

The general stochastic frontier production model is defined as:

$$y_i = f(x_i;\beta)e^{u_i} \tag{5.1}$$

$$u_i = \xi_i - \zeta_i, \quad i = 1,2,3,\ldots,q, \quad -\infty \le \xi_i \le \infty \text{ and } \zeta_i \ge 0.$$

where y_i represents the output of the ith farm, x_i denotes a vector of q inputs, and β denotes the parameters. The error term, u_i, is decomposed into a stochastic random disturbance and an asymmetric non-negative random error term. The stochastic random disturbances, ξ_i, the symmetric random errors, take account of measurement error and capture exogenous shocks and other factors not under the control of the farmers; ξ_i can take any real value and when added to the deterministic frontier, $f(x_i;\beta)$, gives rise to the stochastic frontier. The asymmetric non-negative random errors, ζ_i, which are called technical inefficiency effects, account for technical inefficiency in production. When $\zeta_i = 0$, the production function is the best-practice frontier which yields the maximum output given the inputs; and when $\zeta_i > 0$, output is less than this maximum due to technical inefficiency. The greater the quantity by which the actual output falls short of the stochastic frontier output, the higher the level of technical inefficiency. The observed differences in output can be attributed to either technical inefficiency or stochastic disturbances or both. A model without ζ_i is the average frontier model criticized by Farrell (1957). Further, a model without the random component, ξ_i results in a deterministic or full frontier model and can be estimated by linear programming techniques.

Assuming a probability density function for both ξ_i and ζ_i, we can estimate (5.1) by maximum likelihood methods. This approach yields a means by which we can statistically examine the sources of differences between the farmer's output and the frontier output by calculating the variance parameters which relate the variance of ξ_i to the composed variance of u_i (Kalirajan, 1981).

The variance parameters are expressed as:

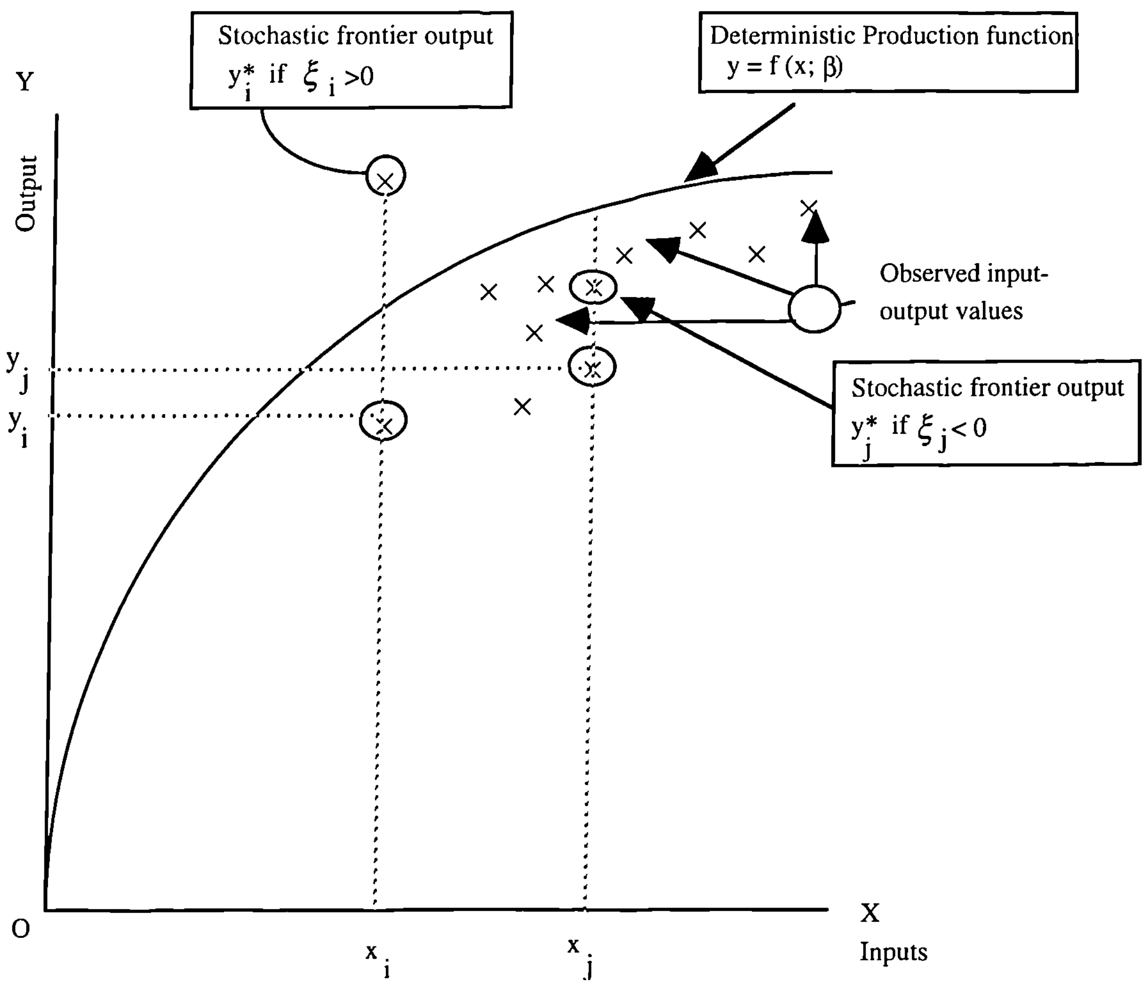
$$\sigma_u^2 = \sigma_\xi{}^2 + \sigma_\zeta{}^2, \qquad \gamma = \sigma_\zeta^2 / \sigma_u^2 \text{ and } 0 \le \gamma \le 1 \tag{5.2}$$

Battese and Corra (1977) define γ as the total variation of output from the production frontier which can be attributed to technical efficiency. If $\gamma \to 0$ then $\sigma_\zeta^2 \to 0$ and $\sigma_\xi^2 \to \sigma_u^2$, which implies that the symmetric error term ξ_i dominates the composed error term and output differs from the frontier output mainly due to measurement errors and the effect of other external factors on production. If $\gamma \to 1$ then $\sigma_\xi^2 \to 0$ and $\sigma_\zeta^2 \to \sigma_u^2$ which indicates that the asymmetric non-negative error term ζ_i dominates the composed error and the differences between output and frontier output can be attributed to differences in technical efficiency.

Figure 5.2 shows the stochastic frontier production function in which the activities of two farms, denoted by i and j, are illustrated following Battese (1992). The inputs are presented on the horizontal axis and the output is on the vertical axis. The frontier output, production function and observed outputs are also shown. The deterministic component of the frontier model is $y = f(x_i;\beta)$. Farm i utilizes inputs x_i to produce output y_i. The frontier output, y_i^*, of this farm exceeds the deterministic production output $f(x_i;\beta)$ because the systematic random error is associated with favourable farming conditions, i.e., $\xi_i > 0$. Farm j obtains output y_j using inputs x_j. The stochastic frontier output of this farm y_j^* is less than the corresponding deterministic output $f(x_i;\beta)$ because of unfavourable farming conditions and the systematic error component, $\xi_i < 0$. For both farms, the observed outputs are less than the corresponding frontier outputs, but the (unobservable) frontier outputs lie around the deterministic production function. The stochastic frontier outputs are of course not observed because the random errors are not observed. The observed outputs may be higher than the deterministic part of the frontier if the random errors are higher than inefficiency term. [5.1]

[5.1] If $\xi_i > \zeta_i$, both the observed and frontier values of output, y_i and $y_i^* = f(x_i;\beta)e^{\xi_i}$, would lie above the corresponding value of the deterministic production function which can easily be shown.

Figure 5.2: Frontier Production Function and Technical Efficiency



The technical efficiency of the ith farm is defined as the ratio of the observed output to the corresponding frontier output, given the levels of the inputs. The farm-specific technical efficiency, φ_i, can be measured as:

$$\varphi_i = \frac{y_i}{y_i^*} = \frac{f(x_i,\beta)e^{(\xi_i - \zeta_i)}}{f(x_i,\beta)e^{\xi_i}} = e^{-\zeta_i} \qquad 0 \le \varphi_i \le 1$$

Alternatively, φ_i is defined as the ratio of the mean of production (given x_i and ζ_i) to the corresponding mean of production if there is no technical inefficiency (Battese and Coelli 1988):

$$\varphi_i = \frac{E(y_i|x_i,\zeta_i)}{E(y_i|x_i,\zeta_i = 0)}$$

Again the systematic random error, ξ_i, is assumed to be independently and identically distributed with mean zero and variance, σ_ξ^2; and ζ_i are non-negative truncations of the $N\left(\mu, \sigma_\zeta^2\right)$ distribution, where:

$$\mu = z_i \delta_i \tag{5.3}$$

where z_i is a $(k \times 1)$ vector of variables which may influence efficiency and δ_i is an $(1 \times k)$ vector of parameters. Furthermore ξ_i and ζ_i are assumed to be independent of each other, i.e., $E(\xi_i, \zeta_i) = 0$ and also independent of the input vector x_i, i.e., $E(\xi_i, x_i) = E(\zeta_i, x_i) = 0$. The probability density function of the symmetric random error, ξ_i, is defined as:

$$f(\xi_i) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\xi_i^2}$$

The probability density function of the truncated normal distribution of technical inefficiency effects term is:

$$f(\zeta_i | \zeta_i \geq 0) = \frac{f(\zeta_i)}{\Pr(\zeta_i \geq 0)} \qquad \zeta_i \geq 0$$

$$= \frac{\left(2\pi\sigma^2\right)^{-1/2} e^{-(\zeta_i - \mu)/(2\sigma^2)}}{\left\{1 - \Pr(\zeta_i \leq 0)\right\}}$$

$$= \frac{1}{\left(\sqrt{2\pi}\right)\sigma_\zeta\left[1 - \Phi\left(-\mu/\sigma_\zeta\right)\right]} e^{-\frac{1}{2\sigma_\zeta^2}(\zeta_i - \mu)^2}$$

$$= \frac{\frac{1}{\sigma_{\zeta i}}\phi\left(\frac{\zeta_i - \mu}{\sigma_{\zeta i}}\right)}{1 - \Phi\left(-\mu/\sigma_{\zeta i}\right)} \tag{5.4}$$

where $\phi(\cdot)$ denotes the standard normal probability density function (pdf) and $\Phi(\cdot)$ represents the cumulative distribution function (cdf) for the standard normal random variable. The mean and variance of the truncated normal distribution of ζ_i are respectively (see Appendix 5.1 for details):

$$E(\zeta_i) = \mu + \frac{\sigma_\zeta \phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)}$$

and:

$$Var(\zeta_i) = \sigma_\zeta^2 \left[1 - \frac{\phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} \left\{ \frac{\mu}{\sigma_\zeta} - \frac{\phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} \right\} \right]$$

Measurements of the farm-specific efficiency, $e^{-\zeta_i}$, depends upon the decomposition of u_i, which is derived from the conditional expectation of $e^{-\zeta_i}$ given u_i, that is:

$$E\left(e^{-\zeta_i|u_i}\right) = \int_0^\infty \zeta_i f(\zeta_i|u_i) d\zeta_i,$$

where the conditional probability density function $f(\zeta_i|u_i) = \frac{f(\zeta_i,u_i)}{f(u_i)}$ and $f(\zeta_i,u_i)$ is the joint probability density function of ζ_i and u_i, and $f(u_i)$ is the probability density function of u_i. $E\left(e^{-\zeta_i|u_i}\right)$ can be re-expressed as:

$$E\left(e^{-\zeta_i|u_i}\right) = \int_0^\infty \zeta_i \frac{f(\zeta_i,u_i)}{f(u_i)} d\zeta_i.$$

Since $u_i = \xi_i - \zeta_i$, a joint probability density function of ξ_i and ζ_i can be derived as:

$$f(\xi_i,\zeta_i) = f(\xi_i)f(\zeta_i) = \frac{1}{\sqrt{2\pi}\sigma_\xi \sqrt{2\pi}\sigma_\zeta \left[1-\Phi(-\mu/\sigma_\zeta)\right]} e^{-\frac{1}{2}\frac{1}{\sigma_\xi^2}\xi_i^2} e^{-\frac{1}{2}\left(\frac{\zeta_i-\mu}{\sigma_\zeta}\right)^2} \tag{5.5}$$

The joint probability density function for ζ_i and u_i, $f(\zeta_i,u_i)$, can be derived by following the joint probability density function of ξ_i and ζ_i in (5.5) as:

$$f(\zeta_i,u_i)=\frac{e^{-\frac{1}{2}\left(\frac{\zeta_i-\mu}{\sigma_\zeta}\right)^2} e^{-\frac{1}{2}\left\{\frac{(u_i+\zeta_i)'(u_i+\zeta_i)}{\sigma_\xi^2}\right\}}}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi\left(-\mu\ \sigma_\zeta\right)\right]\sqrt{2\pi}\sigma_\xi} \tag{5.6}$$

Now $f(u_i)$ can be treated as the marginal probability density function of $f(\zeta_i,u_i)$ which can be expressed as:

$$f(u_i)=\int_0^\infty f(\zeta_i,u_i)d\zeta_i. \quad (\zeta_i\geq 0)$$

Substituting the value of $f(\zeta_i,u_i)$ we obtain:

$$f(u_i)=\int_0^\infty \frac{e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} e^{-\frac{1}{2}\left\{\frac{(u_i+\zeta_i)'(u_i+\zeta_i)}{\sigma_\xi^2}\right\}}}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi\left(-\mu\ \sigma_\zeta\right)\right]\sqrt{2\pi}\sigma_\xi}d\zeta_i$$

$$=\int_0^\infty \frac{e^{-\frac{1}{2}\left\{\frac{\sigma_\xi^2\zeta_i^2-2\zeta_i\mu\sigma_\xi^2+2\zeta_i u\sigma_\zeta^2+\zeta_i^2\sigma_\zeta^2}{\sigma_\xi^2\sigma_\zeta^2}\right\}} e^{-\frac{1}{2}\left\{\frac{u_i'u_i}{\sigma_\xi^2}+\frac{\mu^2}{\sigma_\zeta^2}\right\}}}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi\left(-\mu\ \sigma_\zeta\right)\right]\sqrt{2\pi}\sigma_\xi}d\zeta_i$$

$$=\int_0^\infty \frac{e^{-\frac{1}{2}\left\{\frac{\zeta_i^2(\sigma_\xi^2+\sigma_\xi^2)}{\sigma_\xi^2\sigma_\zeta^2}-2\zeta_i\left(\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2}{\sigma_\xi^2\sigma_\zeta^2}\right)\right\}} e^{-\frac{1}{2}\left\{\frac{u_i'u_i}{\sigma_\xi^2}+\frac{\mu^2}{\sigma_\zeta^2}\right\}}}{\sigma_\xi\sigma_\zeta\sqrt{2\pi}\left[1-\Phi\left(-\mu\ \sigma_\zeta\right)\right]\sqrt{2\pi}}d\zeta_i$$

$$=\int_0^\infty \frac{e^{-\frac{1}{2}\left\{\frac{\zeta_i^2}{\left(\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2\right)}-2\zeta_i.\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}+\left(\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}{\sqrt{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}}\right)^2\right\}}}{\sigma_\xi\sigma_\zeta\sqrt{2\pi}\left[1-\Phi\left(-\mu\ \sigma_\zeta\right)\right]\sqrt{2\pi}}$$

$$\times\ e^{-\frac{1}{2}\left(\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}{\sqrt{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}}\right)^2} e^{-\frac{1}{2}\left\{\frac{u_i'u_i}{\sigma_\xi^2}+\frac{\mu^2}{\sigma_\zeta^2}\right\}}d\zeta_i$$

Thus: $$f(u_i)=\int_0^{\infty}\frac{\dfrac{1}{\sqrt{\dfrac{\sigma_\xi^2\sigma_\zeta^2}{\sigma_\xi^2+\sigma_\xi^2}}\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\zeta_i-\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2}{\sigma_\xi^2+\sigma_\xi^2}}{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}\right)^2}e^{-\frac{1}{2}\left\{\frac{u_i'u_i}{\sigma_\xi^2}+\frac{\mu^2}{\sigma_\zeta^2}-\left(\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}{\sqrt{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}}\right)\right\}}}{\sqrt{\sigma_\zeta^2+\sigma_\xi^2}\left[1-\Phi\left(-\mu\ \sigma_\zeta\right)\right]\sqrt{2\pi}}d\zeta_i$$

(5.7)

We can derive the probability density function of u_i as:

$$f(u_i)=\frac{\left[1-\Phi\left(-\mu_i^*/\sigma_i^*\right)\right]e^{\frac{1}{2}\left\{\left(u_i'u_i/\sigma_\xi^2\right)+\left(\mu/\sigma_\zeta\right)^2-\left(\mu_i^*/\sigma_i^*\right)^2\right\}}}{\sqrt{2\pi}\left[\sigma_\zeta^2+\sigma_\xi^2\right]^{1/2}\left[1-\Phi\left(-\mu/\sigma_\zeta\right)\right]} \tag{5.7a}$$

where $\mu_i^*\equiv\dfrac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2}{\sigma_\xi^2+\sigma_\zeta^2}$ and $\sigma_i^{*2}\equiv\dfrac{\sigma_\zeta^2\sigma_\xi^2}{\sigma_\xi^2+\sigma_\zeta^2}$.

Using (5.6) and (5.7) the conditional distribution of ζ_i, given the random vector, u_i, is:

$$f(\zeta_i|u_i)=\frac{f(\zeta_i,u_i)}{f(u_i)}$$

$$=\frac{e^{-\frac{1}{2}\left(\frac{\zeta_i-\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2}{\sigma_\xi^2+\sigma_\xi^2}}{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}\right)^2}e^{-\frac{1}{2}\left\{\frac{u_i'u_i}{\sigma_\xi^2}+\frac{\mu^2}{\sigma_\zeta^2}-\left(\frac{\mu\sigma_\xi^2-u_i\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}{\sqrt{\sigma_\xi^2\sigma_\zeta^2/\sigma_\xi^2+\sigma_\xi^2}}\right)\right\}}}{\sqrt{2\pi}\left(\sigma_\zeta\sigma_\xi\ \sqrt{\sigma_\zeta^2+\sigma_\xi^2}\right)\left[1-\Phi\left(-\mu_i^*/\sigma_i^*\right)\right]e^{\frac{1}{2}\left\{\left(u'u/\sigma_\xi^2\right)+\left(\mu/\sigma_\zeta\right)^2-\left(\mu_i^*/\sigma_i^*\right)\right\}}}$$

$$=\frac{e^{-\frac{1}{2}\left(\frac{\zeta_i-\mu_i^*}{\sigma_i^*}\right)^2}e^{-\frac{1}{2}\left\{\left(u_i'u_i/\sigma_\xi^2\right)+\left(\mu/\sigma_\zeta\right)^2-\left(\mu_i^*/\sigma_i^*\right)^2\right\}}}{\sigma_i^*\sqrt{2\pi}\left[1-\Phi\left(-\mu_i^*/\sigma_i^*\right)\right]e^{-\frac{1}{2}\left\{\left(u'u/\sigma_\xi^2\right)+\left(\mu/\sigma_\zeta\right)^2-\left(\mu_i^*/\sigma_i^*\right)^2\right\}}}$$

Thus: $$f(\zeta_i|u_i)=\frac{e^{-\frac{1}{2}\left(\frac{\zeta_i-\mu_i^*}{\sigma_i^*}\right)^2}}{\sqrt{2\pi}\sigma_i^*\left[1-\Phi\left(-\mu_i^*/\sigma_i^*\right)\right]} \qquad \zeta_i \geq 0,$$

This is known as the probability density function of the positive truncation of the $N\left(\mu_i^*,\sigma_i^{*2}\right)$ distribution. Now the conditional expectation of $e^{-\zeta_i}$, given u_i, is defined as:

$$E\left[e^{-\zeta_i|u_i}\right]=\int_0^\infty e^{-\zeta_i}f\left(\zeta_i|u_i\right)d\zeta_i$$

$$=\int_0^\infty e^{-\zeta_i}\frac{e^{-\frac{1}{2}\left(\frac{\zeta_i-\mu_i^*}{\sigma_i^*}\right)^2}}{\sqrt{2\pi}\sigma_i^*\left[1-\Phi\left(-\mu_i^* \ \sigma_i^*\right)\right]}d\zeta_i$$

Applying standard integral calculus, the minimum-mean-square-error predictor of the technical efficiency of the ith farm, $\varphi_i = TE_i = e^{-\zeta_i}$ is obtained as:

$$\varphi_i=\int_0^\infty \frac{e^{-\frac{1}{2}\left\{\frac{\zeta_i^2-2\zeta_i\mu_i^*+\mu_i^{*2}+2\zeta_i\sigma_i^{*2}}{\sigma_i^{*2}}\right\}}}{\sqrt{2\pi}\sigma_i^*\left[1-\Phi\left(-\mu_i^* \ \sigma_i^*\right)\right]}d\zeta_i$$

$$=\int_0^\infty \frac{e^{-\frac{1}{2}\left\{\frac{\zeta_i^2}{\sigma_i^{*2}}-\frac{2\zeta_i(\mu_i^*-\sigma_i^{*2})}{\sigma_i^{*2}}+\frac{(\mu_i^*-\sigma_i^{*2})^2}{\sigma_i^{*2}}\right\}}e^{\frac{1}{2}\left\{\frac{(\mu_i^*-\sigma_i^{*2})^2}{\sigma_i^{*2}}-\frac{\mu_i^{*2}}{\sigma_i^{*2}}\right\}}}{\sqrt{2\pi}\sigma_i^*\left[1-\Phi\left(-\mu_i^* \ \sigma_i^*\right)\right]}d\zeta_i$$

$$=\int_0^\infty \frac{e^{-\frac{1}{2}\left\{\frac{\zeta_i-(\mu_i^*-\sigma_i^{*2})}{\sigma_i^*}\right\}^2}e^{\frac{1}{2}\left\{\frac{\mu_i^{*2}}{\sigma_i^{*2}}-\frac{2\mu_i^*\sigma_i^{*2}}{\sigma_i^{*2}}+\frac{(\sigma_i^{*2})^2}{\sigma_i^{*2}}-\frac{\mu_i^{*2}}{\sigma_i^{*2}}\right\}}}{\sqrt{2\pi}\sigma_i^*\left[1-\Phi\left(-\mu_i^* \ \sigma_i^*\right)\right]}d\zeta_i$$

$$=\int_0^\infty \frac{\frac{1}{\sqrt{2\pi}\sigma_i^*}e^{-\frac{1}{2}\left\{\frac{\zeta_i-(\mu_i^*-\sigma_i^{*2})}{\sigma_i^*}\right\}^2}d\zeta_i}{\left[1-\Phi\left(-\mu_i^*/\sigma_i^*\right)\right]}e^{\left\{-\mu_i^*+\frac{1}{2}\sigma_i^{*2}\right\}}$$

$$\therefore \qquad \varphi_i = \left[\frac{1-\Phi\left\{\sigma_i^* - \left(\mu_i^*/\sigma_i^*\right)\right\}}{1-\Phi\left(-\mu_i^*/\sigma_i^*\right)} \right] e^{\left(-\mu_i^* + \frac{1}{2}\sigma_i^{*2}\right)} \tag{5.8}$$

which produces the measure of technical efficiency given the specification of the frontier production function model and the inefficiency effects model. Technical inefficiency is estimated by $1 - E\left\{e^{-\zeta_i|u_i}\right\}$. The efficiency index of each farm, $e^{-\zeta_i}$, is constructed using (5.8). The mean technical efficiency of all farms in the sample, $\overline{\varphi}$, is obtained as:

$$\overline{\varphi} = \left[\frac{1-\Phi\left\{\sigma^* - \left(\mu^*/\sigma^*\right)\right\}}{1-\Phi\left(-\mu^*/\sigma^*\right)} \right] e^{\left(-\mu^* + \frac{1}{2}\sigma^{*2}\right)}.$$

Instead of using the truncated normal distribution defined in (5.4), we can assume that the technical inefficiency term is half-normally distributed, a special case of the truncated normal distribution, so that:

$$f(\zeta_i) = \frac{1}{\sigma_\zeta \sqrt{\frac{1}{2}\pi}} e^{-\frac{1}{2\sigma_\zeta^2}\zeta_i^2} \tag{5.9}$$

The farm-specific technical efficiencies and mean technical efficiency are obtained respectively as:

$$\varphi_i = E\left[e^{-\zeta_i|u}\right] = 1 - \Phi\left(\sigma_i^*\right) e^{\frac{1}{2}\sigma_i^{*2}} \tag{5.10}$$

and

$$\overline{\varphi}_i = 1 - \Phi\left(\sigma^*\right) e^{\frac{1}{2}\sigma^{*2}}$$

(Jondrow et al., 1982), which is equivalent to substituting $\mu = 0$ in (5.8).

The Frontier 4.1 program (Coelli, 1996) calculates the maximum likelihood estimator of the predictor for the technical efficiency that is based on the conditional expectation of

$e^{-\zeta_i}$ given the composed error term of the stochastic frontier production model (Battese and Coelli, 1988). The parameters of the coefficients of stochastic frontier model, β, and the technical inefficiency effects model, δ_i, along with the variance parameters are also estimated. The log-likelihood function for the sample observations, given (5.1), (5.2) and (5.4), is:

$$L(\Omega^*, y) = \sum_{i=1}^{n} \ln\left[1 - \Phi\left(-\mu_i^*/\sigma_{i\zeta}^*\right)\right] - \frac{1}{2}\sum_{i=1}^{n}\left[\{y_i - f(x_i;\beta)\}'\{y_i - f(x_i;\beta)\}/\sigma_\xi^2\right] - \frac{1}{2}n\left(\mu/\sigma_\zeta\right)^2$$

$$+\frac{1}{2}\sum_{i=1}^{n}\left(\mu_i^*/\sigma_{i\zeta}^*\right)^2 - \frac{1}{2}n\ln(2\pi) - \frac{1}{2}n\ln\left(\sigma_\zeta^2 + \sigma_\xi^2\right) - n\ln\left[1 - \Phi\left(-\mu/\sigma_\zeta\right)\right]$$

where $\Omega^* \equiv \left(\beta', \sigma_\xi^2, \sigma_\zeta^2, \mu\right)'$ (see Appendix 5.2 for details).

The principal drawbacks of this approach are assumptions about the distributions of technical inefficiency and the random term and the nonexistence of an *a priori* justification of choosing the distributional form of the random noise (Coelli, 1995).

5.3 Functional Forms of Production Function and Hypothesis Tests

5.3.1. Functional Forms

Cobb-Douglas Production Function: Several specifications of the production function, e.g., Cobb-Douglas, translog, etc. have been developed. The Cobb-Douglas production function has been widely used in econometric analysis:

$$\ln y_i = \beta_0 + \sum_{i=1}^{q} \beta_i \ln x_i \qquad (5.11)$$

where y_i = output, β_0 is an "efficiency parameter", i.e., an indicator of the state of technology, x_i = inputs of production, ln = natural logarithm, β_i $(i = 1,2,3,\ldots,q)$ are the output elasticities with respect each input and the production function is homogeneous of degree $\sum_{i=1}^{n}\beta_i$. Differentiating (5.11) yields the marginal product for input i, for example:

$$\frac{\partial y}{\partial x_i}=\frac{\beta_i y_i}{x_i},$$

which is strictly positive for $x_i > 0$. The marginal rate of technical substitution is:

$$MRTS_{i,j}=\frac{\partial y_i/\partial x_i}{\partial y_i/\partial x_j}=\frac{\beta_i}{\beta_j}\frac{x_j}{x_i}$$

The elasticity of substitution is $\sigma = 1$ for any input combination and all levels of output, which restricts the flexibility of this functional form. The returns to scale is $\sum_{i=1}^{n}\beta_i$.

Translog Production Function: A production function which does not restrict the elasticity of substitution, σ, is the transcendental logarithmic (translog) form of the production function (Christensen et al., 1973) which can be written as:

$$\ln y=\beta_o+\sum_{i=1}^{q}\beta_i \ln x_i+\frac{1}{2}\sum_{i=1}^{q}\sum_{j=1}^{q}\beta_{ij}\ln x_i \ln x_j \qquad (5.12)$$

This function does not presume any restriction on production technology. In (5.12) $\sum_{j=1}^{q}\beta_{ij}$ is included to make the marginal rate of technical substitution homogeneous of degree zero in inputs which yields a Kmenta approximation of CES production function (Kim, 1992). If $\sum_{i=1}^{q}\beta_i = r$ and $\sum_{i=1}^{q}\beta_{ij}=0$, (5.12) is homogeneous of degree r, and if $r=1$ it is linearly homogeneous. The translog function in (5.12) is additively separable if $\beta_{ij}=0\,(i\neq j)$. Cobb-Douglas is a special case of the translog function if $\beta_{ij}=0$.

Differentiating (5.12) yields the marginal product for input i, for example:

$$\frac{\partial y}{\partial x_i} = \frac{y}{x_i}\left[\beta_i + \sum_{i=1}^{q} \beta_{ij} \ln x_j\right]$$

The elasticity of scale = $\partial \ln y \Big/ \sum_{i=1}^{q} \partial \ln x_i$, depends on the factor proportions and the levels of production. The elasticity of substitution of this production function is unbounded.

5.3.2. Hypotheses Testing

Given the specification of the functional form of the production function, such as, the Cobb-Douglas in (5.11) or the translog in (5.12) testing which of the two models fits best is important. In particular, the Cobb-Douglas function is nested in the translog function such that a nested hypothesis restriction can be formulated:

$$\beta_{ij} = 0 \quad (i = 1,2,3,\dots,n, \;\; j = 1,2,3,\dots,n \text{ and } i \neq j)$$

By estimating the translog function, we test the restrictions in (5.12); if this restriction is not accepted we simply obtain (5.11). The null hypothesis is constructed against the alternative hypothesis as:

H_o: = the Cobb-Douglas production model is appropriate (5.13)

H_A: = the translog production model is appropriate

If the null hypothesis is not accepted the translog function is an adequate representation of the sample data.

The standard OLS function assumes that all farms operate on the technical efficient frontier and non-negative technical inefficiency effects are zero (Coelli, 1996). We test the hypothesis of no technical inefficiency against the alternative hypothesis as:

H_o: = no technical inefficiency exists (5.14)

H_A: = technical inefficiency exists

If the null hypothesis is not rejected, the technical inefficiency error term, ζ_i, is removed from the stochastic frontier production model in (5.1). This is equivalent to imposing the restriction that $\delta_i = 0$ $(i = 0,1,2,3,...,n)$ in (5.3) and $\gamma = 0$ in (5.2) and shows that the average production function (standard OLS function) is an adequate representative for the data. This joint hypothesis test also tests if both random and deterministic components of the inefficiency error term are not significant and specifies if the inefficiency effects are non-stochastic.

Now consider whether the farm-specific individual explanatory variables in the stochastic frontier model for the inefficiency effects have no significant effects on the level of inefficiency. The following joint test is specified which is equivalent to imposing the restriction in (5.3) that:

$$H_o: \delta_i = 0 \qquad (i = 1,2,3,...,n) \qquad (5.15)$$
$$H_A: \delta_i \neq 0$$

The null hypothesis states that the coefficients of the explanatory variables in the stochastic model for the inefficiency effects model in (5.3) are zero. If this null hypothesis is not rejected, the farm-specific explanatory variables have no significant effects on the level of inefficiency.

We test the hypothesis about the distribution of the random variable associated with the existence of technical inefficiency. The technical inefficiency component requires a distributional assumption for estimation. The most commonly used distributional forms are half-normal and truncated normal. The generalization of the half-normal distribution is the truncated normal distribution which is derived by the truncation at zero of the normal distribution with mean μ and variance σ^2 (see Appendix 5.1 for details). If the normal distribution is truncated at $\mu = 0$, then it is a half-normal distribution. Given the specifications of truncated normal in (5.4) or half-normal distribution in (5.9) for the technical inefficiency effects, half-normality of the technical inefficiency effects is tested by formulating the null and alternative hypotheses as:

$$H_o\colon \mu = 0 \qquad (5.16)$$
$$H_A\colon \mu \neq 0.$$

If the null hypothesis is not accepted, the half-normal distributional assumption is an inadequate representation for the technical inefficiency effects term.

The null hypotheses in (5.13), (5.14), (5.15) and (5.16) are tested using the generalized likelihood ratio (LR) statistic. This test requires the estimation of the model under both the null and alternative hypotheses and is defined as:

$$LR = -2\ln\left[L(H_0)/L(H_A)\right] \qquad (5.17)$$

where $L(H_0)$ and $L(H_A)$ are the values of the likelihood function under the null and alternative hypotheses respectively. If the null hypothesis is true, then LR has an asymptotic χ^2- distribution with degrees of freedom equal to the number of restrictions imposed under the null hypothesis (Coelli, 1996).

The F statistic can also be used to test the null hypothesis in (5.13) and is defined as:

$$F[J, n-k] = \frac{\left(SSR_C - SSR_T\right)/J}{\left(1 - SSR_T\right)/(n-k)} \qquad (5.17a)$$

where SSR indicates the sum of squared residuals, the subscripts C and T indicate the Cobb-Douglas and translog production functions respectively, n = the number of observations, k = the number parameters of the translog production and J is the number of restrictions imposed. This statistic measures the loss of fit of the regression that results from imposing restrictions; it has some intuitive appeal in this form in that the difference in the fits of the two functions is directly incorporated in the test statistic (Greene, 1997).

5.4. Measures of Technical, Allocative and Economic Efficiency

Our parametric approach allows the decomposition of efficiency into technical, allocative and economic efficiency. We require a functional form of the stochastic frontier production function which is self-dual to obtain the dual frontier cost function. The Cobb-Douglas stochastic frontier production model, [5.2] as self-dual, is specified for cross section farm level data and five inputs as:

$$\ln y_i = \beta_0 + \sum_{i=1}^{5} \beta_i \ln x_i + \xi_i - \zeta_i \tag{5.18}$$

where the definitions of variables are as in (5.1) and (5.2). The maximum likelihood estimation (see Appendix 5.3 for details) of (5.18) produces the estimators for β, $\sigma_u^2 = \sigma_\xi{}^2 + \sigma_\zeta{}^2$ and $\gamma = \sigma_\zeta^2 / \sigma_u^2$. The technical efficiency estimates are obtained using (5.8). If we now replace the parameters in the stochastic frontier production function model in (5.18) and in the technical efficiency predictor in (5.8) by their maximum likelihood estimates, we obtain the estimates for ξ_i and ζ_i. Subtracting e^{ξ_i} from both sides of (5.18) yields:

$$\ln \tilde{y}_i = \ln y_i - \xi_i = \beta_0 + \sum_{i=1}^{5} \beta_i \ln x_i - \zeta_i$$

where $\tilde{y}_i$ now denotes the farm's observed output adjusted for the stochastic random noise captured by ξ_i. This equation constitutes the basis for obtaining the technically efficient input vector x_i^T and algebraically deriving dual frontier cost function which is the basis for calculating the economically efficient (technically and allocatively efficient) vector x_i^E. The dual frontier cost function model is analytically derived from the stochastic frontier production model as (see Appendix 5.4 for details):

[5.2] The dual of the translog stochastic frontier is intractable, that is, it is not feasible to derive the dual cost function from our cost decomposition technique and hence we can not obtain technical, allocative and economic efficienies.

$$C(p_i, y) = \left(1 \Big/ \beta_0^{1/\sum_{i=1}^{5}\beta_i} \right) \left(\sum_{i=1}^{5} \beta_i \Big/ \prod_{i=1}^{5} \beta_i^{\beta_i/\sum_{i=1}^{5}\beta_i} \right) \prod_{i=1}^{5} p_i^{\beta_i/\sum_{i=1}^{5}\beta_i} \tilde{y}^{1/\sum_{i=1}^{5}\beta_i}$$

Differentiating this cost function with respect to each input's price generates the system of input demand equations which provide the economically efficient input vectors x_i^E by applying Shephard's lemma and substituting the input price vectors and output into the system of input demand equations:

$$x_i^E = \frac{\partial C(p_i, y)}{\partial p_i} = \left(1 \Big/ \beta_0^{1/\sum_{i=1}^{5}\beta_i} \right) \left(\sum_{i=1}^{5} \beta_i \Big/ \prod_{i=1}^{5} \beta_i^{\beta_i/\sum_{i=1}^{5}\beta_i} \right) \left(\beta_i \Big/ \sum_{i=1}^{5} \beta_i \right) \prod_{i=1}^{5} \frac{1}{p_i} p_i^{\beta_i/\sum_{i=1}^{5}\beta_i} \tilde{y}^{\frac{1}{\sum_{i=1}^{5}\beta_i}}$$

Alternatively:

$$x_i^E = \frac{\partial C(p_i, y)}{\partial p_i} = \frac{\partial C}{\partial p_i} = C.\frac{\alpha_i}{p_i}$$

where C denotes $C(p_i, y)$ is cost function and $\alpha_i = \beta_i \Big/ \sum_{i=1}^{5} \beta_i$ $(i = 1,2,3,\ldots,5)$. We also solve for the technically efficient input vectors x_i^T using the results from the stochastic frontier production function in (5.18) and the observed input ratios, $x_1/x_i = k_i$ $(i \neq 1)$. Multiplying the observed input vectors x_i, technically efficient input vectors x_i^T and economically efficient input vectors x_i^E by the input price vectors provides the observed, technically efficient and economically efficient costs of production of the ith farm equal to $p_i x_i$, $p_i x_i^T$ and $p_i x_i^E$ respectively which compute the TE, AE and EE indices for the ith farm as: $TE = p_i x_i^T / p_i x_i$, $AE = p_i x_i^E / p_i x_i^T$ and $EE = p_i x_i^E / p_i x_i$ respectively.

5.4. Summary

This Chapter describes the stochastic econometric frontier approach to measuring efficiency. This approach has the advantage over the deterministic approach in that it includes a stochastic error component. We first develop the general stochastic frontier production model which includes the technical inefficiency effects for estimating technical efficiency. We detail the derivation of the technical efficiency estimates with a truncated normal distribution. We also show that the assumption of a half-normal distribution is a special case of the truncated normal distribution.

We explain the two alternative functional forms of production function, namely, the Cobb-Douglas and the translog; the Cobb-Douglas form is nested in the translog form. Moreover, the Cobb-Douglas function is restricted to a unitary elasticity of substitution whereas the elasticity of substitution of the translog function is unbounded.

We discuss statistical tests of selecting the representative frontier production technology for farm efficiency analysis, given the functional forms of the Cobb-Douglas and the translog functions. We highlight a statistical test for choosing the appropriate distributional assumption for the technical inefficiency effects term, given the truncated normal and the half-normal distribution; we also test the existence of overall technical inefficiency effects, and the joint effects of farm-specific explanatory variables included in the technical inefficiency effects model.

We then examine the cost decomposition method to obtain the estimates of technical, allocative and economic efficiency using the self-dual Cobb-Douglas stochastic frontier. We derive the observed output of farms adjusted for the stochastic random noise and explain the dual approach for analytically obtaining the dual frontier cost function from the stochastic frontier production function and hence the economically efficient input vector. From the primal stochastic frontier production model and dual frontier cost function, the technically efficient input vectors can be obtained. These technically and economically efficient input vectors and observed input vectors along with the associated input price vectors yield the technically, economically and observed cost vectors which produce the measures of technical and economic efficiency estimates and hence the allocative efficiency estimates.

Appendix 5.1: Derivation of the Mean and Variance of the Truncated Normal Distribution:

Probability Density Function of a Truncated Normal Variate: If ζ is a continuous random variable with pdf $f(\zeta)$ then the truncated probability density function (pdf) takes the following form:

$$f(\zeta|\zeta \geq 0) = \frac{f(\zeta)}{\Pr\{\zeta \geq 0\}}$$

If the continuous random variable ζ has a normal distribution with mean μ and variance σ_ζ^2, then

$$\Pr\{\zeta \geq 0\} = [1 - \Pr\{\zeta \leq 0\}] = 1 - \int_{-\infty}^{0} \frac{1}{\sigma_\zeta\sqrt{2\pi}} e^{-\frac{1}{2}(-\mu/\sigma_\zeta)^2} d\zeta = 1 - \Phi(-\mu/\sigma_\zeta)$$

where $\Phi(\cdot)$ is the standard normal cumulative density function (cdf). Therefore the probability density function of the truncated normal distribution is:

$$f(\zeta|\zeta \geq 0) = \frac{\frac{1}{\sigma_\zeta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2}}{1-\Phi(-\mu/\sigma_\zeta)} = \frac{\frac{1}{\sigma_\zeta}\phi\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)}{1-\Phi(-\mu/\sigma_\zeta)} = \frac{\phi\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)}{\sigma_\zeta[1-\Phi(-\mu/\sigma_\zeta)]}.$$

Therefore the mean of the truncated normal variable is written as:

$$E(\zeta_i|\zeta_i \geq 0) = \int_0^\infty \zeta f(\zeta|\zeta \geq 0) d\zeta$$

$$= \int_0^\infty \zeta \frac{1}{\sqrt{2\pi}\sigma_\zeta[1-\Phi(-\mu/\sigma_\zeta)]} e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta$$

Let $z = \frac{\zeta-\mu}{\sigma_\zeta}$ or $\zeta = \mu + \sigma_\zeta z$ and $d\zeta = \sigma_\zeta dz$, therefore:

$$\int \zeta \frac{1}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi\left(-\mu/\sigma_\zeta\right)\right]} e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta$$

$$= \frac{1}{\sigma_\zeta\sqrt{2\pi}\left[1-\Phi\left(-\mu/\sigma_\zeta\right)\right]} \int \left(\mu+\sigma_\zeta z\right) e^{-\frac{1}{2}z^2} \sigma_\zeta dz$$

$$= \frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)} \left[\int \frac{\mu}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz + \int \frac{\sigma_\zeta}{\sqrt{2\pi}} z e^{-\frac{1}{2}z^2} dz\right]$$

$$= \frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)} \left[\mu\left(1-\int \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz\right) + \frac{\sigma_\zeta}{\sqrt{2\pi}} \int z e^{-\frac{1}{2}z^2} dz\right] \text{A5.1}$$

So: $$E\left(\zeta_i | \zeta_i \geq 0\right) = \int_0^\infty \zeta \frac{1}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi\left(-\mu/\sigma_\zeta\right)\right]} e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta$$

$$= \frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)} \left[\mu\left(1-\int_{-\infty}^{0} \frac{1}{\sigma_\zeta\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta\right)\right.$$

$$\left. + \frac{\sigma_\zeta}{\sqrt{2\pi}} . e^{-\frac{1}{2}\left\{\left(\zeta-\mu\right)/\sigma_\zeta\right\}^2} \Bigg|_0^\infty \right]$$

$$= \frac{\mu\left[1-\Phi\left(-\mu/\sigma_\zeta\right)\right]}{1-\Phi\left(-\mu/\sigma_\zeta\right)} + \frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)} \frac{\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left\{\left(\zeta-\mu\right)/\sigma_\zeta\right\}^2} \Bigg|_{-\infty}^{0}$$

A5.1 It can be easily shown that $\int z e^{-\frac{1}{2}z^2} dz = -e^{-\frac{1}{2}z^2}$. Let $y = -\frac{1}{2}z^2$, therefore $dy = -dz$, $\therefore \int z e^{-\frac{1}{2}z^2} dz = -\int e^y dy = -e^y$. Now substituting the value of y we can easily show that $\int z e^{-\frac{1}{2}z^2} dz = -e^{-\frac{1}{2}z^2}$.

$$= \mu + \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} \frac{\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2}$$

Thus: $E[\zeta_i | \zeta_i \geq 0] = \mu + \dfrac{\sigma_\zeta \{\phi(-\mu/\sigma_\zeta)\}}{1-\Phi(-\mu/\sigma_\zeta)}$

The variance of the truncated normal distribution of the random variable ζ_i denoted by $Var(\zeta_i | \zeta_i \geq 0)$ is defined as:

$$Var(\zeta_i | \zeta_i \geq 0) = E[\zeta_i^2 | \zeta_i \geq 0] - \left(E[\zeta_i | \zeta_i \geq 0]\right)^2$$

Now $E\left[\zeta_i^2 \middle| \zeta_i \geq 0\right] = \int_0^\infty \zeta^2 f(\zeta | \zeta \geq 0) d\zeta$

$$= \int_0^\infty \zeta^2 \frac{1}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi(-\mu/\sigma_\zeta)\right]} e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta$$

$$= \frac{1}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi(-\mu/\sigma_\zeta)\right]} \int_0^\infty \zeta^2 e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta$$

Let $z = \dfrac{\zeta - \mu}{\sigma_\zeta}$, or $\zeta = \sigma_\zeta z + \mu$, $\therefore\ d\zeta = \sigma_\zeta dz$

Therefore: $\dfrac{1}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi(-\mu/\sigma_\zeta)\right]} \int \zeta^2 e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2} d\zeta$

$$= \frac{1}{\sqrt{2\pi}\sigma_\zeta\left[1-\Phi(-\mu/\sigma_\zeta)\right]} \int (\sigma_\zeta z + \mu)^2 e^{-\frac{1}{2}z^2} \sigma_\zeta dz$$

$$= \frac{1}{\sqrt{2\pi}\left[1-\Phi(-\mu/\sigma_\zeta)\right]} \int \left(\sigma_\zeta^2 z^2 + 2\sigma_\zeta z \mu + \mu^2\right) e^{-\frac{1}{2}z^2} dz$$

$$=\frac{1}{1-\Phi(-\mu/\sigma_\zeta)}\left[\int\frac{\sigma_\zeta^2}{\sqrt{2\pi}}z^2e^{-\frac{1}{2}z^2}dz+2\sigma_\zeta\mu\int\frac{1}{\sqrt{2\pi}}ze^{-\frac{1}{2}z^2}dz+\mu^2\int e^{-\frac{1}{2}z^2}dz\right]$$

$$=\frac{1}{1-\Phi(-\mu/\sigma_\zeta)}\left[\int\frac{\sigma_\zeta^2}{\sqrt{2\pi}}z^2e^{-\frac{1}{2}z^2}dz+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}\left(-e^{-\frac{1}{2}z^2}\right)+\mu^2\{1-\Phi(-\mu/\sigma_\zeta)\}\right]\text{ A5.2}$$

$$=\frac{1}{1-\Phi(-\mu/\sigma_\zeta)}\left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}}\left\{-ze^{-\frac{1}{2}z^2}+\int e^{-\frac{1}{2}z^2}dz\right\}\right.$$

$$\left.+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}\left(-e^{-\frac{1}{2}z^2}\right)+\mu^2\{1-\Phi(-\mu/\sigma_\zeta)\}\right]$$

Substituting the value of $z=\frac{\zeta_i-\mu}{\sigma_\zeta}$ yields:

$$=\frac{1}{1-\Phi(-\mu/\sigma_\zeta)}\left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}}\left\{-\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2}+\int e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2}dz\right\}\right.$$

$$\left.+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}\left(-e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2}\right)+\mu^2\{1-\Phi(-\mu/\sigma_\zeta)\}\right]$$

Therefore:

$$E\left[\zeta_i^2|\zeta_i\geq 0\right]=\int_0^\infty\zeta^2f(\zeta|\zeta\geq 0)d\zeta$$

A5.2 The first part of the integral of the third bracket can be written as by applying the integration by parts. The integration by parts states that $f(y)=f(x)g(x)$ where $f(x)$ and $g(x)$ are two functions, then $\int f(y)dy=f(x)\int g(x)dx-\int\left\{\frac{df(x)}{dx}\int g(x)dx\right\}dx$

$\therefore\ \int z^2e^{-\frac{1}{2}z^2}dz=\int z\left(ze^{-\frac{1}{2}z^2}\right)dz=z\int ze^{-\frac{1}{2}z^2}dz-\int\left\{\frac{d(z)}{dz}\int ze^{-\frac{1}{2}z^2}dz\right\}dz=-ze^{-\frac{1}{2}z^2}+\int e^{-\frac{1}{2}z^2}dz$

$$=\frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)}\left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}}\left\{-\left\{(\zeta-\mu)/\sigma_\zeta\right\}e^{-\frac{1}{2}\left\{(\zeta-\mu)/\sigma_\zeta\right\}^2}\Big|_0^\infty\right.\right.$$

$$\left.+\int_0^\infty e^{-\frac{1}{2}\left\{(\zeta-\mu)/\sigma_\zeta\right\}^2}dz\right\}+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}\left(-e^{-\frac{1}{2}\left\{(\zeta-\mu)/\sigma_\zeta\right\}^2}\right)\Big|_0^\infty$$

$$\left.+\mu^2\left\{1-\Phi\left(-\mu/\sigma_\zeta\right)\right\}\right]$$

$$=\frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)}\left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}}\left\{\left\{(\zeta-\mu)\ \sigma_\zeta\right\}e^{-\frac{1}{2}\left\{(\zeta-\mu)/\sigma_\zeta\right\}^2}\Big|_{-\infty}^0\right.\right.$$

$$\left.\left.+\int_0^\infty e^{-\frac{1}{2}\left\{(\zeta-\mu)/\sigma_\zeta\right\}^2}dz\right\}+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}\left\{(\zeta-\mu)/\sigma_\zeta\right\}^2}\right)\Big|_{-\infty}^0+\mu^2\left\{1-\Phi\left(-\mu/\sigma_\zeta\right)\right\}\right]$$

$$=\frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)}\left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}}\left\{\left(\frac{-\mu}{\sigma_\zeta}\right)e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2}+\int_0^\infty e^{-\frac{1}{2}\left(\frac{\zeta-\mu}{\sigma_\zeta}\right)^2}dz\right\}\right.$$

$$\left.+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2}+\mu^2\left\{1-\Phi\left(-\mu/\sigma_\zeta\right)\right\}\right]$$

$$=\frac{\sigma_\zeta^2}{1-\Phi\left(-\mu/\sigma_\zeta\right)\sqrt{2\pi}}\left(\frac{-\mu}{\sigma_\zeta}\right)e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2}+\sigma_\zeta^2+\frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)}\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2}+\mu^2$$

$$=\mu^2+\sigma_\zeta^2+\frac{1}{1-\Phi\left(-\mu/\sigma_\zeta\right)}e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2}\left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}}\left(\frac{-\mu}{\sigma_\zeta}\right)+\frac{2\sigma_\zeta\mu}{\sqrt{2\pi}}\right]$$

Therefore:

$$Var(\zeta_i|\zeta_i \geq 0) = E[\zeta_i^2|\zeta_i \geq 0] - (E[\zeta_i|\zeta_i \geq 0])^2$$

$$= \mu^2 + \sigma_\zeta^2 + \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2} \left[\frac{\sigma_\zeta^2}{\sqrt{2\pi}} \left(\frac{-\mu}{\sigma_\zeta} \right) + \frac{2\sigma_\zeta \mu}{\sqrt{2\pi}} \right]$$

$$- \left\{ \mu + \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} \frac{\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2} \right\}^2$$

$$= \mu^2 + \sigma_\zeta^2 - \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2} \left[\frac{\sigma_\zeta \mu}{\sqrt{2\pi}} \right]$$

$$-\mu^2 - \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} \frac{2\mu\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2} - \left\{ \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} \frac{\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2} \right\}^2$$

$$= \sigma_\zeta^2 - \frac{\mu}{1-\Phi(-\mu/\sigma_\zeta)} \frac{\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)} - \left\{ \frac{1}{1-\Phi(-\mu/\sigma_\zeta)} \frac{\sigma_\zeta}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{-\mu}{\sigma_\zeta}\right)^2} \right\}^2$$

$$= \sigma_\zeta^2 - \frac{\mu\sigma_\zeta \phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} - \left\{ \frac{\sigma_\zeta \phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} \right\}^2$$

$$= \sigma_\zeta^2 \left[1 - \frac{\mu}{\sigma_\zeta} \frac{\phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} - \left\{ \frac{\phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} \right\}^2 \right]$$

So: $$Var(\zeta_i) = \sigma_\zeta^2 \left[1 - \frac{\phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} \left\{ \frac{\mu}{\sigma_\zeta} - \frac{\phi(-\mu/\sigma_\zeta)}{1-\Phi(-\mu/\sigma_\zeta)} \right\} \right]$$

Appendix 5.2: Log-Likelihood Function

The log-likelihood function for the sample observations, $y \equiv (y_1, y_2, \ldots, y_n)$ can be obtained from the probability density function for y_i for the ith farm. The probability density function for y_i is derived by substituting $\{y_i - f(x_i;\beta)\}$ for u_i in (5.7a), where x_i is the $(1 \times q)$ vector for the ith farm and q is the dimension of the vector β, as:

$$f(y_i) = \frac{\left[1 - \Phi\left(-\mu^*/\sigma_\zeta^*\right)\right] e^{\frac{1}{2}\left[\left(\{y_i - f(x_i;\beta)\}'\{y_i - f(x_i;\beta)\}/\sigma_\xi^2\right) + (\mu/\sigma_\zeta)^2 - (\mu^*/\sigma_\zeta^*)\right]}}{\sqrt{2\pi}\left[\sigma_\zeta^2 + \sigma_\xi^2\right]^{1/2}\left[1 - \Phi\left(-\mu/\sigma_\zeta\right)\right]}$$

Therefore the log-likelihood function is:

$$L(\Omega^*, y) = \sum_{i=1}^{n} \ln\left[1 - \Phi\left(-\mu_i^*/\sigma_{i\zeta}^*\right)\right] - \frac{1}{2}\sum_{i=1}^{n}\left[\{y_i - f(x_i;\beta)\}'\{y_i - f(x_i;\beta)\}/\sigma_\xi^2\right] - \frac{1}{2}n\left(\mu/\sigma_\zeta\right)^2$$

$$+\frac{1}{2}\sum_{i=1}^{n}\left(\mu_i^*/\sigma_{i\zeta}^*\right)^2 - \frac{1}{2}n\ln(2\pi) - \frac{1}{2}n\ln\left(\sigma_\zeta^2 + \sigma_\xi^2\right) - n\ln\left[1 - \Phi\left(-\mu/\sigma_\zeta\right)\right]$$

where $\Omega^* \equiv \left(\beta', \sigma_\xi^2, \sigma_\zeta^2, \mu\right)'$. Using the reparameterization $\sigma_s^2 = \sigma_\xi^2 + \sigma_\zeta^2$ and $\gamma = \sigma_\zeta^2/\sigma_s^2$, suggested by Battese and Corra (1977), the log-likelihood function is written as:

$$L(\Omega, y) = \sum_{i=1}^{N} \ln\left[1 - \Phi\left(-z_i^*\right)\right] - \frac{1}{2}\sum_{i=1}^{n}\{y_i - f(x_i;\beta)\}'\{y_i - f(x_i;\beta)\}/(1-\gamma)\sigma_s^2 - \frac{1}{2}nz^2$$

$$+\frac{1}{2}\sum_{i=1}^{n} z_i^{*2} - \frac{1}{2}n\left\{\ln(2\pi) + \ln\left(\sigma_s^2\right)\right\} - n\ln[1 - \Phi(-z)],$$

where $\Omega \equiv \left(\beta', \sigma_s^2, \gamma, \mu\right)'$, $z \equiv \mu/\left(\gamma\sigma_s^2\right)^{1/2}$ and $z_i^* = \dfrac{\mu(1-\gamma) - \gamma\{y_i - f(x_i;\beta)\}}{\{\gamma(1-\gamma)\sigma_s^2\}^{1/2}}$.

The partial derivatives of the log-likelihood function with respect to the parameters, β, σ_s^2, γ and μ are derived by:

$$\frac{\partial L}{\partial \beta} = \sum_{i=1}^{n} x_i' \{y_i - f(x_i;\beta)\} / (1-\gamma)\sigma_s^2 + \sum_{i=1}^{n} \left[\frac{\phi(-z_i^*)}{1-\Phi(-z_i^*)} + z_i^* \right] \cdot \frac{\gamma x_i'}{\{\gamma(1-\gamma)\sigma_s^2\}^{1/2}},$$

$$\frac{\partial L}{\partial \sigma_s^2} = +\frac{1}{2\sigma_s^2} \sum_{i=1}^{n} \{y_i - f(x_x;\beta)\}' \{y_i - f(x_x;\beta)\} \Big/ (1-\gamma)\sigma_s^2$$

$$-\frac{1}{2\sigma_s^2} \sum_{i=1}^{n} \left\{ \frac{\phi(-z_i^*)}{1-\Phi(-z_i^*)} + z_i^* \right\} z_i^* - \frac{1}{\sigma_s^2} n + n \left\{ \frac{\phi(-z)}{1-\Phi(-z)} + z \right\} z$$

$$= -\frac{1}{2\sigma_s^2} \left[\sum_{i=1}^{n} \left\{ \frac{\phi(-z_i^*)}{1-\Phi(-z_i^*)} + z_i^* \right\} z_i^* - \sum_{i=1}^{n} \{y_i - f(x_x;\beta)\}' \{y_i - f(x_x;\beta)\} \Big/ (1-\gamma)\sigma_s^2 \right.$$

$$\left. + \frac{1}{\sigma_s^2} n - n \left\{ \frac{\phi(-z)}{1-\Phi(-z)} + z \right\} z \right]$$

$$\frac{\partial L}{\partial \gamma} = \sum_{i=1}^{n} \left[\frac{\phi(-z_i^*)}{1-\Phi(-z_i^*)} + z_i^* \right] z_i^* - \frac{1}{2} \sum_{i=1}^{n} \{y_i - f(x_i;\beta)\}' \{y_i - f(x_i;\beta)\} \Big/ \{(1-\gamma)\sigma_s^2\}^2$$

$$+\frac{1}{2} n \left[\frac{\phi(-z)}{1-\Phi(-z)} + z \right] z \gamma^{-1} + \sum_{i=1}^{n} \left[\frac{\phi(-z_i^*)}{1-\Phi(-z_i^*)} + z_i^* \right] \frac{\partial z_i^*}{\partial \gamma}$$

where $\frac{\partial z_i^*}{\partial \gamma} = -\frac{\mu - \{y_i - f(x_i;\beta)\}}{\{\gamma(1-\gamma)\sigma_s^2\}^{1/2}} - \frac{1}{2} \frac{[\mu(1-\gamma) - \gamma\{y_i - f(x_i;\beta)\}](1-2\gamma)}{\sigma_s\{\gamma(1-\gamma)\}^{3/2}}$

$$\frac{\partial L}{\partial \mu} = \sum_{i=1}^{n} \left[\frac{\phi(-z_i^*)}{1-\Phi(-z_i^*)} + z_i^* \right] \times \frac{(1-\gamma)}{\{\gamma(1-\gamma)\sigma_s^2\}^{1/2}} - n \left[\frac{\phi(-z)}{1-\Phi(-z)} + z \right] \times \frac{1}{(\gamma\sigma_s^2)^{1/2}}.$$

Appendix 5.3: Maximum Likelihood Estimators

The principle of maximum likelihood estimation is illustrated in the context of the linear regression which is defined by:

$$y_i = X\beta + u \tag{A5.4.1}$$

where X is a fixed nonstochastic matrix. This model then defines a transformation from u to y. The assumption of a multivariate density function for u implies a multivariate density function for y, which may be written as:

$$f(y) = f(u)\left|\frac{\partial u}{\partial y}\right|$$

where $|\partial u/\partial y|$ denotes the absolute value of the determinant formed from the matrix of partial derivatives:

$$\begin{bmatrix} \partial u_1/\partial y_1 & \partial u_1/\partial y_2 & \dots & \partial u_1/\partial y_n \\ \partial u_2/\partial y_1 & \partial u_2/\partial y_2 & \dots & \partial u_2/\partial y_n \\ \dots & \dots & \dots & \dots \\ \partial u_n/\partial y_1 & \partial u_n/\partial y_2 & \dots & \partial u_n/\partial y_n \end{bmatrix}$$

This matrix appears to be the identity matrix whose determinant is unity in case of (A5.4.1). Thus:

$$f(y) = f(u)$$

If we assume that u is multivariate normal with mean zero and variance $\sigma^2 I$, all the u's are pairwise uncorrelated, then we obtain:

$$f(u) = \frac{1}{\left(\sigma\sqrt{2\pi}\right)^n} e^{-\frac{1}{2\sigma^2}u'u}$$

and so:

$$f(y)=\frac{1}{\left(\sigma\sqrt{2\pi}\right)^{n}}e^{-\frac{1}{2\sigma^{2}}(y-X\beta)'(y-X\beta)} \tag{A5.4.2}$$

Equation (A5.4.2) includes both the observations on y and the unknown parameters β and σ^2. As the observations on y are known and β and σ^2 are not known, the function in (A5.4.2) is termed the likelihood function denoted by L. Taking natural log of the likelihood function in (A5.4.2) yields:

$$\ln L=-\frac{n}{2}\ln(2\pi)-\frac{n}{2}\ln\left(\sigma^{2}\right)-\frac{1}{2\sigma^{2}}(y-X\beta)'(y-X\beta) \tag{A5.4.3}$$

The maximum likelihood (ML) principle consists in estimating the unknown parameters with the values which maximize the likelihood function, given the sample data y. Differentiating (A5.4.3) partially with respect to β and σ^2 and setting equal to zero gives:

$$\frac{\partial(\ln L)}{\partial\beta}=-\frac{1}{2\hat{\sigma}^{2}}\left(-2X'y+2X'X\hat{\beta}\right)=0$$

or:

$$\frac{1}{\hat{\sigma}^{2}}\left(X'y-X'X\hat{\beta}\right)=0$$

and:

$$\frac{\partial(\ln L)}{\partial\sigma^{2}}=-\frac{1}{2\hat{\sigma}^{2}}+\frac{1}{2\hat{\sigma}^{4}}\left(y-X\hat{\beta}\right)'\left(y-X\hat{\beta}\right)=0$$

where $\hat{\beta}$ and $\hat{\sigma}^2$ are maximum likelihood estimators. The solution of these equations simultaneously gives:

$$\hat{\beta}=(X'X)^{-1}X'y$$

and:

$$\hat{\sigma}^{2}=\frac{e'e}{n}$$

where $e=y-X\hat{\beta}$. The ML $\hat{\beta}$ is identical with OLS estimator and the estimates of σ^2 is asymptotically unbiased.

Appendix 5.4: Derivation of the Cost Function

We now explain the mathematical model from which the cost function for profit maximizing farms are derived:

Minimize $\quad C = \sum_{i=1}^{q} p_i x_i$

Subject to: $y = f(x_1, x_2, \ldots, x_q)$, $\; x_i > 0$ and $y > 0$

where the p_i's are input prices, y is a parametric output value, $f(x_1, x_2, \ldots, x_q)$ is the production function of the farm. Assume that the farms minimizes the total cost of producing any specific output level. For simplicity, we begin with the three variable case and the production function of the Cobb-Douglas type. Hence the Lagrangian function is constructed as follows:

$$L_{x,\lambda}(p, y) = p_1 x_1 + p_2 x_2 + p_3 x_3 + \lambda\left(y - \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} e^{-u}\right)$$

where λ is the Lagrange multiplier.

The first-order conditions of this function are written as:

$$p_1 = \lambda \beta_0 \beta_1 x_1^{\beta_1 - 1} x_2^{\beta_2} x_3^{\beta_3} \qquad \text{(A5.3.1)}$$

$$p_2 = \lambda \beta_0 \beta_2 x_1^{\beta_1} x_2^{\beta_2 - 1} x_3^{\beta_3} \qquad \text{(A5.3.2)}$$

$$p_3 = \lambda \beta_0 \beta_3 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3 - 1} \qquad \text{(A5.3.3)}$$

$$y = \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} \qquad \text{(A5.3.4)}$$

From (A5.3.1) and (A5.3.2) we get:

$$\frac{p_1}{p_2} = \frac{\lambda \beta_0 \beta_1 x_1^{\beta_1 - 1} x_2^{\beta_2} x_3^{\beta_3}}{\lambda \beta_0 \beta_2 x_1^{\beta_1} x_2^{\beta_2 - 1} x_3^{\beta_3}} = \frac{\beta_1 x_2^{\beta_2}}{\beta_2 x_1^{\beta_1}}$$

$$\therefore \qquad x_2 = \frac{p_1\beta_2}{p_2\beta_1} x_1$$

And from (A5.3.2) and (A5.3.3) we derive that:

$$\frac{p_3}{p_2} = \frac{\lambda\beta_0\beta_3 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3 - 1}}{\lambda\beta_0\beta_2 x_1^{\beta_1} x_2^{\beta_2 - 1} x_3^{\beta_3}} = \frac{\beta_3 x_2^{\beta_2}}{\beta_2 x_3^{\beta_3}}$$

$$\therefore \qquad x_3 = \frac{p_2\beta_3}{p_3\beta_2} x_2$$

Substituting the value of x_2 and x_3 into (A5.3.4) yields:

$$y = \beta_0 x_1^{\beta_1} \left(\frac{p_1\beta_2}{p_2\beta_1} x_1 \right)^{\beta_2} \left(\frac{p_2\beta_3}{p_3\beta_2} x_2 \right)^{\beta_3}$$

$$y = \beta_0 x_1^{\beta_1} x_1^{\beta_2} \left(\frac{p_1\beta_2}{p_2\beta_1} \right)^{\beta_2} \left(\frac{p_2\beta_3}{p_3\beta_2} \right)^{\beta_3} x_2^{\beta_3}$$

$$y = \beta_0 x_1^{\beta_1+\beta_2+\beta_3} \left(\frac{\beta_2}{\beta_1} \right)^{\beta_2} \left(\frac{\beta_3}{\beta_1} \right)^{\beta_3} \left(\frac{p_1}{p_2} \right)^{\beta_2} \left(\frac{p_1}{p_3} \right)^{\beta_3}$$

$$x_1 = \frac{1}{\beta_0^{\frac{1}{\beta_1+\beta_2+\beta_3}}} \frac{1}{\left(\frac{\beta_2}{\beta_1} \right)^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}} \left(\frac{\beta_3}{\beta_1} \right)^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}} \frac{1}{\left(\frac{p_1}{p_2} \right)^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}} \left(\frac{p_1}{p_3} \right)^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}} y^{\frac{1}{\beta_1+\beta_2+\beta_3}}$$

$$x_1 = \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_1 \beta_1^{-\frac{\beta_1}{\sum_{i=1}^{3}\beta_i}}}{(\beta_2)^{\beta_2/\sum_{i=1}^{3}\beta_i} (\beta_3)^{\beta_2/\sum_{i=1}^{3}\beta_i}{}^{\beta_3/\sum_{i=1}^{3}\beta_i}} \frac{p_2^{\beta_2/\sum_{i=1}^{3}\beta_i} p_3^{\beta_3/\sum_{i=1}^{3}\beta_i}}{(p_1)^{\beta_2/\sum_{i=1}^{3}\beta_i} (p_1)^{\beta_3/\sum_{i=1}^{3}\beta_i}} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

$$x_1 = \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_1}{\prod_{i=1}^{3} \beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \prod_{j=2}^{3} \left(\frac{p_j}{p_1} \right)^{\beta_j/\sum_{i=1}^{3}\beta_i} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

Similarly we can get the input demand functions for x_2 and x_3 as:

$$x_2 = \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_2}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \prod_{j=2}^{3}\left(\frac{p_j}{p_2}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

And

$$x_3 = \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_3}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \prod_{j=2}^{3}\left(\frac{p_j}{p_3}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

In general, the input demand functions are:

$$x_i = \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{n}\beta_i}}} \frac{\beta_i}{\prod_{i=1}^{n}\beta_i^{\beta_i/\sum_{i=1}^{n}\beta_i}} \prod_{j=2}^{n}\left(\frac{p_j}{p_i}\right)^{\beta_j/\sum_{i=1}^{n}\beta_i} y^{\frac{1}{\sum_{i=1}^{n}\beta_i}}$$

Now the cost function is derived on the basis of the production function as follows:

$$C(p,y) = p_1x_1 + p_2x_2 + p_3x_3$$

$$= p_1 \cdot \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_1}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \prod_{j=2}^{3}\left(\frac{p_j}{p_1}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}} + p_2 \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_2}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \prod_{j=2}^{3}\left(\frac{p_j}{p_2}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

$$+ p_3 \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{3}\beta_i}}} \frac{\beta_3}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \prod_{j=2}^{3}\left(\frac{p_j}{p_3}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

$$= \frac{1}{\beta_0^{1/\sum_{i=1}^{3}\beta_i}} \frac{1}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \left[p_1\prod_{j=2}^{3}\left(\frac{p_j}{p_1}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} + p_2\prod_{j=2}^{3}\left(\frac{p_j}{p_2}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} + p_3\prod_{j=2}^{3}\left(\frac{p_j}{p_3}\right)^{\beta_j/\sum_{i=1}^{3}\beta_i} \right] y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

$$= \frac{1}{\beta_0^{1/\sum_{i=1}^{3}\beta_i}} \frac{1}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \left[\frac{\beta_1 p_1 p_2^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}} p_3^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}}{p_1^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}} p_1^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}} + \frac{\beta_2 p_2 p_1^{\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_3^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}}{p_2^{\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_2^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}} + \frac{\beta_3 p_3 p_1^{\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_2^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}}}{p_3^{\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_3^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}}} \right] y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

$$= \frac{1}{\beta_0^{1/\sum_{i=1}^{3}\beta_i}} \frac{1}{\prod_{i=1}^{3}\beta_i^{\beta_i/\sum_{i=1}^{3}\beta_i}} \left[\frac{\beta_1 p_1 p_2 p_3 + \beta_2 p_1 p_2 p_3 + \beta_3 p_1 p_2 p_3}{p_1^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}} p_1^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}} p_2^{\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_2^{\frac{\beta_3}{\beta_1+\beta_2+\beta_3}} p_3^{\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_3^{\frac{\beta_2}{\beta_1+\beta_2+\beta_3}}} \right] y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

$$= \frac{1}{\beta_0^{1/\sum_{i=1}^{3}\beta_i}} \frac{1}{\prod_{i=1}^{3}\beta_i^{\beta/\sum_{i=1}^{3}\beta}} \left\{ \frac{(\beta_1+\beta_2+\beta_3) p_1 p_2 p_3}{p_1 p_1^{-\frac{\beta_1}{\beta_1+\beta_2+\beta_3}} p_2 p_2^{-\frac{\beta_2}{\beta_1+\beta_2+\beta_3}} p_3 p_3^{-\frac{\beta_3}{\beta_1+\beta_2+\beta_3}}} \right\} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

Therefore the derived cost function which is a function of factor prices and output is:

$$C(p,y) = \frac{1}{\beta_0^{1/\sum_{1}^{}\beta}} \frac{\sum_{i=1}^{3}\beta_i}{\prod_{i=1}^{3}\beta_i^{\beta/\sum_{1}^{}\beta}} \prod_{i=1}^{3} p_i^{\beta_i/\sum_{i=1}^{3}\beta} y^{\frac{1}{\sum_{i=1}^{3}\beta_i}}$$

In a similar fashion, we obtain the input demand functions and the cost function for our five input case as follows:

The input demand functions: $x_i = \frac{1}{\beta_0^{\frac{1}{\sum_{i=1}^{5}\beta_i}}} \frac{\beta_i}{\prod_{i=1}^{5}\beta_i^{\beta_i/\sum_{i=1}^{5}\beta_i}} \prod_{j=2}^{5}\left(\frac{p_j}{p_i}\right)^{\beta/\sum_{i=1}^{5}\beta_i} y^{\frac{1}{\sum_{i=1}^{5}\beta_i}}$

The cost function: $C(p,y) = \frac{1}{\beta_0^{1/\sum_{i=1}^{5}\beta_i}} \frac{\sum_{i=1}^{5}\beta_i}{\prod_{i=1}^{5}\beta_i^{\beta_i/\sum_{i=1}^{5}\beta_i}} \prod_{i=1}^{5} p_i^{\beta_i/\sum_{i=1}^{5}\beta_i} y^{\frac{1}{\sum_{i=1}^{5}\beta_i}}$

These functions can easily be generalized for n inputs.

Chapter 6

The Stochastic Econometric Frontier Approach to Measuring Efficiency: Results

6.1. Introduction

This Chapter discusses the results of estimating the stochastic econometric frontier models analyzed in Chapter 5. We focus on the estimation of technical efficiency using the translog stochastic production frontier model in a single-stage estimation procedure by maximum likelihood methods, given the specification of the technical inefficiency effects model. Technical inefficiency is modelled as a function of socioeconomic characteristics and other factors and thus this single-stage method simultaneously identifies the factors associated with technical inefficiency. We estimate the technical inefficiency effects model using both half-normal and truncated normal distributional assumptions; we then test the restriction implied by the half-normal distribution. Next, technical, allocative and economic efficiency measures are obtained using the self-dual Cobb-Douglas stochastic frontier applying the Kopp and Diewert (1982) cost decomposition procedure. We also identify and quantify the factors affecting efficiency and provide some policy implications regarding the introduction of new technologies and in particular those policies which aim to increase the productivity of farmers.

The structure of the Chapter is as follows: Section 2 presents a description of the variables and summary statistics of the data; Section 3 describes the sources which affect inefficiency; Section 4 specifies the stochastic frontier production model for the technical inefficiency effects; Section 5 discusses the estimation of technical, allocative and economic efficiency and Section 6 concludes.

6.2. Summary Statistics and Description of Variables

The cross-section data are collected from two villages in the High Barind part of Bangladesh by a survey conducted in August-September 1997 and described in Chapter 3. The two villages were selected to represent different irrigation infrastructures: in one village irrigation is operated by diesel pumps and in the other by electricity. In addition, we identify different degrees of environmental degradation and, in particular, land degradation. The questionnaire was administered to 150 farms for the period of one year covering three growing seasons; 75 farms buy irrigation water from diesel pumps, 75 farms buy irrigation water from electricity pumps and 92 farms are identified as environmentally degraded. Cropping is dominated by rice which covers 95 per cent of the total cultivated lands. Other crops (wheat, potatoes, papaya, pulses and vegetables) are grown but only account for a small proportion of farm revenue. Thus we restrict our analysis to rice only. The overall cropping intensity of this region is 175 per cent.

Summary statistics are presented in Table 6.1. The average revenue of farms is Tk.50555 per annum (£1 = 70 Bangladeshi taka) and the coefficient of variation is 105 which indicates a high variability in revenues and income. For the purpose of efficiency analysis, the single output is rice and five inputs are land, labour, irrigation, fertilizer and pesticides. Land is the most important factor. Fertilizer cost represents 48 per cent of average total variable cost (ATVC) and the coefficient of variation (C.V.) indicates high variability of fertilizer use and price among the farmers followed by irrigation costs which reflect expenditure on irrigation per acre. Irrigation costs constitute 33 per cent of ATVC with a C.V. of 113.22. Labour costs and pesticides cost represent 14 and 5 per cent of ATVC with C.V. of 160.85 and 130.96. The average schooling of the sample farmers is about seven years and the mean value of the farm plot size is only 0.26 acres. On average, the land price per acre is 49,853 taka with a C.V. of 13.63; the wage per manday is 45 taka; the irrigation price is 15.75 taka per day with a C.V. of 14.33; the fertilizer price acre is 806.08 taka per acre with a C.V. of 79.71; and the pesticides price is 74.43 taka per acre.

Table 6.1: Summary Statistics of Variables

Variable	Notations	Sample mean	Minimum	Maximum	C.V.
Input values					
Revenue (taka)	y	50555.09	1600.38	289939.68	105.21
Land (taka)	x_1	316509.07	14210.00	1899500.00	105.72
Labour (taka)	x_2	3072.86	7.65	34191.00	160.85
Irrigation (taka)	x_3	6946.59	229.50	46764.00	113.22
Fertilizer (taka)	x_4	10301.68	7.25	397893.64	316.08
Pesticides (taka)	x_5	951.26	2.81	8357.8	130.96
Input prices					
Land price (per acre)	p_1	49853.33	6794.58	70000.00	13.63
Labour price (per manday)	p_2	45.00	0.00	45.00	0.00
Irrigation price (per day)	p_3	15.75	2.26	18.00	14.33
Fertilizer price (per acre)	p_4	806.08	750.5	1040.6	79.71
Pesticides price (per acre)	p_5	74.43	25.31	110.00	34.00
Farm-specific characteristics					
Age (years)		38.59	20.00	70.00	31.02
Schooling (years)		6.81	0.00	18.00	73.42
Plot size (acres)		0.26	0.00	0.73	42.31

Note: All prices are in Bangladeshi taka.

Output (y) is defined as the market value of the observed rice production during the survey period. It is measured in Bangladeshi taka. Land (x_1) represents the total market value of land used for rice production and the price of land (p_1) is the price per acre of land. Labour (x_2) includes both family and hired labour and represents the total costs of labour measured at the market price and the price of labour (p_2) indicates the wage per manday. Irrigation (x_3) is the total irrigation cost for rice production and is estimated from the total rice land irrigated and the market price of irrigation for each acre during the survey period. The price of irrigation (p_3) is the irrigation price each day. Although farmers are charged an irrigation price per acre, but if water sellers are unable to deliver water due to power failures or mechanical breakdowns. Then a portion of money is returned to the farmers. In effect, farmers are charged a price per day of irrigation. Fertilizer (x_4) includes all organic and inorganic fertilizer and the total cost of fertilizer is measured at market prices; the fertilizer price (p_4) indicates the average price of all fertilizer used per acre. Pesticides

(x_5) is the market costs of pesticides and the pesticides price (p_5) is the price of pesticides per acre. Schooling is the years of attending schools.

6.3. Factors Determining/Affecting Farm Inefficiency

The literature indicates that a range of socio-economic and demographic factors determine the efficiency of farms (Seyoum et al., 1998; Coelli and Battese, 1996; Wilson et al., 1998). These include land use, credit availability, land tenure, and the education level of farmer (Kalirajan and Flinn, 1983; Lingard et al., 1983; Shapiro and Muller, 1977; Kumbhakar, 1994). Techniques of cultivation, share tenancy, farm holding size may also influence efficiency (Ali and Choudhury, 1990; Coelli and Battese, 1996; Kumbhakar, 1994). Some environmental factors and non-physical factors like farming experience and extension services may affect the capability of a producer to utilize the available technology efficiently (Parikh and Shah, 1995; Kumbhakar, 1994). We now consider the variables which may affect efficiency in agriculture in Bangladesh.

There is no proper guidelines in the literature as to which variables are to be included in the stochastic frontier production function and which in the technical inefficiency effects model. For example, Wilson et al. (1998) included, among others, the cultivated potato area in the production function and the proportion of the cultivated potato area that is irrigated in the technical inefficiency effects model. Coelli and Battese (1996) included land variable, among others, in the production function and land size, among other, in the technical inefficiency effects model. Parikh and Shah (1994) and Parikh et al. (1995) included off-farm work, farm assets, nonfarm assets and credit in the technical inefficiency effects model. On the basis of this literature we include in the technical inefficiency effects model socioeconomic, infrastructure and environmental degradation variables which have not traditionally been included as input variables in the production function.

In the context of farms within the High Barind, the age of the farmer, the years of schooling, and plot size are considered relevant. The age of the farmer, *a priori*, may have a positive or negative effect on inefficiency. Farming experience can be achieved with increasing age but this may reduce inefficiency. However some older farmers are less receptive to new technologies and practices. There is an interaction between age and education of farmers because younger farmers tend to be more educated than older farmers due to gradual improvements in the educational system in the Barind over recent years.

A priori, we expect that more years of formal education will increase efficiency because education enables farmers to acquire and process relevant information more effectively. Basic literacy enables farmers to use modern fertilizer and pesticides and choose input combinations. Farmers can be exposed to new technologies and improved techniques with education and extension services. Levels of increased education and extension services are related to the allocative efficiency of Indian farmers by Ram (1980). Extension services availability and education level were found by Huffman (1977) to be important explanatory variables of the rate of adjustment in fertilizer use in response to price changes.

Land fragmentation, that is, the average plot size, is likely to have a negative effect upon efficiency. Average plot size is used as a measure of land fragmentation, thus the smaller the plot size the greater is the land fragmentation. The greater the plot size (less fragmentation) of a farm, the greater is the opportunity to apply new technologies such as tractors and irrigation, and hence farmers with less land fragmentation are expected to be more efficient. The average distance of plots from home is half mile and the average interplot distance is also half mile. Plot communication from home and from other plots is through land boundaries.

The demand for irrigation is increasing as the cropping pattern in Bangladesh shifts from Aman to Boro which requires intensive irrigation. There has been a rising dependence on groundwater because surface water sources have been silting up. The number of DTWs under the control of BMDA (Barind Multipurpose Development Authority) is increasing. However, the area has low potential for groundwater exploitation, and the over-use of

DTWs results in STWs drying up. There are no watershed scale water management systems to improve water conservation and recharge. Irrigation management and infrastructure are differentiated by irrigation fuel, that is, by diesel and electricity: irrigation schemes are powered either by diesel pumps or by electricity-operated pumps. Diesel pumps incur higher costs and lower water extraction capacity than electricity-operated pumps and may reduce the efficiency of agricultural production by reducing the availability of water during critical periods in the growing season.

Environmental degradation is increasing because of the dependence on crop residues, animal dung, wood, leaves and twigs for household fuel. If recycled back to the soil, these sources of organic matter would reduce the rate of soil erosion, and soil structure degradation. Population pressure and the consequent intensification of rice-based agriculture has increased soil degradation which occurs through runoff of heavy rainfall in the rainy season. Some farmers have land which has low water retention capacity and low fertility. Low fertility arises because of a fall in the organic matter. Soil degradation is attributed to low moisture availability in the soils, soil structural deterioration due to high bulk density, low aeration capacity in the soils and a reduction of soil pH and base saturation through soil organic matter reduction and cropping intensification (Idris, 1994). All are hypothesized to reduce production efficiency.

6.4. Translog Stochastic Frontier and Technical Efficiency: Results

The stochastic frontier production models are represented by specifying both the Cobb-Douglas model (for example, Seyoum et al., 1998; Son et al., 1993; Tadesse and Krishnamoorthy, 1997) and the translog production model (for example, Wilson et al., 1998; Hallam and Machado, 1996; Greene, 1980; Parikh and Shah, 1994; Khmbhakar, 1989). *A priori*, the Cobb-Douglas model restricts the flexibility of the functional form on the farm's production technology by imposing the elasticity of scale to be constant and the elasticity of input substitution to be unity. The flexible translog stochastic frontier model assumes no such restrictions. The Cobb-Douglas stochastic frontier model is nested in the translog model. A representative functional form can be selected on the basis of statistical tests. We specify a translog stochastic frontier production model:

$$\ln y_i = \beta_0 + \sum_{i=1}^{5} \beta_i \ln x_i + \sum_{i=1}^{5} \sum_{i=1}^{5} \beta_{ij} \ln x_i \ln x_j + \xi_i - \zeta_i \tag{6.1}$$

where y_i represents the value of rice output, x_1 is the total value of land utilized, x_2 is the total labour costs during the production period, x_3, x_4 and x_5 represent the irrigation, fertilizer and pesticides costs respectively and ln represents the natural logarithm. The symmetric error components, ξ_i, are assumed to be independently and identically distributed random errors having normal distribution with mean zero and variance σ_ξ^2, i.e., $\xi_i \sim N(0, \sigma_\xi^2)$ and the technical inefficiency effects, ζ_i, are assumed to be independently distributed of ξ_i such that ζ_i is satisfied by the truncation (at zero from below) of the $N(\mu_i, \sigma_\zeta^2)$ where μ_i can be specified and defined as:

$$\mu_i = \delta_0 + \delta_1 z_{1i} + \delta_2 z_{2i} + \delta_3 z_{3i} + \delta_4 z_{4i} + \delta_5 z_{5i} \tag{6.2}$$

where z_1 = the age of farmers, z_2 = the years of schooling, z_3 = the fragmentation of land, z_4 is a dummy variable where $z_4 = 1$ for farmers buying irrigation water diesel operated pumps and $z_4 = 0$ otherwise, and z_5 is a dummy variable where $z_5 = 1$ for farmers with undegraded lands and $z_5 = 0$ otherwise.

6.4.1 Model and Distribution Selection

The maximum-likelihood estimates of the parameters of both the Cobb-Douglas (presented latter in Table 6.9) and the translog stochastic frontiers are obtained using the computer program Frontier version 4.1 (Coelli, 1996). This program follows a three step process in estimating the maximum-likelihood estimates of the parameters of the stochastic frontier production models. The first step estimates the OLS parameters of the function which are unbiased with the exception of the intercept, which is biased because of the non-zero expectation of the technical inefficiency component. The second step uses a two-phase grid search of γ in (5.2) that evaluates the log-likelihood function for $0 \leq \gamma \leq 1$ setting the parameters of the β_i coefficients equal to the OLS values, and the intercept term and σ^2 are adjusted according to corrected ordinary least squares (Coelli, 1996). All other parameters are set to zero in this grid search. The third step uses the estimates corresponding to the largest log-likelihood value in the second step as starting values in the Davidon-Fletcher-Powell iterative maximization procedure to obtain the final maximum likelihood estimates.

The generalized log-likelihood ratio test is applied to test which of the Cobb-Douglas and translog frontiers is the best model. Results are reported in Table 6.2. The value of the log-likelihood of the Cobb-Douglas production function is 72.84 which is less than that of the translog frontier of 113.13; this allows to conduct a LR test (see 5.13) The LR = 80.59 which exceeds the critical value from χ^2 distribution and we reject the null hypothesis of the appropriateness of the Cobb-Douglas production technology in favour of the alternative hypothesis of the translog production technology. This is confirmed by the result of the $F_{15,135}$ test (see 5.17a). The estimated value of $F_{15,135}$ test is 5.16 and the critical value is 2.94. Thus, we reject the null hypothesis that the Cobb-Douglas model is appropriate and accept the translog production function model. Accordingly, we discuss only the results obtained using the translog stochastic functional form.

Table 6.2: Hypothesis Test

Model	Log-likelih d value	LR statistic	χ^2 critical value*	Decision
Cobb-Douglas	72 8365 (113.1305)	80.5881	24 9958	Reject H_0 Accept H_A

N te: The figure in the parenthesis is fr m the translog fr ntier and * at 5 per cent significance level.

We estimate the technical inefficiency effects model for the translog stochastic frontier using both the half-normal and truncated normal distributions. The null hypothesis of the half-normal distribution in 5.9 , is rejected as shown by the generalized LR test, reported in Table 6.3, suggesting that the half-normal distribution is not an appropriate distribution for the inefficiency effects model. Thus we present the results assuming that the technical inefficiency effects model follows a truncated normal distribution.

Table 6.3: Test of Hypothesis for the Distribution for Inefficiency Effects Models

Null Hypothesis	Log-likelihood value	LR statistics	Critical value *	Decision
H_0: $\mu = 0$	63.21	99.85	3.84	Reject H_0 Accept H_A

Note: * indicates at 5 per cent significance level

Stochastic Frontier Results

Consider the maximum likelihood estimates of the translog stochastic frontier model in Table 6.4. Five of the coefficients are significant at the 5 per cent level and fifteen are significant at the 10 per cent level suggesting that the model is a good fit. [6.1]

[6.1] One would expect multicollinearity in a production function, especially in its translog form with many parameters. However our real interest is in the technical efficiency.

Table 6.4: Maximum-Likelihood Estimates of the Translog Frontier Model

Name of Variables	Parameters	Coefficients	t-ratios
Stochastic frontier			
Constant	β_0	-2.7628	-0.3366
Land	β_1	0.5274	0.3239
Labour	β_2	0.1639	0.1570
Irrigation	β_3	2.9314	3.0670
Fertilizer	β_4	-0.7186	-1.1193
Pesticides	β_5	-1.3128	-1.2879
Land × Land	β_{11}	0.0356	0.4565
Labour × Labour	β_{22}	0.0695	1.4422
Irrigation × Irrigation	β_{33}	0.0584	1.0610
Fertilizer × Fertilizer	β_{44}	0.0310	1.6263
Pesticides × Pesticides	β_{55}	0.0287	1.6241
Land × Pesticides	β_{12}	-0.1007	-2.5528
Labour × Pesticides	β_{13}	0.0449	0.8118
Irrigation × Pesticides	β_{14}	-0.1626	-2.1923
Fertilizer × Pesticides	β_{15}	0.2177	1.6758
Land × Labour	β_{23}	-0.1135	-1.1102
Land × Irrigation	β_{24}	-0.2984	-2.9154
Land × Fertilizer	β_{25}	0.1190	1.4924
Labour × Irrigation	β_{34}	0.1124	1.6571
Labour × Fertilizer	β_{35}	0.0519	0.9261
Irrigation × Fertilizer	β_{45}	-0.1254	-2.0529
Inefficiency model			
Constant	δ_0	0.2238	1.9331
Age of farmers (z_1)	δ_1	0.0013	0.8863
Land fragmentation (z_2)	δ_2	-0.3716	-1.8193
Year of schooling (z_3)	δ_3	0.0005	0.1407
Irrigation infrastructure dummy (z_4)	δ_4	0.1837	3.6880
Environmental degradation dummy (z_5)	δ_5	-0.1621	-3.0552
Variance parameters			
	$\sigma^2 = \sigma_\xi^2 + \sigma_\zeta^2$	0.0178	4.9155
	$\gamma = \left(\sigma_\zeta^2 / \sigma^2\right)$	0.8340	7.1564
	σ_ξ^2	0.0148	
	σ_ζ^2	0.0030	
Log-likelihood		113.1305	

We now assess the economic plausibility of the estimated coefficients in the translog model. Since they do not yield any direct interpretation, the elasticities of output for land, labour, irrigation, fertilizer and pesticides and returns to scale are calculated and reported in Table 6.5. All elasticities except for fertilizer are positive and the perverse fertilizer elasticity can be attributed to overutilization of fertilizer in this region. The elasticity of output for land is highest which indicates that land is the dominant factor of production; this is consistent with land being scarce in Bangladesh. The policy implication of this is that farm households should be provided with incentives to maintain their existing cultivated land and protect productive land from degradation. With a growing population, a reduction in farm size/land size not only has a direct effect on agricultural output, as land is the major influence on output, but also has an indirect effect through decreasing the marginal productivity of other inputs if land and the other inputs are complementary.

Table 6.5: Output Elasticities of the Translog Frontier Model

Inputs	Elasticity	Inputs	Elasticity
Land	0.48	Labour	0.13
Irrigation	0.24	Fertilizer	-0.04
Pesticides	0.08	Returns to Scale	0.89

Irrigation is the second most important factor of production followed by labour. Irrigation is a land-augmenting factor of production in a sense that it increases the fertility/quality of existing land and hence enhances yield per acre. In the High Barind, soil degradation is major technical constraints on production and their effects can be reduced by efficient utilization and combination of fertilizer and irrigation. Pesticides have a relatively small effect. The returns to scale is 0.89 which indicates slightly decreasing returns to scale.

Technical Inefficiency Results

The overall technical inefficiency effects are evaluated in terms of σ_u^2 and γ, reported in Table 6.4. The estimated value for γ-parameter is 0.83 which is highly significant and indicates that the random component of the technical inefficiency effects has a significant contribution in determining the level and variability of output; this also indicates that the technical inefficiency effect dominates the error u_i. This result is in conformity with those of Sharma et al. (1997), Hjalmarsson et al. (1996), Coelli and Battese (1996), Kalirajan (1981), Ajibefun et al. (1996), Ali and Flinn (1989). The estimate of σ^2 is also significant at the 5 per cent level which conforms with the results of Hjalmarsson et al. (1996). We conduct the generalized LR tests to examine if the overall effects have a significant contribution in explaining the level and variability of output. Results of these hypotheses tests are provided in Table 6.6.

Table 6.6: Tests of Hypothesis for the technical Inefficiency Effects

Null Hypothesis	Log-likelihood value	LR statistic	Critical value*	Decision
H_0: $\gamma = \delta_0 = \ldots = \delta_5 = 0$	67.1686	91.9238	14.067	Reject H_0
H_0: $\delta_1 = \ldots = \delta_5 = 0$	88.2917	49.6776	11.070	Reject H_0

Note: * indicates 5 per cent significance level.

We test the null hypothesis of no technical inefficiency effects by testing $\gamma=\delta_0=\delta_1=\delta_2=\delta_3=\delta_4=\delta_5=0$. The estimated LR statistic of 91.92, is greater than the χ^2_7 critical value, and the null hypothesis of no technical inefficiency effects is therefore rejected, that is, the technical inefficiency effects are significant and their inclusion is appropriate. This suggests that the OLS response function is an inadequate representation for agricultural production in the High Barind.

Now consider the estimates of the δ_i-coefficients associated with the farm-specific technical inefficiency effects model. Our model specification allows these farm-specific factors to shift the mean of the technical inefficiency error component and we examine whether they have a significant effect on technical inefficiency. The coefficients of δ_i-parameters are presented in Table 6.4. The signs of the estimated coefficients need to be analyzed carefully because variation in technical efficiency of farms arises due to these variables and these affect the capability of farms to utilize adequately the existing infrastructure and technology. We test the null hypothesis that the farm-specific variables have no significant effects on the level of inefficiency by testing $\delta_1 = \ldots = \delta_5 = 0$. The estimated LR statistic, in Table 6.6, of 49.68, is greater than the χ^2_5 critical value, and the null hypothesis is rejected, i.e., the farm-specific variables jointly have significant effects on inefficiency.

The coefficient of the age of the farmers is positive which implies that the older farmers are less technically efficient than the younger ones. However the coefficient is insignificant. This conforms with results obtained by Ajibefun et al. (1996), Seyoum et al. (1998) and Coelli and Battase (1996) and could be explained in terms of lower credit availability for older farmers. Further, although older farmers are likely to be more experienced in farming, they are likely to be more conservative and less receptive to newly introduced agricultural technology and practices, thereby being less efficient. The coefficient of the farmer's years of schooling is positive indicating that the farmers with greater years of schooling are less technically efficient which is unexpected; however it is very insignificant. This result conforms with result obtained for the Kanzara village in India by Coelli and Battese (1996). Land fragmentation measured by average plot size has a negative coefficient as expected which shows that the technical inefficiency effects are lower for farmers with greater land plot size. Again this conforms with results for Indian farmers by Coelli and Battese (1996).

The coefficient on the irrigation infrastructure dummy is positive and significant and indicates that farmers buying irrigation water from schemes operated with diesel are less technical efficient than those buying from electricity schemes. This may be caused by the water extraction capacity of diesel pumps being lower than for electricity-operated pumps; diesel costs are also higher than electricity costs. The price of irrigation water from a diesel pump is higher than from an electric pump. Farmers are not likely to reduce the amount of cultivated land even though the cost of irrigation water from diesel pumps is higher. This is likely to affect the overall input management of farmers. The expansion of the electrification programme by the government of Bangladesh, which converts diesel pumps into electricity-operated pumps in this region, could decrease farming costs and increase efficiency, thereby increasing farm revenue and household welfare.

The estimate of the coefficient on the environmental degradation dummy, i.e., soil degradation dummy is negative and significant, as is expected, implying that the farmers with undegraded land have greater technical efficiency. In this region, top soils degrade through runoff due to heavy rainfall during the rainy season and hence the fertility of soils decreases. Since soil fertility is defined as the productive capacity of land - the higher the productivity of a land the greater the fertility - a reduction of environmental degradation, and in particular, soil degradation and quality, will increase farm efficiency.

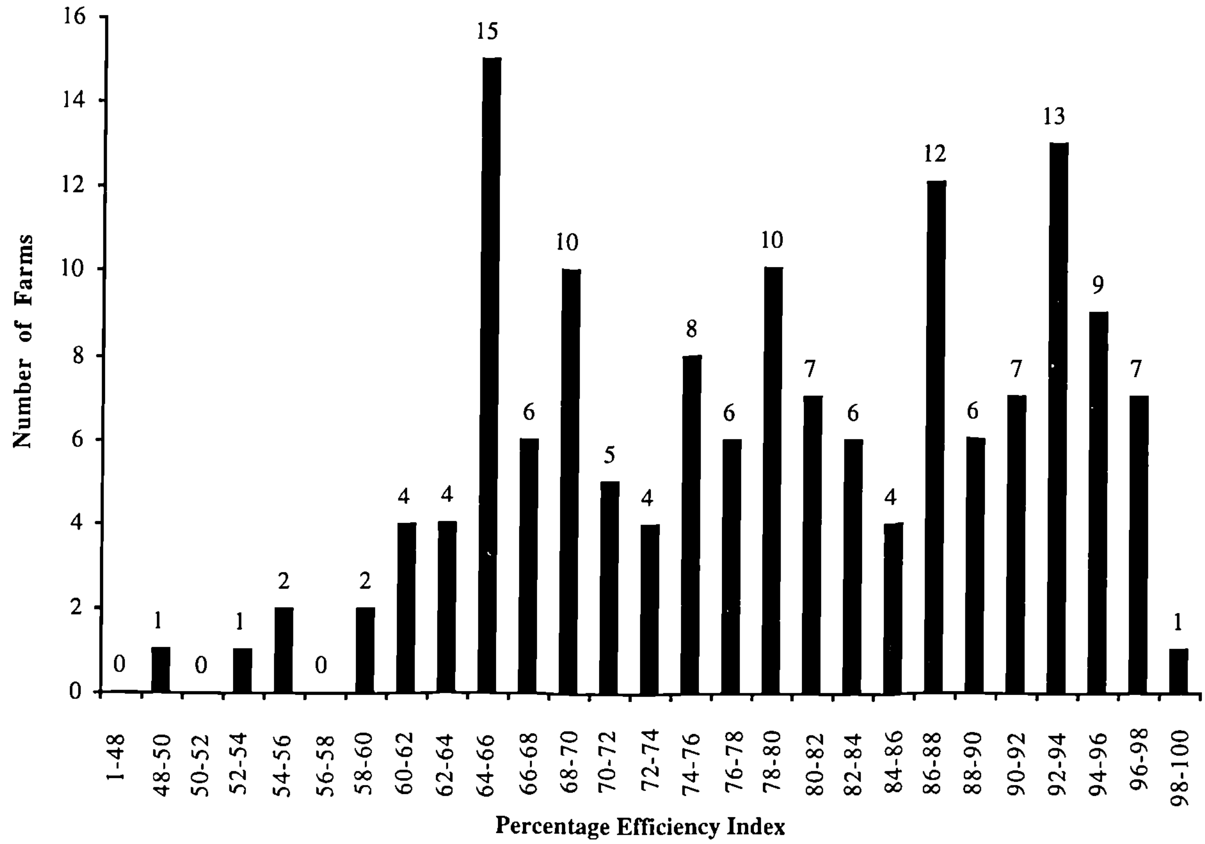
6.4.2. Farm-specific Technical Efficiency

The estimated farm-specific technical efficiencies show substantial variability, ranging between 49 - 98 per cent with a mean value of 79 per cent and a standard deviation of 12 per cent. The frequency distribution of technical efficiency estimates and their summary statistics are presented in Table 6.7. The associated histogram of the efficiency index is presented in Figure 6.1. The majority of farms, 26 per cent are 60 - 70 per cent technically efficient; 25 per cent of farms are between 90 - 100 per cent technically efficient; 23 per cent of farms are between 80 - 90 per cent technically efficient, 22 per cent of farms are between 70 - 80 per cent technically efficient; 3 per cent of farms are between 50 - 60 per cent technical efficient; only one per cent of farms are between 1 - 50 per cent technical efficient; however no farm is fully efficient. Therefore it appears that there is considerable room for improvement in productivity through increased technical efficiency.

Table 6.7: Frequency Distribution of Farm-Specific Efficiencies from Stochastic Translog Frontier

Efficiency Index (%)	Number of farms	Percentage of farms	Cumulative frequency	Cumulative percentage
1-48	0	0.00	0	0.00
48-50	1	0.67	1	0.67
50-52	0	0.00	1	0.67
52-54	1	0.67	2	1.33
54-56	2	1.33	4	2.67
56-58	0	0.00	4	2.67
58-60	2	1.33	6	4.00
60-62	4	2.67	10	6.67
62-64	4	2.67	14	9.33
64-66	15	10.00	29	19.33
66-68	6	4.00	35	23.33
68-70	10	6.67	45	30.00
70-72	5	3.33	50	33.33
72-74	4	2.67	54	36.00
74-76	8	5.33	62	41.33
76-78	6	4.00	68	45.33
78-80	10	6.67	78	52.00
80-82	7	4.67	85	56.67
82-84	6	4.00	91	60.67
84-86	4	2.67	95	63.33
86-88	12	8.00	107	71.33
88-90	6	4.00	113	75.33
90-92	7	4.67	120	80.00
92-94	13	8.67	133	88.67
94-96	9	6.00	142	94.67
96-98	7	4.67	149	99.33
98-100	1	0.67	150	100.00
Mean	79			
Minimum	49			
Maximum	98			
Standard Deviation	12			

Figure 6.1: Distribution of Efficiency Index of the Translog Technology



6.5. Cobb-Douglas Production Frontier and Technical, Allocative and Economic Efficiency: Results

6.5.1 Selection of Distribution

We estimate the Cobb-Douglas stochastic frontier model with a technical inefficiency effects model for both the half-normal and truncated normal distributions. The half-normal distribution given in (5.9) is an inadequate representation for the distribution of the inefficiency effects in the model as indicated by the generalized LR tests in Table 6.8. Accordingly we explain the results obtained using the truncated normal distribution.

Table 6.8: Test of Hypothesis for the Distribution of the Technical Inefficiency Effects Models

Null Hypothesis	Log-likelihood value	LR statistic	Critical value*	Decision
H_0: $\mu = 0$	47.1282	51.4166	3.841	Reject H_0

Note: * indicates 5 per cent significance level.

6.5.2. Cobb-Douglas Frontier Results

The maximum likelihood estimates of the parameter coefficients using the Cobb-Douglas are presented in Table 6.9. The signs of the β-coefficients are all positive as expected and five out of the six coefficients are significant. The highest elasticity of output is for land which indicates that land is the dominant factor of production and is consistent with land being scarce. Irrigation is the next important input followed by labour. Fertilizer and pesticides have relatively small effects. The Cobb Douglas model provides the same order of output elasticities as in the translog model in Table 6.5. The returns to scale of 0.87 indicates slightly decreasing returns to scale as in the translog case.

Table 6.9: Maximum-Likelihood Estimates of the Cobb-Douglas Frontier Model

Name of Variables	Parameters	Coefficients	t-ratios
Stochastic frontier			
Constant	β_0	2.7152	7.5704
Land	β_1	0.2922	7.1224
Labour	β_2	0.2060	6.9401
Irrigation	β_3	0.2784	7.1607
Fertilizer	β_4	0.0078	0.4164
Pesticides	β_5	0.0810	3.5982
Inefficiency model			
Constant	δ_0	0.0245	0.1468
Age of farmers	δ_1	0.0040	1.3481
Land fragmentation	δ_2	-0.5043	-1.5919
Year of schooling	δ_3	0.0031	0.5347
Irrigation infrastructure dummy	δ_4	0.2996	3.3441
Environmental degradation dummy	δ_5	-0.2364	-2.3314
Variance parameters			
	$\sigma_u^2 = \sigma_\xi^2 + \sigma_\zeta^2$	0.0377	4.3312
	$\gamma = \left(\sigma_\zeta^2 / \sigma_u^2\right)$	0.8146	10.6860
	σ_ξ^2	0.0069	
	σ_ζ^2	0.0307	
Log-likelihood		72.8365	

We calculate the overall technical inefficiency effects in the stochastic frontier with respect to the coefficients of the parameters associated with σ_u^2 and γ reported in the last section of Table 6.9. The coefficients of the parameters, σ_u^2 and γ, are estimated to 0.04 and 0.81 respectively and both are significant. These indicate that the technical inefficiency effects are a significant component of the total variability of farm output. We test whether the technical inefficiency effects are significant and the results are shown in Table 6.10. The generalized LR test strongly rejects the null hypothesis of no technical inefficiency effects (first null hypothesis in Table 6.10). The joint effects of the farm-specific individual explanatory variables of the technical inefficiency effects model in the stochastic frontier are also significant as indicated by the generalized LR test (second null hypothesis in Table 6.10). These suggest that the farming activities are affected by the factors determining technical efficiency.

Table 6.10: Hypothesis Tests for the Technical Inefficiency Effects

Null Hypothesis	Log-likelihood value	LR statistic	Critical value*	Decision
H_0: $\gamma = \delta_0 = ... = \delta_5 = 0$	33.1590	79.3537	14.067	Reject H_0
H_0: $\delta_1 = ... = \delta_5 = 0$	38.6189	68.4352	11.070	Reject H_0

Note: * indicates 5 per cent significance level.

The dual cost frontier, analytically derived from the stochastic production frontier for the truncated normal distribution of the inefficiency component shown in Table 6.9, is:

$$C = 1.2 p_1^{0.3377} p_2^{0.2380} p_3^{0.3217} p_4^{0.009} p_5^{0.0936} \tilde{y}^{0.0936}$$

From this dual cost function, we estimate the technically and economically efficient input vectors, technically and economically costs and hence technical and economic efficiency.

6.5.3. Estimates of Technical, Allocative and Economic Efficiency

The frequency distribution of the TE, AE and EE estimates and their summary statistics are presented in Table 6.11; the corresponding frequency histograms are plotted in Figures 6.2, 6.3 and 6.4. The TE, AE and EE estimates range from 40 - 99 per cent, 49 - 99 per cent and 30 - 89 per cent with the mean efficiencies of 80, 77 and 61 per cent respectively.

Table 6.11: Frequency Distribution of Farm-Specific Efficiency Estimates

Efficiency	Number of farms			Percentage of farms		
Index (%)	TE	AE	EE	TE	AE	EE
1-48	2	0	27	1.33	0.00	18.00
48-50	1	2	3	0.67	1.33	2.00
50-52	1	2	5	0.67	1.33	3.33
52-54	1	6	10	0.67	4.00	6.67
54-56	0	2	6	0.00	1.33	4.00
56-58	2	5	7	1.33	3.33	4.67
58-60	4	2	4	2.67	1.33	2.67
60-62	2	6	14	1.33	4.00	9.33
62-64	6	8	12	4.00	5.33	8.00
64-66	9	6	8	6.00	4.00	5.33
66-68	6	8	7	4.00	5.33	4.67
68-70	4	6	9	2.67	4.00	6.00
70-72	7	4	5	4.67	2.67	3.33
72-74	8	4	7	5.33.	2.67	4.67
74-76	6	6	9	4.00	4.00	6.00
76-78	4	12	5	2.67	8.00	3.33
78-80	8	5	2	5.33	3.33	1.33
80-82	4	6	4	2.67	4.00	2.67
82-84	4	7	3	2.67	4.67	2.00
84-86	9	3	1	6.00	2.00	0.667
86-88	4	5	1	2.67	3.33	0.67
88-90	8	8	1	5.33	5.33	0.67
90-92	16	6	0	10.67	4.00.	0.00
92-94	13	4	0	8.67	2.67	0.00
94-96	14	4	0	9.33	2.67	0.00
96-98	7	3	0	4.67	2.00	0.00
98-100	0	20	0	0.00	13.33	0.00
Mean	80	77	61			
Minimum	40	48	30			
Maximum	99	99	89			
Standard Deviation	13	15	12			

This indicates that there are considerable inefficiencies in agricultural production in Bangladesh, especially *EE*. Thus there is much scope for increasing farm income and welfare of farm households. The mean value of *TE* is the highest and the mean value of

EE is the lowest as expected. The majority of farms, 33 per cent, are between 90 - 100 per cent technically efficient; 19 per cent of farms are between 80 - 90 per cent technically efficient; 22 of farms are between 70 - 80 per cent technically efficient; 18 per cent of farms are between 60 - 70 per cent technically efficient; 5 per cent of farms are between 50 - 60 per cent technically efficient; and 2 per cent of farms are between 1 - 50 per cent technically efficient.

In the case of allocative efficiency, 25 per cent of farms are between 90 - 100 per cent efficient; 19 per cent of farms are between 80 - 90 per cent efficient; 21 per cent of farms are between 70 - 80 per cent efficient; 23 per cent of farms are between 60 - 70 per cent efficient; 11 per cent of farms are between 50 - 60 per cent efficient; and only one per cent farm is between 1 - 50 per cent efficient.

In the case of economic efficiency, no farms are greater than 90 - 100 per cent efficient; 7 per cent of farms are between 80 - 90 per cent efficient; 19 per cent of farms are between 70 - 80 per cent efficient; 33 per cent of farms are between 60 - 70 per cent efficient; 21 per cent of farms are between 50 - 60 per cent efficient; and 20 per cent of farms are between 1 - 50 per cent efficient.

The relationship between TE, AE and EE is that the product of TE and AE measures provides the measure of EE. The technical and allocative efficiency gives rise to four ways for describing the relative success of farms. First, a farm may be both technically and allocatively efficient; second, a farm may show technical efficiency but allocative inefficiency; third, a farm may display technical inefficiency but allocative efficiency; fourth, a farm may be both technically and allocatively inefficient. We calculate Spearman rank correlation coefficient reported in Table 6.12.

Table 6.12: Spearman Rank Correlation Coefficients between TE, AE and EE

	TE	AE	EE
TE	1		
AE	-0.36	1	
EE	0.46	0.62	1

The rank correlation coefficient between technical and allocative efficiency is negative which implies that farms on average are either technically efficient and allocatively inefficient or technically inefficient and allocatively efficient. This result is counter intuitive in that it is expected that farmers with a high level of technical efficiency will also achieve a high allocative efficiency and has not clear explanation without further analysis.

Figure 6.2: Frequency Histogram of Technical Efficiency Index using the Cobb-Douglas Frontier

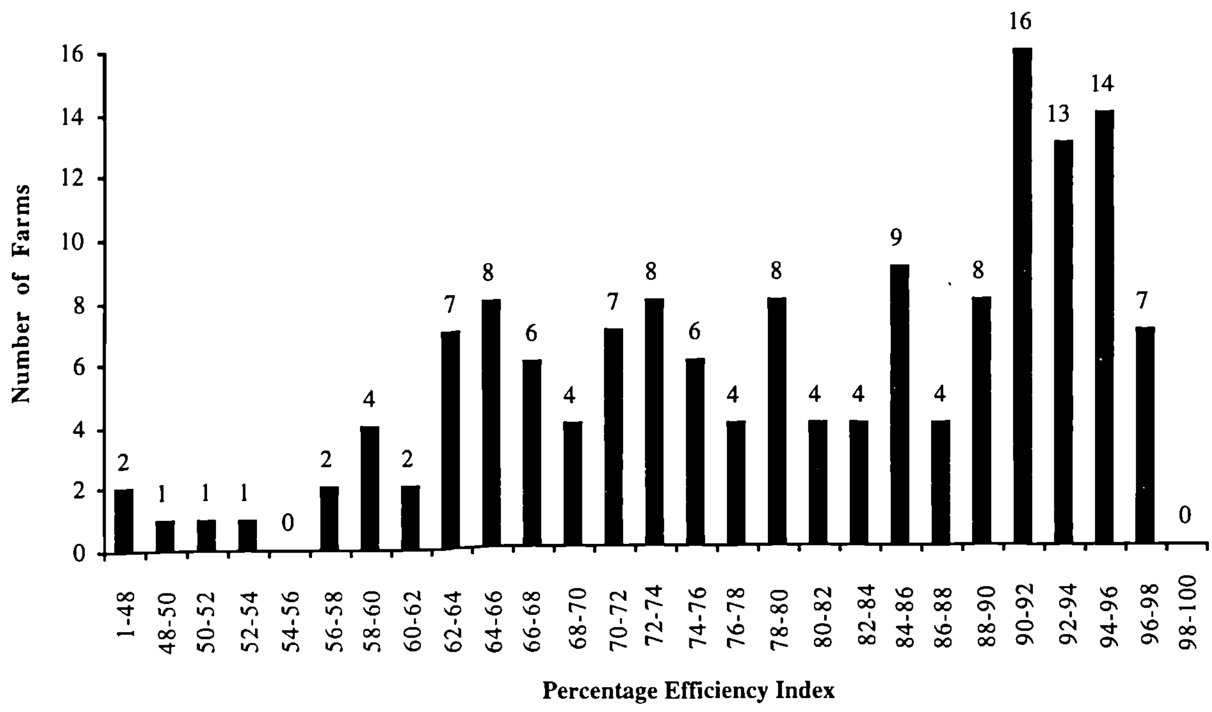


Figure 6.3: Frequency Histogram of Allocative Efficiency Index using the Cobb-Douglas Frontier

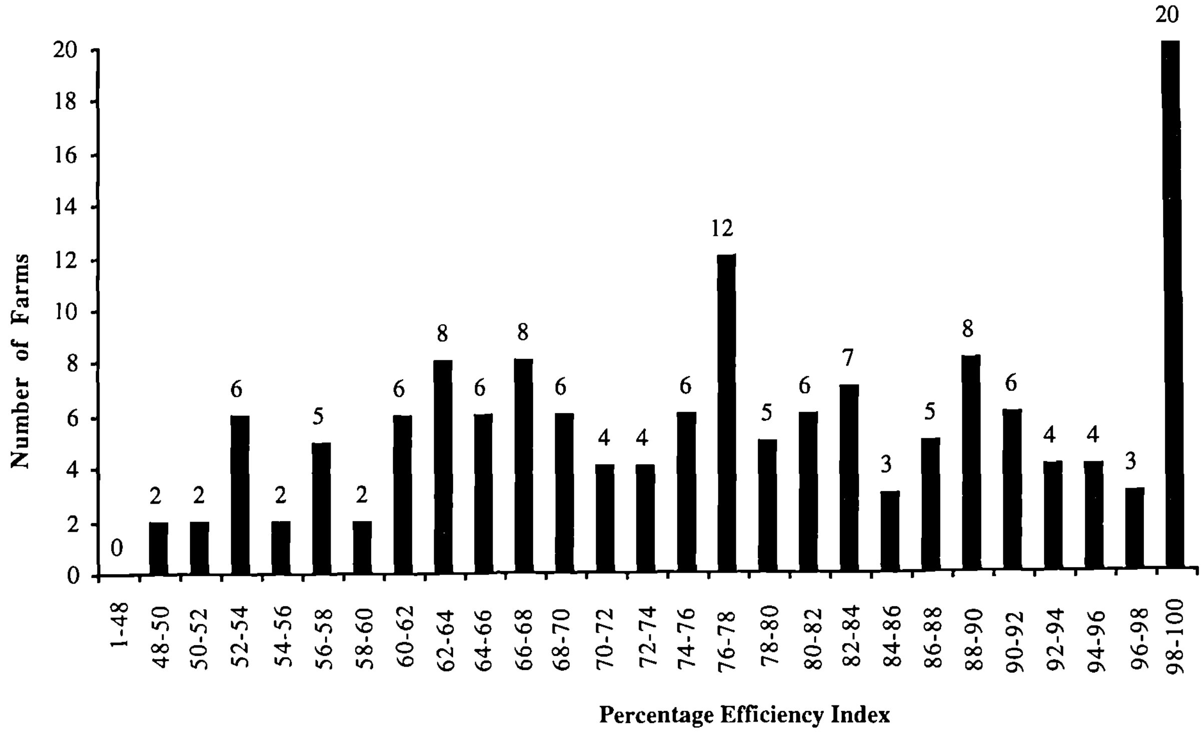
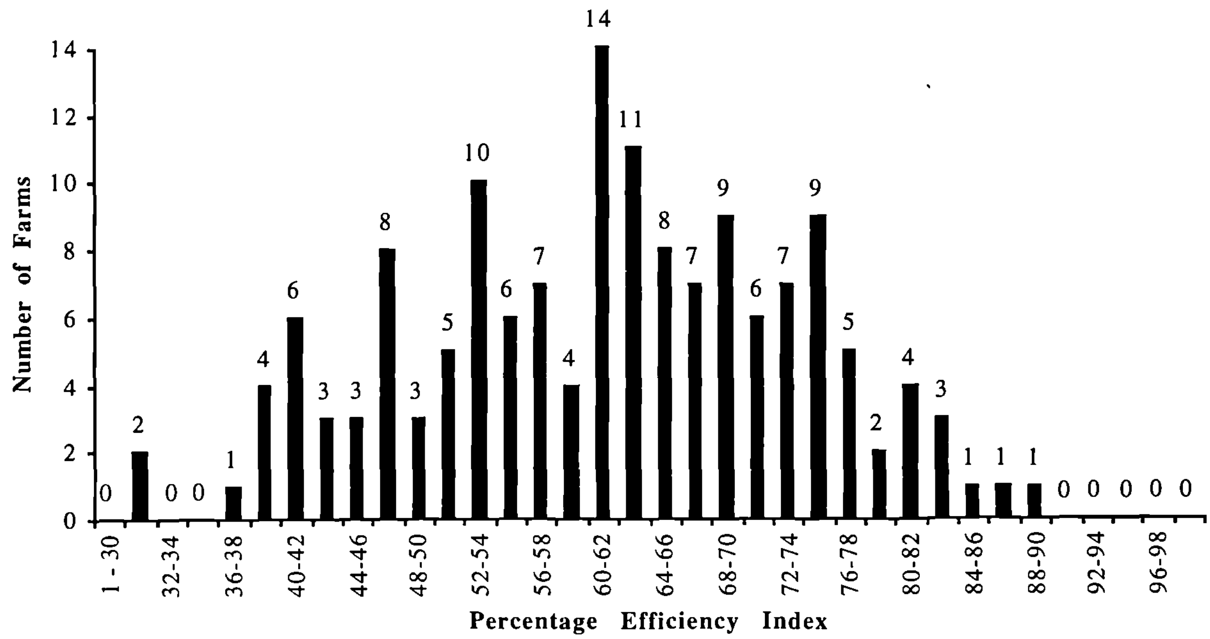


Figure 6.4: Frequency Histogram of Economic Efficiency Index using the Cobb-Douglas Frontier



6.5.4. Factors Affecting Efficiency Estimates

Differences in efficiencies may be due to factors that vary among farmers. An analysis of efficiency by socioeconomic, infrastructure and environmental factors may provide some explanation of these factors which affect efficiencies. Inefficiency is hypothesized to be determined by socioeconomic, irrigation infrastructure and environmental degradation factors, that is:

$$IE_i = \delta_0 + \delta_1 z_{1i} + \delta_2 z_{2i} + \delta_3 z_{3i} + \delta_4 z_{4i} + \delta_5 z_{5i} + w_i$$

where IE denotes farm inefficiency, z_i's are as previously defined and w_i is a stochastic random error assumed to be normally distributed. As IE is a measure of inefficiency, the dependent variable with a positive (negative) coefficient will have a negative (positive) effect on the level of efficiency.

We now turn to explain the farm-specific inefficiencies by the farm-specific socioeconomic, infrastructure and environmental degradation variables. Results are shown in Table 6.13. The coefficient for the age of farmers for technical inefficiency is positive as in the translog frontier case in Table 6.4. Again this indicates that the younger farmers are more technically efficient than older farmers. The estimated coefficients for age of allocative inefficiency and economic inefficiency are negative and significant which implies that older farmers are more capable in choosing input mixes at minimum cost. The coefficient of the years of schooling for technical inefficiency is positive which is unexpected but insignificant as in the translog case in Table 6.4 and those for allocative inefficiency and economic inefficiency are negative and significant as expected which indicates that farmers with greater years of schooling are more allocatively and economically efficient than those with fewer years of schooling. Thus farmers with more education respond more readily in adjusting input combinations with changing input prices and in using new technologies and produce closer to the frontier output.

Table 6.13: Factors Affecting Inefficiency

Factors	TI		AI		EI	
	Coefficients	t-ratios	Coefficients	t-ratios	Coefficients	t-ratios
Constant	0.0245	0.1468	0.6289	14.446	0.6631	16.669
Age of farmers	0.0040	1.7481	- 0.0031	- 3.9882	-0.0015	- 2.1094
Land fragmentation	-0.5043	-1.5919	- 0.4878	- 5.2525	-0.5002	- 5.8941
Year of schooling	0.0031	0.5347	- 0.0047	- 2.5027	-0.0034	- 1.9893
Irrigation infrastructure dummy	0.2996	3.3441	0.2215	9.6910	0.0609	2.9155
Environmental degradation dummy	-0.2364	-2.3314	- 0.0379	- 1.6351	-0.0880	- 4.1496
R^2			- 0.4912		0.4112	

Note: TI results do not have an R^2 because they are estimated along with the production frontier reported in Table 6.9. TI = technical inefficiency, AI = allocative inefficiency and EI = economic inefficiency.

The estimated coefficients for land fragmentation, i.e., land plot size, for the three inefficiencies are negative which shows that, on average, farms with greater land plot size, i.e., less fragmentation, operate at higher levels of efficiency. The coefficients for allocative and economic inefficiencies are significant. Better performance among farms with larger land size is attributable to better application of new technologies like tractors etc. and better irrigation management. This corresponds with the results from the translog stochastic frontier for technical efficiency.

The coefficient for the irrigation infrastructure dummy for technical inefficiency effects indicates that TE performance of farmers buying water from diesel pump irrigation schemes is significantly lower than of those from electricity-operated irrigation scheme as is case from the translog function in Table 6.4. The estimated coefficients for allocative inefficiency and economic inefficiency are positive and significant which indicates that farmers buying irrigation water from diesel pumps are less allocatively and economically efficient and farmers buying water from electricity-operated ones are more allocative and economic efficient. This is explained by the water extraction capacity of electricity-operated pumps being greater and irrigation cost for electricity pumps being lower. The coefficients of environmental degradation dummy variable for all inefficiencies are negative as expected which implies that the effect of environmental degradation on efficiency is positive: farmers with less degraded lands operate at higher levels of efficiency; the coefficients for technical and economic inefficiencies are significant.

6.5.5. Comparison of Technical Efficiency Estimates from both Translog and Cobb-Douglas Frontiers

This section compares the technical efficiency estimates from both the translog and Cobb-Douglas models. We test whether both models produce the same efficiency estimates. The results are reported in Table 6.14. The normal test rejects the differences in average technical efficiency obtained from the frontier production models at the five per cent level of significance. We also conduct non-parametric Wilcoxon signed-ranks test (Freund and Walpole, 1987) which implies no differences in average technical efficiency obtained from the Cobb-Douglas and translog models. We conduct both parametric F-test and Bartlett's test (Kanji, 1993) to check the homogeneity of variances of efficiency estimates. The test statistics support the hypothesis that both the frontiers produce technical efficiency estimates of equal variance at the five per cent level of significance.

Table 6.14 Tests of Hypotheses for Estimates of Technical Efficiency

	Hypothesis	Estimated value	Critical value*	Decision
Normal test	H_0=Equal means of efficiency	0.52	1.96	Accepted
F-test	H_0=Equal variances of efficiency	1.18	1.38	Accepted
Bartlett's test	H_0=Equal variances of efficiency	1.03	6.63	Accepted

Note: * indicates 5 per cent significance level.

To examine the agreement between the Cobb-Douglas and translog models and the sensitivity of estimates of technical efficiency to the choice of functional form, the Spearman rank correlation coefficient is calculated. The coefficient of rank correlation is 0.92 and significant. This indicates that both the Cobb-Douglas and translog models are in agreement on the technical efficiency rankings; the rankings of the farms along the technical efficiency spectrum are not affected by the choice of functional form. Therefore we can assume that the Cobb-Douglas frontier provides AE and EE estimates for this data set which could be obtained from the translog frontier.

6.6. Summary and Conclusions

This Chapter first estimates the translog stochastic frontier production model, with technical inefficiency effects being determined by age and education of the farmers, land fragmentation, irrigation infrastructure and environmental degradation, applying maximum likelihood single stage estimation methodology. The estimates of the output elasticities have the expected signs except for the fertilizer which is insignificant. This may be a consequence of the overuse of fertilizer in the High Barind. Farms are characterized by slightly decreasing returns to scale. The technical efficiencies among the farms range from 49 - 98 per cent with a mean of 79 per cent.

We then estimate technical, allocative and economic efficiencies following a cost decomposition technique specifying a self-dual Cobb-Douglas stochastic production model. This specification of the technical inefficiency effects model also involves the farm-specific variables as in the translog case. Again this model is estimated by maximum likelihood methods. The estimated output elasticities all have the expected signs. Farms are again characterized by slightly decreasing returns to scale. Economic efficiency and hence allocative efficiency are derived from the dual frontier cost function model and from cost decomposition. The estimates of technical, allocative and economic efficiencies vary from 40 - 99 per cent, 49 - 99 per cent and 30 - 89 per cent with the mean efficiencies of 80, 77 and 61 per cent respectively. Thus there are considerable inefficiencies in agricultural production in Bangladesh, especially, economic inefficiency and hence considerable room for improving output levels and thereby enhancing farm income and welfare of the farm households.

Hypotheses tests indicate that the technical inefficiency effects have a significant effect on farm output and the explanatory variables included in the inefficiency models have significant influences upon the technical inefficiencies for both the translog and Cobb-Douglas specifications. Further, hypotheses tests confirm no differences in averages and variances of technical efficiency estimates, an insensitivity of technical efficiency estimates to the choice of functional form, and no significant differences in the technical efficiency rankings obtained from both the translog and Cobb-Douglas specifications. The results of the analysis of technical inefficiency by socio-economic factors show that the younger farmers are more receptive to new technology, and those with more education are more likely to operate farming activities efficiently and older farmers are more allocatively and economically efficient than their younger counterparts. Moreover plot size is inversely related technical, allocative and economic inefficiencies. Thus greater land plot size, that is, less land fragmentation, contributes significantly to increasing farm efficiency. Land management and land tenure policies in reducing the land fragmentation are beneficial in generating gains in efficiencies.

An important feature of our analysis of the inefficiency models is that it includes irrigation infrastructure and environmental factors to examine their effects on farm efficiency. The results show that the irrigation infrastructure, i.e., diesel-operated irrigation schemes have positive influences upon technical inefficiency effects; and electricity-operated irrigation schemes have positive influences upon allocative and economic efficiencies. Thus, electrification programmes in rural areas are critical in reducing the inefficiency effects in production. Soil degradation, as an environmental factor, is found to be positively related to inefficiencies. Policies which reduce soil degradation, for example, terracing, drainage etc., would be effective in reducing inefficiency in production, thereby increasing productivity and household welfare for the rice farmers in Bangladesh.

Chapter 7

Efficiency Measurement and Data Envelopment Analysis

7.1. Introduction

Production frontiers can be constructed using the mathematical programming approach of Data Envelopment Analysis (DEA) and the DEA approaches are summarized in Figure 5.1. The stochastic econometric frontier approach specifies a function for the production function, assumes a distribution for the error term and then fits it to observed data by minimizing some measure of their distance from the estimated frontier; deviations from the efficient frontier are measures of technical efficiency. This parametric method produces a consistent framework for analyzing efficiency by segregating variations from the frontier technology into a stochastic error component and an asymptotic non-negative random term which reflects inefficiency. However the approach has an important drawback in that the maintained hypothesis of the functional form can not be observed (Varian, 1984; Banker and Maindiratta, 1988) and thus it imposes restrictions on the frontier production technology that may not hold; this affects the distribution and estimation of the efficiency measures (Chavas and Aliber, 1993).

The deterministic nonparametric mathematical programming approach to efficiency measurement, attributable to Farrell (1957), is an alternative technique to analyze and measure efficiency. This Chapter discusses the construction of production frontiers using mathematical programming. We discuss DEA methods starting with a simple model and progressing to a more general model.

DEA is a nonparametric mathematical programming approach to frontier estimation which has been developed independently of the stochastic frontier approach over the past two decades. Farrell's (1957) approach to efficiency measurement consists of a conical hull of input-output vectors based on a production possibility set. The conical hull of vectors is constructed by linear programming (LP) techniques, for the single output case, with a subset of the sample lying on the production possibility set and the rest lying above. Charnes, Cooper and Rhodes (1978) reformulated this piecewise-linear convex hull approach to the estimation of technical efficiency and frontier models, to incorporate multiple-output multiple-input technologies. Their approach assumes constant returns to scale (CRS) and is referred to as the CRS DEA model. This model is used here to assess the relative efficiency of homogeneous farms in transforming inputs into outputs.

Banker, Charnes and Cooper (1984) extended the CRS model by relaxing the assumption of constant returns to scale to variable returns to scale (VRS). The model is known as the VRS model. The VRS DEA model differs from the CRS DEA model in that it envelops the data more closely, thereby producing technical efficiency estimates greater than or equal to those from the CRS DEA model.

Coelli (1995) provides a review and critique of different DEA approaches. [7.1] DEA is both nonparametric and nonstochastic since it does not impose any *a priori* parametric restrictions on the underlying frontier technology (because it does not necessitate any functional form to be specified for the frontier methodology) and it does not require any distributional assumption for the technical inefficiency terms. Therefore the method avoids the imposition of unwarranted structures on both the frontier technology and the inefficiency component that might create a distortion in the measures of efficiency (Färe et al. 1985). DEA has the added advantages of evaluating scale, allocative and economic efficiency. The minimum assumptions required for this DEA frontier methodology is monotonicity and convexity of the efficient frontier (Banker et al., 1984).

DEA estimates efficiency relative to the Pareto-efficient frontier which estimates best performance (Murthi et al., 1997). Furthermore, DEA can obtain target values based on the best practice units (peers) for each inefficient farm that can be used to provide guidelines

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[7.1] Seiford and Thrall (1990), Bjurek, et al. (1990), Lovell (1993, 1994), Charnes et al. (1995), Seiford (1996) and Ali and Seiford (1993) also review the nonparametric DEA approach.

for improved performance. However the major shortcoming of DEA is that it is deterministic and assumes a zero value for the stochastic random error component; thus technical inefficiency reflects all unexplained variations of agricultural production and the inefficiency of the observed farm is therefore biased upwards. Moreover, since there is no measurement error or other random noise and since it is nonparametric, efficiency measures can not be subjected to statistical tests.

DEA can be employed to estimate both technical and economic efficiency depending on the type of data available (cross section or panel) and the kind of variables available (quantities only, or quantities and prices). Technical efficiency can be measured from quantity data for inputs and outputs while measures of economic efficiency require both quantity and price data. Estimation of technical and allocative efficiency assumes behavioural goals, such as cost minimization or profit maximization and a two stage procedure. The first estimates technical efficiency and the second measures economic efficiency; allocative efficiency is calculated from economic and technical efficiency.

The DEA frontier gives either the maximum output for a given input level or uses the minimum input for a given output level. Thus the analysis of efficiency can have an input-saving or an output-augmenting interpretation.

This Chapter is organized as follows: Section 7.2 explains the construction of DEA to measure technical efficiency; Section 7.3 provides a description of DEA to measure technical, allocative and economic efficiency simultaneously; Section 7.4 explains a Tobit model to quantify the sources of farm-specific inefficiency; and a summary follows in Section 7.5.

7.2. DEA Frontier

Assume that there are n farms to be evaluated and that each uses q inputs to produce r outputs. The ith farm uses $x_i = \{x_{ki}\}$ of inputs $(k = 1,2,3,\ldots,q)$ and produces $y_i = \{y_{mi}\}$ of outputs $(m = 1,2,3,\ldots,r)$. Assume that $x_{ki} > 0$ and $y_{mi} > 0$. The $(k \times n)$ input matrix is denoted by X and the $(m \times n)$ output matrix is denoted by Y for all n farms. The column vectors x_i and y_i represent inputs and outputs for the ith farm respectively. The DEA frontier can be expressed as follows:

$$F(y_1, y_2, y_3, \ldots, y_r) = \left\{ (x_1, x_2, x_3, \ldots, x_q): \right.$$

$$y_m \leq \sum_{i=1}^{n} \omega_i y_{mi},$$

$$x_k \geq \sum_{i=1}^{n} \omega_i x_{ki},$$

$$\left. \omega_i \geq 0;\ \sum_{i=1}^{n} \omega_i = 1 \right\}$$

where $\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n)$ is an intensity vector that forms convex combinations of observed input vectors and output vectors and represents the percentages of other farms used to construct the virtual efficient farmers. For instance, if the efficient farm A is capable of producing output $y_{(A)}$ using inputs $x_{(A)}$, then other farms should also be capable of the same production schedule. Similarly, if the efficient farm B produces output $y_{(B)}$ using inputs $x_{(B)}$, then other farms should also be able to produce the same if they were to produce efficiently. Farmers A, B and others can be combined to form a composite farmer with composite outputs and inputs. Since this composite farmer does not necessarily exist, it is sometimes called a "virtual farmer".

7.2.1. Input-oriented DEA Models

The input-oriented DEA frontiers address the issue: by how much can quantities of inputs be proportionally contracted without altering the amount of output produced? We introduce the CRS DEA model via the ratio form. Its basic characteristic is the reduction of the multiple-output/multiple-input situation for every farm to a single virtual output and a single virtual input ratio. The ratio of all outputs to all inputs provides a measure of efficiency for the ith farm that is a function of the multipliers. This forms the objective function for the ith farm which is maximized:

$$Maximize \quad \left(\sum_{m-1}^{r} \vartheta_m y_{mi} \Big/ \sum_{k=1}^{q} v_k x_{ki} \right) \tag{7.1}$$

where ϑ_m and v_k are the weights assigned to each output and input. These weights are optimally assigned to the DEA, subject to the constraints that no other farm has an efficiency larger than unity if it applies the same weights, indicating that the efficient farm has an efficiency value of unity and the derived weights are non-negative. Equation (7.1) is unbounded unless additional constraints are imposed. The mathematical programming problem for the CRS input-oriented ratio form to measure the efficiency of the ith farm relative to set of peer units can be written as:

$$Maximize \quad \left(\sum_{m\ 1}^{r} \vartheta_m y_{mi} \Big/ \sum_{k=1}^{q} v_k x_{ki} \right) \tag{7.2}$$

$$subject\ to \quad \left(\sum_{m\ 1}^{r} \vartheta_m y_{mj} \Big/ \sum_{k-1}^{q} v_k x_{kj} \right) \leq 1$$

$$\vartheta_m \geq 0 \text{ for } m = 1,2,3,...,r \text{ and } v_k \geq 0 \text{ for } k = 1,2,3,...,q.$$

The optimal weights can be obtained by solving this mathematical programming problem. In matrix notation, (7.2) can be expressed as:

$$\underset{\vartheta, \upsilon}{Maximize} \quad \left(\vartheta y_i / \upsilon x_i \right)$$

$$subject\ to \quad \vartheta y_j / \upsilon x_j \leq 1 \qquad (i = 1,2,3,....,n)$$

$$\vartheta, \upsilon \geq 0$$

where ϑ and υ represent $(m \times 1)$ vector of output weights and $(k \times 1)$ vector of input weights respectively. This ith farm is known to be efficient $\left(\vartheta y_i / \upsilon x_i = 1\right)$ if no other farm can produce more of at least one output than the ith farm without requiring more of at least one input or producing less of some output. The LP in (7.2) provides values of ϑ and υ to measure the efficiency of the ith farm subject to the constraints that each efficiency estimate will be less than or equal to unity. This ratio form produces an infinite number of optimal solutions: if (ϑ^*, v^*) is optimal, then $(B\vartheta^*, B\upsilon^*)$ is also optimal for $B > 0$. This fractional programming problem needs to be transformed to an LP model to be solved. This can be done by scaling the denominator of the objective function equal to a constant such as unity [7.2] that yields the following LP problem:

$$\underset{\theta,\upsilon}{Maximize} \ (\theta y_i)$$

$$subject\ to \quad \upsilon x_i = 1,$$

$$\theta y_j - \upsilon x_j \leq 0 \qquad (j = 1,2,3,\ldots,n)$$

$$\theta, \upsilon \geq 0.$$

This is termed the multiplier form of the LP problem. Its dual is:

$$\underset{\varphi_i^{I,CRS},\omega}{Minimize} \quad \varphi_i^{I,CRS} \tag{7.3}$$

$$\text{subject to} \quad -y_i + Y\omega \geq 0,$$

$$\varphi_i^{I,CRS} x_i - X\omega \geq 0,$$

$$\omega \geq 0$$

where Y is an $(m \times n)$ matrix of outputs, X is an $(k \times n)$ matrix of inputs and ω is an $(n \times 1)$ dimensional vector of intensity weights. The scalar, $\varphi_i^{I,CRS}$ $(\varphi_i^{I,CRS} \leq 1)$ is the technical efficiency score for the ith farm with $\varphi_i^{I,CRS} = 1$ implying that it is technically efficient and on the frontier (Farrell, 1957). The farm-specific values for φ_i are obtained by solving the LP problems n times where φ_i is the fraction by which a farm can multiply its input vector and still produce the same output. The first constraint states that the

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[7.2] This can also be done by scaling the numerator of the objective function equal to a constant such as unity that yields the following LP problem:

$$\underset{\theta_1,\vartheta}{Min} \ (\theta_1 x_i), \quad subject\ to \quad \vartheta y_i = 1;\ \upsilon x_j - \theta_1 y_j \geq 0 \quad (j = 1,2,3,\ldots,n) \text{ and } \theta_1, \upsilon \geq 0.$$

quantity of output produced by ith farm must be less than or equal to the quantity of output produced by the reference farm. The second constraint implies that efficiency-corrected inputs utilized by ith farm must at least equal inputs used by the reference farm.

The application of CRS technology is appropriate if all farms operate at an optimal scale. In practice, farms do not because of, say, constraints on finance or the gradual adjustment of capital and labour. The CRS LP problem can be extended to account for variable returns to scale by including the convexity constraint, $\Omega'\omega = 1$, in (7.3), where Ω is an $(n \times 1)$ vector of constants. If we compute the LP in (7.3) with this convexity constraint n times for n farms, the solution $\varphi_i^{I,VRS}$ is a measure of TE of the ith farm under VRS in an input-oriented framework. If $\varphi_i^{I,VRS} = 1$, the farm is technically efficient and lies on the frontier; the farm is technically inefficient and lies outside the frontier if $\varphi_i^{I,VRS} < 1$.

Non-increasing Returns to Scale (NIRS) is modelled by adding the constraint $\Omega'\omega \le 1$ to the LP model in (7.3) where the solution $\varphi_i^{I,NIRS}$ is a measure of TE of the ith farm under NIRS in an input-oriented methodology.

7.2.2. Output Orientations of DEA Frontier

The output-oriented DEA models address the issue: by how much can the amounts of output be proportionally expanded without changing the quantities of inputs applied? We can formulate CRS output-oriented mathematical programming problem in ratio form by considering the ratio of virtual input to virtual output as follows:

$$\textit{Minimize} \quad \left(\sum_{k=1}^{q} v_k x_{ki} \Big/ \sum_{m=1}^{r} \vartheta_m y_{mi} \right)$$

$$\textit{subject to} \quad \left(\sum_{k=1}^{q} v_k x_{kj} \Big/ \sum_{m=1}^{r} \vartheta_m y_{mj} \right) \ge 1$$

$$\vartheta_m \ge 0 \text{ for } m = 1,2,3,\ldots,r \text{ and } v_k \ge 0 \text{ for } k = 1,2,3,\ldots,q.$$

Scaling the denominator of the objective function equal to a constant such as unity, we obtain the LP problem as follows:

$$Minimize \left(\sum_{k=1}^{q} v_k x_{ki} \right)$$

$$subject\ to \quad \sum_{m=1}^{r} \vartheta_m y_{mi} = 1$$

$$\sum_{k=1}^{q} v_k x_{kj} - \sum_{m=1}^{r} \vartheta_m y_{mj} \geq 0$$

We write the problem in matrix notation to facilitate our discussion as:

$$\underset{\theta_i', \vartheta}{Minimize} \quad \theta_i' x_i$$

$$subject\ to \quad \vartheta y_i = 1,$$

$$\theta_i' x_j - \vartheta y_j \geq 0,$$

$$\vartheta \geq 0 \text{ and } \theta_i' \geq 0 \ ,$$

The corresponding dual is:

$$\underset{\varphi_i^{O,CRS}, \omega}{Maximize} \quad \varphi_i^{O,CRS} \tag{7.4}$$

$$subject\ to \quad -\varphi_i^{O,CRS} y_i + Y\omega \geq 0,$$

$$x_i - X\omega \geq 0,$$

$$\omega \geq 0$$

where $\varphi_i^{O,CRS}$ is a scalar which measures farm-specific efficiency under the output-oriented CRS method; $\varphi_i^{O,CRS} = 1$ indicates that the farm is efficient and lies on the frontier and $\varphi_i^{O,CRS} < 1$ implies that the farm is inefficient and lies outside the frontier.

The first constraint states that the efficiency-corrected amount of output must be less than or equal to the quantity of output produced by the reference farm. The second constraint implies that the quantity of input utilized by ith farm must at least equal the quantity of input used by the reference farm. The single-output, output-oriented DEA frontier maximizes the proportional increase in the output vector while remaining within the efficient frontier.

In the same way as input-oriented model, the output-oriented VRS DEA model can be formulated by adding the convexity constraint $\Omega'\omega = 1$ to (7.4); the resulting frontier is estimated n times for n farms to provide measures of TE. Similarly, the constraint $\Omega'\omega \leq 1$ is added to (7.4) to formulate the NIRS DEA model.

The output-oriented VRS approach yields efficiency scores forming a convex hull of intersecting planes and envelops the data more tightly than the CRS conical hull, thus gives technical efficiency scores greater than or equal to those achieved from the CRS model.

7.2.3. Economies of Scale in DEA

Measures of scale efficiency for each farm can be obtained using both the CRS and VRS DEA. Technical inefficiency scores from the CRS DEA (CRS TI) can be decomposed into pure technical inefficiency (VRS TI) and scale inefficiency. The CRS TI is greater than that of VRS TI and thus the difference in the CRS and VRS technical inefficiency scores for a particular farm provides a measure of scale inefficiency.

The CRS, VRS and NIRS technologies are illustrated in Figure 7.1. In an input-oriented framework, the CRS approach measures the input-oriented technical inefficiency of the farm operating at point D by the distance BD. However, the VRS approach estimates technical inefficiency as CD which is smaller than the technical inefficiency BD from the

CRS approach since the VRS approach envelops the data more closely. The difference, BC, measures scale inefficiency $\left(SE_i^I\right)$. These notions can be expressed as:

$$\varphi_i^{I,CRS} = \frac{AB}{AD} \qquad \left(0 \le \varphi_i^{I,CRS} \le 1\right)$$

$$\varphi_i^{I,VRS} = \frac{AC}{AD} \qquad \left(0 \le \varphi_i^{I,VRS} \le 1\right)$$

and

$$SE_i^I = \frac{AB}{AC} \qquad \left(0 \le SE_i^I \le 1\right)$$

Further:

$$\varphi_i^{I,VRS} \times SE_i^I = \frac{AC}{AD} \times \frac{AB}{AC} = \frac{AB}{AD} = \varphi_i^{I,CRS}$$

$$\therefore \qquad \varphi_i^{I,CRS} = \varphi_i^{I,VRS} \times SE_i^I$$

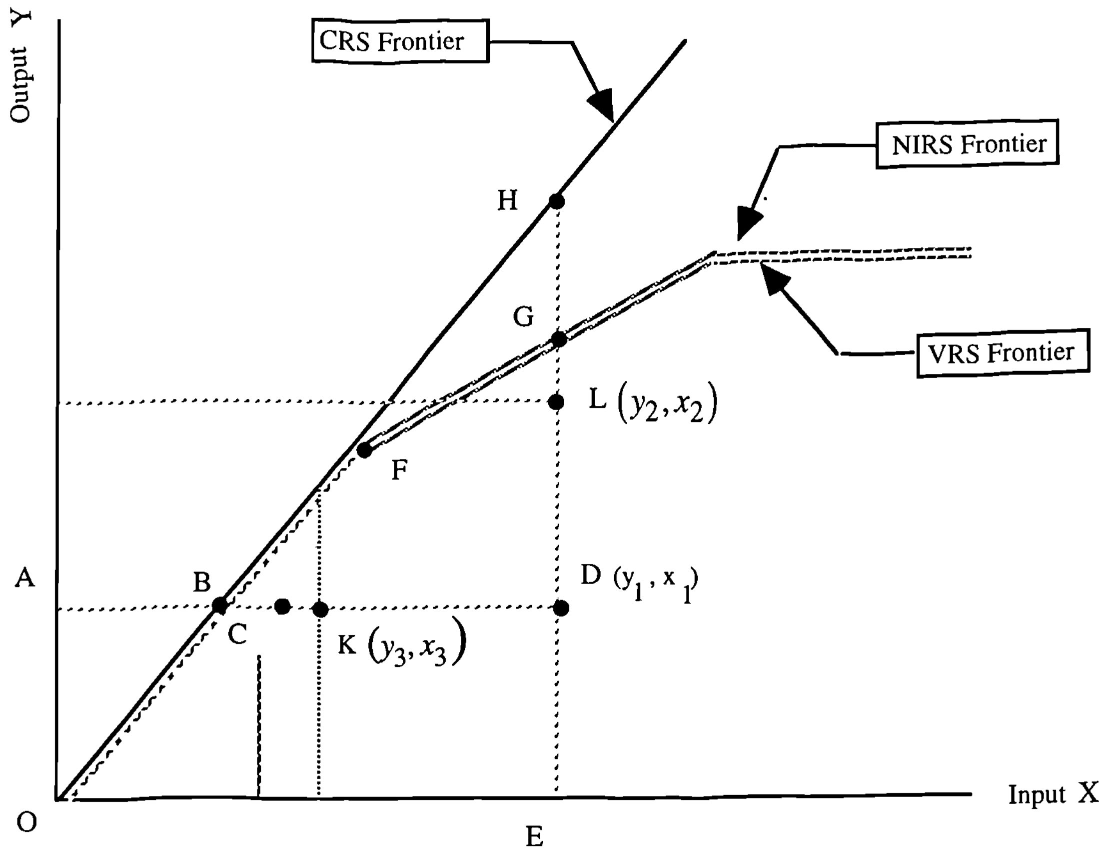
or

$$SE_i^I = \frac{\varphi_i^{I,CRS}}{\varphi_i^{I,VRS}} \tag{7.5}$$

Thus, scale efficiency is obtained by decomposing the CRS technical efficiency into pure technical efficiency and scale efficiency. This scale efficiency measure does not predict whether the farm is operating at decreasing or increasing returns to scale which can be determined by running a DEA problem with the additional constraint imposed of NIRS.

The efficiency scores obtained for each farm from the three DEA frontiers (CRS, VRS and NIRS) can be ordered relative to each other and this ordering provides information regarding the existence of scale economies at any observed output.

Figure 7.1: Economies of Scale in DEA model



For the farm (y_1,x_1) at point D, the CRS and NIRS technologies provide the same measure of efficiency scores, but the VRS technology yields a higher level indicating that the VRS technology envelops the data more closely than the CRS and NIRS technologies at output vector y_1. So increasing returns to scale (IRS) prevail. If we consider the farm (y_2,x_2) at point L, the efficiency measures are equal relative to both the VRS and NIRS technologies, but lower for the CRS technology which implies that the CRS technology does not envelop the data as closely as the other two predicting decreasing returns to scale (DRS) at output vector y_2.

In an output-oriented framework, the CRS DEA estimates technical inefficiency of the farm operating at D is measured by the distance DH and the VRS by the distance DG. The distance GH is due to scale inefficiency SE_i^O.

Therefore the measures of efficiency are:

$$\varphi_i^{O,CRS} = \frac{ED}{EH}$$

$$\varphi_i^{O,VRS} = \frac{ED}{EG}$$

and

$$SE_i^O = \frac{EG}{EH}$$

Further:

$$\varphi_i^{O,VRS} \times SE_i^O = \frac{ED}{EG} \times \frac{EG}{EH} = \frac{ED}{EH} = \varphi_i^{O,CRS}$$

$$\therefore \ \varphi_i^{O,CRS} = \varphi_i^{O,VRS} \times SE_i^O$$

or $$SE_i^O = \frac{\varphi_i^{O,CRS}}{\varphi_i^{O,VRS}} \qquad (7.6)$$

The scale efficiency itself does not indicate if decreasing or increasing returns to scale exist. The presence of potential economies of scale at any input is predicted by observing the ordering of efficiency scores of CRS, VRS and NIRS frontiers.

Consider the farm (y_1, x_1) at point D in Figure 7.1 where measures of efficiency are equivalent for both VRS and NIRS technologies, but less for CRS technology. This indicates that CRS technology does not envelop the data as closely as the other two technologies at input x_1 and hence DRS exist. Now consider the farm (y_3, x_3) at point K where the efficiency measures are equivalent for both the CRS and NIRS technologies, but greater relative to the VRS technology. This implies that the VRS technology envelops the data more closely than the other two technologies at input vector x_3 and thus IRS exist.

To summarize:

Input orientation: $\varphi_i^{I,CRS} \leq \varphi_i^{I,NIRS} \leq \varphi_i^{I,VRS}$

Output orientation: $\varphi_i^{O,CRS} \leq \varphi_i^{O,NIRS} \leq \varphi_i^{O,VRS}$

For both orientation: $\varphi_i^{NIRS} < \varphi_i^{VRS}$ implies IRS

$\varphi_i^{CRS} < \varphi_i^{NIRS}$ implies DRS

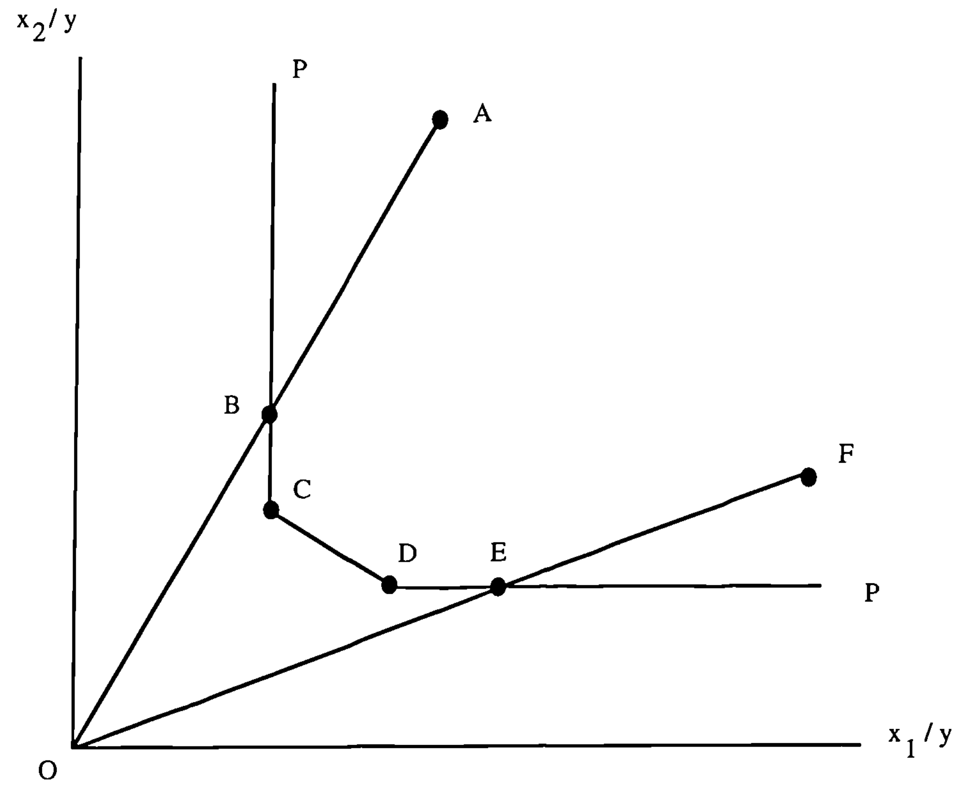
and: $\varphi_i^{CRS} = \varphi_i^{NIRS} = \varphi_i^{VRS}$ implies the restrictive property of NIRS.

Alternatively, scale economies arises due to either increasing or decreasing returns to scale and can be determined by inspecting the sum of the weights $S = \sum_{j=1}^{n} \omega_j$ with CRS technology (Banker, 1984). Therefore $S = 1$ implies constant returns to scale (optimal scale), $S > 1$ implies decreasing returns to scale (super-optimal scale) and $S < 1$ implies increasing returns to scale (sub-optimal scale) (Löthgren and Tambour, 1996; Banker and Thrall, 1992; and Førsund and Hernaes, 1994).

7.2.4. Efficiency Measurement and Slacks

The nonparametric DEA frontier, consisting of a piecewise linear conical hull and piecewise linear frontiers running parallel to the axes, causes a problem in measuring efficiency. To illustrate, consider Figure 7.2 where the farms applying input mix C and D are efficient; the frontier is PP and farms using input combinations B and E are inefficient.

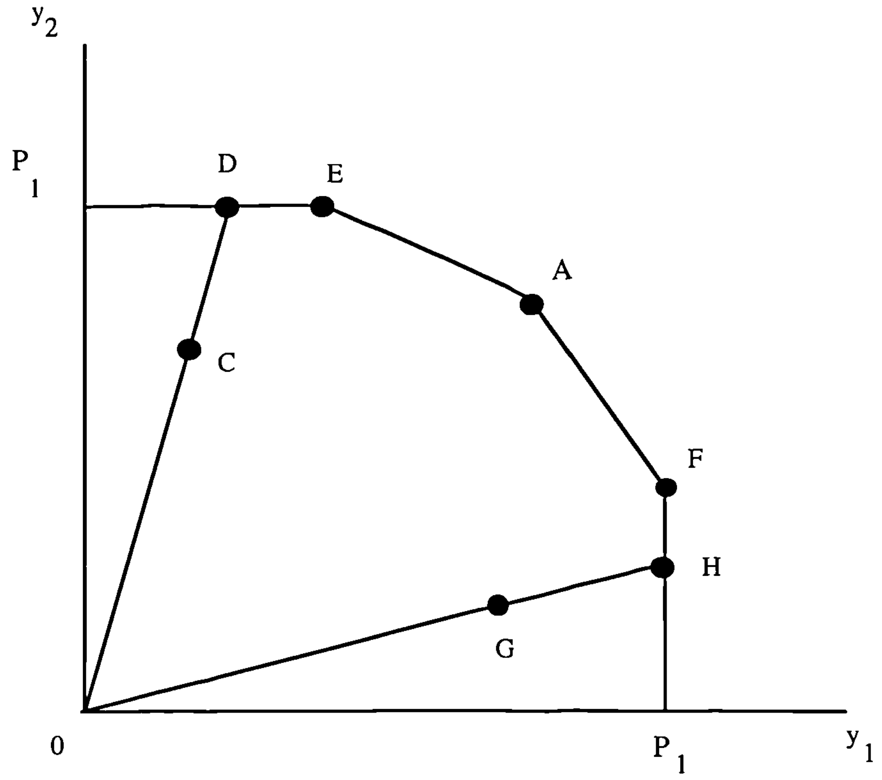
Figure 7.2: Input Slacks and Efficiency Measurement



Farrell's definition of technical efficiency provides the efficiency of farms operating at points A and F with input mix B and E as OB/OA and OE/OF respectively. However the farm operating at point A with input combination B can reduce input x_2 by the amount BC and the farm operating at point F with input mix E can reduce input x_1 by the amount DE and both farms remain capable of producing the same output; the amount BC is input slack of farm operating at point A and the amount DE is input slack of farm operating at point F and hence these farms are inefficient. Thus the amounts of inputs which can be reduced while remaining on the same level of output are called "input slacks".

Similarly, consider the analogous "output slack". An output-oriented DEA approach with two-outputs is shown in Figure 7.3. The piecewise linear production possibility curve is $P_1DEAFHP_1$. An output slack occurs for the farm which lies below the production possibility curve and remains at a right angle of the section of the curve to the axes when a radial expansion in output projects the farm onto those parts of the curve. Consider the farm with production point C.

Figure 7.3: Output Slacks and output-oriented DEA



The production point C can be projected to the point D which lies on the frontier but not on the efficient frontier, because without reducing the output y_2 and applying any more inputs, the production of output y_1 could be increased by the amount DE. In the same manner, for the farm at G, production of output y_2 could be increased by HF without increasing the amount of inputs and decreasing the level of output of y_1. Therefore the output slack of the farm with production point C is DE in output y_1, and that of G is HF in output y_2.

Note that both the output- and the input-oriented DEA models estimate the same frontier and therefore, by definition, determine the same set of farms as being efficient, but the efficiency measures associated with the inefficient farms may vary between the two models (Coelli et al., 1998). So an appropriate direction of technical efficiency could be provided by reporting the Farrell measure of technical efficiency and any non-zero input or output slacks.

The VRS DEA models can be reexpressed with input and output slacks as follows:

Input-oriented

$$\underset{\varphi_i^{I,VRS},\omega}{Minimize} \quad \varphi_i^{I,VRS}$$

$$subject\ to \quad -y_i + Y\omega - S_O = 0,$$

$$\varphi_i^{I,VRS} x_i - X\omega - S_I = 0,$$

$$\Omega'\omega = 1$$

$$\omega \geq 0$$

Output-oriented

$$\underset{\varphi_i^{O\,VRS},\omega}{Maximize} \quad \varphi_i^{O,VRS}$$

$$subject\ to \quad -\varphi_i^{O,VRS} y_i + Y\omega - S_o = 0,$$

$$x_i - X\omega - S_I = 0,$$

$$\Omega'\omega = 1$$

$$\omega \geq 0$$

where S_I and S_O are $(k \times 1)$ and $(m \times 1)$ vectors of input and output slacks respectively.

The identification of nearest efficient frontier point and the estimation of slacks are not straightforward if there are multiple inputs and outputs. A second-stage LP problem can be formulated to identify the nearest efficient point which maximizes the sum of slacks required to shift from the first stage projected point (inefficient point, such as point B in Figure 7.2) to a second stage efficient point (such as point, C in Figure 7.2). This second-stage LP problem is formulated as:

$$\underset{\omega_I, S_O, S_I}{Minimize} \quad -\left(H'S_O + R'S_I\right) \tag{7.7}$$

$$subject\ to \quad -y_i + Y\omega - S_O = 0,$$

$$\varphi' x_i - X\omega - S_I = 0,$$

$$\omega \geq 0, \quad S_O \geq 0, \quad S_I \geq 0,$$

where H and R are $(m \times 1)$ and $(k \times 1)$ unit vectors respectively. In this second stage, LPs are solved for each farm where the first-stage gives the value of φ which is used in the second-stage. However in this second-stage, the projected point obtained is not invariant to the units of measurement (Lovell and Pastor, 1995): a change of the unit of measurement, say for land from acres to hectares, ceteris paribus, might identify various efficient boundary points and hence various values of ω and slacks. As a result, many studies solve the first-stage, which does not explicitly include slacks, for the measures of Farrell technical efficiency for each farm and report the values of technical efficiency and the residual slacks as $S_O = -y_i + Y\omega$ and $S_I = \varphi x_i - X\omega$. This removes the problem relating to the units of measurement and involves less programming.

7.2.5. Multi-stage DEA Method

Coelli (1998) proposes a multi-stage DEA methodology to remove the problems in the two-stage DEA method. This multi-stage DEA method results in a sequence of radial DEA models and identifies more representative efficient points. Further, it is invariant to the units of measurement.

Consider the LP model in (7.3) which constructs the input-oriented CRS frontier; [7.3] we omit the subscript and superscript on φ for notational brevity. This is solved for each of the n farms and the estimate of the φ's for each farm results. In the second stage, the radial DEA method maximizes the sum of remaining slacks; this is presented in (7.7) where $\varphi' x_i$ is the vector of inputs of the ith farm which has been contracted by being multiplied by the φ's from the first stage. This second-stage LP is solved for each of the n farms which yields the set of all farms with no slacks and with technical efficiency scores of $\varphi = 1$. In a Koopmans's (1951) sense, this is the "efficiency set" among all the farms. Moreover, the set of all farms with at least one non-zero slack variable is also identified as the "have-slacks set" (Coelli, 1998).

[7.3] An output-oriented multi-stage DEA method can also be formulated in an analogous manner.

The standard DEA approach in Section 7.2.3 uses two steps. But the projected point achieved in the Step 2 LP is not applied in this multi-stage technique where Step 2 attempts to find the efficient set and the have-slack set. The remaining steps of this multi-stage methodology undertake a sequence of radial movements from Step 1 projected point having each farm in the have-slacks set to obtain a projected point on the efficient frontier. We use the efficient set as the reference set in all the LPs in the remaining steps.

In Step 3, a sequence of q LPs is undertaken allowing contraction in only one of the inputs in each LP that attempts to find all input dimensions in which some slacks may occur for the ith farm in the have-slacks set. We find the existence of a potential input slack in that input if contraction is obtained. The LP for the q_1th input of the ith farm is defined as follows:

$$\underset{\varphi,\omega}{Minimize} \quad \varphi$$

$$subject\ to \quad -y_i + Y_e\omega \geq 0,$$

$$\varphi\varphi' x_i^{q} - X_e^{q_1}\omega \geq 0,$$

$$\varphi' x_i^{q_1'} - X_e^{q'}\omega \geq 0$$

$$\omega \geq 0$$

where $X_e^{q_1}$ represents the $(1 \times n_e)$ vector of the q_1th input of all efficient farms, n_e is the number of efficient farms identified in Step 2, Y_e is the matrix of outputs of these efficient farms, ω is of dimension $(n_e \times 1)$, $\varphi' x_i^{q_1}$ indicates the q_1th input of the ith farm which has been contracted by being multiplied by φ from Step 1, $\varphi' x_i^{q'}$ is the $\{(k-1) \times 1\}$ vector of inputs of the ith farm (excluding the q_1th input) which has been contracted by being multiplied by φ from Step 1, and $X_e^{q'}$ is the $\{(k-1 \ \times n_e\}$ matrix of inputs of all efficient farms excluding the q_1th farm. Step 3 is only used to determine all input dimensions in which potential slacks prevail because the slacks identified in Step 2 need not identify all dimensions in which potential slack exists. Therefore Step 3, leaving the projected point obtained unchanged, determines the input dimensions in which slacks may remain.[7.4]

In Step 4, we conduct a LP, for the ith farm in the "have-slack" set, which attempts to find a radial contraction in all inputs getting potential slack (in Step 3). This is specified as:

$$\underset{\varphi,\omega}{Minimize} \quad \varphi$$

$$subject\ to \quad -y_i + Y_e\omega \geq 0,$$

$$\varphi\varphi' x_i^r - X_e^r \omega \geq 0,$$

$$\varphi' x_i^{rs} - X_e^{rs} \omega \geq 0$$

$$\omega \geq 0$$

where the superscript r denotes the subset of inputs having potential slacks and rs indicates the remaining inputs. The projected point, $(y_i, \varphi' x_i)$, found in Step 1, becomes the starting point for this radial input contraction.

Step 5 repeats Step 3 and 4 until no slack remains in any input by taking the projected point identified in Step 4 if some input slacks remain in some dimensions after conducting the radial reduction in Step 4. If slacks only exist in Step 3; Step 4 and 5 are omitted.

Having obtained the projected point from Step 5 for the ith farm and repeating Steps 3-5, Step 6 undertakes the radial enhancement in output slack dimensions until no slack exists in output. The final projected point now remains on the efficient frontier, determines the peers of the farm from the ω vector, and estimates the slacks by subtracting it from the projected point in Step 1. This final projected point will be independent of the units of measurement since this result comes from the observation of a single radial movement which is unit invariant.

[7.4] Step 3 breaks down if some of inputs are absent.

7.3. Measures of Technical, Allocative and Economic Efficiency

Simultaneous estimation of allocative and technical efficiencies can be obtained if a behavioural objective, such as cost minimization or profit maximization is assumed and price data are available. This involves two sets of LPs, one to measure technical efficiency and the other to measure economic efficiency and hence allocative efficiency. If the farm's goal is to minimize cost, its objective function can be expressed by the following input-oriented LP:

$$\underset{x_i,\omega}{Minimize} \quad p_i x_i \tag{7.8}$$

$$\begin{aligned} subject\ to \quad & -y_i + Y\omega \geq 0, \\ & x_i - X\omega \geq 0, \\ & \Omega'\omega = 1, \\ & \omega \geq 0, \end{aligned}$$

where p_i represents the vector of input prices for the ith farm and the solution vector x_i^* is the cost-minimizing input vector, given the prices of the inputs and the levels of output y_i. This cost minimization LP problem is solved separately for each farm. The economic efficiency or cost efficiency (EE) of the ith farm is measured by the ratio of minimum cost to observed cost:

$$EE = \frac{p_i x_i^*}{p_i x_i}$$

Once the technical and economic efficiency are estimated, allocative efficiency can be estimated residually using:

$$TE \times AE = EE.$$

This cost minimizing LP computes efficiency scores under VRS; CRS efficiency estimates can be obtained by computing the LP without the convexity constraint, $\Omega'\omega = 1$. Thus, $\left(p_i x_i^* / p_i \varphi_i^{CRS} x_i\right)$ and $\left(p_i x_i^* / p_i \varphi_i^{VRS} x_i\right)$ compute AE estimates from CRS and VRS models where φ_i^{CRS} and φ_i^{VRS}measures CRS and VRS TE estimates. As in the stochastic econometric frontier approach, the total cost or economic inefficiency of the ith farm $\left(p_i x_i - p_i x_i^*\right)$ is decomposed into its technical efficiency component $\left(p_i x_i - p_i \varphi_i^{VRS} x_i\right)$, and allocative component $\left(p_i \varphi_i^{CRS} x_i - p_i x_i^*\right)$. In addition a scale efficiency component, $\left(p_i \varphi_i^{VRS} x_i - p_i \varphi_i^{CRS} x_i\right)$, is calculated from the DEA.

Like (7.5) where scale efficiency is computed in terms of TE, it can also be estimated in terms of economic efficiency as:

$$SE = \frac{EE_i^{CRS}}{EE_i^{VRS}}$$

where EE_i^{CRS} and EE_i^{VRS} are economic efficiency under constant and variable returns to scale respectively (Chavas and Aliber, 1993; Lund et al., 1993).

7.4. Estimating the Determinants of Inefficiency

Nonparametric LP methods cannot incorporate farm-specific effects directly into the estimation of an efficient frontier. Instead, we first calculate efficiency estimates as discussed above using DEA and then regress them against a set of farm-specific factors to analyze and quantify the effects of these farm-specific factors on inefficiency. We postulate the regression equation:

$$IE_i = \delta' z_i + w_i \qquad\qquad w_i \sim N\left(0, \sigma_w^2\right)$$

where δ_i denotes a $(k \times 1)$ vector of unknown parameters, z_i is a $(k \times 1)$ vector of variables defined in Chapter 6 and w_i is a $(k \times 1)$ vector of residuals that are independently and normally distributed with mean zero and variance σ_w^2. However there are a number of farms for which inefficiency is zero and hence the estimation of δ and σ_w^2 using OLS produces biased and inconsistent estimates. Tobin (1958) developed the censored regression model which can be specified as:

$$IE_i = \delta' z_i + w_i \quad \text{if } (\delta' z_i + w_i) > 0\text{, i.e., inefficiency is not zero}$$

and:

$$IE_i = 0 \qquad \text{otherwise, i.e., inefficiency is zero}$$

Assume that N_0 be the number of farms for which $IE_i = 0$ and N_1 be the number of farms for which $IE_i > 0$. We can define the following for convenience:

$$\phi_i^t = \phi^t\left(\delta' z_i, \sigma_w^2\right) = \left(1/\sigma_w \sqrt{2\pi}\right) e^{-\left(1/2\sigma_w^2\right)\left(\delta' z_i\right)^2}$$

and:

$$\Phi_i^t = \Phi^t\left(\delta' z_i, \sigma_w^2\right) = \int_{-\infty}^{\delta' z_i} \left(1/\sigma_w \sqrt{2\pi}\right) e^{-\left(1/2\sigma_w^2\right)\left(\delta' z_i\right)^2} d\delta' z_i$$

where ϕ_i^t and Φ_i^t are respectively the density function and distribution of the standard normal evaluated at $\delta' z_i / \sigma_w$ (see Maddala, 1983, p.151 for details). For the inefficiencies IE_i that are zero, we know that:

$$\Pr\left(IE_i = 0\right) = \Pr\left(w_i < -\delta' z_i\right) = \int_{-\infty}^{-\delta' z_i} f_t\left(w_i\right) dw_i = \int_{\delta' z_i}^{\infty} f_t\left(w_i\right) dw_i = 1 - \Phi_i^t$$

and:

$$\Pr\left(IE_i > 0\right).f_t\left(IE_i \mid IE_i > 0\right) = \Phi_i^t\left\{f_t\left(IE_i - \delta_i' z_i \sigma_w^2\right)/\Phi_i^t\right\}$$

Hence the log likelihood function is:

$$\log L = \sum_0 \log\left(1 - \Phi_i^t\right) + \sum_1 \log\left(1/\sqrt{2\pi\sigma_w^2}\right) - \sum_1 \left(1/2\sigma_w^2\right)\left(IE_i - \delta' z_i\right)^2$$

where the first summation is over N_0 farm units for which $IE_i = 0$ and the second summation is over N_1 farm units for which $IE_i > 0$. This Tobit model is estimated using maximum likelihood methods.

7.5. Summary

DEA methodology has the advantage over the stochastic frontier approach in that it does not require an assumption of a functional form for the production function or a distribution for technical inefficiency term; moreover it estimates the efficiency relative to the Pareto-efficient frontier which estimates best performance. This Chapter discusses the DEA method from the simplest form to a more complex multi-stage model. We explain the construction of DEA frontiers, namely, the input- and output-oriented CRS and VRS models; the VRS model relaxes the assumption of CRS from frontier technology and envelops the data more closely than does the CRS frontier.

We also examine how technical and scale efficiencies can be measured applying these models; scale efficiency is obtained from the relationship between CRS and VRS models. Moreover, we discuss how economies of scale are identified using the NIRS DEA frontier with the CRS and VRS DEA frontiers. Inequality of technical efficiency scores from the CRS and NIRS implies decreasing returns to scale and the CRS frontier estimate of technical efficiency is lower than that from NIRS frontier as CRS frontier envelops the date less closely. On the other hand, inequality of technical efficiency estimates from the VRS and NIRS methods implies increasing returns to scale as the VRS frontier envelops the data more closely than does the NIRS frontier; hence the VRS frontier calculates efficiency estimates greater than or equal to those relative to the NIRS frontier.

The standard DEA method uses a two-stage procedure which is not invariant to the units of measurement. In order to avoid this problem, we discuss the input-oriented multi-stage DEA frontier to measure technical efficiency. Further, we provide a DEA model with cost minimizing behaviour which is input-oriented to measure simultaneously technical and economic efficiency and hence allocative efficiency of each farm. This simultaneous estimation procedure reflects the ability of farmers both to utilize existing technologies properly and choose cost-minimizing input combinations. The Tobit model can be used to identify and quantify the effects of farm-specific factors on inefficiencies.

Chapter 8

DEA Frontier Results

8.1. Introduction

Nonparametric analysis of efficiency of our sample of rice farmers in Bangladesh is conducted in this Chapter using Data Envelopment Analysis (DEA). The linear programs (LPs) for estimating DEA frontiers in a multi-stage methodology are specified to measure technical efficiency. Constant returns to scale (CRS) and variable returns to scale (VRS) input-oriented and output-oriented DEA frontiers are estimated. The CRS frontier produces measures of overall technical efficiency and the VRS frontier produces measures of pure technical efficiency. *Scale efficiency* is obtained as the ratio of the two and we obtain farms operating at optimal, sub-optimal or super-optimal level by comparing efficiency estimates from CRS, VRS and NIRS technologies. We use Tobit analysis to analyze factors which affect technical inefficiency.

We then estimate a cost minimization input-oriented DEA frontier which involves two sets of LPs to estimate technical, allocative and economic efficiency simultaneously: one measures technical and the other measures economic efficiency; allocative efficiency is calculated residually. The CRS and VRS input-oriented cost minimization DEA frontiers are estimated. These frontiers assume that farmers produce output at minimum cost whereas the frontier for estimating technical efficiency only assumes that farmers produce maximum output from a given input mix, given existing technology. The VRS frontier envelops data more closely than the CRS does so that efficiency measures derived from the former are greater than or equal to those obtained from the latter. A Tobit model is then estimated to identify and quantify farm-specific determinants associated with technical inefficiency, allocative inefficiency and economic inefficiency.

The outline of this Chapter is as follows: Section 2 discusses the DEA frontier results of technical efficiency and estimates the effects of farm-specific factors on technical inefficiency; Section 3 discusses the DEA frontier results of technical, allocative and economic efficiency simultaneously, and quantifies and discusses factors affecting these inefficiencies; and Section 4 concludes.

8.2. DEA Frontier Results for TE Estimates

Our sample consists of 150 farms producing rice, from five inputs, land, labour, irrigation, fertilizer and pesticides measured in value terms. Both the output-oriented and input-oriented CRS and VRS DEA models are estimated using the DEAP program (Coelli, 1996). A series of 150 LPs, one for each farm, is run for each of CRS and VRS input-oriented frontiers in (7.3), and for the CRS and VRS output-oriented frontiers in (7.4). The frequency distribution of technical efficiency estimates is presented in Table 8.1 and summary statistics of technical and scale efficiency are provided in Table 8.2. The corresponding frequency histograms are given in Figures 8.1-8.4.

The frequency distribution in Table 8.1 and Figures 8.1 and 8.3 show that both output and input orientation of DEA predict very similar TE scores for CRS; most farms (48 per cent) are over 80 per cent technically efficient and 1 per cent are below 50 per cent efficient. The rest of the farms are technically efficient between 50 - 80.

Table 8.1: Frequency Distribution of TE and SE from DEA Frontiers (n = 150)

Effici-ency Score (%)	Output Orientation						Input Orientation					
	CRS		VRS		SE		CRS		VRS		SE	
	No. of farms	% of farms	No. of farms	% of farms	No. of farms	% of farms	No. of farms	% of farms	No. of farms	% of farms	No. of farms	% of farms
1-48	1	0.67	1	0.67	0	0.00	1	0.67	0	0.00	0	0.00
48-50	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00
50-52	1	0.67	0	0.00	0	0.00	1	0.67	1	0.67	0	0.00
52-54	1	0.67	0	0.00	0	0.00	1	0.67	1	0.67	0	0.00
54-56	4	2.67	3	2.00	0	0.00	4	2.67	2	1.33	0	0.00
56-58	6	4.00	2	1.33	0	0.00	6	4.00	3	2.00	0	0.00
58-60	6	4.00	0	0.00	0	0.00	6	4.00	4	2.67	0	0.00
60-62	5	3.33	4	2.67	0	0.00	5	3.33	1	0.67	0	0.00
62-64	6	4.00	4	2.67	1	0.67	6	4.00	3	2.00	1	0.67
64-66	5	3.33	3	2.00	0	0.00	5	3.33	9	6.00	0	0.00
66-68	6	4.00	2	1.33	3	2.00	6	4.00	3	2.00	2	1.33
68-70	5	3.33	2	1.33	1	0.67	5	3.33	2	1.33	0	0.00
70-72	6	4.00	5	3.33	2	1.33	6	4.00	5	3.33	0	0.00
72-74	4	2.67	7	4.67	0	0.00	4	2.67	4	2.67	3	2.00
74-76	2	1.33	4	2.67	0	0.00	2	1.33	1	0.67	0	0.00
76-78	10	6.67	4	2.67	3	2.00	10	6.67	5	3.33	2	1.33
78-80	10	6.67	5	3.33	2	1.33	10	6.67	5	3.33	1	0.67
80-82	4	2.67	7	4.67	7	4.67	4	2.67	7	4.67	4	2.67
82-84	7	4.67	7	4.67	6	4.00	7	4.67	9	6.00	7	4.67
84-86	7	4.67	5	3.33	10	6.67	7	4.67	8	5.33	2	1.33
86-88	16	10.67	12	8.00	8	5.33	16	10.67	13	8.67	9	6.00
88-90	2	1.33	8	5.33	9	6.00	2	1.33	3	2.00	9	6.00
90-92	5	3.33	7	4.67	6	4.00	5	3.33	3	2.00	13	8.67
92-94	3	2.00	3	2.00	12	8.00	3	2.00	5	3.33	8	5.33
94-96	2	1.33	5	3.33	15	10.00	2	1.33	3	2.00	10	6.67
96-98	4	2.67	6	4.00	11	7.33	4	2.67	6	4.00	17	11.33
98-100	22	14.67	44	29.33	54	36.00	22	14.67	44	29.33	62	41.33

The output orientation of DEA frontier for VRS in Table 8.1 and Figure 8.2 results in technical efficiency measures that show that 43 per cent of farms (most farms) are between 90 - 100 per cent efficient and 1 per cent are less than 50 per cent efficient. Thus the output-oriented VRS DEA model generates higher levels of efficiency estimates than its

CRS counterpart as expected. The input orientation of VRS DEA frontier technology in Table 8.1 and Figure 8.4 shows that 41 per cent of farms are between 90 - 100 per cent efficient and no farm is less than 50 per cent efficient. Again the input-oriented VRS DEA method computes technical efficiency estimates greater than those of its CRS counterpart as expected. Both orientations of VRS DEA efficiency estimates are very similar.

The frequency distribution in Table 8.1 and Figures 8.5 and 8.6 show scale efficiency measures for both orientations; for output orientation, 65 per cent of farms are between 90 - 100 per cent efficient whereas for the corresponding input orientation the figure is 73 per cent. Similarly, 27 per cent of farms are 80 - 90 per cent for output orientation and 21 per cent for input orientation. For both orientations, no farm is less than 70 per cent scale efficient. This implies that the differences between CRS and VRS technical inefficiency is lower.

The mean overall TE in Table 8.2 is 79 per cent for both orientations; the estimated means of pure TE are 86 per cent for output orientation and 85 per cent for input orientation; and means of scale efficiency are 92 per cent and 93 per cent respectively. Thus the mean TE measure from the VRS DEA is greater than that obtained from the CRS DEA approach, and the estimated mean scale efficiency from the input-oriented frontier is greater than that obtained from the output-oriented frontier.

Table 8.2: Summary Statistics of Efficiency Estimates from DEA Model (in percentage)

Statistics	Output Orientation			Input Orientation		
	CRS	VRS	SE	CRS	VRS	SE
Mean	79	86	92	79	85	93
Minimum	45	46	62	45	52	62
Maximum	100	100	100	100	100	100
Standard Deviation	14	13	9	14	14	8

The overall TE ratings range for output- and input-oriented CRS is the same from 45 - 100 per cent with same mean and standard deviation of 79 and 14 per cent respectively. Pure TE estimates vary from 46 - 100 per cent and 51 - 100 per cent with values of standard deviations of 13 and 14 per cent respectively for both output and input orientations. The scale efficiency index for the sample ranges from 62 - 100 per cent with a sample mean and standard deviation of 92 and 9 per cent respectively from output orientation and input-oriented scale efficiency ranges from 62 - 100 per cent with a mean value and standard deviation of 93 per cent and 8 per cent respectively. Thus DEA reveals substantial inefficiencies in rice production in the High Barind. Forty one farms are fully technically efficient under the VRS and 20 farms are identified as fully technically efficient for the CRS from both input- and output-oriented frontiers; this accords with the theory that both input orientation and output orientation of DEA frontiers identify the same number of farms as efficient.

The scale efficiency index for farms is estimated using (7.5) for input orientation and (7.6) for output orientation where the efficiency scores estimated under the VRS DEA frontier are equal to or greater than those calculated under the CRS DEA; results are presented in Table 8.3.

Table 8.3: Optimal, Sub-optimal and Super-optimal Outputs

Scale	Output Orientation			Input Orientation		
	No. of farms	Mean Output	Output Range	No. of farms	Mean Output	Output Range
Optimal scale	22	31256	1600-188121	25	31159	1600-188122
Sub-optimal scale	21	13248	3744-39125	31	10710	1604-36101
Super-optimal scale	107	70421	1998-289940	94	68854	2257-289940

In terms of scale efficiency from the output orientation model, 107 farms are characterized by decreasing returns to scale (super-optimal scale), 22 farms have constant returns to scale (optimal scale) and 21 farms have increasing returns to scale (sub-optimal scale). From the input orientation model, 94 farms are characterized by decreasing returns to scale, 25 farms have constant returns to scale and 31 farms have increasing returns to scale. The mean output at the super-optimal scale is greater and the mean value at the sub-optimal scale is the lowest for output orientations as expected. This result conforms with

Hjalmarsson et al. (1996), but contrasts with Sharma et al. (1997) who found surprisingly that the mean value at the optimal scale is the highest. For input orientation, the mean output at sub-optimal scale is the lowest and the mean output at super-optimal scale is the highest as expected. The results indicate that the optimal output levels overlap a great portion of the sub-optimal and super-optimal output values which conform with both Hjalmarsson et al. (1996) and Sharma et al. (1997). If all farms are using the same technology, then we would expect returns to scale to be increasing for farms with a relatively low output and decreasing returns to scale for farms with a relatively high output. Constant returns to scale would be expected for farms with a output level equal to mean output (Silberberg, 1990).

Figure 8.1: Frequency Histogram of TE Scores from Output-Oriented CRS DEA

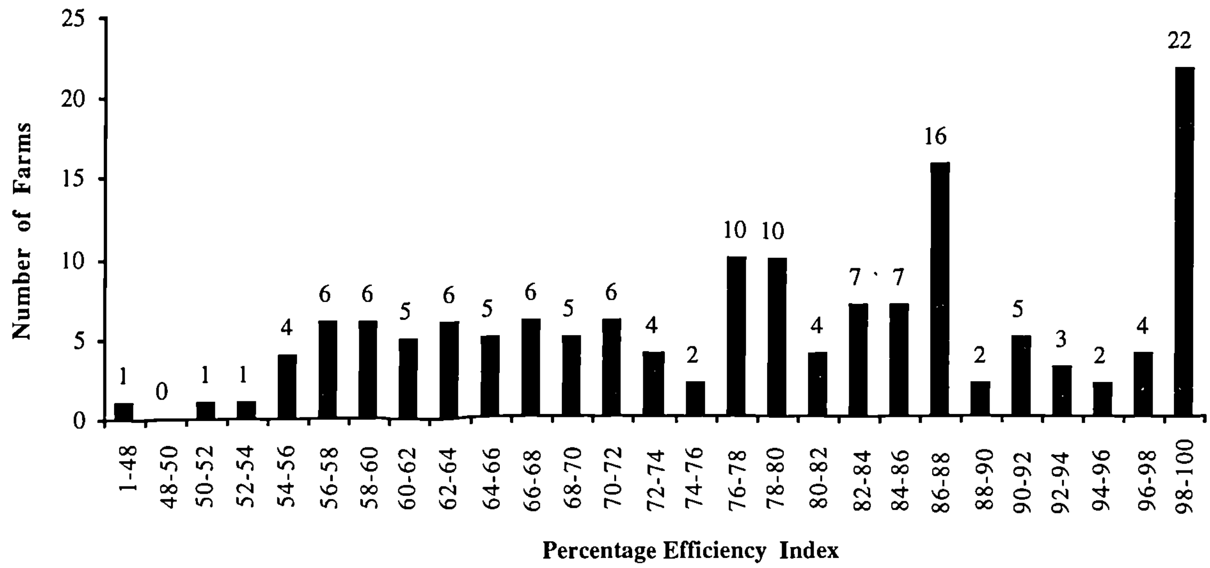


Figure 8.2: Frequency Histogram of TE Scores from Output-Oriented VRS DEA

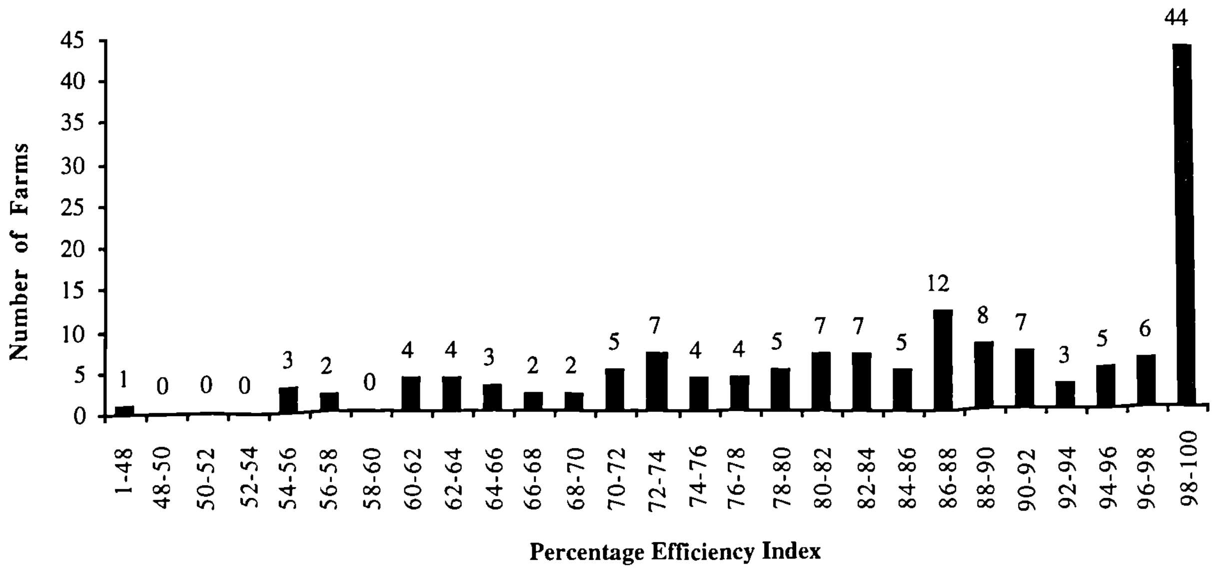


Figure 8.3: Frequency Histogram of TE Estimates from Input-Oriented CRS DEA Frontier

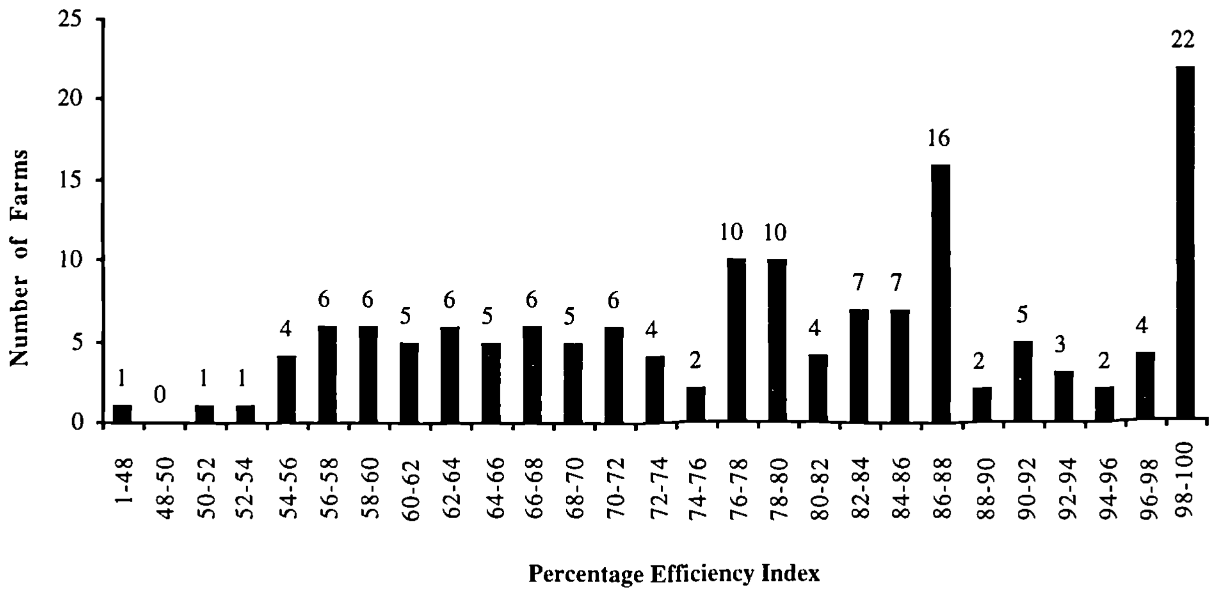


Figure 8.4: Frequency Histogram of TE Estimates from Input-Oriented VRS DEA Frontier

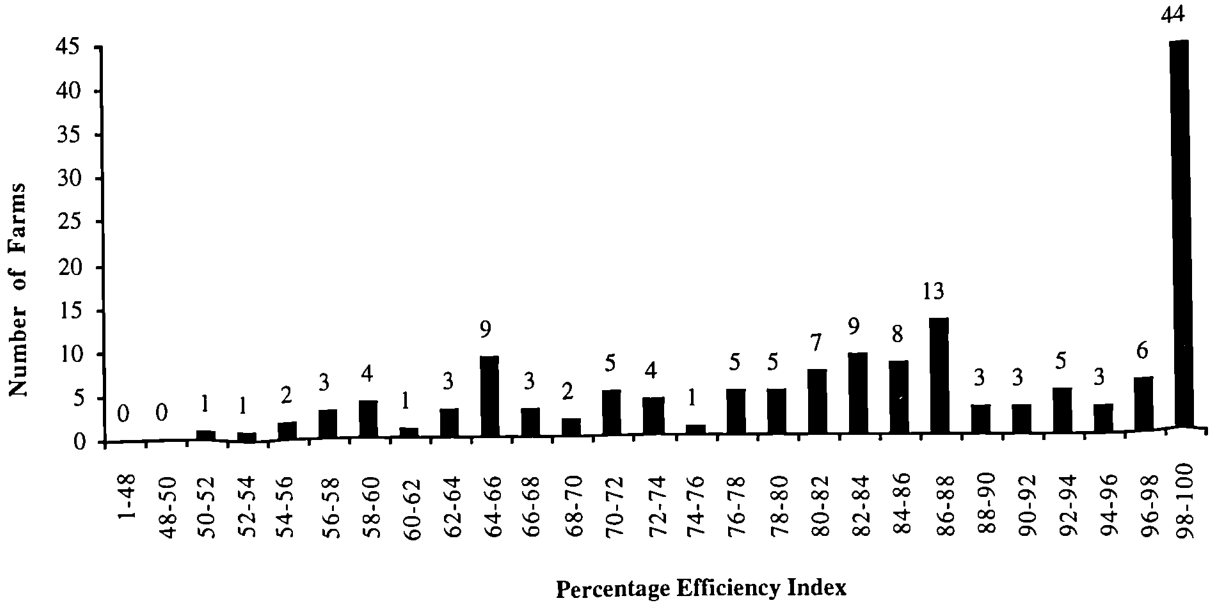


Figure 8.5: Frequency Histogram of SE Estimates form Intput-Oriented DEA Frontier

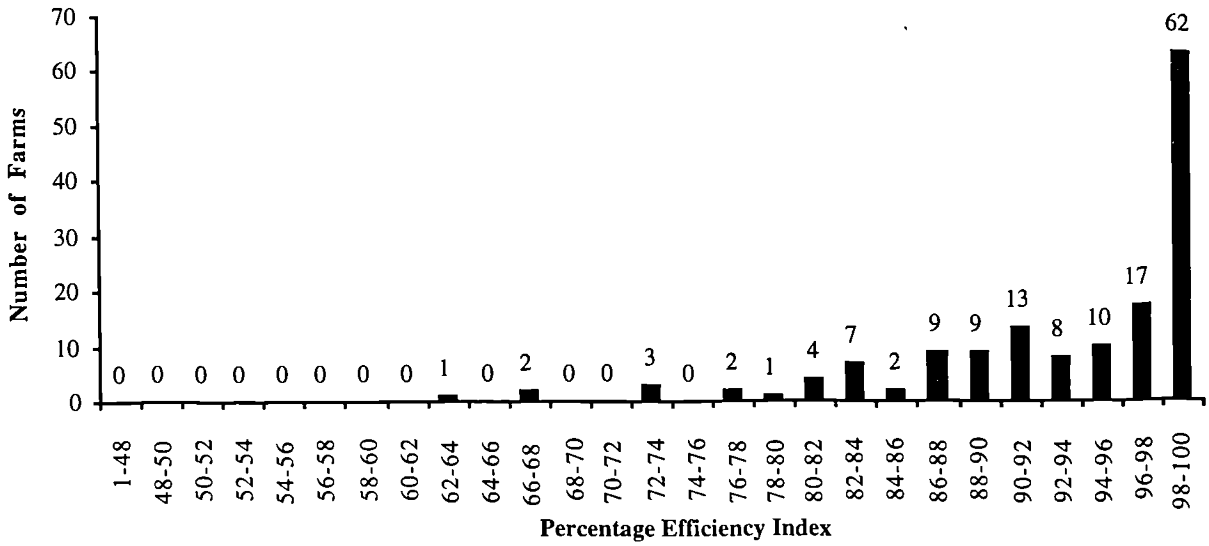
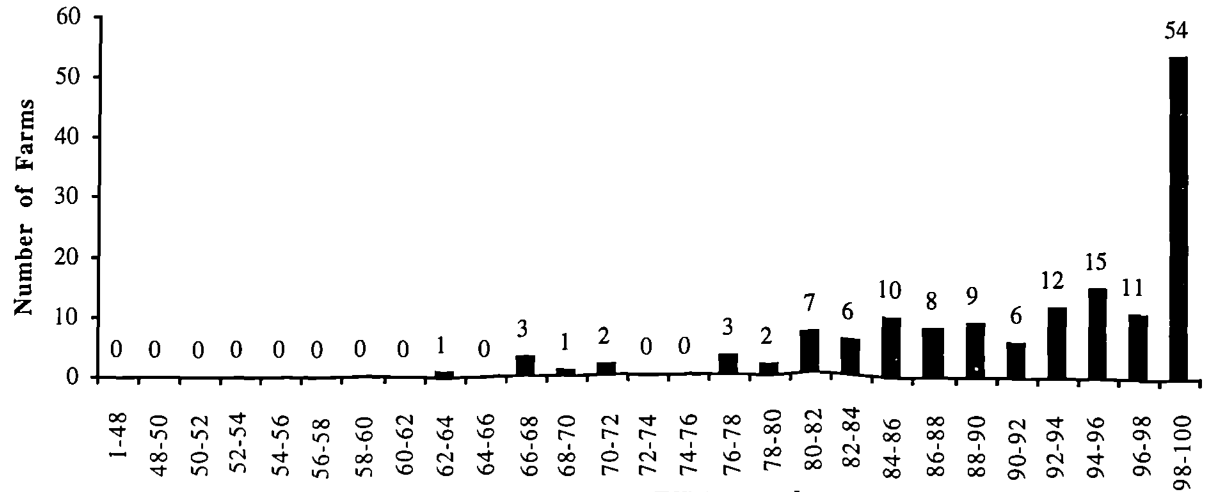


Figure 8.6: Frequency Histogram of SE Estimates form Output-Oriented DEA Frontier



8.2.1. The Sources of Technical Inefficiency

The CRS and VRS DEA TE estimates derived from the DEA are now regressed on farm-specific explanatory variables considered in Chapter 6, that is, the age of the farmer, land fragmentation, the years of schooling of the farmer, irrigation infrastructure and environmental degradation to identify and quantify possible factors associated with inefficiency. Tobit analysis is used to investigate the effects of these explanatory variables on the technical inefficiency of each farm. Analysis of TE scores reveals that there are a number of farm units for which inefficiency is zero. We specify the following regression to conduct the Tobit analysis:

$$IE_i = \delta_1 x_{1i} + \delta_2 x_{2i} + \delta_3 x_{3i} + \delta_4 x_{4i} + \delta_5 x_{5i} + w_i$$

if $\left(\delta_1 x_{1i} + \delta_2 x_{2i} + \delta_3 x_{3i} + \delta_4 x_{4i} + \delta_5 x_{5i} + w_i\right) > 0$, i.e., inefficiency is not zero,

and

$IE_i = 0$ otherwise, i.e., inefficiency is zero

Results are presented in Table 8.4. CRS technical inefficiency (CRS TI), VRS technical inefficiency (VRS TI) and scale inefficiency (SI) are corresponding efficiencies subtracted from 100.

The signs of the estimated coefficients associated with the age of farmers for output- and input-oriented CRS TI and SI and input-oriented VRS TI are positive which implies that the older farmers are more technically inefficient and scale inefficient than the younger farmers. This accords with results obtained by Ajibefun et al. (1996), Seyoum et al. (1998) and Coelli and Battase (1996) and can be explained in terms of credit availability and the willingness to adopt new technology: older farmers are likely to be more experienced in farming, but they are likely to be more conservative and less receptive to modern and newly introduced agricultural technology and practices. The coefficient associated with age from the output-oriented VRS model however is negative indicating that the farmers with perhaps more farming experience are more technically efficient.

Table 8.4: Regression Analysis Testing Factors Influencing Technical and Scale Inefficiency

Variables	CRS TI [8.1]		VRS TI		SI	
	Coefficients	t-ratios	Coefficients	t-ratios	Coefficients	t-ratios
Output Orientation						
Constant	0.0712	1.6422	0.1475	3.3117	-0.0846	-2.8032
Age of farmers	0.0012	1.6022	-0.0001	-0.0979	0.0014	2.6555
Land fragmentation	0.0732	0.7908	-0.1257	-1.3228	0.2254	3.5027Y
Year of schooling	0.0013	0.6849	0.0017	0.9038	-0.0006	-0.4268
Irrigation infrastructure dummy	0.1686	7.4018	0.0922	3.9439	0.0935	5.9028
Environmental degradation dummy	-0.0528	-2.2827	-0.0722	-3.0403	0.0190	1.1789
Log Likelihood	123.6384		119.7095		178.1430	
Input Orientation						
Constant	0.0712	1.6422	0.1258	2.7058	-0.0559	-1.9150
Age of farmers	0.0012	1.6022	0.0001	0.1655	0.0011	2.1857
Land fragmentation	0.0732	0.7908	-0.0653	-0.6584	0.1469	2.3587
Year of schooling	0.0013	0.6849	0.0014	0.7151	-0.0001	-0.0625
Irrigation infrastructure dummy	0.1686	7.4018	0.1126	4.6150	0.0685	4.4689
Environmental degradation dummy	-0.0528	-2.2827	-0.0700	-2.8261	0.0169	1.0862
Log Likelihood	123.6383		113.2992		183.0993	

[8.1] Both the output- and input-oriented frontier produce identical coefficients and t-ratios for CRS TI as they calculate identical efficiency estimates.

The coefficients associated with years of schooling for both output- and input-oriented CRS and VRS models are positive implying that the farmers with more schooling are more technically inefficient; this is unexpected, but the coefficients are insignificant. This result accords with that obtained for the Indian village of Kanzara by Coelli and Battase (1996). Similar estimated coefficients to explain scale inefficiency are negative which indicates that farmers with more formal education are scale efficient. The land fragmentation (plot size) coefficients are negative, as expected, under output- and input-oriented VRS frontiers which shows that technical inefficiency effects are lower for farmers with greater land size because farmers with greater land size can operate modern equipment and manage irrigation more effectively. Again this accords with Coelli and Battese (1996). But the CRS TI show positive but insignificant relationships; SI reveals positive and significant relationships.

The coefficients associated with the irrigation infrastructure dummy for all output- and input-oriented CRS and VRS technical inefficiency and scale inefficiency are positive and significant. This indicates that irrigation schemes operated with diesel have significant positive contributions to technical inefficiency and scale inefficiency; thus the diesel-operated pumps lower technical and scale efficiency. This can be attributed to diesel prices and water extraction capacity of such irrigation schemes: the overall water extraction capacity of diesel-operated irrigation schemes is lower than their electricity counterpart and diesel costs are higher than electricity costs. Irrigation in Bangladesh is mainly concentrated on rice crops and in particular, irrigation entirely supports rice crops in Season III; cultivation of rice crops in this season is not possible without irrigation because of a lack of rainfall. The benefits of irrigation include increased crop yields, quality and continuity of supply. A policy which leads to the conversion of irrigation infrastructures from diesel pumps to electricity-operated pumps could enhance crop yields and hence farm revenues through the improvement of farmers' efficiency.

The estimates of the coefficients on the environmental degradation dummy for both output- and input-oriented CRS and VRS technical inefficiency are negative and significant, as is expected, implying that the farmers with less degraded land have smaller technical inefficiencies. The estimated coefficient for scale inefficiency is positive but insignificant. A policy which helps reduce land degradation in this region could increase production and farm welfare.

8.3. Frontier Results for Estimates of TE, AE and EE

We use the input-oriented DEA model in (7.8) in Chapter 7 to estimate TE, AE and EE scores. These measures are calculated using DEAP 2.0 (Coelli, 1996). This estimation procedure solves a series of 150 LPs one for each of the 150 farms. The frequency distributions of TE, AE and EE measures under CRS and VRS frontier technologies and their summary statistics are reported in Tables 8.5 and 8.6. Also presented in Figures 8.7 - 8.12 are their respective frequency histograms.

The efficiency groupings reported in Table 8.5 and the corresponding Figures 8.7-8.9 show that under CRS DEA, 47 per cent of farms (most farms) are 90 - 100 per cent technically efficient, most of farms (66 per cent) are 90-100 per cent allocatively efficient and most of economically efficient farms (28 per cent) are between 60 - 70 per cent; 1 per cent farms are 50 - 60 per cent technically efficient and economically efficient respectively, but no farm falls within this allocatively efficient and only 1 per cent farms are 1 - 50 per cent economically efficient.

Under variable returns to scale, Table 8.5 and Figures 8.10-8.12 show that the 90 - 100 per cent TE and AE interval includes most of technically efficient and allocatively efficient farms (66 and 43 per cent respectively) and the 70 - 80 per cent EE interval includes most of economically efficient farms (27 per cent). The least number of farms (1 per cent) are between 50 - 60 per cent technically efficient; the least number of allocatively efficient farms (3 per cent) are between 60 - 70 per cent; and the 1 - 50 per cent EE interval includes the least number of farms (1 per cent). Therefore there is room for improving efficiency of farmers. It is also evident that the VRS DEA frontier produces efficiency estimates greater than those calculated from the CRS DEA frontier conforming with the theory that the CRS frontier least envelops the data set.

Table 8.5: Frequency Distribution of TE, AE and EE from DEA Frontiers (in percentage)

Efficiency Index	CRS						VRS					
	No. of farms			% of farms			No. of farms			% of farms		
	TE	AE	EE	TE	AE	EE	TE	AE	EE	TE	AE	EE
1-48	0	0	1	0.00	0.00	0.67	0	0	1	0.00	0.00	0.67
48-50	0	0	0	0.00	0.00	0.00	0	0	0	0.00	0.00	0.00
50-52	1	0	0	0.67	0.00	0.00	0	0	0	0.00	0.00	0.00
52-54	0	0	1	0.00	0.00	0.67	1	0	1	0.67	0.00	0.67
54-56	0	0	0	0.00	0.00	0.00	0	0	0	0.00	0.00	0.00
56-58	0	0	1	0.00	0.00	0.67	0	0	0	0.00	0.00	0.00
58-60	1	0	0	0.67	0.00	0.00	0	0	1	0.00	0.00	0.67
60-62	0	0	12	0.00	0.00	8.00	0	0	8	0.00	0.00	5.33
62-64	1	0	4	0.67	0.00	2.67	0	2	6	0.00	1.33	4.00
64-66	7	0	2	4.67	0.00	1.33	2	0	3	1.33	0.00	2.00
66-68	4	1	11	2.67	0.67	7.33	1	1	10	0.67	0.67	6.67
68-70	2	3	13	1.33	2.00	8.67	2	4	10	1.33	2.67	6.67
70-72	7	1	10	4.67	0.67	6.67	4	4	12	2.67	2.67	8.00
72-74	8	0	10	5.33	0.00	6.67	3	11	10	2.00	7.33	6.67
74-76	7	3	10	4.67	2.00	6.67	2	9	10	1.33	6.00	6.67
76-78	6	4	7	4.00	2.67	4.67	5	3	4	3.33	2.00	2.67
78-80	7	3	3	4.67	2.00	2.00	6	4	4	4.00	2.67	2.67
80-82	4	3	6	2.67	2.00	4.00	3	9	6	2.00	6.00	4.00
82-84	7	6	3	4.67	4.00	2.00	11	7	4	7.33	4.67	2.67
84-86	8	8	11	5.33	5.33	7.33	4	9	13	2.67	6.00	8.67
86-88	4	11	8	2.67	7.33	5.33	4	8	8	2.67	5.33	5.33
88-90	5	8	8	3.33	5.33	5.33	3	14	4	2.00	9.33	2.67
90-92	16	13	10	10.67	8.67	6.67	11	10	11	7.33	6.67	7.33
92-94	9	17	7	6.00	11.33	4.67	8	6	4	5.33	4.00	2.67
94-96	12	26	3	8.00	17.33	2.00	16	15	5	10.67	10.00	3.33
96-98	8	20	7	5.33	13.33	4.67	7	9	6	4.67	6.00	4.00
98-100	26	23	2	17.33	15.33	1.33	57	25	9	38.00	16.67	6.00

Under CRS and VRS technology, the mean values of TE ratings, given in Table 8.6, are 86 and 91 per cent with ranges from 52 - 100 per cent and 53 - 100 per cent; and standard deviations are 12 and 10 per cent respectively. This indicates that farmers on average can produce observed output levels as the most technically efficient farm using 86 per cent and 91 per cent of their observed quantities of inputs respectively. Conversely, on average, farm input use is 14 per cent and 9 per cent higher than the most technically efficient farms. Although the mean CRS TE ratings is slightly lower than that of VRS TE ratings, their ranges and standard deviations are similar.

Table 8.6: Summary Statistics of Efficiency Estimates (in percentages)

	CRS DEA Frontier			VRS DEA Frontier		
Statistics	TE	AE	EE	TE	AE	EE
Mean	86	91	78	91	87	79
Minimum	52	67	46	53	63	46
Maximum	100	100	100	100	100	100
Standard Deviation	12	7	12	10	10	12

The averages of AE ratings under CRS and VRS DEA frontiers are 91 and 87 per cent respectively which implies that farms on average can increase their observed levels of output as the most allocatively efficient farm spending 91 and 87 per cent of their observed costs respectively. Alternatively, production costs on average are 9 and 13 per cent greater than if they were allocatively efficient. The ranges of AE ratings from both frontiers are 67 -100 per cent and 63 - 100 per cent respectively and their standard deviations are 7 and 10 per cent which indicate little variation in AE ratings between the assumed technologies.

The mean values of 78 and 79 per cent for the EE scores calculated from CRS and VRS technologies show that farms on average can reduce their production costs by 22 and 21 per cent respectively if production is as efficient as the most cost efficient farm. The EE ranges and standard deviations are similar which shows that there is no significant variation across technologies.

Figure 8.7: Frequency Histogram of TE Estimates from CRS DEA Frontier

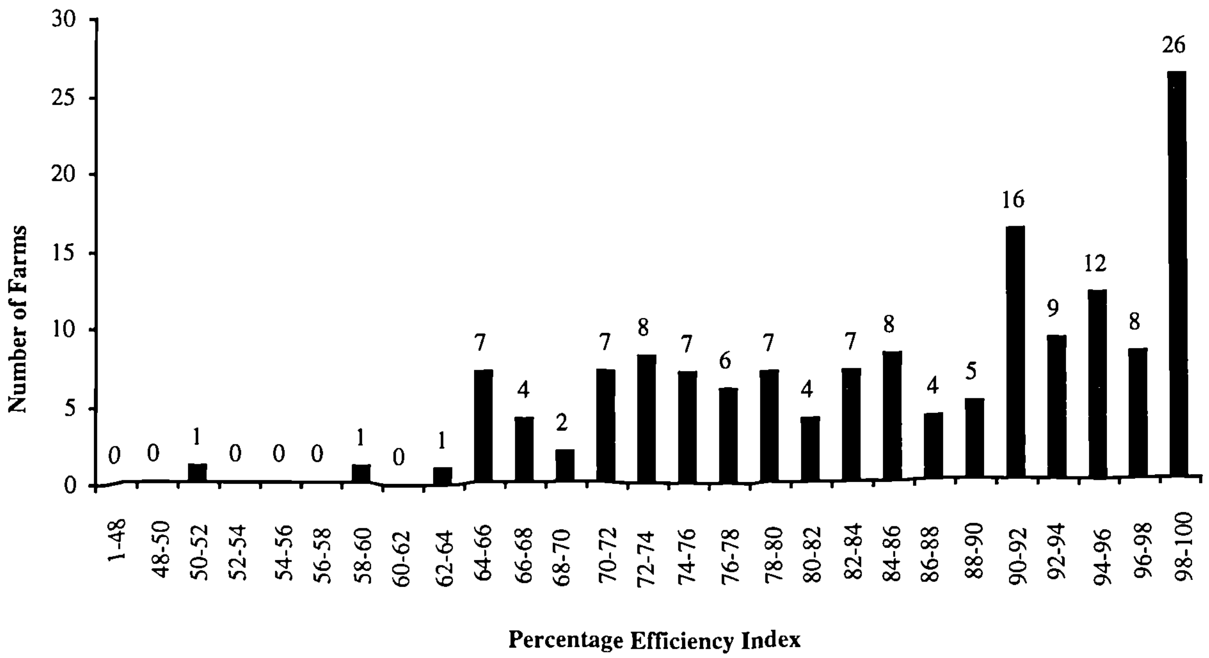


Figure 8.8: Frequency Histogram of AE Estimates from CRS DEA Frontier

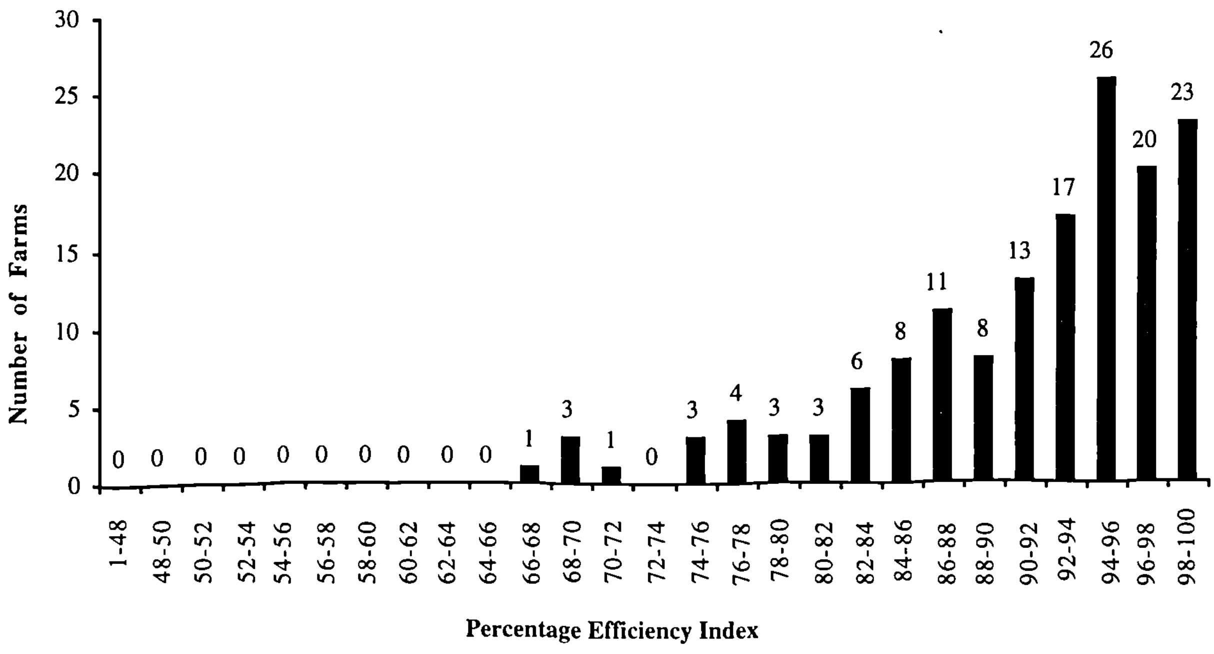


Figure 8.9: Frequency Histogram of EE Estimates from CRS DEA Frontier

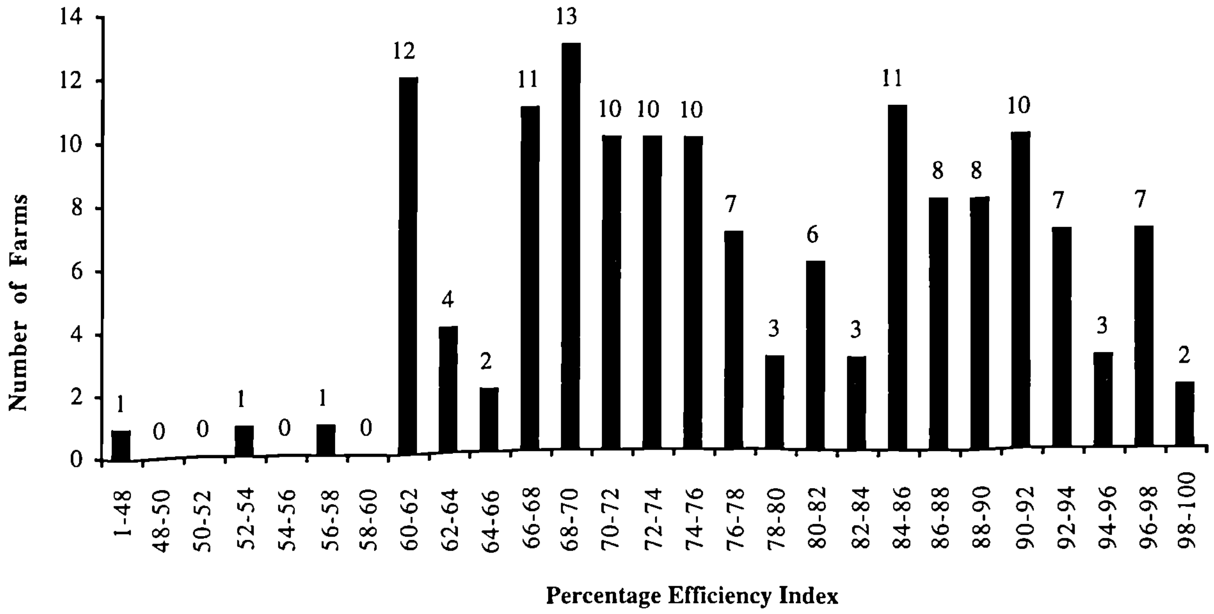


Figure 8.10: Frequency Histogram of TE Estimates from VRS DEA Frontier

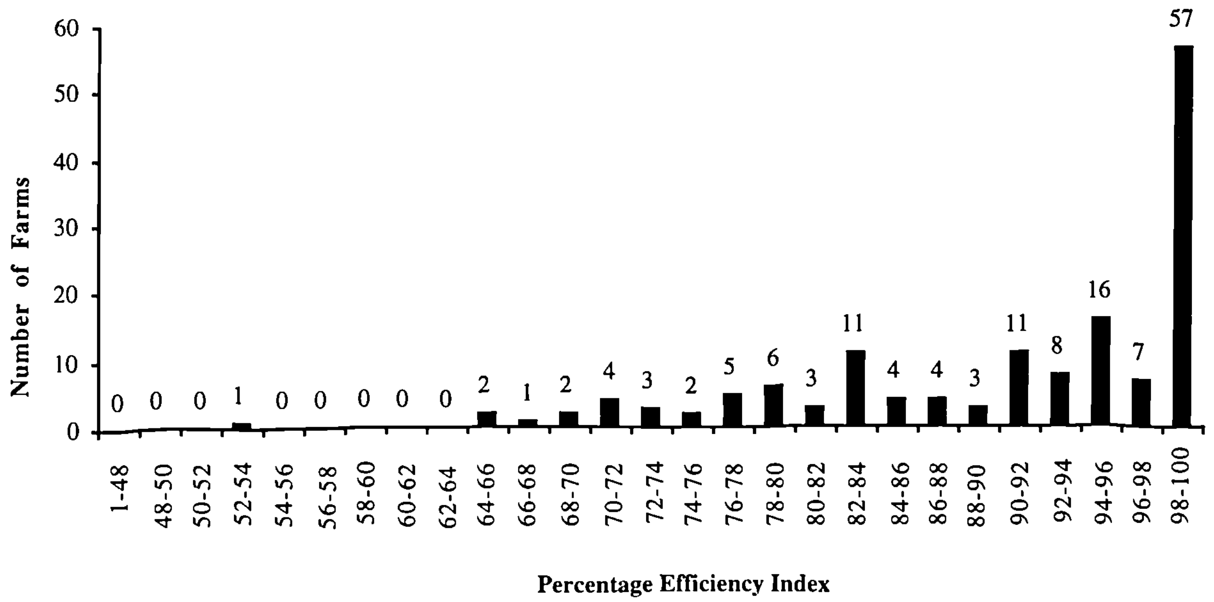


Figure 8.11: Frequency Histogram of AE Estimates from VRS DEA Frontier

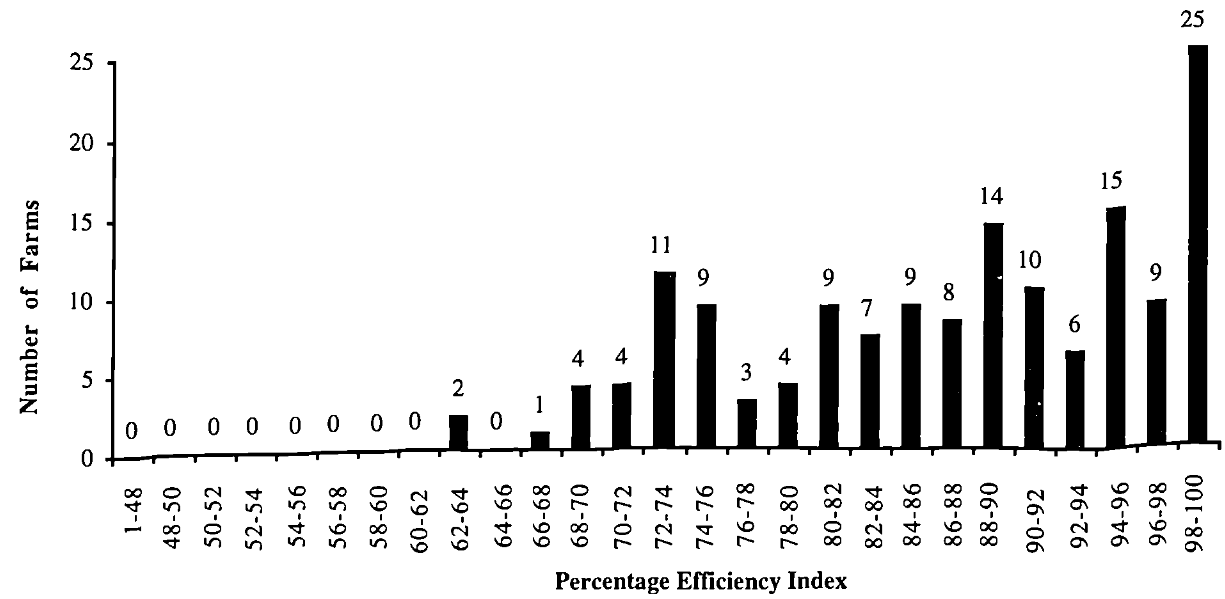
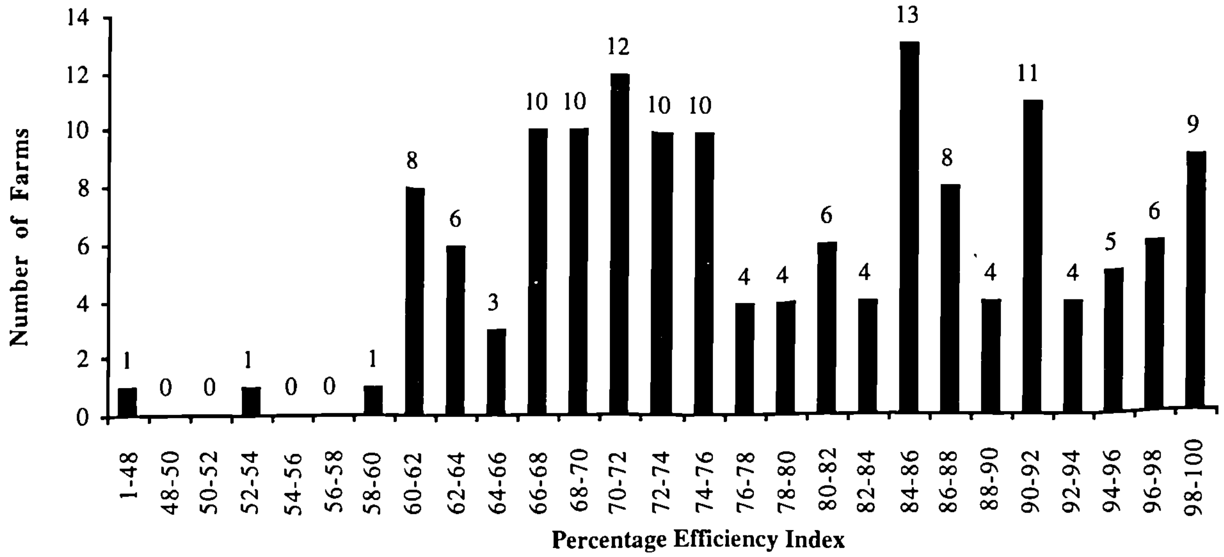


Figure 8.12: Frequency Histogram of EE Estimates from VRS DEA Frontier



8.3.1. Factors Associated with Inefficiency

We now quantify the effects of sources of technical inefficiency (TI), allocative inefficiency (AI) and economic inefficiency (EI) using the Tobit model, discussed in Section 8.2.1; results are given in Table 8.7. The estimated coefficients on the age of the farmers for CRS TI, EI and VRS TI are positive but insignificant which implies that younger farmers tend to have higher levels of TE than their older counterparts. This may be explained if younger farmers adopt recent technological advances which typically have more credit privileges from banks and merchants. However, the coefficient signs on age for CRS AI and VRS AI and EI are negative but insignificant; this indicates that older farmers are experienced when choosing cost-minimizing input combinations. Schooling is positively and insignificantly related with TI, is negatively related with AI and EI and the coefficients on AI are significant; this indicates that education is positively related with cost-minimizing input combinations. The estimated coefficients of land fragmentation for CRS TI, AI and EI, and for VRS AI and EI are all negative, as expected, and the coefficients for CRS AI, VRS AI and EI are significant; this indicates that smaller plot size is associated with higher level of relative TI, AI and EI. The exception is the VRS TI where the coefficient is positive but insignificant.

Table 8.7: Tobit Regression Results of Factors Affecting Inefficiencies

Factors	TI		AI		EI	
Constant Returns to Scale	Coefficients	t-ratio	Coefficients	t-ratio	Coefficients	t-ratio
Constant	0.0747	2.0858	0.1571	5.8854	0.2207	7.9267
Age of farmers	0.0007	1.0744	-0.0005	-0.9719	0.0002	0.3068
Land fragmentation	-0.0005	-0.0061	-0.1167	-2.0490	-0.0982	-1.6525
Year of schooling	0.0014	0.9246	-0.0035	-3.0282	-0.0019	-1.5536
Irrigation infrastructure dummy	0.1121	5.9643	0.0179	1.2828	0.1217	8.3255
Environmental degradation dummy	-0.0608	-3.1876	-0.0149	-1.0485	-0.0722	-4.8666
Log Likelihood	152.3666		196.5172		190.1654	
Variable Returns to Scale						
Constant	0.0457	1.2033	0.1831	6.4496	0.2214	7.7698
Age of farmers	0.0003	0.4904	-0.0005	-0.9531	-0.0001	-0.2461
Land fragmentation	0.0300	0.3710	-0.1530	-2.5269	-0.1231	-2.0258
Year of schooling	0.0021	1.2610	-0.0039	-3.1885	-0.0019	-1.5438
Irrigation infrastructure dummy	0.0532	2.6735	0.0869	5.8331	0.1293	8.6416
Environmental degradation dummy	-0.0447	-2.2119	-0.0279	-1.8421	-0.0673	-4.4326
Log Likelihood	143.7501		187.2714		186.6935	

Positive and significant coefficients on the irrigation infrastructure dummy for both CRS TI and EI; and VRS TI, AI and EI imply that irrigation infrastructure, i.e., diesel-operated irrigation pumps, positively affect technical, allocative and economic inefficiency; the coefficient for CRS AI is also positive but insignificant. This may be attributed to lower water extraction capacity of diesel pumps and there are constraints to allocation decisions since diesel costs are higher which reduces TE, AE and EE. The estimated coefficients on environmental degradation dummy for both CRS and VRS TI, AI and EI are negative implying that environmental degradation not only creates obstacles in applying technology efficiently but also hinders the cost-minimizing input utilization in rice production in the High Barind in Bangladesh; the coefficients for CRS TI, EI and VRS TI and EI are significant but the coefficients for CRS and VRS AI are insignificant. Therefore policies leading to improving irrigation infrastructures and reducing environmental degradation could enhance the efficiency of farms, thereby increasing farm production, revenue and welfare.

8.5. Summary and Conclusions

This Chapter estimates DEA frontiers to measure output- and input-oriented CRS and VRS technical efficiency; it also estimates cost minimization input-oriented DEA frontiers to calculate input-oriented CRS and VRS technical, allocative and economic efficiency simultaneously. These two frontiers are estimated because estimation of technical efficiency alone measures only the ability of a farm to use existing technology efficiently; in contrast, technical, allocative and economic efficiency simultaneously measure the cost-minimizing production capability of farmers which in turn measures the ability to use existing technology efficiently *and* choose cost-minimizing input combinations. This Chapter reports the results of efficiency estimates from both DEA frontiers and the results of Tobit regression analyses associated with factors affecting inefficiency.

The procedure of technical efficiency estimation allows the relative technical efficiency for each farm to be determined and for technical efficiencies to be decomposed into pure and scale efficiency. The output-oriented and input-oriented frontiers are computed to predict overall technical efficiency assuming CRS and pure technical efficiency assuming VRS by comparing each farm's individual performance with the respective frontier. We then use their ratios to estimate scale efficiency. Evaluating farms for output and input orientation shows that, on average, overall technical efficiencies are both 79 per cent while pure technical efficiencies are both about 86 per cent. This accords with the theory that the VRS DEA frontier produces efficiency estimates equal to or greater than those calculated the CRS DEA frontier. Average scale efficiencies are both about 93 per cent. Scale properties are also estimated and results show that the farms are characterized by decreasing returns to scale.

Our analysis also examines the farm-specific factors which may determine the technical inefficiency of farms. The technical inefficiency effects are examined as a function of various farm-specific socio-economic factors, environmental factors and irrigation infrastructure. We conduct Tobit analysis to quantify the effects of farm-specific factors on technical inefficiency. Results show that both input-oriented and output-oriented measures of overall, pure technical inefficiency and scale inefficiency are affected positively by factors associated with irrigation infrastructure; overall and pure technical inefficiency are negatively affected with environmental degradation.

The input-oriented CRS and VRS DEA frontier model is calculated simultaneously to measure technical, allocative and economic efficiency. Assessing farms for input orientation reveals that the CRS frontier produces TE, AE and EE ratings with means of 86, 91 and 78 per cent respectively, and the VRS frontier calculates TE, AE and EE scores with mean values of 91, 87 and 79 per cent respectively. Again the estimate of the mean technical efficiency from the VRS frontier is greater than that from the CRS DEA frontier model. The variability of AE estimates as measured by the standard deviation are lower than those of TE and EE estimates.

Tobit analysis is used to evaluate factors associated with TI, AI and EI; it shows that irrigation infrastructure and environmental degradation are the most statistically significant factors associated with technical, allocative and economic inefficiency from both the CRS and VRS frontiers. Results indicate that all types of inefficiency are positively influenced by the irrigation infrastructure, i.e., diesel-operated irrigation schemes create more inefficiencies than do electric ones, which implies that irrigation infrastructure is not only creating obstacles to obtaining the maximum output from given inputs and technology but is also causing sub-optimal cost-minimizing input decisions. Results also indicate that soil degradation as an environmental factor is negatively associated with TI, AI and EI; these imply that soil degradation lowers farmer's ability to utilize existing technology efficiently and hinders the allocation of inputs in a cost-minimizing way.

Evaluating efficiency suggests that there is a considerable amount of inefficiency among our sample farms and a substantial potential for increasing rice output through the improvement of technical, allocative and economic efficiency without resort to technological improvements. In assessing factors associated with inefficiency, we have two main conclusions. First, government electrification programmes which convert diesel pumps into electricity-operated pumps for irrigation in rural areas would reduce inefficiency, thereby increasing rice production and the welfare of farm households. Second, policies which aim to reduce soil degradation would be beneficial for similar reasons.

Chapter 9

Summary and Conclusions

9.1. Introduction

Since the mid-1960s the government of Bangladesh has promoted the introduction of Green Revolution technologies which aim to increase yields. However the growth rate of food production lags behind population growth. The annual food deficit over recent years is about 1.5 million tonnes of rice. Farm household production is hampered by illiteracy, land fragmentation, weak irrigation infrastructure and soil degradation. The aim of this thesis is to investigate the factors associated with inefficiency through different methods of efficiency measurement and the implication of the variability of efficiency for agricultural development policy.

9.2. Summary and Main Results

We conduct a survey of 150 farm households from two villages in the High Barind, Bangladesh. The villages are selected by applying purposive sampling by irrigation infrastructure. Farm households are stratified according to their land holdings as the distribution of land holding is skewed. Finally a simple random sampling technique is applied to each stratum. The cross-section primary data are collected using a face-to-face questionnaire. The survey collects data mainly on farm output and output prices, input and input prices, socioeconomic characteristics, irrigation infrastructure, environmental degradation and other information. Participatory Rural Appraisal techniques are also applied and it complements the main survey by helping to identify factors affecting efficiency.

The survey results show that the distribution of land holding is skewed. Inequality in land holding is high and land is fragmented. Eighty eight per cent of the total land area has already been exploited for cultivation. This indicates that increasing cropping frequency and increasing yields per acre are the only options available to increase output. The farming system is dominated by rice which accounts for 95 per cent of the cultivated area. Irrigation technology has increased productivity making the production of Boro paddy successful. The relatively high prices of agricultural inputs and low prices of agricultural products are identified as major problems by the farmers.

Two approaches are adopted to measure the efficiency of the survey farms: the stochastic frontier and Data Envelopment Analysis (DEA). The stochastic frontier method as applied here uses two approaches: first the translog stochastic frontier with the technical inefficiency effects model estimates technical efficiency and second the Cobb-Douglas stochastic frontier using a cost decomposition technique estimates technical, allocative and economic efficiency. DEA adopts first the input-oriented and output-oriented constant returns to scale (CRS) and variable returns to scale (VRS) multi-stage DEA models to measure technical efficiency. Scale efficiency is determined from the relationship between CRS and VRS models. Second a cost minimizing input-oriented CRS and VRS DEA frontier is used to estimate technical, economic and allocative efficiency. A Tobit model is used to identify and quantify the effects of farm-specific factors associated with inefficiency.

The translog stochastic frontier results show that the estimates of the coefficients of the frontier have the expected sign except the coefficient of the fertilizer which is insignificant. The farm households appear to be characterized by slightly decreasing returns to scale. The technical efficiency among the farm households ranges from 49 - 98 per cent with the mean value of the technical efficiency of 80 per cent given the specification of the translog stochastic frontier production model. Results of the technical inefficiency effects model show that the technical inefficiency effects dominate the composed error term. Results of the analysis of technical inefficiency by socio-economic factors show that better educated, younger farmers are most likely to operate farming activities efficiently. Plot size is also positively related with technical efficiency, thus the larger the plot size the greater the technical efficiency. Technical inefficiency effects are positively influenced by the irrigation infrastructure, i.e., diesel-operated irrigation schemes. Land degradation is negatively associated with technical inefficiency.

Results of output elasticities from the Cobb-Douglas stochastic frontier give similar results to the translog stochastic frontier. The variance parameters are also significant as in the translog stochastic frontier. The technical, allocative and economic efficiency estimates of farms are on average 80, 77 and 61 per cent. The results of the analysis of inefficiency by socioeconomic factors indicate that age of farmers and years of schooling are negatively associated with allocative and economic inefficiency, as expected. Further, land fragmentation is negatively related with the technical, allocative and economic inefficiency estimates. The coefficients on the irrigation infrastructure and environmental degradation dummies have expected signs and are significant. The differences in average technical efficiency and in variances of technical efficiency estimates from the translog and Cobb-Douglas frontier are rejected. The ranking of technical efficiency by farm households are not sensitive to the choice of functional form.

Results from DEA frontier for measuring technical efficiency show that the CRS and VRS technical efficiency and scale efficiency are, on average, 79, 86 and 92 per cent from the output-orientation and are 79, 85 and 93 from the input-orientation. An analysis of factors associated with technically efficiency by Tobit model shows that both input-oriented and output-oriented measures of technical and scale efficiency are affected positively by factors associated with irrigation infrastructure and overall and pure technical inefficiency are negatively related with environmental degradation. The input-oriented DEA frontier for measuring technical, allocative and economic efficiency simultaneously shows that the averages of technical, allocative and economic efficiency produced by CRS frontier are 86, 91 and 78 per cent respectively and those produced by VRS frontier are 91, 87 and 79 per cent respectively. The mean technical efficiency from the VRS frontier is greater than that from the CRS frontier and the standard deviation of technical efficiency estimates from the CRS frontier is higher than that from the VRS frontier. A Tobit analysis to evaluate factors associated with technical, allocative and economic inefficiency reveals that irrigation infrastructure and environmental degradation are significant factors in explaining technical, allocative and economic efficiency.

The stochastic econometric frontier decomposes the composed error term into a stochastic random noise component and a technical inefficiency component. It attempts to distinguish the effects of stochastic noise from the effects of inefficiency. Thus this approach produces a consistent framework for analyzing efficiency by segregating variations from the frontier technology into a stochastic error component and an asymmetric non-negative random component which reflects inefficiency. Addressing the stochastic noise problem, associated with the deterministic frontier, and statistical hypothesis testing are the main strengths of the stochastic econometric frontier. However, the major drawbacks of this approach are the maintained hypotheses of the functional form and distributional assumption which can not be observed. This affects the distribution and estimation of efficiency estimates.

DEA, a nonparametric mathematical programming approach, consists of a conical hull of input-output vectors based on a production possibility set. The conical hull of vectors is constructed by linear programming techniques, for the single output, with a subset of the sample lying on the production possibility set and the rest lying above. The approach estimates efficiency relative to the efficient frontier which estimates best performance. Further, it can obtain target values based on the best practice unit for each inefficient farm that can be used to provide guidelines for improved performance. DEA is both nonparametric and nonstochastic since it does not impose any *a priori* parametric restrictions on the frontier technology and it does not require any distributional assumption for the technical inefficiency term. Thus this method avoids the imposition of unwarranted structures on both the frontier technology and inefficiency component that might create a distortion in the measures of efficiency. However the major drawback of this method is that it is deterministic and assumes a zero value for the stochastic random error term; thus technical inefficiency reflects all unexplained variations and the inefficiency of the observed farm is therefore biased upwards. Moreover, since there is no measurement error or other stochastic random noise and since it is nonparametric, efficiency measures can not be subjected to statistical testing.

The stochastic econometric and DEA frontiers are used here to estimate the technical, allocative and economic efficiency estimates. The averages of technical, allocative and economic efficiency estimates from the stochastic frontier and DEA are presented in Table 9.1. Table 9.1 shows that the mean values of efficiency estimates based on CRS and VRS DEA frontier are higher than those based on the stochastic frontier because the DEA frontier fits tighter to the data set. The stochastic frontier exhibits greater variability in technical, allocative efficiency estimates than the DEA frontier but has similar variability of economic efficiency.

Table 9.1: Mean and Standard Deviation (Std Dev.) of Efficiency Estiamtes (in percentages)

	Stochastic Frontier (SF)			DEA CRS			DEA VRS		
	TE	AE	EE	TE	AE	EE	TE	AE	EE
Mean	80	77	61	86	91	78	91	87	79
Std Dev.	13	15	12	12	7	12	10	10	12

Spearman's rank correlation coefficients between efficiency scores are estimated to assess the agreement between the stochastic econometric and DEA frontier and results are reported in Table 9.2. All the TE, AE and EE rank correlations are positive. Technical efficiency shows the strongest correlation and allocative efficiency the weakest correlation.

Table 9.2: Spearman Rank Correlation of Efficiency Ranking based on SF and DEA

	DEA CRS			DEA VRS		
SF	TE	AE	EE	TE	AE	EE
TE	0.83			0.61		
AE		0.36			0.05	
EE			0.54			0.56

Few studies compare the performance of the stochastic frontier and DEA method in predicting the technical, allcoative and economic efficiency estimates. Based on the analysis of Guatemalan farmers, Kalaitzandonakes and Dunn (1995) reported a significantly higher level of technical efficiency under CRS DEA than under the stochastic frontier. This result conforms with our results. Analyzing a sample of swine farm in Hawaii Sharma et al. (1999), based on the analysis of US banks Ferrier and Lovell (1990) and based on the study of UK building societies Drake and Weyman-Jones (1996) reported both similar and dissimilar results. The dissimilarities in empirical results in comparing the two methods can be attributed to difference in the characteristics of the data analyzed, choice of input and output variables, measurement and specification errors and estimation procedures (Sharma et al., 1999).

An assessment of factors associated with inefficiency from the stochastic frontier and input-oriented CRS and VRS frontiers, given in Table 6.12 and 8.7, shows that younger farm households are more technically efficient and older farm households are more efficient in allocating least-cost inputs. Results from the stochastic frontier imply that farmers with greater schooling are more allocatively and economically efficient and DEA frontiers imply that farmers with more years of schooling are efficient in choosing cost-minimizing input combinations. Both frontiers, except VRS TI, show that small land plot size reduces the technical, allocative and economic efficiency of farm households. The assessment of factors affecting efficiency shows that inefficiency estimates from both the stochastic and DEA frontiers are positively related to irrigation infrastructure implying that farm households buying irrigation water from electricity-operated pumps are more technically, allocatively and economically efficient. Moreover, results from both the stochastic and DEA frontiers show that soil degradation not only imposes a constraint on using technology efficiently but also constrains the cost-minimizing input utilization; thus farm households with less soil degraded lands, on average, operate at higher levels of technical, allocative and economic efficiency.

9.3. Conclusions and Policy Implications

This thesis considers both the stochastic econometric frontier and Data Envelopment Analysis (DEA) method. Each method has its strengths and weaknesses. Therefore the choice between the methods should be on a case-by-case basis. Results from the stochastic frontiers imply that farms on average operate at super-optimal scale and there is room for efficiency improvement. There are considerable technical, allocative and economic inefficiencies in agricultural production in Bangladesh, especially, economic efficiency and hence considerable room for increasing output levels through efficiency improvement and thereby enhancing farm income and the welfare of the farm households. Farms could reduce total variable costs by, on average, 21 - 31 per cent if they could utilize their inputs in a technically and allocatively efficient manner. Both the translog and the Cobb-Douglas stochastic frontiers produce the same pattern of technical efficiency estimates and hypothesis tests do not accept any differences in the averages and variances of technical efficiency estimates; and the rankings of technical efficiency estimates are highly correlated.

The estimates of the technical inefficiency effects model from both the translog and the Cobb-Douglas stochastic frontier imply that the level and variability of output are determined by the components of the technical inefficiency effects and the traditional OLS response function does not adequately explain the input-output relationship for rice production. An analysis of technical inefficiency effects by socioeconomic, infrastructure and environmental degradation factors reveals that these factors jointly determine the variability of rice output.

We emphasize the need of education to improve the ability of farm households to receive and understand information regarding modern technology. Thus the government could implement an agricultural education policy so that the younger fermers can obtain appropriate knowledge. Older farm households, who have had limited educational opportunities, can be assisted with adequately trained extension advisers who are committed to implement new production technologies. Extension programmes could be used to reorient the application of methods and timing of application of inputs and production methods. Extension policy could also be reformed to reorganize the duties of extension officials so as they spend more time on field visits, thereby improving farmers' understanding. This would reduce the extent of variation in output from the maximum output. At the same time, learning-by-doing may help farm households in adapting better to prevailing as well as new technologies. In general, policies which improve the education of younger farm households and promote extension training to assist older farm households would help them better understand the requirements of HYV cultivation.

Land consolidations are beneficial in generating gains in technical, allocative and economic efficiency. Thus land tenure and management policies could be designed to reduce fragmentation. Since this research shows that land is the dominant factor of production, irrigation is a land-augmenting factor of production in the sense that it increases the productivity of land and hence increases yield per acre. Land fragmentation not only causes obstacles to utilizing existing technology efficiently but also creates problems in allocating inputs in a cost minimizing way. Farm households could be encouraged to consolidate their land especially in Season III for better utilization of irrigation, fertilizer and land preparation methods using tractor in particular. This will increase farm output and farm households could share output in proportion to their lands. Here again extension officials can play an important role in demonstrating the consolidation effects of land on output to farm households.

Irrigation infrastructural development policy which aims to convert diesel pumps into electricity pumps will reduce the technical, allocative and economic inefficiency. This may be one of the easiest and cheapest ways to improve the efficiency. Electricity cost is lower than diesel cost and the water extraction capacity of electricity-operated pumps is greater than diesel pumps. Irrigation in Bangladesh entirely supports rice crops in Season III. Cultivation of rice in this season is not possible without irrigation because of a lack of rainfall. The benefits of irrigation include increased crop yields, improved crop quality and reduced yield variability. Thus irrigation policy which recommends greater use of electricity for irrigation could enhance farm income and the welfare of the farm households through improvements in efficiency.

Land degradation is major technical constraint on production and we find that it creates obstacles in applying technology efficiently and hinders the cost-minimizing input allocation. Policies leading to reduced land degradation would also be effective in reducing technical, allocative and economic efficiency in production, thereby increasing productivity and household welfare for the rice farmers in Bangladesh. Land degradation can be slowed down through the joint efforts of farm households and the government given the present circumstances in Bangladesh. We make two suggestions for government policy. First, the lack of organic matter, which is used for domestic fuel, causes soil structural deterioration, soil erosion and hence land degradation. Since Bangladesh has natural gas, development policy could be oriented to transfer it to rural areas including the High Barind. This will allow farm households to reduce the use of organic matter for domestic fuel and allow them to recycle back it to the soil which would reduce the rate of soil structural deterioration, soil erosion and land degradation, and this will enhance farm output. Second, land degradation occurs through runoff of heavy rainfall during Season II. Farm households could build on their lands small canals and terraces which carry rainfall water away to main canals, which are provided by the government or by farm households jointly. This would also reduce land degradation, increase land quality and enhance land productivity and farm output. Efficient utilization and combination of fertilizer and irrigation can reduce the effects of constraints because of land degradation. In all respects, the most efficient farms could be encouraged to disseminate best practices to and share experience with other farms.

However all these activities regarding policies are not costless and the added cost should be weighed against the added benefits when deciding which activities to pursue in further reducing farm production costs. Given technological development and efficiency improvement in raising productivity, efficiency improvement may offer more immediate gains at a relatively low cost. These policy recommendations could be applied to the entire area of the High Barind and to other similar parts of Bangladesh.

9.4. Further Research

The small sample survey in the High Barind Bangladesh is the basis of results of the inefficiency model. Results of the model drawn from a large survey covering different agro-ecological zone and the comparison of the results across various zones can further improve the predictive power of the model. Efficiency across years and across zones can be predicted using data collected at the government level. The inclusion of several other factors affecting inefficiency would increase the precision of this model.

Inefficient management and utilization is a problem common to almost every sector of Bangladesh economy. This research, the first of its kind for Bangladesh, presents a snapshot of inefficiency in Bangladesh agriculture. Given the momentous changes taking place during the Green Revolution, research using panel data would be helpful in tracking changes in efficiency over time.

We can not do much beyond the study with this current data set. Extension services expose farmers to new technologies, new techniques and practices which would contribute to improvements in efficiency. Farmers can purchase inputs with credit availability which relaxes cash constraints. A lack of credit unavailability can seriously hamper a farmer as failure to purchase inputs, like fertilizer and irrigation water, for his standing crops may cause irretrievable output loss. Therefore an efficiency analysis should incorporate extension services and credit facilities. Productivity growth involves two major components: technical change and technical efficiency (Good et al., 1993). A study on the decomposition of output growth into technical change and efficiency can show the contribution of each to productivity and then a study of factors associated with inefficiency could produce better policy directions.

Finally, there is no scientific method introduced in Bangladesh to measure environmental degradation. Scientific techniques like Geographical Information System, remote sensing and soil surveys can be applied to measure land degradation. The inclusion of the resulting measure of land degradation can improve the prediction power of our efficiency models. Local water markets are important in irrigating lands and its inclusion in the models in some form may provide better analysis of farm efficiency. The stochastic profit frontier approach may be one alternative to measure technical and allocative efficiency and a comparison between the efficiency estimates from the stochastic production frontier and the stochastic profit frontier may add further insight.

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